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PREFACE AND ACKNOWLEDGMENTS

The chief object of this book is to describe the fundamental theories, equipment and field techniques of the recognized exploratory geophysical methods, and to illustrate their application to problems of economic geology. The wide-spread use of geophysics has resulted in a voluminous quantity of domestic and foreign literature, in addition to some three thousand patents on geophysical methods, apparatus and allied equipment. The dynamic condition of the art has necessitated continual revision of the manuscript during the period of its preparation. As new and better techniques became accepted by the profession, their inclusion in the text relegated previously written material to a position of less importance. Constant revision of the manuscript has been necessary to keep abreast of developments.

A careful attempt has been made throughout the text to give full credit to the proper investigators for all original ideas and applications. It often is difficult to differentiate between original developments of one individual and developments that occur over a period of time, and which are due to the collaboration of many workers. Especially is this true in the field technique governing the application of the various exploratory methods and the development of the practical equipment.

An effort has been made to cite patent references whenever possible as well as literature references, because, due to the highly competitive nature of modern exploration geophysics, patent specifications oftentimes tend to be more authoritative than the reports in contemporary literature.

During final compilation of the manuscript, each chapter was submitted to reviewers who had extensive experience in the particular phases of geophysics treated in that chapter. In practically all cases, the reviewers reworked portions of the chapter or added new material. The present form of the book has been determined in large measure by the criticisms and additions supplied by these collaborators. In many cases where controversial views were held, all the reviewers naturally did not agree as to the relative advantage or disadvantage of certain methods or equipment and therefore did not subscribe to all statements of the text. In such cases, the author has attempted to present both views, and he must necessarily assume full responsibility for errors of omission, or over emphasis, of this controversial material.

Grateful acknowledgment is given to the various consulting geophysicists and officials of oil companies who so generously supplied certain material and assisted in the revision of the manuscript. The excellent cooperation of these collaborators and critics is one of the pleasant memories associated with the compilation of the text.

COLLABORATORS AND CRITICS

Geologic and Economic Background of Exploration Geophysics:

C. H. Wilson, M.S., International Geophysics, Inc., Los Angeles, California.

H. W. Hoots, Ph.D., Richfield Oil Corp., Los Angeles, California.

Howard C. Pyle, M.S., Union Oil Co., Los Angeles, California.

Magnetic:

C. H. Wilson, M.S., International Geophysics, Inc., Los Angeles, California.

Noel H. Stearn, Ph.D., W. C. McBride, Inc., St. Louis, Missouri.

V. G. Gabriel, Sc.D., Los Angeles, California.

Gravity:

W. M. Rust, Jr., Ph.D., Humble Oil and Refining Co., Houston, Texas.

L. M. Mott-Smith, Ph.D., Mott-Smith Corporation, Houston, Texas.

Paul Weaver, Gulf Oil Corporation, Houston, Texas.

L. F. Athy, Ph.D., Continental Oil Co., Ponca City, Oklahoma.

E. V. McCollum, M.S., Mott-Smith Corporation, Houston, Texas.

Donald C. Barton, Ph.D., Humble Oil and Ref. Co., Houston, Texas

V. G. Gabriel, Sc.D., Los Angeles, California.

Seismic:

M. B. Widess, Ph.D., Western Geophysical Company, Los Angeles California.

W. M. Rust, Jr., Ph.D., Humble Oil and Refining Co., Houston Texas.

Beno Gutenberg, Ph.D., Professor of Geophysics, California Institute of Technology, Los Angeles, California.

L. F. Athy, Ph.D., Continental Oil Company, Ponca City, Oklahoma

Henry Salvatori, M.S., Western Geophysical Company, Los Angeles California.

N. A. Haskell, Ph.D., Western Geophysical Company, Los Angeles California.

Frank Rieber, B.Sc., Frank Rieber, Inc., Los Angeles, California.

Wm. Miller, Miller Corporation, Pasadena, California.

Explosives:

A. R. Ely, B.Sc., in collaboration with A. E. Forster, E.M., H. E. Nash B.Sc., and J. M. Martin, B.Sc., Hercules Powder Company, Wilmington, Delaware.

Drilling Exploration:

J. A. Noyes, M.S., Sullivan Drill Company, Dallas, Texas.

Soil Analysis:

W. B. Lewis, Ph.D., Humble Oil and Refining Company, Houston, Texas.

Eugene McDermott, M.S., Geophysical Service, Inc., Dallas, Texas.

E. E. Rosaire, Ph.D., Subterrex, Houston, Texas.

Geothermal Methods:

J. N. A. van den Bouwhuijsen, Ph.D., International Geophysics, Inc., Los Angeles, California.

Drill Hole Measurements:

E. G. Leonardon, M.S., Schlumberger Oil Well Surveying Corporation, Houston, Texas.

R. R. Henquet, E.M., Schlumberger Oil Well Surveying Corp., Los Angeles, California.

J. C. Stick, Jr., B.Sc., Lane-Wells Company, Los Angeles, California.

Physical Principles Applied to Production Problems:

Howard C. Pyle, M.S., Union Oil Company, Los Angeles, California.

Everett G. Trostel, M.S., Union Oil Company, Los Angeles, California.

Daniel F. Elam, B.Sc., Halliburton Oil Well Cementing Company, Duncan, Oklahoma.

Permit and Trespass Practices, and Insurance:

Howard Painter of Reynolds and Painter, Los Angeles, California.

History of Geophysics:

Katharine F. Jakosky, A.B., Los Angeles, California.

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The original subject matter of this text is based upon lectures delivered at and a manuscript submitted to the faculty of the College of Mines Engineering of the University of Arizona, in 1933. The author is indebted to Dean G. M. Butler and the faculty of that college for permission to use much of the material included in that manuscript.

Lawrence, Kansas, J. J. JAKOSKY.
July, 1940.

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EXPLORATION GEOPHYSICS

CHAPTER I

INTRODUCTION

Geophysics is a study of the physics of the earth with special reference to its physical properties, structure, and composition. It is commonly divided into two general branches. The first includes the subdivisions of the physical sciences such as electricity, magnetism, chemistry, heat, elasticity, etc., while the second branch pertains to the study of the three major components of the earth: solid (lithosphere), liquid (hydrosphere) and gas (atmosphere). The wide diversity of subject matter embraced in the field of geophysics may be more thoroughly appreciated by reference to Table 1.†

Exploration Geophysics is the art of applying the physical sciences to the study of the structure and composition of those layers of the earth that are sufficiently shallow to be exploited by man. This book is primarily concerned with the application of geophysics to the solid or rock components of the outer portion of the earth's crust, especially in the solution of problems of structural and economic geology. This application may embrace extremely practical engineering techniques at one extreme and the methods of mathematical physics at the other.

Exploration geophysics has evolved from the practical application of the cumulative knowledge of many independent investigators working in different parts of the world. Many of the fundamental laws and theories were developed by scientists of the past centuries, most of whom probably had no thought of the direct application to economic geology. Due chiefly to stiff commercial competition, this branch of applied geophysics is a dynamic art continually changing and bristling with personalities. Many different methods are being applied and it is the purpose of this book to outline the fundamental theory and general application of these various scientific methods.

The demand for metals in the latter part of the nineteenth century contributed much to the early development of mining geophysics. The chief incentive to modern geophysics, however, was the search for oil during the early part of the twentieth century. No longer can the geologist go forth to look for oil using only such signs as gas bubbles on water, petroleum seepage along creek beds, outcropping oil bearing strata or other surface evidence of subsurface oil accumulation. To these methods of finding oil have been added surface geology with its cartography, micro paleontology, lithology, electrophysics, geophysics and geochemistry.

† B. Gutenberg, "Geophysics as a Science," *Geophysics*, Vol. 2, July, 1937.

TABLE 1

MAJOR PROBLEMS OF GEOPHYSICS †

(a) General problems, (b) Applied problems.

	Solid Body	*Hydrosphere*	*Atmosphere*
Mechanics	(a) Forces and stresses; gradual and sudden movements; earthquakes, earthquake waves, elastic and viscous movements; tides; movements of the poles; figure; density; volcanism (belongs also in other sections and geochemistry); mechanical effects of ice, water, wind (b) Reduction of earthquake damage; seismic prospecting	(a) Tides, waves, currents; hydrology (b) Investigation and prediction of tides and currents for navigation and fisheries; hydrology; echo sounding	(a) Tides, waves including sound; currents (b) Weather forecasting
Gravitation	(a) Gravity; layering; pressure; isostasy; sedimentation (b) Use of pendulum, torsion balance, etc., in prospecting	(a) Layering; sedimentation	(a) Distribution of gases; layering
Electricity	(a) Electric currents and electric waves (b) Electric prospecting	(a) Electric phenomena	(a) Electric phenomena; ionosphere; aurora
Magnetism	(a) Earth's magnetism (b) Magnetic prospecting	(b) Compass; magnetic charts	
Optics		(a) Color and transparency of lakes, oceans	(a) Meteorological optics, halos, etc.; color of sky; polarization; turbidity; colors of clouds (b) Visibility for aviation
Composition of matter	(a) Composition of the earth; radioactivity; state of the interior	(a) Radioactivity of hydrosphere; salt content	(a) Composition of the atmosphere, ozone, etc.
Heat	(a) Temperature in the earth and its changes; crystallization and melting (b) Thermal prospecting	(a) Temperature in lakes, rivers and oceans; glaciers; icebergs; thermal currents	(a) Thermodynamics of the atmosphere; temperature; climates (include other factors) (b) Climatology

During the period of early petroleum production only six per cent of the wildcat wells came in as producers. An increased demand for oil and the inadequacy of the haphazard methods of the early "wildcatter" in meeting this demand forced the development of the new science of petroleum geology, and with it the effective tools of geophysical exploration so well adapted to the study of the remaining unexplored areas. Even with these aids only approximately 14% to 19% of the wells now drilled are producers. During the entire life of the American petroleum industry an average of about 180,000 barrels of oil have been discovered for each dry hole drilled. During the past three years, with geophysical exploration as a guide in a large portion of the wells drilled, discovery of oil has been at the rate of about 300,000 barrels for each dry hole.

† Gutenberg, *Geophysics.*

Geophysical methods have been successful in the search for oil primarily because of their ability to determine the geologic structure of buried formations. Although each method is undergoing constant revision to improve its accuracy and broaden its field, several of them have had phenomenal success in mapping subsurface features. The seismograph in particular has been successful in mapping hidden structures favorable for the occurrence of new oil and gas fields.

Whereas once oil was found in shallow wells, it is now produced in wells from 6,000 to 10,000 feet in depth, and in 1939 there were 33 wells producing at depths in excess of 12,000 feet. The cost of the exploratory drilling of these deep-seated oil deposits is a prohibitive gamble without reliable data as to the existence of favorable geologic structure.

The estimated proved oil reserves in the United States are considerably in excess of those known fifteen years ago in spite of the fact that more petroleum has been produced than was previously assumed to exist. An estimated total of about 38,000 million barrels of oil have been discovered (1938), of which about 21,000 million barrels have been produced, leaving 17,000 million barrels in the ground as proved oil reserves. Of this reserve, the state of Texas has 54.5 per cent, California 18.4 per cent, Oklahoma 6.7 per cent, Louisiana 6.0 per cent and New Mexico 4.1 per cent. From present knowledge it appears that the balance is chiefly in Kansas, Wyoming, Illinois, Arkansas and Pennsylvania. It is probable that many new fields will be found to augment this supply materially. Exploration geophysics has played an important part in the discoveries of the last 15 years, and it will be only by the improvement of present methods or the development of new methods that the recent past rate of discovery can be maintained.

Each geophysical method has had its period of initial trial, followed by its peak commercial application over a period of a few years. As the areas amenable to a particular method have been studied and covered, the application or popularity of that method decreased as it was replaced by other methods. The succession of one method after another sometimes followed rapidly and sometimes a considerable time period elapsed between the peak application of succeeding methods. The period of peak commercial activity of each method has been limited because either it covered the areas where it was most amenable, or else was succeeded by a newer technique which had economic advantages.

HISTORY OF DEVELOPMENT

Magnetic Methods.—Probably the first geophysical instrument had its beginning in the discovery that a lodestone, or a piece of a certain kind of iron that has contacted a lodestone, will orient itself in an approximately north-south direction. Early Chinese literature indicates that this orientation property of lodestone or magnetized iron was known and utilized some time during the period from 2637 B.C. to 121 A.D. Pereginus in 1269 discovered the "magnetic poles" and named them North and South. His investigations further proved that unlike poles attract and like poles repel each other. He also made the important discovery that the fragments of broken magnets behaved as the original magnets. In 1492, on his voyage to America, Columbus noted that his compass deviated from astronomical north. Hartmann in 1544 discovered the in-

clination of the compass. In 1581, Robert Norman proved that magnetization of a steel needle had no effect upon its weight. This prepared the way for further studies of magnetic inclination because it showed that the angle assumed by a magnetized needle was due to the inclination of the earth's magnetic field and not, as originally believed, to an additional mass which had been added during its magnetization.

William Gilbert, a physician and physicist contemporary of Queen Elizabeth of England, conducted various scientific investigations and experiments on magnets and magnetic bodies. His conception of the earth as a giant magnet was very advanced for his time. His book "De Magnete" was published in 1600.

In 1722 George Graham discovered that the orientation of the compass varies slightly through the day. Also by the end of the 18th century it became known that the earth's total magnetic intensity varies laterally over its surface and it became vaguely evident that there is some relationship between structural geology and the inclination of the earth's magnetic field. These observations were made by Humbolt, a noted scientist and traveler in 1798-1803. The diurnal variations in the intensity of the earth's magnetism were first noted by Arago in 1827. To Gauss, who carried out magnetic investigations in Göttingen in 1834, must be given the credit for working out methods of measuring terrestrial magnetism. The electromagnetic unit of magnetic force is named for him.

As late as 1616, in Magnetical Advertisements, the compass was spoken of "as the most useful and admirable instrument of the whole world, yet blundering." In 1820, Peter Barlow reported that "half of the compasses of the British navy are mere lumber and should be destroyed." He built an improved one which remained in use until 1876, when the present type was adopted. As the compass has developed so have its uses.

Exploration geophysics probably had its beginning about 1640 when the compass was first used in the search for iron ores in Sweden. Theoretical aspects of the work were propounded by Von Wrede in 1843 who advocated that local variations in the earth's field might be indicative of buried magnetic materials. Magnetic measurements were conducted in the Michigan iron district in 1873 by T. B. Brooks, in 1875 by H. Smock, and in 1899 by H. L. Smyth. Numerous measurements were made with these early, insensitive instruments. In 1879, Thalen published a detailed account of magnetic technique and results. Investigations on placer gold deposits were conducted in 1914 by Gibson, who carried out surveys in Butte and Shasta Counties in California, using a Thalen-Tiberg magnetometer. Hotchkiss used a dip needle for exploratory work over the iron ore deposits of Wisconsin in 1915.

Since the beginning of the century, numerous large scale *geomagnetic** investigations have been made by various organizations throughout the world. The purpose of the earlier investigations was to obtain information on the distribution of the earth's magnetism with particular reference to navigation problems, any geological conclusions being purely secondary. An impetus to practical geomagnetic investigations for exploration purposes was given by the construction of the Schmidt field magnetometer in 1915. This was the first rugged and portable instrument capable of detecting local magnetic anomalies of small magnitude.

* The term *geomagnetics* as used here refers to the study of the magnetic properties of the earth.

Gravitational Methods.—The variation of gravity at various locations on the surface of the earth was discovered by Jean Richer in 1672 when he noted that a clock whose pendulum was calibrated to beat seconds in Paris lost about two and one-half minutes a day in Cayenne. It was known that the time of swing of a simple pendulum is a function of its length and the pull of gravity. Richer concluded that since the length had not altered, the variation in time must be due to the fact that the gravitational value in Cayenne differed from that in Paris. This explanation indicated that the gravitational attraction might be expected to vary at different locations on the earth and suggested a method of determining this variation, namely, the method of pendulum observations.

The next important advance was the invention of the reversible pendulum by H. Kater in 1818. This type of pendulum is still employed for precision measurements of absolute gravity. The difficulty of calibrating a reversible pendulum accurately and the tediousness of the extreme precautions required in determinations of the absolute gravity led to the use of the so-called invariable pendulums. Invariable pendulums are suitable for determinations of the relative gravity at a large number of stations. The invariable pendulum comprises a bob rigidly attached to a "massive" rod, in place of the "weightless" rod of the simple pendulum. The type of invariable pendulum most widely employed at present is patterned after that of the Austrian geodesist, von Sterneck.

Sir John Herschel, in a book published in 1833, suggested the first gravity meter wherein the relative displacements of a bob, suspended by a spring, would serve as a measure of the relative pull of the earth, i.e., the gravity, at various locations.

During the latter part of the 19th century and the early part of the present century, various gravity meters having an accuracy higher than the Herschel instrument were proposed. The Threlfall-Pollack gravity meter and the other meters proposed prior to 1918 were not widely used, due to the fact that their accuracy was even less than that of the pendulum. Ising in 1918 was probably the first investigator to propose a practical astatic gravity meter design. The intensive development of gravity meters began during the years 1928-1930 when various American oil companies became interested in developing new geophysical instruments. At present there are several different types of gravity meters having an accuracy greater than 1 millidyne per gram, including a few with an accuracy greater than 0.1 millidyne per gram. (Measurements to within 0.1 millidyne correspond to an accuracy of one ten millionth of the total value of gravity.)

The search for the answer to the question of how much the earth weighs led to a study of the force of gravity by Cavendish, who began his work early in the 18th century. Cavendish used the torsion balance, previously employed by Coulomb for studies of magnetic and electrical attraction, to measure the force of attraction between two masses. Further

studies were pursued by Baron Roland von Eötvös (1848-1919) who refined and applied the principles of the original Coulomb balance, primarily to the field of geodetic research. With the Eötvös instrument, geophysicists have been able to obtain concrete evidence of the variation of density with depth in the outer crust of the earth.

The characteristic feature of the Eötvös balance which renders it a practical and useful instrument for geophysical exploration is that by placing two equal masses at different levels and by proper choice of instrumental constants the balance may be made extremely sensitive to variations in the rate of change of gravity in a horizontal plane and to the curvature values.

The double beam balance introduced by Eötvös in 1902 is essentially the same as the torsion balance employed at present. In addition to designing a practical field instrument, Eötvös investigated various fundamental principles of gravitational prospecting and applied his method to certain regions in Hungary. Schweydar modified the Eötvös balance by using a "Z-beam". Following him Shaw and Lancaster Jones introduced a gradiometer which measured the gradient of gravity in a horizontal plane.

The application of gravity surveys as an aid in prospecting for petroleum was investigated in 1914 by E. de Golyer, who took an active part in subsequent exploration. The correlation of gravity variations and geology was due largely to Hugo V. Boeckh who, in 1917, gave geological reasons explaining why anticlines and domes, with cores of different density, produce gravity anomalies of magnitudes which are measurable by the Eötvös balance. E. W. Shaw, about the same time, suggested the possibility of locating salt domes by gravity anomalies. In 1918, Schweydar under the guidance of Boeckh made successful measurements over a German salt deposit. In 1920, de Golyer contracted for two balances to be built by F. Süss of Budapest, and in 1922, D. C. Barton went to Budapest to receive the instruments and instructions for their use. Barton occupied an important position in subsequent gravity work in the United States. More recent developments are described later in the text.

Initial tests over the Spindletop dome in Texas gave a definite gravity maximum, but subsequent work over various prospects gave indefinite results, until a survey of the Nash area in Texas gave a good anomaly. A well was drilled in 1924 and proved the presence of the structure. Oil was discovered on the flank in 1926, and this is probably the first oil pool to be discovered by geophysical methods in America. Large oil fields mapped by the torsion balance method that have now been developed include: Thomson (Rabbs Ridge), Sugarland, Tomball, Manvel, Dickinson, Hastings, Anahuac, Cedar Point, Fairbanks, Friendswood, Mykawa, Roanoke, Iowa, Gillis and English Bayou. It is estimated that the total ultimate recovery from fields located by gravitational methods will exceed one billion barrels of oil. In June, 1939, there were approximately 60

gravimeter and 15 torsion balance crews operating in various areas of the United States.

Electrical Methods.—As early as 1720, Gray and Wheeler made electrical studies of rocks and tabulated their electrical conductivities. Watson, in 1746, discovered that the ground was a conductor and noted that current passed between electrodes imbedded in the ground at a separation of 10,600 feet acted erratically and differently than when wire was used to complete the circuit.

It was the ore-finding problems of the mining industry which led to the early study of electrical phenomena in relation to mineral deposits. Robert Fox in 1815 and later Dr. Carl Barus in 1882 were convinced that the phenomenon of spontaneous polarization in rocks and minerals had a possibility in locating ore bodies. Subsequent to this discovery by Fox, the phenomenon of spontaneous polarization of ore bodies was studied by several investigators. The self-potential method used by Barus on the Comstock Lode in Nevada is essentially the same technique as employed today. The first commercial use of the effects produced at the surface of the earth by spontaneous polarization as the basis of a prospecting method was made by C. Schlumberger which, applied in 1913 at Bor, probably was the world's first geophysical discovery of a non-magnetic ore deposit. The first published account of the results of these studies (deferred on account of the World War) appeared in 1920.

A second factor in the development of electrical methods was the general concept that subsurface bodies of relatively high conductivity should in some way affect conductivity measured between two points at the surface. Williams and Daft in 1897 attempted to determine differences in conductivity by an alternating current method which consisted in passing the current through the ground and observing the variations in the intensity of sound in a telephone receiver connected to two grounded electrodes. Professor James Fisher first experimented with electrical conductivity devices in the Quincy Mine in Michigan in 1893, attempting to locate copper-bearing lodes. This was followed by the work of Osborn who experimented with methods of mapping electric equipotential lines on the Mesabi iron range and in the Lake Superior copper areas.

At the beginning of the twentieth century F. H. Brown carried out investigations and secured patents on a method wherein the resistance between two grounded points was measured. Several patents on modifications of this method were secured by Brown and McClatchey about 1900. Resistance measurements, as proposed by Brown, have the marked disadvantage that the ground in the immediate neighborhood of the electrodes exerts a large and undefinable effect on the measurements. In practice, resistance measurements are useful only in cases where the material surrounding the electrodes is homogeneous. This condition is ful-

filled for measurements made in bore-holes filled with water. In 1912 and 1913 C. Schlumberger proposed the direct current equipotential line method and Bergstrom the alternating current equipotential methods. The first significant proposal regarding the use of electromagnetic methods was made in 1913 by Schilowsky who investigated the magnetic effects produced by subsurface anomalies when the ground was energized by alternating current.

Schlumberger's studies from 1912 to 1914 in the Calvados Silurian basin (France) probably constitute the first tectonic studies by geophysical methods using artificial fields of force. Additional studies of the French Sain-Bel and Serbian Bor ore deposits, carried out during the same period, showed that the qualitative concepts of locating ore deposits by their electrical conductivity had been transformed into an effective technique. Wenner's work about 1916 did much to simplify the calculation of resistivity data obtained by passing a current between two electrodes and measuring the potential between two auxiliary electrodes.

The concept of "apparent resistivity", which greatly simplified interpretation, was introduced in 1922 and led to the possibility of systematic geophysical studies over large areas. The first large scale petroleum survey was carried out by C. and M. Schlumberger in 1923 and led to the proving of the existence and the mapping of the Aricesti dome in the Roumanian plain.

Prior to the twentieth century only a few isolated instances of successful application of geoelectrical methods for the discovery of commercially valuable ore bodies were recorded. With the advancement in the art and utilization of metals, more attention was directed toward the discovery of ore bodies. The more obvious mineralized areas were approaching depletion, and this furnished an incentive for means of detecting the presence of ores in unknown areas. The application of geophysical principles to mining problems reached its maximum during and immediately following the World War. Electrical methods were developed concurrently in France, Germany and Sweden, as well as in America where men engaged in this work soon far surpassed previous accomplishments by perfecting highly specialized methods and equipment. Geophysical methods applied to mining have not resulted in any spectacular developments, although a large number of smaller ore bodies, aggregating a most important economic value, have been located. In America there have been few major mineral discoveries since 1915, and in Europe none since 1850.

Recognition of the influence of subsurface structure on the current distribution at the earth's surface was followed by the use of electrical methods for structural mapping, with many successful applications to oil prospecting and civil engineering problems. Deep structural mapping by electrical methods has been most successful in the Permian basin New Mexico and West Texas. The geologic section comprises one

to two thousand feet of recent depositional material, underlain with a salt and anhydride section of variable thickness from a few feet to over a thousand feet. Beneath the anhydride section lie the sandstones and limestones containing the petroliferous source beds and commercial accumulations in closed structures and stratigraphic traps.

The variable thickness of the anhydrite section has prevented successful application of the seismic and gravitational methods, while the electrical methods have proved to be more applicable due to the large differences in electrical conductivity of the lithologic units constituting the section.

Electrical logging for determining the character and thickness of the strata penetrated by a drill hole was developed commercially by C. and M. Schlumberger in 1928. This technique with modifications has now become an accepted step in oil well drilling and structure correlation. In exploration work for mapping of subsurface structure the electrical logs are correlated between drill holes.

Popular interest in radio has focused attention on electrical methods of prospecting, and the rapid progress in radio communication has led to repeated attempts to locate ore bodies and oil by radio. It will be seen later that, to date, these methods have not been of commercial value due to the poor penetrating power of high frequency currents and electromagnetic fields. Recent theories linking near surface mineralization with deeper seated petroleum deposits and structural conditions has revived interest in resistivity measurements for shallow stratigraphic studies. At the present time the economic utility of these shallow penetration methods is controversial.

Seismic Methods.—Perhaps the person most worthy of being called the first seismologist was John Michell. In 1761, Michell published a paper in which he stated that the motion of the ground produced by earthquakes was transmitted as elastic vibrations through the earth's crust. In the same memoir, Michell suggested that observations on the time of shock at several places would permit the determination of the place of origin of the earthquake. As early as 1855, a number of seismographs using electromagnetic recording had been constructed in Italy by Palmieri. Robert Mallet, the first investigator to suggest the term "seismology", did much to create a widespread interest in the study of earthquakes. In addition to his studies of natural earthquakes, Mallet produced artificial earthquakes by exploding gunpowder and investigated the effects with a crude seismometer comprising a tray of mercury and a small telescope.

In 1888, A. Schmidt, on the basis of the work of Mallet, suggested that time distance graphs of artificial earthquake waves could be used to study the variation of velocity with depth.

One of the early papers which is of special interest is that of Fouque and Levy in 1889 in which they describe various experiments whereby they determined the velocity of seismic waves in various kinds of rocks. They used gunpowder and dynamite to create seismic waves in some of their experiments, while in others they utilized a large stamping machine weighing 100 tons. A dish of mercury served as their seismometer. Photographic registration on a moving plate was accomplished by focusing a concentrated beam of light on the surface of the mercury from which it was reflected to the photographic plate. They state that telephonic communication wa-

used in many of their experiments. Electrical firing of their explosive charge was accomplished by discharging a Leyden jar through a spark gap in proximity to gunpowder.

In the early part of the twentieth century, Belar, Von dem Borne, Benndorf, Galitzin and others, suggested the use of artificial explosions for studying subsurface structure. Prince Boris Galitzin perfected the galvanometric seismometer with magnetic damping which bears his name. The results of Galitzin's investigations were published in a volume, *Seismometrie,* in 1911. A revised edition of Galitzin's work, translated into German, has become one of the classics of the science.

L. P. Garrett, apparently the first to see the possibility of using refraction methods for locating salt domes, conducted investigations during the years 1905-1906 and made the first successful commercial application of refraction technique in 1923. Mintrop experimented with a mechanical seismograph during the World War. Working with Wiechert in Göttingen, Mintrop perfected his instrument and technique so that in 1919 he was able to secure a basic patent, since revoked, on the seismic method for determining subsurface structure. The first discovery of oil resulting from use of this method was on the Orchard Dome in Fort Bend County, Texas, in 1924. This work was done by a crew of Dr. L. Mintrop under the direction of Alexander Deussen. This seemed, and later proved to be, an ideal method to use on the Gulf Coast in the search for shallow salt domes. There the rocks possess fairly uniform and low velocities which are in marked contrast with the high velocities of the intruding salt dome. The rapidity of this method and the positive character of its results surpassed those of the earlier torsion balance methods.

The refraction method proved to be a rapid type of exploring tool and an effective means of locating salt domes. The method became obsolete as a leading exploration tool after about six years. As stated by De Golyer,† "the refraction technique in prospecting for salt domes in Coastal Texas and Louisiana is one of the most brilliant successes of applied geophysics in oil prospecting as well as the most clean cut example of the life cycle of a technique."

The reflection method of seismic geophysics was preceded by the studies of iceberg detection and measurement of ocean depths. Reginald Fessenden used it in 1914 and patented both refraction and reflection methods for use in locating geologic formations. Although Fessenden is usually credited as being the discoverer of the reflection method, J. C. Karcher, and others, notably E. A. Eckhardt and Burton McCollum, later improved the technique and made its application more practical. The first attempt to prove the applicability of this method was made in 1927 in Oklahoma. In 1928 drilling of a seismic high yielded a small producing well. The professionally accepted proof came in 1930 when three geophysical indications (seismic highs) found in the Seminole Oklahoma area were tested and yielded producing wells. Later the Edwards and the Chase areas in Oklahoma confirmed previous reflection predictions and definitely established the method. In 1929 the dip method of reflection shooting was developed. The seismic method today stands pre-

† E. De Golyer, "The Development of the Art of Prospecting," *The Guild of Brackett Lectures* (Princeton Univ. Press).

eminently as the most successful and widely used geophysical method for oil exploration. A great majority of all fields discovered from 1936 to 1939 are wholly or in part due to seismic work.

During the past three years over 25 new producing fields at depths from 8000 to 14,000 feet below the surface have been discovered by seismic methods. Today, successful seismic prospecting is being carried to depths of 15,000 feet as routine procedure in many surveys, and some surveys are being conducted to depths exceeding 25,000 feet. The trend toward deeper drilling is well-shown by the fact that some 315 wells have been drilled to depths exceeding 10,000 feet and about 250 of these deep wells have been drilled during the past two years.

CONTEMPORARY WORKERS AND PRESENT DEVELOPMENTS

Time and achievement must evaluate the methods being advocated and utilized today. This chapter covering only the early history of development is followed in later chapters by a record of current development and research in the several methods of exploration geophysics. The contributions of many individuals to the progress and enviable success achieved by these methods are recorded throughout the following text. More than 3,000 odd patents and literature references indicate the diversity of thought which has contributed to the development of modern geophysics.

TRENDS IN DEVELOPMENT OF FUTURE METHODS

The generally dependable character of the geological data obtained by geophysical methods and the advantage of operating cost which these methods have over exploration by drilling have established the value of geophysics in evaluating prospective lands. Geophysical exploration will decline in value and popularity, however, unless the resolving or detecting power of its methods is increased in proportion to the diminishing size of the undiscovered deposits. This may be illustrated by a brief review of the several techniques that have been employed in the exploration for oil.

Early prospectors who located the first wells drilled for oil were guided entirely by the occurrence of surface seepages. Soon after the discovery of the first fields by this method and by even more haphazard prospecting, it became recognized that the occurrence of oil and gas is commonly limited to the structurally high parts of anticlines. The anticlinal or structural theory, advanced to explain this occurrence, soon became established as sound and of tremendous value in the search for oil. Throughout subsequent years the search for favorable exposed anticlines by geologists using surface methods was feverish until the supply of favorable structures of this type became exhausted.

Petroleum geologists later recognized the probable existence of additional favorable anticlines hidden beneath extensive areas of shallow allu-

vium or poorly exposed surface rocks. The necessity of extending knowledge of structure into such areas led to pit-digging and trenching; a very old method of prospecting. As knowledge increased, it became apparent that suitable structural conditions may exist in the older strata unconformably underlying those exposed at the surface. To map the structure underneath the unconformity, core drilling was introduced in Oklahoma in 1919. This method of exploration was new to the petroleum industry in this country. Core drilling gave the ultimate in exactness of data, but its application was limited by its high cost, with resultant restriction to relatively shallow depths. Core drilling was extended to most of the petroleum districts of the country and is still used in areas where structural relief is small or where more recent methods are not effective.

The use of core-drilling for structural mapping had made considerable progress in Oklahoma, Kansas, and some parts of the Gulf Coast when it was abruptly superseded by the more rapid and economical reflection seismic method, introduced in 1927. (The difficulties of obtaining small cores made that type of drilling quite expensive.) During the past few years increasing use has been made of another type of drilling method, termed "slim-hole" drilling.

Present trends of development indicate that increasing application may be expected from: (a) slim-hole exploratory drilling, (b) a type of electric method possessing much greater resolving power than the present methods utilizing near-surface potential studies, and, (c) soil analysis.

Direct exploratory methods, such as drilling, are a means of obtaining the most definite and irrefutable type of data. The information from such a direct method possesses three components that are most desirable: (1) direct determination of the structural conditions existing at a specific point or localized area, as contrasted to all other geophysical methods which involve measurements over areas of relatively large extent; * (2) specific information regarding the strata traversed by the bore hole, by means of cores or electrical logs; and (3) accurate depth measurements.

In present commercial usage, slim-hole drilling constitutes the drilling and electrical logging of a small diameter hole. Successful work of this type is being conducted to depths as great as 8,000-10,000 feet. The subsurface structure is determined by the correlation of the electrical logs from holes properly spaced in the area under investigation. Any desired portion of a hole may be cored to obtain samples for lithologic, paleontological, and

* Reflection seismic work has a relatively good resolving power because its zone of measurement is small as compared to gravitational, magnetic, electrical, and other potential methods which are decidedly "weighted" types of measurements involving a zone of measurement of indefinite lateral extent and depth.

porosity studies and to determine the presence or absence of oil. Substitution of an electrical log for the complete core effects a saving which makes the method economically feasible in many areas, especially the Gulf Coast where soft strata permit rapid drilling at low cost. There is a possibility of obtaining greater advantages and of effecting still further economy by the use of some form of electrical or mud logging technique while drilling is in progress. The latter procedure would permit further economy by allowing much smaller holes to be drilled since the present diameter of the hole is limited by a size sufficient to allow the electrical logging electrodes to be lowered to bottom. Present commercial practice indicates that a 4½″ hole is the minimum diameter if the drilling operation is to be followed by electrical logging. If a usable log could be obtained while drilling, the diameter of the hole could be reduced, with a material saving in drilling time and cost. Work now under way indicates that the logging-while-drilling techniques may be able to utilize holes of about 2 to 2½ inches in diameter. The limiting minimum diameter appears to be governed by the ability to circulate sufficient water or mud to clear the hole of cuttings.

Slim-hole drilling with coring at desired intervals, may be combined with core analysis and orientation to produce an exploration procedure far surpassing in accuracy any technique in use today. If greater economy can be achieved in the drilling and logging technique, the cost of such work may become commercially feasible for general exploration. Considerable experimental work has already been done using slim-hole drilling as control, with holes spaced one to five miles apart, and other lower cost geophysical methods for structural mapping between holes. Successful development of such a combined technique would allow its application in many areas not amenable to the more common geophysical techniques.

The diversity of the electrical methods and the relative ease with which a measurable field of force may be established give these methods undeniable advantages over other potential type methods.* However, the definiteness of the electrical methods utilizing surface potential measurements has been relatively poor due to the masking and disturbing effect of the near-surface materials on the potential distribution. Work during the past few years indicates that many of these near-surface effects can be minimized by some direct type measurement (such as the measurement of the magnetic field associated with the flow of current) instead of the indirect measurements dependent upon the surface distribution of potential. Proper utilization of this type of measurement

* The close agreement between soil analysis data on lateral distribution of mineralization and electrical data on variations in lateral resistivity over structure adds increasing importance to the relatively low cost constant-depth type of resistivity measurement.

may produce a method giving sufficient accuracy in many areas where other geophysical methods have not proved applicable.

Soil analysis, including the determination of both the hydrocarbon and mineralization content, is receiving considerable publicity and has temporarily captured the popular fancy due to its ability to detect and measure diagnostic variables which appear to be directly associated with known oil fields. Unless this method proves useful in directly determining the location of new oil fields its value is limited since its use adds nothing to the geologist's knowledge of the structure and stratigraphy of the area surveyed. The usual data must be treated from a statistical viewpoint to evaluate the areal distribution of the hydrocarbons or minerals, after which allowance must be made for the effects of surface or near-surface conditions not necessarily related to the deeper existence of an oil field. Careful consideration must also be given ground water movements and its effect on the distribution of hydrocarbons and secondary mineralization at the surface of the earth. Present indications seem to point to the use of soil analysis methods for furnishing confirmatory evidence as an adjunct to exploratory methods capable of mapping structure directly and quantitatively.

Exploration geophysics will continue to move forward. Increasing applications will be found for the instruments and fundamental techniques. Many of these developments will be employed in other lines of commercial endeavor and for military operations.

Numerous adaptations to military problems are being made. The location of guns can be determined accurately by employing seismic detecting and recording equipment to record sound waves or a combination of sound waves and earth waves. Analysis of such a record commonly will indicate by wave frequency and other characteristics the type of gun and its size. Seismic equipment, with slight modification, will record direction and distance to troop and field equipment movement. Sound wave recording equipment is being successfully employed for detection and location of aircraft. The characteristics of the record are often a clue to the type of plane and motor.

Other applications of seismic methods are being made to buildings and foundations, dam sites, etc., to determine their natural period of vibration as an aid in minimizing possible earthquake damage.

CHAPTER II

GEOLOGIC AND ECONOMIC BACKGROUND OF EXPLORATION GEOPHYSICS

INTRODUCTION

Exploration for mineral resources, which are bountifully but irregularly distributed within the earth's crust, has engaged the attention of men since earliest time.† From these early beginnings the frontiers of mineral exploration have advanced step by step ‡ from a search for those mineral deposits whose presence was suggested or indicated by evidence at the surface of the ground (certain characteristic types of rock and mineral outcrops or surface structure) to the present-day search for hidden mineral substances which are deeply buried below the earth's surface. Modern exploration methods have made it possible to discover these deeply buried deposits even though indicative surface outcrops are completely absent.* This advance in the science and art of exploration has in general followed steps in the advancement of civilization and improved methods for refining and conserving our resources. Before the advent of geophysical exploration the chief hope of a continuing supply of necessary mineral resources was provided by improvements in exploitation and refining to permit increasingly greater recovery and utilization of the raw product. The recent application of geophysical exploration methods on a world-wide scale, while in no way insuring a permanent supply of mineral resources, has resulted in a vast increase in estimated known reserves.

General Factors Governing Application and Choice of Geophysical Methods.—The application of geophysical methods to the various geological problems involved in the search for mineral resources** is

† T. A. Rickard, *Man and Metals*, 2 Vols. (Whittlesey House, McGraw-Hill Book Co., Inc., New York, 1932.)

‡ A. J. Ellis, "The Divining Rod—A History of Water Witching," U.S.G.S. Water Supply Paper No. 416.

* An example of the successful culmination of modern exploratory effort is the recent discovery and production of oil from strata buried over 13,000 feet below the blanketing alluvium of the San Joaquin Valley, California. H. W. Hoots, "Additions to California Oil Reserves," *California Oil World*, April, 1939. See, also, the March, 1938 issue of *The Petroleum World*.

** The term mineral resources is used here to include any substance whose natural habitat is the earth's crust and whose beneficial exploitation requires the use of the various well known mining, drilling, and engineering methods. Practically, the term includes metallic and non-metallic minerals, petroleum, and water.

governed primarily by two factors: (1) the technical factor which is associated with mode of geologic occurrence or type of geologic structure and (2) the economic factor of cost of operation.

Regardless of the specific field of exploration in which it is proposed to do geophysical work, there are certain factors of general scope which will indicate: first, whether use of a geophysical method is technically justified and second, whether its use is economically sound. Technical justification for the use of a geophysical method depends primarily on the possibility of coordinating the known and/or probable subsurface geological conditions in any area with the data which can be supplied by a specific geophysical method. Certain of the methods are uniquely applicable to certain geological and terrain conditions. The selection of a suitable method requires, therefore, a study of the theoretical applicability of the various methods to the specific problem.

Economic justification for the use of a geophysical method is mainly a function of unit cost in terms of expected results; particularly as to the extent these results will decrease the financial risk of subsequent exploration. Due to the variable and sometimes unique nature of geological conditions in different areas, however, strict standardization of cost is not possible in geophysical work. Generally, the cost factor can be evaluated only in conjunction with geological and other technical considerations. The interrelation of the cost factor and the technical factor may be expressed by defining the purpose of a geophysical survey to be the securing of optimum geologic information at minimum cost.

Theoretically, the applicability of geophysical methods of exploration may extend to any area in which the subsurface geological relations are obscure and where additional subsurface information may serve to reduce the financial risk or engineering hazard involved. Practically, specific fields of application as well as choice of proper methods to employ depend upon a great many considerations, some of the more important of which may be listed as follows: (1) amount of surface and/or subsurface geological information already available, (2) local geology (probable type of structure, etc.), (3) purpose of survey, (4) depth of investigation required, (5) terrain conditions, (6) theoretical applicability of specific methods, (7) cost of geophysical survey, (8) cost of alternative means of securing required information (core drilling, etc.), and (9) local land ownership. These various factors are discussed in greater detail in the following sections in which are outlined the specific applications of the several methods in the fields of petroleum geology, mining geology, civil engineering, and water supply engineering.

Classification of Exploration Methods.—(In order of increasing resolving power for their usual applications.)

Natural Field of Force	Artificial Field of Force
Geothermal	Electrical (inductive)
Magnetic	Electrical (conductive)
Gravitational	Seismic (refraction)
Geochemical: surface soil or gas analysis	Seismic (reflection)
Radioactive	Slim-hole drilling, with electrical logging
Electrical self potential	
Slim-hole drilling with mud logging	
Core-hole drilling, with paleontological studies and core orientation	

In this tabulation the various methods are listed in the generally recognized order of increasing resolving power;† i.e., their ability to obtain data diagnostic of subsurface conditions. The term resolving power relates to the general use and applicability of the methods only, because a method of relativity low resolving power might under specific circumstances become a method of superior resolving power. For example, magnetic methods while generally of low resolving power for sedimentary structural investigations achieve a high resolving power when applied to the location of highly magnetic subsurface bodies under shallow depth of cover.

Generally, the resolving power of the methods utilizing natural fields of force is less than that of methods in which the force field is artificially created and subject to control both as regards direction and intensity. It is characteristic of the group of methods utilizing a natural field that the field of force measured at the surface is the resultant of near-surface irregularities and regional variations as well as the effect of local geological structure concerning which information is desired. The practical difficulty of separating the composite effect into its component parts is a serious disadvantage of these methods. The methods utilizing an artificial field are also subject to several component effects; in the case of these methods, however, the extraneous effects usually may be evaluated or minimized by controlling the points of application of the force fields and their magnitudes and configurations. The particular feature of greatest importance is the ability to control the effective depth range of the artificially-created force fields.

† See also E. E. Rosaire, "On the Strategy and Tactics of Exploration for Petroleum," *Journal of the Society of Petroleum Geophysicists,* Vol. VI, No. 1, July, 1935, pp. 11-26.

The technique of slim-hole drilling as now used commonly incorporates a subdivision of the electrical methods and/or the geochemical methods. It differs from other exploratory methods in that measurements are made in previously drilled deep core holes, rather than on the surface of the ground. Ordinary electrical logging comprises those methods utilizing electrical measurements in wells or holes drilled primarily for discovery and production of oil. Slim-hole drilling on the other hand employs the technique of making electrical measurements in small-diameter, low-cost holes drilled expressly for this purpose. In the electrical logging and slim-hole drilling techniques, the electrical measurements are ordinarily conducted after completion of drilling. The most recent technique, however, obtains electrical and mud-logging data during the process of drilling.* Another recently developed method of recording subsurface data is that of constructing drilling-time logs which commonly reveal any important change in lithology by the change in rate of drilling.

The geochemical method, as subsequently explained, holds promise of being able to locate petroleum deposits directly, in contrast to the usual use of petroleum geophysical methods merely as a means for determining favorable structure for oil accumulation. For this reason, the geochemical method, although possibly of relatively minor importance in its present stage of development, is potentially a method of higher detectability for oil deposits than other surface methods now in use.

Fundamental principles and general procedure are essentially the same for all of the geophysical methods. All of the methods depend for their *modus operandi* upon the following basic facts: (1) The subsurface is composed of rock formations possessing certain differing physical properties. (2) Natural or artificially-created force fields within the subsurface are affected to different degrees by these physical properties. (3) The extent to which certain force fields are affected depends, among other factors, on the magnitude of particular physical properties and on the sizes, masses, and arrangements of the subsurface rock materials. (4) The effects produced by the subsurface variations in physical properties upon certain force fields can be measured at the surface of the ground.** (5) The data obtained can be interpreted or translated into terms of probable geological structure. The basic procedure in all methods, therefore, consists in (a) measuring variations in force fields at the surface of the ground and (b) predicting, on the basis of a knowledge of the influencing physical properties, the probable configuration of subsurface materials (geological structure) which would cause the measured effects.

* See chapter on Drill Hole Investigations.

** An exception is noted in electrical logging and slim-hole drilling wherein the measurements are made below the surface.

The following classification shows the fundamental properties and relationships which form the basis for the various methods:

Method	*"Field of Force"*	*Physical or Chemical Property*
Magnetic	Earth's magnetic field	Magnetic permeability
Gravitational	Earth's gravitational field	Density
Electrical	Natural potential field	Spontaneous polarization
	Artificially created electric or electromagnetic field	Electrical conductivity or its reciprocal resistivity; path of current flow
Seismic	Artificially created seismic waves	The velocity of transmission of seismic waves, as affected by density, elasticity, etc.
Radioactive	Radioactive radiation	Radioactivity, i.e., the emission of electrically charged particles from the nuclei of the atoms of radioactive materials
Geothermal	Earth temperature gradients	Thermal conductivity
Geochemical	Emanation of hydrocarbon vapors	Hydrocarbon content of the earth
	Ascending or descending aqueous solutions.	Mineral content of the earth

As is shown later in this chapter, the relationship of these various physical properties of the rock materials to problems of economic geology is usually the main factor dictating applicability and choice of methods for exploring any given area.

Technique of Applying a Geophysical Method.—In applying a geophysical method the method of attack may be either direct or indirect.† In the direct attack, the purpose of the geophysical measurements is to locate a subsurface body or geological condition by means of an anomaly which is directly associated with, and the direct result of, this body or condition. Examples of the direct method are the location of bodies of sulphide ore by means of their high electrical conductivity, the location of bedrock in a dam site problem, the detection of a petroleum accumulation by an increased concentration of hydrocarbons in the soil, etc. In one type of the indirect mode of attack, the probable location of the desired subsurface deposits is inferred from the similarity of the geological structure determined by the geophysical survey to those structures known to commonly contain such deposits. An example of the indirect attack is the locating of a petroleum deposit by the detection of a subsurface structure or trap suitable for petroleum accumulation and which, when

† Compare R. Ambronn, *Elements of Geophysics*, p. 5 (McGraw-Hill, N. Y. 1928).

drilled, was found to contain petroleum. An example in mining exploration is the indirect location of placer gold concentrations associated with black sand concentrations by means of the magnetic anomalies produced by these black sands.

In mining exploration the direct method is of predominant importance, but there are many applications of the indirect type. A combination of both techniques, where possible, has produced the most satisfactory results in many cases.

The choice of one particular type of geophysical method is commonly dictated by existing conditions. The mapping of basic igneous dikes that intrude sedimentary formations is an example. For this particular problem, the use of magnetic methods combined with available surface geological data will usually give optimum information at minimum cost. In a majority of cases, however, the controlling factors are not so obvious, and judicious safe exploration may demand the use of more than one geophysical method.

Initial exploratory studies of a reconnaissance nature commonly are conducted to indicate, in a general way, the major geological features of probable importance. Investigations may be carried out with a method of greater selectivity to obtain confirmatory and more detailed evidence in certain parts of the area. An example of this procedure in mining exploration is the preliminary use of a magnetic survey to outline the plan location of certain magnetic dikes or ore-bearing mineral veins followed by an electrical survey to determine the occurrence, location, and depth of sulphide mineralization in the veins, or associated with the dikes. A similar example in petroleum exploration is the common procedure of supplementing preliminary magnetic or gravimetric results by detailed seismic surveys.

Outline of an Exploration Program

A general exploration program which, with minor revisions, can be applied to a wide variety of exploratory problems may be formulated. The initial step for such a program in any given area, whether for ore deposits, petroleum, or other resources, should be a preliminary study of available geological literature pertaining to the region and to the specific area. This will be concerned principally with regional geology, manner of occurrence of ore or petroleum deposits, and the history of their discovery and exploitation in this and adjacent areas.

The second step is the securing of a temporary legal control and right-of-way on the land to be investigated. This may be of various forms. In mining, the common form is the bond and lease with option to purchase.

In oil prospecting, however, the geophysical work is commonly done with nothing but right-of-way permission to trespass on the property, leaseholds being obtained during or after completion of the survey and the discovery of favorable prospects.

The third step in this general program is the preliminary geological study which is undertaken for two purposes: (1) to determine to what extent geophysical methods will be required to secure information necessary for economical development of the property and (2) to provide information which will determine the choice of geophysical methods and aid in subsequent interpretation of the geophysical data. Since the geophysical survey is to be conducted to gather geological data pertaining to matters such as structure and the location and depth of certain rock formations, it is essential that all available geological information from rock outcrops, drill holes, pits, and mining excavations, be correlated and appraised in advance of the relatively costly geophysical survey. By this means the geophysical survey may be planned to greater advantage and the time and money spent may be materially reduced. In many cases, the geological study alone will indicate the character and amount of geophysical work needed to complete the desired geological picture.

The preparation of preliminary maps is an important part of this step in the exploration program. Topographic maps may be important for several reasons: (1) They aid in planning any exploration program and in interpreting surface geology. (2) They provide the topographic control essential to the interpretation of geophysical data obtained from areas of appreciable surface relief. (3) Topographic relief frequently is related to surface structure and is a further aid, therefore, in accomplishing the purpose of the survey: namely, the determination of local subsurface structure. Under certain conditions, the use of aerial photographs will be of assistance and will be justified economically.† Careful and systematic inspection of them, particularly with a stereoscope, provide valuable clues to the dip of strata, the position of faults and critical rock ledges, and the location of native vegetation and crops which may impede geophysical work. The use of aerial photography, however, is more readily justified for relatively extensive exploration and difficult terrain conditions. Considerable discrimination is necessary at all times to insure that the economic value of the technical data secured justifies the expense. All preliminary maps should be prepared with due regard to their proposed use in conjunction with the planning of a geophysical survey and the plotting and interpretation of the geophysical results.

The fourth step in this general exploration program is the determination of the applicability of the various geophysical methods to the

† Report of symposium on "Aerial Geologizing." (*Tech. Pub. No. 756, A.I.M.E.*, 1936.)

problem at hand and the selection of one or more methods. (This step may, of course, be contemporaneous with preceding steps.) If the previously acquired geological information is comprehensive and requires for completion of the study only an extension of known surface features to greater depths, it is probable that the need of a detailed survey by a particular geophysical method in certain limited parts of the area will be clearly indicated. The choice of method commonly is controlled by the geological setting, the information desired, and the cost. Where the area is extensive and available geological information is meager, economic considerations may dictate a preliminary reconnaissance by one of the more rapid and less costly geophysical methods before certain parts of the area are selected for detailed survey by another more costly method.

The fifth step is the performance of the geophysical field work. This is an engineering as well as a scientific task. The engineering work of geophysical surveys has much in common with that of other land surveys requiring the compilation of a series of observations at designated locations. The taking of notes, the proper recording of station locations, and the plotting of stations and recorded data on maps are similar in all types of land surveys, except for differences arising from the requirements of individual methods.

The tabulated geophysical measurements are next subjected to computations by means of appropriate formulas and corrections, so that the computed results will show the variations, if present, in some physical property of the subsurface such as magnetic permeability, electrical conductivity, density, and the depth and altitude of reflecting beds, elastic wave velocity, etc. Usually the computed results are plotted in a manner which lends itself readily to subsequent analysis and correlation with known geology and surface features.

The sixth step is the analysis and interpretation of the observed and recorded data. The interpretation consists primarily of translating the computed results into a graphic three-dimensional picture of the subsurface. Graphs, cross sections, contour maps, and vector diagrams are the usual means of illustrating these final results.

In actual field operation, the computation and interpretation of results proceeds almost contemporaneously with the securing and recording of field data. Preliminary interpretations are used to guide the survey as it progresses and to expedite land acquisition and/or the performance of lease requirements, prior to completion of the survey.

The compiled information provides the basis for recommendations for either additional exploration or for development. If the survey has been successful and complete in gathering the desired information, such recommendations will take one of three forms: (1) test drilling or mining, (2) abandonment of the venture, or (3) holding the property for later development. The decision to abandon an enterprise is fully as important

and oftentimes more important than the decision to mine or drill. The history of mining and oil exploration clearly indicates that the greatest single factor causing loss of capital has been the common human trait of "not knowing when to stop."

The interpretation of the geophysical data is the most important step in this general exploration program. The technique of interpretation covers a wide range of procedures, from that relying predominantly on mathematical analysis to that relying predominantly on empirical methods built up through extensive experience in actual field problems and supplemented by empirical data from small scale laboratory experiments. In most cases, successful interpretation includes both a mathematical and an empirical treatment of the field data, the relative predominance of one over the other usually being a function of the amount of geological information available and the relative complexity of the subsurface structure. Pure mathematical analysis alone is seldom adequate, principally because of the many complex variations in the structure and composition of underground rocks. Successful interpretation usually is based on theoretical considerations tempered by experience gained during prior similar studies.

The interpretation of geophysical data is a highly involved task requiring ingenuity, imagination and adequate training in geology and physics. It is not uncommon that the task is divided in such a manner that the observed and computed data are first translated into basic geologic terms by a geologist trained in the theories and methods of analyzing geophysical data and thoroughly familiar with the regional and local structural details of the areas under investigation. Experience teaches that it is only by the intelligent correlation of geophysical data with all available geologic data that the greatest ultimate practical value of a geophysical survey can be realized. At present, only relatively few men are qualified by adequate training and experience to handle without assistance the many problems of geophysical interpretations.†

The responsibility of directing exploration rightfully falls to the geologist or mining engineer. To him geophysical data are of practical value only to the extent that they improve his understanding of existing geologic conditions; hence, if these data are to have value, they must be interpreted and the results described in geologic terms.‡

GEOPHYSICAL METHODS IN PROSPECTING FOR PETROLEUM

General Field of Application.—The initial impetus to the development of geophysical exploration methods was provided by their demonstrated usefulness in the search for certain types of ore bodies. The

† "Round Table on Geophysical Education," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. No. 950.

‡ O. L. Brace, "Interrelation of Geology and Geophysics," *The Oil Weekly*, April 26, 1937.

greatest application, development, and utility of the methods, however, has been in the field of petroleum exploration rather than in mining.

Until the advent of geophysical methods, the search for, and discovery of, new deposits of petroleum were accomplished by the geologist's mapping of surface features, conduct of core drilling, and correlation of subsurface data made available from drilled wells. Exploration for possible oil-bearing structures was first extended to those areas in which surface evidence of one sort or another indicated the probability of subsurface oil accumulation. Favorable surface signs included: (1) direct evidence of petroleum occurrence such as oil seeps, gas seeps, bituminous rocks or dikes, "paraffin dirt," asphalt pits, oil-impregnated shales, etc.; (2) exposed anticlinal folds and other structural features favorable for petroleum accumulation at depth; (3) topographic and physiographic evidence of favorable structure not otherwise reflected at the surface of the ground. Extensive exploration in the known oil-bearing provinces of the world between 1860 and 1920 rapidly depleted the supply of favorable prospects apparent from the use of methods then available. Subsurface information from drilled wells and exploratory core holes became increasingly important after 1915, but, although still of indispensable value, these data took a subordinate place in aggressive exploration after the advent of successful geophysical methods. The need for new methods was obvious since the demand for larger supplies of petroleum was increasing rapidly.

In many of the known oil-bearing provinces, large areas contiguous to producing oil fields are covered by soil, alluvium, glacial deposits, sand dunes, or water which, to date, have prevented or seriously impeded the determination of deeper structure. Regional geologic considerations of such areas may indicate the presence of probable petroleum source beds, and the possible existence of favorable traps for oil below the concealing mantle of surface materials. Notable examples of such areas in the United States are the glacial drift covered parts of Michigan, Indiana, Illinois, Missouri, Nebraska and Iowa, and the extensive alluvium covered San Joaquin and Sacramento valleys in California.† In such areas, the possibility of additional future discoveries of petroleum rests largely on the application of improved techniques in geophysical exploration. Even in cases where structure of the surface formations can be mapped satisfactorily, the projection of exposed structure to the depths at which oil accumulation might be expected often introduces a high element of risk because of obvious structural asymmetry, or the possible occurrence of unconformities or appreciable variations in the thickness of subsurface beds.

Many known accumulations of petroleum occur in sand lenses and other types of stratigraphic traps not necessarily associated with closed anticlinal

† W. G. Osborn, (1) "Geological Complex of Iowa," *The Oil and Gas Journal*, May 13, 1937; (2) "Geologic Aspects of the Forest City Basin," *The Oil and Gas Journal*, January 5, 1939.

structures. The discovery of such stratigraphic traps is a difficult problem, even with the aid of modern geophysical methods. Present methods of attack include the application of a thorough knowledge of the regional and local geologic history, slim-hole drilling, and geochemical surveys.

Geophysical exploration for petroleum structures consists primarily in locating and delineating those types of subsurface structure which experience has shown to be favorable for oil accumulation. The successes of the methods to date may be measured, therefore, by the extent to which they have disclosed favorable subsurface structures for which geologic evidence was previously uncertain or lacking. By this criterion and the many prolific new fields so discovered, petroleum geophysics has become established as a necessity in most exploration programs.

The several geophysical methods listed in the first part of this chapter have been utilized in varying degrees in the search for oil. By far the greatest success on a commercial scale, however, has been attained by the seismic and gravitational methods. Seismic methods have been the most successful, generally, and currently are enjoying the widest application. Recently, however, the application of slim-hole drilling with its accompanying electrical logging has increased as a result of the decreased costs and the direct application of this method to the search for stratigraphic traps and structures of small closure.

Relative Utility and Importance of the Methods

Magnetic Methods

Magnetic methods have been widely employed in some areas for specific problems in oil exploration. Their greatest success has been in certain cases of uniquely favorable geological conditions wherein strong magnetic anomalies are directly associated with the oil-bearing structure. As an example, magnetic methods have been used successfully to map the trends of buried igneous ridges and other features of basement topography, which, from previous experience are known to have, commonly, a definite relation to oil-bearing structures in overlying sedimentary rocks. Another practical example is the Hobbs oil field of New Mexico which occurs on a structure overlying a pronounced uplift in the basement rocks. The location of the discovery well of this field was based on the results of a magnetic survey. (See Figure 65.)

As an independent exploration technique, magnetic methods have several serious disadvantages: (1) lack of depth control, (2) difficulty in separating the unrelated magnetic components due to near-surface materials from those which reflect the deeper structure, (3) difficulty in distinguishing between anomalies due to structural variation and those due to lithologic and mineralogic variations, (4) difficulty in relating the areal location of a magnetic anomaly to a definite location of the structure

causing the anomaly. The two greatest deterrents to successful interpretation of magnetic data are: (a) magnetic anomalies may be due to structure or to depositional variations of magnetic permeability and oftentimes it is impossible to specify which of these factors is responsible for the anomaly; (b) even when the magnetic anomalies are structural in origin, there may be no definite relationship between the type of anomaly and the structure, e.g., subsurface structural "highs" may, under different geologic settings, be associated either with magnetic "highs" or magnetic "lows." These unfavorable factors arise in the use of magnetic methods in searching for favorable structure in sedimentary rocks, but are not present in many local problems of specific nature, such as the location of igneous dikes.

Offsetting these unfavorable factors are the relatively high speed and low cost of magnetic surveys. These latter factors are sufficiently pronounced to indicate a definite field of usefulness for the magnetic methods for reconnaissance preliminary to more detailed studies by other methods. These factors, together with the value of the magnetometer under certain circumstances, commonly justifies a magnetic reconnaissance preliminary to further detailed studies by other methods, particularly in areas where information is available on the controlling magnetic properties of the subsurface.

Gravitational Methods

In fundamental theory and in practical limitations the gravitational or gravimetric methods are quite similar to the magnetic methods. In general, however, the gravitational methods have been used more extensively and with relatively greater success in locating anticlinal folds and faults. These methods are useful in mapping the relative topographic relief of the comparatively dense basement rocks and thus oftentimes are valuable in locating associated structure in the overlying sedimentary formations. They are valuable, also, in locating major structural deformations in the overlying sediments. The outstanding economic use of gravitational methods in the United States has been the locating of the deep, lower density, salt plugs which form the cores of the numerous salt domes in the Gulf Coast region.

Precisely as with magnetic methods, the chief deterrent to more general application of gravimetric methods is the difficulty of interpretation, particularly in the resolution of differential depth effects. However, the relationship between structure and variations in gravity is commonly more definite than similar relationships between structure and the earth's magnetic field. For example, salt dome structures are usually associated with gravity minima while ordinary domal or anticlinal structures are more often associated with gravity maxima. (The gravity maxima existing over certain shallow salt domes such as the Spindletop dome, the Nash dome, and a few others, constitute an exception to this rule.)

Extensive use of gravitational methods for reconnaissance was pre-

viously restricted by their limited application to areas outside the salt dome province and by the relatively high cost of torsion balance work. Recent development and improvement of the gravimeter has resulted in wider use of this method both for reconnaissance preliminary to detailing by methods with greater resolving power and for detailed gravitational work.

Electrical Methods

Several different types of electrical methods have been employed in petroleum exploration. All of them may be classed as either conductive or inductive. The former type has had the most successful application for deep investigations.

The theoretical applicability of electrical methods in oil structure mapping is extensive, while the practical applicability is generally conditioned by limitations as to depth and their adaptation to local conditions, and by appreciable cost for moderate depth of exploration. A technique which permits controlled variation of the characteristics of the current field, configuration and spacing of electrodes, etc., allows a depth control which is not enjoyed either by magnetic or gravitational methods. Electrical methods may be successfully adapted to the requirements both of reconnaissance and detailed work on geological problems in areas where the character of subsurface rocks are favorable for their operation. For reconnaissance work, the cost of electrical profiling is generally less than that of any other method, with the exception of the magnetic methods and the improved gravimeter methods. Reconnaissance electrical profiling consists in measuring lateral variations in resistivity, generally at a constant electrode spacing, in order to obtain an indication of the location of hidden structural features. In detailed work a depth variable is employed which permits the mapping of underground structure by determining the depth and configuration of some electrically prominent member of the stratigraphic section.

Electrical methods employing depth control are generally comparable to the refraction seismic method in resolving power, but are inferior, in this regard, to the reflection seismic method. Although the cost of continuous electrical profiling to depths of 4,500-5,000 feet is usually less than that for reflection seismograph work, the more general nature of electrical results restricts its adaptation in oil exploration to those areas in which reliable data cannot be obtained with the seismograph. For example, in the Permian Basin of New Mexico and West Texas, geological conditions are generally favorable for the electrical method but provide almost insurmountable difficulties for most other geophysical methods.

The electrical methods commonly have distinct advantages in the mapping of near-surface features. In many cases, they can be used as an indirect means of locating faults, principally because fault zones generally have a relatively high electrical conductivity.

Electrical logging of oil wells has recently assumed considerable importance in correlation work. This method measures the vertical varia-

tion in the electrical characteristics of the geologic section traversed by drill holes. The measurements are made in the drilled well or while drilling and the data are plotted as a continuous log. From a study and comparison of the electrical logs of wells in an area it is generally possible: (1) to differentiate between beds of shale and sand, (2) to distinguish sands containing oil or gas from those carrying salt water, and (3) to make stratigraphic correlations and thus determine the structure throughout an area. These methods are discussed in detail in Chapter X.

Seismic Methods

The seismic methods have proved generally the most applicable and uniformly successful of the various geophysical methods used in oil exploration. Two general types of seismic technique are employed: (1) refraction surveying and (2) reflection surveying.

The refraction seismic method has been used most extensively and successfully in the search for shallow salt domes in the Gulf Coast of Texas and Louisiana. The characteristic "higher velocity" of the salt cores of the domes renders this type of structure readily amenable to refraction technique. In some regards, the general limitations of refraction technique are similar to those of electrical methods, particularly in that increasing depth results in rapidly increasing costs and loss in detectability.

Reflection seismic surveys are currently the most popular and successful of the geophysical techniques used in exploration for oil structures. They have been employed in areas containing practically all types of geological structures important in oil exploration, and in most of the major oil-producing provinces of the world. They are of special value in mapping geological structure in sedimentary beds to depths of 15,000 feet. This is usually accomplished by one of three methods: (a) by correlating corresponding reflections from the same formation over an area, (b) by computing isolated dips (dip-shooting) of the formation or formations from which reflections are obtained, or (c) by continuous correlation of short profiles obtained from closely spaced shot points. The presence of suitable reflecting formations in the subsurface sedimentaries and the absence of near-surface masking layers (glacial fill, deeply weathered zones and other material of irregular velocity) are essential to the success of the method.

The results of actual drilling have shown that the reflection seismic method has mapped oil structures with required accuracy to depths of approximately 15,000 feet, and current data indicate that reliable fragmentary records are commonly obtained below that depth.

In common with the other methods, the reflection technique has inherent limitations which hinder or prevent its application in certain areas.† These limitations are especially pronounced: (1) in extensively faulted areas, (2) in steeply dipping areas, (3) in areas which are covered

† Report of symposium on "Seismograph Prospecting for Oil," *A.I.M.E. Geophysical Prospecting, Tech. Pub. No. 1059,* April, 1939.

by thick layers of material having an irregular velocity such as several hundred feet of alluvium, glacial till, deep weathering, etc., (4) in areas in which the reflecting properties of the subsurface formations are so mediocre that difficulty is encountered in recognizing corresponding reflections from the same formation throughout the area.

Choice of Methods for Particular Geologic Problems.—The various general factors to be considered in selecting the proper geophysical method for exploration have already been discussed briefly. Perhaps the most important factors to be considered in particular cases are the geological and terrain conditions of the area. Commercial accumulations of petroleum are always associated with extensive areas of sedimentary rocks, and with very few exceptions the oil-bearing rock is of sedimentary origin—most commonly porous sand, sandstone, limestone, or fractured shale. Accumulation of petroleum is governed by structural, stratigraphic, and lithologic conditions. The essential requirements are: (1) petroleum "source beds," (2) a porous and permeable reservoir rock such as sandstone to contain the oil, (3) an impervious overlying bed, such as shale, to prevent upward escape of the oil, and (4) a favorable structural or stratigraphic trap for the accumulation of oil. The selection of regions favorable for all of these conditions is the responsibility of petroleum geology, but in its search for conditions fulfilling the last requirement this applied science leans heavily upon petroleum geophysics.

The search for certain types of geologic features as favorable sites for new oil fields results entirely from knowledge of the manner of occurrence of oil in existing fields throughout the world, and an understanding of the geologic factors which control these occurrences. The general conditions governing oil accumulations have become well established since the anticlinal theory † first achieved prominence some 80 years ago. Traps which cause accumulation of important oil deposits have been discussed by De Golyer. ‡ "Geologically, the trap may be of structural, stratigraphic or chemical origin. If of structural origin, it may be the result of folding, of faulting, of a combination of the two, of differential compaction of sediments over buried hills, of the intrusion of igneous rocks, or of the formation of salt domes. If of stratigraphic origin, it may be the result of the lensing out or shaling out of sands, of old shore lines, of the deposition of sands against old land masses, of the removal of sand by erosion and subsequent roofing by deposition of relatively impervious formations or of various combinations of stratigraphic processes. If of chemical origin, the trap may be the result of solution, of cementation, or of metamorphic alteration which may involve both processes."

The more common forms of oil-bearing structures are listed in Table 2.

Throughout the world there are certain areas or regions (generally sedimentary basins) in which major accumulations of oil occur and in which have been developed the major producing oil fields of the world. The petroleum accumulations (oil fields or districts) in each region are usually related genetically, and structural conditions in the various districts are often similar and typical of the region as a whole. Such regions are known as petroliferous provinces. Examples of petroliferous provinces

† For additional information see W. H. Emmons, *The Geology of Petroleum* (McGraw Hill, 1921).

‡ De Golyer, *loc. cit.*, p. 10.

TABLE 2

CLASSIFICATION OF OIL STRUCTURES WITH RESPECT TO APPLICATION OF GEOPHYSICAL METHODS

I. STRUCTURAL TRAPS

A. *Folded Structures*

1. Anticlines and domes
2. Salt domes
3. Domes or anticlines over igneous intrusions
4. Domes or anticlines over buried ridges
   a. Folds due to uplift
   b. Folds due to differential compaction of sedimentary beds
5. Noses

B. *Monoclinal Structures*

1. Monoclines cut by igneous dikes
2. Outcropping monoclines sealed by paraffin or asphalt
3. Monoclines sealed by local cementation of reservoir rock

C. *Faulted Structures*

1. Faulted anticlines
2. Faulted monoclines

II. STRATIGRAPHIC TRAPS

A. Unconformities
B. Lateral variations in lithology (sand lenses)
C. Lateral variations in porosity and permeability

III. MISCELLANEOUS

A. Combinations of two or more of the above structural features
B. Differential porosity on the flanks of folded structure
C. Accumulation in faults, fissures, joints, etc.

in the United States are the Permian basin of southeastern New Mexico and the Gulf Coast province of Texas and Louisiana. †

Frequently some certain type or types of oil-bearing structure are characteristic of a particular province or of particular districts within a province. Even more frequently, the stratigraphic sequence and accompanying lithology within a province are characteristic of that province and serve to distinguish it from other provinces.

The topography and vegetation of a province has some influence upon the choice of a geophysical method. Because of the wide geographical distribution of the petroliferous provinces, there is usually a corresponding diversity in terrain conditions. In the Permian basin, high, flat, treeless plains are characteristic of practically the entire province. In the Gulf Coastal province, low, marshy ground is characteristic of a large part of the area. In certain parts of California, and in many other provinces, the rural terrain is characterized by high topographic relief and thick growths of trees and underbrush.

Some areas are too thickly populated or too intensely cultivated to permit the use of some geophysical methods. In the selection of the geophysical methods that will be most appropriate and efficient for any one

† Walter A. Ver Wiebe, *Oil Fields in the United States* (McGraw-Hill, 1930).

area, one must consider the theoretical applicability of the methods as related to these various structural, stratigraphic, and topographic factors.

Extensive application and the comparative successes of the various geophysical methods throughout the world provide a useful criterion in the selection of methods best adapted to the geological conditions and to the specific problems under consideration.

The usual purpose of reconnaissance by means of magnetic or gravitational methods is to locate any prominent anomalies in the buried surface of the basement rocks and to detect by this means the possible occurrence of favorable structures in the overlying sedimentary rocks. Suggestive anomalies are subsequently detailed by methods more adapted to obtaining precise data as to the actual structure in the sedimentary section. Reconnaissance may be conducted for the purpose of determining directly the major structural character of the sedimentary rocks. Success in such reconnaissance by use of magnetic or gravitational methods requires the presence of appreciable stratigraphic variations in the magnetic or density properties of the sediments, and the absence of lateral variations which might be interpreted erroneously as evidence of subsurface structure. A favorable condition would be the presence in the subsurface section of a formation of abnormally high magnetic permeability or density in which case structural highs are located by corresponding magnetic or gravitational anomalies. Ordinarily, no depth measurements are obtainable from these methods and the problem may justify the additional cost of other methods supplying quantitative structural data.

Gravitational and magnetic methods are not adapted to all conditions. For example, the magnetic method clearly cannot be used to map the structure of deep beds in an area where near-surface magnetic deposits (alluvium, lava flows, etc.) are unconformable with the underlying structure. Gravitational methods are greatly hampered in areas of near-surface lateral density variations and in some areas of large topographic relief.

The relative applicability of the various geophysical methods in the location of folded structures often depends more on the geologic setting (stratigraphic and lithologic properties of a petroliferous province) than upon the type of structure. For example, the seismic reflection method has given excellent results in most of the petroliferous provinces of the United States. In other provinces, the relative success of the seismic reflection results has been moderate to mediocre. Difficulty has been experienced: (a) in areas of steep dips and faults, as in California, (b) in areas covered by thick surface deposits of alluvial gravel or glacial fill, as in certain parts of California, Iowa, Illinois, Michigan, and (c) in areas in which lithologic continuity is poor and formations of high elastic wave velocity intervene between the surface and the oil-bearing structure, as exemplified by the salt beds of parts of Kansas and the Permian basin and the highly indurated limestone which caps the Edwards Plateau area of Texas. In certain instances of detrimental near-surface conditions, the drilling of deep shot holes might improve the success of this method, but the increased costs usually are not justified by the quality of the results. Areas of this type generally may be more satisfactorily explored by slim-hole drilling.

Certain areas and problems found to be difficult for reflection seismic work sometimes have proved amenable to continuous electrical profiling, the outstanding cases

being the successful mapping of structure near the top of the San Andreas Lime and at the base of the salt in the Permian basin, and the mapping of faults in California and South Texas. Electrical methods have also been successful in mapping faults and folded structure beneath a mantle of 500 to 800 feet of unconsolidated alluvium in the San Joaquin valley of California, and have been employed to determine structure beneath the glacial drift in parts of Iowa, Illinois, and Michigan. A small amount of electrical work has been conducted in the Edwards Plateau area with some evidence that the surface cover of Edwards limestone may not prevent the mapping of underground structure by this method. The usefulness of electrical methods in petroleum exploration appears to be confined to: (1) detecting the location of faults, (2) mapping of shallow structure, (3) detailed structural mapping to depths of about 5000 feet in certain districts where underground stratigraphy is favorable.

Salt dome structure is a special case of folded structure. The outstanding early success of refraction seismic and gravitational methods was in the location of relatively shallow salt dome structure, and this still constitutes the special field of applicability of refraction seismic methods. Reflection seismic methods, however, are the first choice in modern exploration for *deep-seated* salt domes.

The discovery of monoclinal oil-bearing structures by geophysical methods generally has been only moderately successful. (It is of interest to note that the usual type of stratigraphic trap is associated with monoclinal structure.) If the monoclinal structure is sealed by a fault or by an igneous dike, or other intrusive, the location of possible oil accumulations may be determined indirectly by locating the impounding fault, dike or intrusive. Magnetic, electrical, gravitational, and seismic methods have been employed in fault location. However, direct location of faults in sedimentary sections *generally* is most economically accomplished by use of electrical methods. If the impounding structure is an igneous dike or other type of igneous intrusion, the magnetic, and in some cases the gravitational, methods are clearly the most applicable.

As indicated earlier, the application of geophysical methods to the solution of the general problem of locating stratigraphic traps has not been very successful. Oil accumulation may occur in the up-dip edges of sand lenses and beds truncated by an uncomformity on broad structural arches or plunging anticlinal noses. The reflection seismograph generally can reveal the location and depth of these features but, except in cases of large divergence, can seldom provide an indication of the occurrence and location of the stratigraphic trap itself. Slim-hole drilling, although commonly expensive, offers the most positive method of exploring for such traps. Geochemistry or soil analysis may ultimately provide a direct method of locating traps of this type that contain oil. Stratigraphic traps are responsible for the East Texas field, the East Coalinga field, much of the accumulation in the Midway-Sunset, and many other important producing areas scattered throughout the United States. Accumulations of oil in traps frequently are extensive and profitably recoverable. The future of petroleum exploration depends to a large extent upon the development of methods to locate the stratigraphic traps that still remain hidden.

The possible use of shallow structural characteristics as a guide to deeper structure has been discussed by Rosaire. † He lists the following shallow stratigraphic changes as factors which may indicate, in some cases, the presence of uplift in the deeper part of the section: (1) induration of overlying shallow sediments, (2) mineralization of shallow ground waters over structure, (3) locally increased seismic velocities in shallow sediments over structure, (4) local variations in electrical properties of the shallow sediments overlying structure, and (5) "haloes" of hydrocarbon and secondary mineral concentrations. An additional factor which may be added is: (6) variations in geothermal gradient.

Measurements of the hydrocarbon and secondary mineral content by means of soil analysis have been discussed. Induration of sediments, mineralization by shallow ground waters, and geothermal gradient variations may sometimes be detected by measuring related variations in the near-surface electrical properties. It is evident that any measurement of these possible variations over structure will necessarily employ the electrical, seismic, or geochemical techniques. Success in the search for stratigraphic traps and the remaining structures of low relief by geophysical exploration probably will depend on one or a combination of several of the various techniques discussed in the immediately preceding paragraphs.

In predicting the future tasks of exploration geophysics it may be assumed that the oil traps still to be found are similar to those known to contain accumulations. Hence, if present methods and future improvements in geophysical techniques are to be successful in the search for the yet undiscovered accumulations of oil, they must be adapted to locating: (1) anticlinal structures of low relief, (2) structural closures against faults, and (3) stratigraphic traps whether they be depositional sand lenses, unconformities, or lenticular zones of porosity resulting from chemical action.

Economics of Petroleum Geophysics.—Emphasis in the preceding sections has been placed mainly upon the geological considerations influencing choice of methods with only brief reference to cost. Cost of geophysical surveys is mainly determined by: (1) equipment and personnel of crew, (2) quantity of information required, (3) terrain and other conditions affecting speed of operations. The amount of detail required is obviously a function of the amount of geological information already available, the complexity of the problem, and the general purpose of the survey. Speed of operation and coverage is determined principally by the nature of the geophysical technique employed, terrain conditions, local land ownership, etc.

It is obvious from these factors that specific cost is a characteristic of individual surveys and that only an approximate general tabulation of relative costs can be made. Table 3 is such a tabulation listing the methods in their order of increasing cost.

Rosaire ‡ has presented an interesting summary of the various factors which are important in formulating and applying geophysical exploration programs. The economics of geophysical exploration is but one phase of the

† E. E. Rosaire, "Shallow Stratigraphic Variations over Gulf Coast Structures," *Geol* Vol. III, No. 2, March, 1938.

‡ E. E. Rosaire, "On the Strategy and Tactics of Exploration for Petroleum," *Jour. So Geoph.*, Vol. VI, No. 1, July, 1935; *Geophysics*, Vol. III, No. 1, Jan., 1938.

larger sphere of petroleum exploration economics, albeit an increasingly important one. The sole purpose of petroleum geophysics is to furnish essential information which can be used in the evaluation of prospective oil land. Current evaluation of oil land, however, may have alternative purposes: (1) discovery of prospects for immediate drilling and (2) discovery of prospects suitable for maintenance of reserve. In the first class, search is for prospects which, by reason of location, shallow depth, and other factors, can be exploited rapidly and profitably in modern markets under modern proration requirements. Under maintenance of reserve, on the other hand, will fall prospects which because of greater depth, high drilling and production costs, poor location with reference

TABLE 3

APPROXIMATE COST OF GEOPHYSICAL PROSPECTING

Method	*Cost per month (in dollars)*
Magnetic	500- 750
Gravitational	
Gravimeter	3000- 5000
Torsion Balance	3500- 4500
Electrical	3000- 5000
Geochemical	3000- 6000
Reflection Seismic	6000-11,000
Refraction Seismic	6000-15,000
Slim-hole Drilling	0.25 to 1.75 per foot of hole drilled

to accessibility or markets, proration requirements, etc., are unsuitable for current exploration. These prospects are therefore considered as a future reserve on the assumption that increasing price due to diminishing supply will eventually allow profitable production.

Continual improvement of geophysical prospecting techniques is perhaps the chief requisite for the maintenance of a continuing reserve of petroleum. It is undoubtedly true that were it not for the widespread development and application of geophysical methods in recent years, present oil reserves would more nearly approximate the pessimistic estimates of a few years ago. However, despite the fact that geophysical methods of prospecting now have considerably greater resolving power than they had a few years ago, continued exploration is again showing

the influence of the law of diminishing returns. The cost of prospecting and the cost of drilling of wells for the many fields yet to be discovered probably will be considerably in excess of an amount that would permit profitable operation today. †

These conditions will probably furnish an impetus to the search for relatively shallow occurrences in new areas and for shallow stratigraphic traps and low closure structures in or adjacent to present producing areas. For example, the methods of slim-hole drilling will probably find increasing application in areas where the other geophysical methods give ambiguous data. Electrical methods are already being used in investigating structure under the glacial mantle covering parts of Michigan, Nebraska, Illinois, and Iowa. Detailed reflection seismic work is being applied to the search for low relief structures in Kansas, Oklahoma, and Northern Texas. Geochemical or soil analysis methods are being applied to various traps of stratigraphic and chemical origin.

Comparison of Wells Located with and without Technical Evidence

The records of wildcat drilling are of interest particularly in attesting the importance of modern geophysical methods. Lahee ‡ has given a preliminary report covering wildcat activities in the states of the Coastal Plain area from Texas and southeastern New Mexico to Georgia and Florida. During 1938, 1471 holes were drilled; of these 200 were producers and 1271 dry holes, a percentage of 13.6% of producers. Of these, 68 producers and 531 dry holes were located with geological information (surface, subsurface, trend, core drilling), a percentage of 11.3% of producers; 50 producers and 174 dry holes were located on geophysical evidence (seismic, gravimetric, electrical, magnetic), a percentage of 22.3% of producers; 10 producers and 36 dry holes were located on combined geological and geophysical evidence, a percentage of 21.7% of producers; 3 dry holes were located on other technical evidence; 14 producers and 292 dry holes were located for non-technical reasons (lease requirements, hunches, shows in old wells, etc.), a percentage of 4.6% of producers.

Summarizing: 14.7% of the wells located on technical evidence were producers, while of those located without technical advice only 4.6% were producers. The wells located on technical advice were therefore 3.2 times as successful as those drilled without such advice. It is also noteworthy that the holes drilled on geophysical evidence alone were 2.0 times as successful as those drilled solely on geological evidence. The fact that less than 15% of the wildcats drilled on technical evidence found oil or gas is an illuminating commentary on the uncertainties and complexities of geology and the resultant difficulty of interpretation of available technical data. Although these figures are for the Gulf Coastal Plain area only it is likely that they are typical of many areas.

GEOPHYSICAL METHODS IN MINING

General Field of Application.—The use of geophysical methods in mining exploration is influenced by the same general economic factors that have been described for petroleum exploration. From a technical

† Gerald H. Westby, "Practical Problems in Oil Exploration," *Oil and Gas Jour.*, August 25, 1938, p. 39.

‡ Frederic H. Lahee, "Wild Cat Drilling in 1938," *Oil and Gas Journal*, March 23, 1939.

viewpoint, mining exploration differs from petroleum exploration chiefly in the scale and complexity of the geological problems involved. Whereas in petroleum exploration the geophysicist deals with large extended areas and structures; in mining he is concerned with complex local structures of small areal extent, generally in more rugged terrain. Consequently, greater detail is necessary in the mining geophysical observations, and closer correlations are required between geophysical data, surface geology, and history of local ore occurrence. Usually, therefore, costs per acre for mining geophysical exploration are higher than those for petroleum exploration. Also, because the relatively small size of the ore bodies imposes depth limitations on the various geophysical methods, the feasible exploration range may not include the entire depth range which may be amenable to economic ore exploitation in any one mining district. These factors together with other economic conditions associated with the recent history of metal production have acted to delay a more general adoption of geophysics as a prerequisite in mining exploration. Even so, geophysical methods have been adapted to a wide variety of mining geological problems.†

The complex geology of mining districts and the variable nature of mineral deposits, however, offer an extensive and fertile field for the application of geophysical methods. The more prominent modes of mineral occurrence (particularly in the case of metallic deposits) are abrupt discontinuities in local geology, and the magnitude of the associated changes in physical properties of the rocks and ores may be correspondingly large; for example, the electrical conductivity of a pyrite vein may be a million times greater than that of the adjacent country rock. Such large differences in physical properties can often offset the adverse influence of structural complications and irregularities.

The main utility of geophysical methods in mining is in preliminary evaluation of prospects. The methods may be employed for the location of new prospects or for the re-evaluation of old ones. The usefulness of geophysics does not necessarily end with discovery, however, for it may well furnish additional information as mining development proceeds.*

Some of the practical problems encountered in the various phases of mining geophysics may be described briefly as follows:

† V. G. Gabriel, "Geophysical Prospecting—Its Part in American Mining," *Eng. and Min. Jnl.*, April, 1939.

V. G. Gabriel, "Geophysical Prospecting—Its Value in the Canadian Metal Mining Industry," *Canadian Min. Jnl.*, August, 1939.

* An important role of geophysical work in mining is to minimize exploration in barren areas—particularly, in areas where the ore occurs in, or in association with, highly conductive zones or highly magnetic zones which cause readily measurable anomalies.

Metal Mining: Location of new ore bodies, extensions of old ore bodies; depths of oxidized zones; location and determination of general extent of sulphide deposits below oxidized cappings; determination of length and width of mineralized areas; extensions of partially exposed ore bodies; location of faulted segments of veins; delineation of ore shoots in veins; etc.

Non-Metallic Mining: Determination of thickness of overburden; presence and location of faults and other structural features; extent of particular formations or rock masses; depth, size, and extent of gravel deposits; etc.

Choice of Methods for Particular Geologic Problems.—In general, the selection of geophysical methods for mining exploration is governed by considerations similar to those that influence the selection for petroleum exploration: namely, geological information available, type of mineral occurrence, resolving power of the geophysical method, etc. Magnetic, electrical, seismic, gravitational, and thermal methods have been employed successfully. The magnetic and electrical methods have enjoyed the greatest general application and success because of the relatively prominent variations in magnetic and electrical properties typical of the majority of types of ore and rock deposits commonly encountered. Complex geology and rough terrain usually hinder or prevent the application of seismic and gravitational methods. However, seismic methods have been used successfully in determining thickness of overburden and bedrock contours. Successful location of ledges of pyrite and other relatively heavy materials by means of the torsion balance and gravity meter have also been reported.† Rugged terrain characteristic of most mining areas is a specific deterrent to wider application of the torsion balance, because under these conditions extraneous near-surface anomalies are excessive.‡

The magnetic or electrical effects produced at the surface by an ore body or a structure depend primarily upon: (1) the *difference* between the magnetic susceptibility or electrical conductivity of the ore body, or the structure, and the same property (susceptibility or conductivity) of the surrounding country rock; (2) the size, form, and orientation of the subsurface ore body or structure; and (3) the effective depth of the subsurface ore body or structure.

The general applicability of geophysical methods and the selection of the best method for a particular case depend chiefly upon the geologic factors of mineralogical composition and mode of occurrence. The usual

† "Studies of Geophysical Methods, 1930," *Memoir 170*, p. 108 (Geological Survey, Canada Department of Mines, Ottawa, Canada). Helmer Hedstrom, "A New Gravimeter for Ore Prospecting," *A.I.M.E. Geophysical Prospecting, Tech. Pub.* No. 953, Feb., 1938.

‡ P. W. George, "Experiments with Eötvös Torsion Balance in the Tri-State Zinc and Lead District," *A.I.M.E. Geophysical Prospecting*, 1929.

classification of ore deposits comprises the genetic divisions: (1) mechanical concentration deposits, (2) chemical concentration deposits. †

TABLE 4

CLASSIFICATION OF ORE DEPOSITS WITH RESPECT TO THE APPLICATION OF GEOPHYSICAL METHODS

A. PLACER DEPOSITS
   1. Gold Placers
      a. Eluvial placers
      b. Stream placers
      c. Bench or terrace placers
      d. Marine placers
      e. Buried stream placers
   2. Placer deposits of other heavy minerals
   3. Detrital deposits

B. VEINS, LODES, AND DIKES (REGULAR DEPOSITS)
   1. Lodes in which the valuable ore mineral has diagnostic physical properties, e.g., sulphide ore bodies having high electrical conductivity
   2. Lodes in which the valuable ore minerals are genetically and structurally associated with minerals having diagnostic physical properties, e.g., auriferous pyrite veins containing electrically conductive pyrite
   3. Gold-quartz veins
   4. Pegmatite dikes
   5. Lodes in which the ore mineral is a refractory type (non-magnetic, non-conductive, etc.), e.g., sphalerite veins

C. IRREGULAR CONCENTRATED DEPOSITS
   1. Shear zones
   2. Pipes and stocks
   3. Replacement deposits
   4. Contact-metamorphic deposits
   5. Magmatic segregations

D. DISSEMINATED DEPOSITS
   1. Disseminations in igneous rocks, e.g., "porphyry coppers"
   2. Disseminations in sedimentary rocks

E. BEDDED DEPOSITS

   Metallic
   1. Gold-bearing conglomerates (South African)
   2. Sedimentary iron ores
   3. Residual deposits
   4. Copper-bearing conglomerates and lavas

   Non-Metallic
   1. Sedimentary rocks
   2. Saline residues
   3. Residual deposits
   4. Coal and lignite
   5. Miscellaneous non-metallic deposits

† Waldemar Lindgren, *Mineral Deposits* (McGraw-Hill).

However, in determining the applicability of geophysical methods, the form, size, and mineralogical content of an ore body are usually more important than its genesis. Table 4, while incomplete from a genetic viewpoint, illustrates all the common forms of ore or mineral deposits which normally need be considered in studying the applicability of geophysical methods.

Placer Deposits

Geophysical methods have been extensively and successfully employed in placer exploration.† Gold placers are the best known of this type of deposit and will serve as an example for the broad class of placer deposits. The purpose and utility of geophysical methods in gold placer exploration are usually twofold: (1) indirect location of gold concentrations, (2) determination of depth to and contour of the underlying bedrock.

No geophysical method can locate placer gold directly, in the normal concentrations usually found in placer deposits. In many placers, however, concentrations of "black sands" (magnetite, ilmenite, etc.) are associated with gold, thereby providing an indirect means for its location, for, under favorable conditions, these magnetic "black-sand" concentrations can be located by magnetic methods. However, such magnetic data are capable of a unique interpretation only when simple conditions prevail in the area. When the bedrock and the deeper basement complex have fairly uniform or low magnetic permeabilities, the magnetic studies will usually show the main concentrations of placer materials by magnetic "highs" superimposed over the regional average of the underlying rocks.

Such favorable conditions do not usually exist. Variations in thickness of the gravel will cause magnetic anomalies. These variations may be caused by irregular surface topography (which can be seen) and irregular bedrock contour (that can not be seen). Similarly, the unknown variations in bedrock permeability will produce anomalous magnetic variations. Most of these factors cannot be properly evaluated or recognized by geological and magnetic studies alone, so that much supplementary work is generally necessary.

Magnetic methods may also be employed for the various other types of placer deposits. They usually are not applicable, however, when the placer deposits are capped by lava flows—as is typically the case in the Sierra Nevada placers of California, in Australia, and in other parts of the world. In such cases, magnetic methods may sometimes be utilized for evaluating the thickness and extent of the lava flows, but they are seldom useful for locating "black-sand" concentrations.

Electrical and seismic methods have both been employed for determining the depth of bedrock in placer deposits; the former far more extensively than the latter. These methods usually furnish a reliable

† A. Gibson, "Magnetometric Determinations Applied to Placer Mining," *Eng. and Min. Jnl.* (1922), vol. 114.

K. C. Laylander, "Magnetometric Surveying as an Aid in Exploring Placer Ground," *Eng. and Min. Jnl.* (1926), vol. 121.

C. A. Heiland and W. H. Courtier, "Magnetometric Investigations of Gold Placer Deposits near Golden, Colorado," *A.I.M.E. Geophysical Prospecting* (1929).

and rapid means of determining the thickness of gravels and the contour of the underlying bedrock. The electrical methods will also indicate important structural features in the bedrock, such as faults and dikes, as well as the subsurface distribution of water.

Determining bedrock depth and contour is an important part of the preliminary exploration of placer ground. Ordinarily, depth determinations are made over a grid work of stations or along sectional traverses. From these studies it is possible: (1) to plot bedrock contours, and thereby locate and trace stream channels in the bedrock, (2) to estimate yardage of gravel, overburden, etc., and (3) to determine suitable locations for mining and sluicing operations to take advantage of existing slope of bedrock. Electrical methods have been employed successfully in locating and tracing gold-bearing stream channels below lava caps. Two criteria serve to indicate the subsurface location of such channels: † (1) the difference in electrical conductivity between the water-bearing gravels and the bedrock and (2) the lowest point in the bedrock profile.

Veins and Lodes

Magnetic and electrical methods have been employed extensively in exploration for commercial mineralization in veins and lodes, while gravimetric methods have been employed to a much lesser extent. A great variety of minerals and ores occur in the form of veins and lodes. Prominent in these classes are: (1) *sulphide veins* (usually intermixed sulphides of copper, iron, lead, zinc, etc.), (2) *gold-pyrite veins*, and (3) *gold-quartz veins*. Electrical methods are useful for locating all of these types of deposits, but are especially applicable for deposits of electrically conductive base metal sulphides (ores of chalcopyrite, pyrite, galena). In the latter cases, the methods are used to locate the vein directly or to outline shoots or other concentrations within the vein. A problem which is frequently encountered is the location of possible zones of primary sulphides or of secondary enrichment beneath oxidized cappings. The electrical methods have been especially useful for locating non-outcropping veins in areas adjacent to outcropping veins of known characteristics and for locating veins beneath coverings of alluvium, glacial till, tundra, etc. ‡

The use of electrical methods in prospecting for gold-bearing veins and lodes is an indirect process. Such deposits may be located indirectly by means of the occurrence of pyrite or other metallic sulphides in genetic association with the gold. Gold-

† J. J. Jakosky and C. H. Wilson, "Geophysical Studies in Placer and Water Supply Problems," *A.I.M.E. Geophysical Prospecting, Tech. Pub. No. 515*, 1934.

‡ "Summary of Results from Geophysical Surveys at Various Properties," *A.I.M.E. Geophysical Prospecting*, 1932.

quartz veins, under favorable conditions, may be located by reason of their poor electrical conductivity relative to the country rock. †

Igneous dikes: Ore bodies are found in association with igneous dikes only infrequently. However, because such dikes have a structural significance in many mineralized areas, their occurrence is mentioned here. Dikes of basic material may be located by magnetic methods due to their high content of minerals of large magnetic susceptibility. ‡

Pegmatite dikes: Valuable ore and mineral concentrations are frequently associated with pegmatic dikes. Where such dikes have a sufficiently lower electrical conductivity than the country rocks, they may be located by electrical methods, as in the case of quartz veins. Magnetic methods are applicable when, as is frequently the case, magnetite or other magnetic material is an accessory mineral in the pegmatite. The literature of geophysical methods contains many descriptions of successful application of geophysical methods to the discovery and mapping of veins, lodes, and dikes. §

Irregularly Concentrated Deposits

Magnetic and electrical methods have found extensive application in prospecting for irregular ore deposits such as those found in shear zones, pipes and stocks, replacements, contact-metamorphic deposits, and magmatic segregations. †† Spontaneous polarization methods are particularly applicable to vertical pipe-like bodies of sulphide ore.* In cases where the pipe-like form of the ore bodies is the result of their deposition in shattered volcanic plugs, magnetic methods often may be used to locate the intrusive plug. ‡‡

Replacement ore bodies are of many types and are widely distributed in occurrence and varied in mineralogical assemblage. Replacement processes occur to a certain extent in all cases of ore deposition. Common types are replacement bodies of base metal sulphides. These are often accompanied by gold and silver values deposited in limestone by ascending solutions ** or by percolating meteoric waters.*** The forms of these replacement ore bodies are usually very irregular, depending, in many cases, upon the texture and other properties of the host rock. In certain cases, replacement in fairly flat dipping limestones has extended laterally for sufficient distances to result in a bedded form of deposit.

Electrical methods are preferred for locating the various types of replacement ore bodies which have a high electrical conductivity. Occasionally, however, this

† Folke H. Kihlstedt, "Electrical Methods in Prospecting for Gold," *A.I.M.E. Geophysical Prospecting*, 1934.

Sherwin F. Kelly, Theodor Zuschlag and Bela Low, "Discovering Gold-Quartz Veins Electrically," *Mining and Metallurgy*, June, 1934.

‡ Noel H. Stearn, "Geomagnetic Exploration with the Hotchkiss Superdip," *A.I.M.E. Geophysical Prospecting*, 1932.

§ *A.I.M.E. Geophysical Prospecting*, 1929, 1932, 1934.

†† Hans Lundberg, "Recent Results in Electrical Prospecting for Ore," *A.I.M.E. Geophysical Prospecting*, 1929.

* See Chapter V.

‡‡ Compare Lindgren, *loc. cit.*, pages 153 and 183.

** E.g., lead-silver ores of Park City, Utah.

*** E.g., lead-zinc ores of the Mississippi Valley.

method has not been successful, as, for example, in the upper Mississippi Valley where there is a large percentage of poorly conductive sphalerite in the ores. †

Contact-metamorphic deposits frequently contain the ore minerals: magnetite and chalcopyrite. Magnetic methods have been used for direct location of the magnetite ores of contact deposits and for outlining the intrusive igneous rocks responsible for the ore deposition. Electrical methods are applicable when the conductive base metal sulphides are the prominent ore constituents.

Ore deposits resulting from *segregation and separation of ore minerals from magmas* during the processes of solidification are common in many parts of the world. The Sudbury Canada nickel ores are prominent in this group. The chief application of geophysical methods to this class of deposit consists in locating and outlining the igneous rock in which the segregation may have taken place. When such deposits have a high content of magnetite or metallic sulphides, they can sometimes be located directly by magnetic or electrical methods. ‡

Disseminated Deposits

The class of disseminated deposits covers many genetic occurrences. They are important chiefly to the extent that concentration or enrichment has taken place subsequent to original deposition. Important representatives of this class of deposit are the so-called "porphyry coppers." * In the typical "porphyry copper" deposit, primary mineralization consists of copper and iron sulphides widely distributed through a large mass of intrusive rock, generally monzonitic in character. Oxidation processes in the upper portion of this mass have removed a large part of the original metallic content and redeposited it at greater depth in the form of secondary enrichment. Electrical methods may be employed to locate such zones of secondary sulphide concentration. In addition, they may sometimes be employed to outline the lateral extent of low-grade primary mineralization.

Bedded Deposits

Bedded mineral deposits are probably of greater commercial importance than any other general class. These deposits are found in all parts of the world and include a wide range of metallic and non-metallic ores. The most prominent auriferous deposits belonging to this class are the well-known gold conglomerates of the Witwatersrand in South Africa. Important representatives of the base metal ores are the sedimentary iron and sulphide ores.

† Hans Lundberg, *loc. cit.*

‡ Max Mason, "Geophysical Exploration for Ores," *A.I.M.E. Geophysical Prospecting*, 1929.

* E.g., in Bingham, Utah, and Ely, Nevada.

Bedded Deposits — Metallic

Gold-Bearing Conglomerates.—Magnetic methods have been extensively applied in prospecting for the gold-bearing conglomerates in the Witwatersrand and other areas of South Africa. In the Witwatersrand area alone, magnetic surveys have been conducted over several thousand square miles. † From the standpoint of geophysical exploration, the steeply dipping Witwatersrand conglomerates are similar to veins and lodes. Because of their steep dips and large lateral extents, the outcropping deposits have been called reefs. Even though magnetite is practically absent in the conglomerates, the magnetic methods have been useful in locating certain magnetic formations which are genetically and structurally associated with the gold series.

Sedimentary Iron Ores.—The use of magnetic methods in the exploration for iron ores may be either direct or indirect. Direct location of such ore deposits can be accomplished only when magnetite or other magnetic material is a constituent of the ore. Only certain iron ores are magnetic. (Compare Chapter III.) In particular, some of the ores of the Lake Superior district contain sufficient magnetic materials to allow their direct detection by means of magnetic measurements. In the majority of cases, however, the ores do not differ sufficiently from the adjacent formations and hence an indirect means of locating favorable areas is used: viz., the mapping of structure in the iron-bearing formations. ‡

In general, the chief constituent of sedimentary iron ores is hematite; hence, the ores are often less magnetic than the adjacent iron formations from which they may have been derived. In the limonite and hematite ores of direct sedimentary origin, for example the Clinton iron ores, the magnetic methods ordinarily can only be applied indirectly: that is, to determine structural conditions associated with the ore occurrence.

Electrical and gravitational methods have also been used in some of the iron ore districts, particularly at Lake Superior, but to date they have had little utility in direct iron ore location.

Residual Deposits.—Residual iron ores are representative of this class of ore deposits. The deposits have various forms and composition and are generally the result of rock decay and weathering. Frequently the individual deposits are quite irregular, although occurrence and form are generally somewhat controlled by bedding in sedimentary rocks. Geophysical methods have not been applied very extensively or successfully to this class of deposit. In certain cases, it is

† Noel H. Stearn, "Geomagnetic Exploration in 1938," *Geophysics*, Vol. IV, No. 2, March, 1939.

‡ Noel H. Stearn, *loc. cit.*

C. O. Swanson, "Use of Magnetic Data in Michigan Iron Ranges," *A.I.M.E. Geophysical Prospecting*, 1934.

probable that magnetic and electrical methods may be useful in determining structure related to these deposits.

Copper-Bearing Conglomerates and Lavas.—The Lake Superior copper ores are a unique occurrence and are mentioned here to illustrate the use of magnetic methods for the location of ore bodies occurring in lava flows. In one type of ore body in this district, native copper occurs in amygdaloidal basalt flows which dip from 20° to 40°. Magnetic methods have been used to trace: (1) the flows in which the copper ore is concentrated and (2) certain flows which bear structural and genetic relationships to the copper deposits.†

Bedded Deposits — Non-Metallic

Geophysical prospecting in the field of non-metallic mining has expanded greatly in the past few years. The general class of sedimentary, bedded deposits includes representatives of most of the valuable non-metallic deposits. Because many non-metallic minerals and ores extend through a broad genetic range, it is not possible to restrict them to a strict classification as to form or mode of occurrence. Phosphate deposits, by way of illustration, may occur in the following ways:‡ (1) marine concretionary beds; (2) disseminations in igneous rocks; (3) pegmatite dikes; (4) guano deposits; (5) replacements of limestone; (6) residual concretions. A summary of prior literature dealing with exploration for various non-metallic materials is contained in an early publication.§

Applications of geophysical methods in the non-metallic field are governed by the factors outlined for other mineral resources earlier in this chapter. For the sake of brevity, the following discussion will comprise a resume of applications to bedded and residual deposits. The applications are primarily of the indirect type, i.e., the mapping of structure associated with the occurrence of commercially valuable deposits. The location of faults, dikes, and folds and the determination of thickness of overburden constitute the important problems. For the solution of these problems, magnetic, electrical, seismic, and gravitational methods are all used.

Sedimentary Rocks.—Examples of sedimentary rocks or rock materials which are of commercial importance are limestone, sandstone, clay, sand, gravel, etc. Electrical methods are ordinarily the most applicable in exploration for deposits of these materials.

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† N. H. Stearn, *loc. cit.*

‡ Lindgren, *loc. cit.*, p. 277.

§ C. A. Heiland, "Geophysical Prospecting in the Non-Metallic Field," *A.I.M.E. Geophysical Prospecting*, 1934.

In addition to structural studies, it is possible in some cases to locate the various deposits directly by utilizing differences in electrical conductivity relative to adjacent rocks. The degree of alteration or weathering, which is often important in evaluating limestones and other rocks employed for building purposes, may be determined by electrical methods. Oftentimes, sand and gravel deposits may be located directly by electrical methods. †

Magnetic methods may be useful (1) for determinating structural conditions governing deposits of these types of rock materials and (2) for locating buried stream gravels having an appreciable content of magnetite.

Saline Residues.—Saline residues are accumulations of certain minerals which have been deposited as the result of evaporation in closed or partially closed basins. Some of the commercially important minerals or ores in this class are rock salt, gypsum, anhydrite, sodium nitrate, borax, and potash.

Under favorable conditions, distinctive physical properties may permit direct detection of these deposits. For example, salt, relative to some sedimentary rocks, has a low density, poor electrical conductivity and transmits seismic waves at a high velocity. These properties have been utilized in exploration for salt domes by gravitational, electrical, and refraction seismic methods, respectively. To date, however, utilization of these properties in the direct location of salt and other saline deposits has been extremely limited.

Magnetic and electrical methods have been employed to a limited extent in determining local structural conditions. An example is the search for borax deposits in California. The magnetic and electrical work located faults and determined the subsurface distribution of lake bed sediments and buried lava flows that are genetically related to the accumulations of borax in the Mojave desert area.

Residual Deposits.—The most common non-metallic ores or rock materials which occur as residual deposits are clay, barite, phosphate, and bauxite. Geophysical methods have not been extensively applied to these deposits, but the field for indirect application of the methods is

† M. King Hubbert, "Results of Earth Resistivity Survey on Various Geologic Structures in Illinois," *A.I.M.E. Geophysical Prospecting*, 1934.

Karl S. Kurtenacker, "Some Practical Applications of Resistivity Measurements to Highway Problems," *A.I.M.E. Geophysical Prospecting*, 1934.

similar to that for other non-metallic resources. Stearn † has given an example of the indirect use of magnetic methods in the search for bauxite deposits in Arkansas.

Coal and Lignite.—Measurements have been conducted in different parts of the world to investigate the applicability of geophysical methods in exploration for coal deposits. Due to the variable chemical and physical properties of coal, as well as the variety of structural conditions associated with the deposits, its direct location by geophysical methods has not been very successful. In a few cases of shallow occurrence, experimental electrical surveys have directly indicated the presence of coal seams. ‡ Hawkins § reports generally unsatisfactory results in attempts to locate lignite directly by resistivity measurements.

The geophysical work that has been done in exploring for coal indicates that direct location of coal beds may be accomplished only under especially favorable conditions of coal composition and structural setting. Anthracite deposits in some cases exhibit high electrical conductivity relative to the surrounding sediments. Under such conditions, these deposits undoubtedly can be located directly by electrical methods. Similarly, the relatively poor conductivity of lignite beds in certain areas suggests the possibility of locating such deposits by electrical methods.

In general, however, it may be concluded that the most successful applications of geophysics in coal exploration will be in the field of structural and stratigraphic determinations. Electrical, seismic, gravitational, and magnetic methods have all been tried. At the present time, electrical methods promise to be the most valuable.

Miscellaneous Non-Metallic Deposits.—Various kinds of igneous rocks are used commercially in the form of building stones, road building materials, etc. Magnetic methods are usually the preferred choice in exploration for these materials because of the relatively high magnetic susceptibility of most igneous rocks. Electrical methods are useful also—particularly to outline the extents of the rock deposits, locate contacts, and give qualitative information regarding the amount of weathering or alteration.

† N. H. Stearn, *loc. cit.*

‡ Maurice Ewing, A. P. Crary, J. W. Peoples and J. A. Peoples, Jr., "Prospecting for Anthracite by the Earth-Resistivity Method," *A.I.M.E. Geophysical Prospecting, Tech. Pub.* 683.

§ R. H. Hawkins, "Application of Resistivity Methods to Northern Ontario Lignite Deposits," *A.I.M.E. Geophysical Prospecting*, 1934.

In some cases, geophysical methods are useful in indirect exploration for gems and precious stones by reason of their genetic relationship to certain types of rocks or formations. Stearn † gives an example of the location of diamond-bearing peridotites by magnetic methods. Gems, semi-precious stones, and certain non-metallic minerals of industrial use occur in pegmatites. The possibility of locating a pegmatite dike by use of electrical methods has been mentioned in a preceding section.

GEOPHYSICAL METHODS IN WATER SUPPLY ENGINEERING

Engineering, economic, and geological problems arising in the general field of water supply are very extensive and can be discussed here only briefly. ‡ The sources of water are: (1) surface water in lakes, streams, rivers, etc., and (2) subsurface water (ground water). Control, conservation, storage, and use of surface water is accomplished by building dams and other engineering works. Similar utilization of ground water requires storage in subsurface basins and the drilling of wells in proper locations to produce water by natural flow (artesian wells) or by pumping wells. The original source of the ground water for which drilling exploration is conducted is run-off water and rainfall. The *available supply* of ground water in any area is therefore mainly determined by: amount of precipitation, character of topography, extent of water-sheds and drainage basins, etc.

Local accumulation of ground water, however, is controlled primarily by the geological factors of local structure, petrology, porosity, etc. Detailed geological information is essential to aid the engineer in the proper location of water wells. He should know the extent, thickness, depth, composition and relative position of water-bearing strata or subsurface basins, the depth to the ground water table, the location of buried stream channels, faults, dikes and other subsurface structural features. Oftentimes some of this information may be obtained by surface geological observations. In recent years, geophysical methods have been applied in an increasing extent to secure much of the subsurface geological information which is necessary for the intelligent direction of subsurface water-supply development.

Information concerning accumulation and distribution of ground water may be desired for several purposes: (1) to locate a supply suitable for domestic uses, irrigation, mining, or other purposes; (2) to determine ground water conditions insofar as they may affect engineering construction such as dams and drainage projects; (3) to determine the distribution of ground water insofar as it may affect conservation of crops or encroachment of sea water, etc. The geological and geophysical problems involved in these determinations are generally resolved into: (1) location of permanent water table or perched bodies of water; (2) location of subsurface structure favorable to accumulation, impounding, and storage of water; (3) location of boundaries between fresh and saline waters.

† Noel H. Stearn, *loc. cit.*
‡ C. F. Tolman, *Ground Water* (McGraw-Hill Book Company, New York, 1938).

Geophysical methods as applied to the various problems of water supply are usually of the indirect type wherein structural conditions influencing ground water distribution are determined. In certain relatively simple problems, the contour of the water-table may be mapped directly and isolated accumulations of water may be located.* Structural conditions that most commonly influence accumulation and impounding of subsurface water are: (1) subsurface basins and channels in bedrock underlying unconsolidated porous materials; (2) subsurface structural barriers such as erosional relief in the bedrock, igneous dikes, etc.; (3) faults.

Choice of Methods.—Electrical, magnetic, and seismic methods have been used to secure structural information. Electrical methods are being used more extensively than the other methods and are generally applicable to most phases of the structural problem: viz., determination of depth of unconsolidated alluvium, etc., mapping bedrock contour, and location of faults. Magnetic methods are useful in cases in which impounding of subsurface water is caused by igneous dikes. Seismic work has been employed to map bedrock boundaries. Improvements are being made in the adaptation of seismic methods to shallow work and it is likely, therefore, that the use of these methods in problems of water supply will increase. †

The actual location of the ground-water table is important in determining favorable locations for wells and in evaluating conditions prior to construction of dams, tunnels, etc.** Electrical methods have been used successfully to locate water impounded above bedrock in quaternary gravels and alluvium and to map the upper boundary of ground water at greater depths in the more consolidated sedimentary rocks. ‡ Various investigators have employed the seismic method in the direct location of water tables. The method depends upon increase in velocity of the elastic wave with increase in water content.

GEOPHYSICAL METHODS IN CIVIL ENGINEERING

The application of geophysical methods in civil engineering and construction work has increased rapidly in recent years because of the recognition of the importance of geological structure in all phases of engineering concerned with excavation of material or selection of sites for dams and other earth works. The geophysical methods and procedures employed, and the nature of information desired, are similar to those described in the preceding section. Literature dealing with

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* See Chapter V.

† F. L. Partlo and Jerry H. Service, "Seismic Refraction Methods as Applied to Shallow Overburdens," *A.I.M.E. Geophysical Prospecting*, 1931.

** See following section on Geophysical Methods in Civil Engineering.

‡ J. J. Jakosky, C. H. Wilson and J. W. Daly, "Geophysical Examination of Meteor Crater, Arizona," *A.I.M.E. Geophysical Prospecting*, 1932.

J. J. Jakosky and C. H. Wilson, "Geophysical Study of Fort Peck Dam Site and Reservoir," paper now in press.

engineering applications of geophysics is fairly extensive.† Electrical methods have been employed almost to the exclusion of the other methods, although application of seismic methods is increasing and magnetic methods are often useful for special structural problems.

Examination and Location of Dam Sites.—Excellent discussions of the relationship between geology and engineering for dams and reservoirs have been given in various publications.‡

In evaluating the location of a dam, five geological factors are of primary importance: (1) the depth of overburden or fill materials which must be removed, (2) the presence of faults or other structural defects which might constitute a failure or leakage hazard, (3) the elevation or height of the water table in the abutments and the reservoir rims, (4) the strength of the foundation rocks, and (5) the perviousness of the foundation rocks.

Ordinarily, surface geological work will not provide sufficient detailed information along these lines and in such cases proper geophysical work combined with a small amount of confirmatory core drilling will usually provide the needed information. This is particularly true of the first three factors enumerated above. Bedrock depth determinations by geophysical methods are relatively simple and inexpensive. The accuracy obtainable is well within the necessary limits, especially when occasional drillholes are used for control.

The location of major faults in the vicinity of the dam site is of great importance, not only from the standpoint of possible movement along them which might endanger the dam, but also from the standpoint of leakage through the reservoir rocks. The location of faults from surface evidence is a well-known geological technique. However, important faults may be covered by alluvial material, and they must therefore be identified by geophysical methods.

Ground water conditions at dam sites are important chiefly from the standpoint of leakage. The depth to the water table in the dam abutments and reservoir rims usually can be determined satisfactorily by electrical methods.

In regard to the strength and perviousness of the foundation rocks, geophysical work will often give useful information for correlation with

† Irving B. Crosby and E. G. Leonardon, "Electrical Prospecting Applied to Foundation Problems," *A.I.M.E. Geophysical Prospecting, Tech. Pub., No. 131* (1928).

E. G. Leonardon and Sherwin F. Kelly, "Some Applications of Potential Methods to Structural Studies," *A.I.M.E. Geophysical Prospecting, Tech. Pub. No. 115* (1928).

S. F. Kelly, "Engineering Uses for Geophysics," *Civil Engineering*, October, 1932.

G. G. Stipe and Sherwin F. Kelly, "Geophysical Methods Aid Construction Work," *Civil Engineering*, April, 1937.

Karl S. Kurtenacker, "Some Practical Applications of Resistivity Measurements to Highway Problems," *A.I.M.E. Geophysical Prospecting*, 1934.

E. G. Leonardon, "Electrical Exploration Applied to Geophysical Problems in Civil Engineering," *A.I.M.E. Geophysical Prospecting*, 1932.

‡ "Geology and Engineering for Dams and Reservoirs," *A.I.M.E. Geophysical Prospecting, Tech. Pub. 215.* (Papers and discussions presented at the New York Meeting, February, 1929.)

Douglas Clark, "Application of Geology to Civil Engineering," *California Journal of Mines and Geology*, Vol. 29, January and April, 1933, pp. 161-173.

surface evidence, core holes, and laboratory tests. In limestone regions, the foundation rocks may be characterized by solution channels and caves at certain horizons, and such conditions can often be detected by changes in the apparent resistivity or the subsurface current distribution.

Several examples are cited below to illustrate the need for detailed geological and geophysical work.†

At the location of the O'Shaughnessy dam on the Tuolumne River at the lower end of the Hetch Hetchy Valley in Tuolumne County, California, the depth of the stream gravels overlying the granite bedrock was found to be excessive, the maximum depth being 101 feet. Owing to this condition a greater amount of excavation was required and a higher dam built than was originally planned, which added greatly to the cost of the project.

At the site of the St. Francis dam near Saugus, Los Angeles County, mica schist occurs on the east slope of the canyon and conglomerate of the Sespe formation on the west side. The contact between the two formations is a fault which parallels the canyon on the west slope about 60 feet above its base. Within the fault zone there are numerous seams of clay gouge and fractures filled with gypsum. Furthermore, the conglomerate, where fractured, completely disintegrates when immersed in water. A commission investigating the failure of this dam concluded that it was due to disintegration of the fractured conglomerate.

During the construction of the Lafayette earth-fill dam in Contra Costa County, California, by the East Bay Municipal Utility District, difficulty was encountered because of the occurrence of plastic clay beneath the dam. The strata underlying the dam site include beds of clays and sands of the Orinda formation, dipping at high angles and overlain by alluvial fill from 71 to 91 feet thick, the bedrock in part being sandy clay. When the dam was completed to within 20 feet of its final height the crest of the dam sank vertically 24 feet, causing a movement of the downstream face. The clay below the dam readily absorbed water and became a soft mud which moved out under the weight of the dam. This condition made it necessary to complete the dam with a height of 40 feet less than that originally intended and with greatly flattened slopes.

The foregoing are typical examples of adverse conditions unexpectedly revealed after the expenditure of large sums had made relocations inexpedient but which would very probably have been disclosed by preliminary geophysical surveys.

Electrical, seismic, and magnetic methods are now being widely used to secure the geological information outlined at the start of this section. This no doubt will result in less frequent failure of dams, and subsequent discovery of unfavorable geological conditions that might have been disclosed by preliminary geophysical surveys. Geophysical methods are being utilized by the Bureau of Reclamation, U. S. Army Corps of Engineers, Tennessee Valley Authority, by various Municipal Agencies such as the Metropolitan Water District of Southern California, and by private engineering agencies.*

Location of Reservoir Sites.—Several years ago the U. S. Federal Government constructed the Hondo reservoir near Roswell, New Mexico, in connection with

† Douglas Clark, *loc. cit.*

* Typical examples of geophysical investigation of dam sites are given in the chapter on Electrical Methods.

an irrigation project at a cost of about $500,000. The rocks underlying the reservoir area consist of alternating beds of gypsum and limestone and the structure of the formation is that of a faulted and collapsed anticline, the fault traversing the center of the reservoir. Underground cavities caused by the dissolving out of the relatively soluble gypsum apparently caused the collapse of the structure and rendered the reservoir useless for water storage. Proper geophysical and geological examinations would probably have saved the wasted expenditures.

Selection of proper reservoir sites is, of course, closely allied to that of dam sites. The main geological factor to be considered in the selection of reservoir sites is the perviousness of the reservoir rims. Leakage hazards may consist of (1) faults or solution channels in the rim formation and (2) insufficient elevation or breadth of bedrock along the rims relative to the pool level of the proposed reservoir. In many cases, these problems may be readily solved by electrical, seismic, and magnetic methods.*

Highway Engineering.—Many of the subsurface geological problems which are encountered in highways, railroads, aqueduct and pipe line routes, etc., may be solved by geophysical methods. Usually, the required depth of study is relatively shallow (less than 100 feet) and geophysical surveys can be conducted rapidly and economically. As in other phases of civil engineering, electrical methods are proving most applicable to these general problems.

Several fields of investigation may be listed in this general class of engineering: (1) quarries, (2) pipe line and aqueduct routes, (3) tunnel sites, (4) bridge foundations, (5) cut and fill determinations.

Application of geophysical prospecting to quarry deposits was discussed briefly in the earlier section devoted to non-metallic mining. Quarrying operations are carried on extensively as a part of most civil engineering and construction projects. Selection of suitable quarry sites is based upon the accessibility of the quarry deposit, cost of quarrying and crushing the rock, quality of the deposit, and extent of the deposit. The last three factors are partially dependent upon geological conditions which may be evaluated by geophysical methods. A problem common to all quarrying operations is the determination of the thickness of the overburden, and this is readily accomplished by electrical methods. Electrical methods can be used also to locate certain kinds of quarry deposits such as gravel deposits, igneous rocks, limestone deposits, etc., and to afford qualitative information as to degree of weathering. The presence of solution channels and sink holes is sometimes important in selecting suitable limestone deposits for quarrying, and such defects are readily located by electrical methods in many cases.

Magnetic and electrical work can be advantageously used prior to excavation to locate faults, changes in formation, degree of weathering,

* Examples of electrical and magnetic work in examination of reservoir rims are given in a later chapter.

ıd aqueduct routes. If the work requires tunnel ... of thickness of overburden can be made in con- ... studies.*

tion of material to be excavated classification. Electrical surveys of highway grade settlement in swamps. the depth to firm material at the bottom of and in some cases whether or not newly constructed fill is properly settled on a firm foundation.†

Applications of electrical work in bridge construction problems is similar to that for dam sites: namely, the determination of depth to bedrock or to a material of proper characteristics to support piling.

Seismic studies have also proved valuable in work of this nature and increasing applications of these methods for determining the bearing capacity of soils ‡ and the natural period of vibration of foundations and dams may be expected.

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* Extensive electrical work of this kind was employed by the Metropolitan Water District of Southern California in planning aqueduct routes from Boulder Dam to the city of Los Angeles.

† Karl S. Kurtenacker, *loc. cit.*

‡ R. K. Bernhard, "Geophysical Study of Soil Dynamics," *A.I.M.E. Geophysical Prospecting* Tech. Pub. 834, 1938. See also chapter on Seismic Methods.

CHAPTER III

MAGNETIC METHODS

The magnetic exploration methods are one of the oldest of the applied geophysical methods. They utilize a natural field of force, viz., the earth's magnetic field, and are applicable economically in regions where the magnetic properties of the rocks have some known relationship to the economic geology.

Geomagnetic studies are of greatest value in reconnaissance and preliminary work in oil prospecting, and in reconnaissance and oftentimes in detailed work in exploration for magnetite deposits, gold-bearing placers, nickel ore bodies, and ilmenite and pyrrhotite deposits. They can be conducted rapidly with a small personnel at a resultant low cost. In oil prospecting, magnetic reconnaissance work usually is employed for determining the location and detail necessary for the application of the other geophysical methods of better resolving power.

Interpretation of magnetic data is based on the fact that the earth's normal magnetic field is uniform over areas of magnetically homogeneous composition but is distorted in certain regions of inhomogeneous composition, the amount of distortion depending on the relative magnetic susceptibilities of the subsurface materials and the relative masses and configurations of these component materials. Most magnetic anomalies are due to igneous rocks, iron ores, and those sedimentary deposits which contain magnetite derived from igneous rocks. Magnetic methods, therefore, are directly applicable where the mineral whose presence is being explored is itself magnetic or is associated in its occurrence with magnetic materials.

Applied to petroleum exploration, magnetic methods allow indirect information to be obtained about formations, usually non-magnetic, which may be oil-bearing. Work of this type includes detecting and mapping intrusions of igneous or crystalline rocks, buried igneous ridges, simple anticlines, anticlines underlain with igneous masses, anticlines underlain with salt, and faults which have displaced magnetic formations or sedimentary beds which are detectable by magnetic means.*

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* Although the susceptibility of sedimentary beds is usually small, such beds may be detected in certain cases by magnetic measurements. For example, they may be detected when they lie close to the surface or when the susceptibility of the adjacent beds is negligible.

PHYSICAL CONCEPTS

The Magnetic Field of the Earth.—The earth is surrounded and permeated by a magnetic field. This field is distributed irregularly and is changing constantly. † An approximation to the distribution of the earth's magnetism is obtained by regarding it as due to a relatively short bar magnet placed at the earth's center. This magnet is oriented along the line joining the earth's magnetic poles. (Figure 1.) The positions of the magnetic poles as determined in 1906 and 1909 were:

North Pole 70° 30′ N., 97° 40′ W.
South Pole 72° 25′ S., 155° 16′ E.

The earth's magnetic field may also be approximated by regarding the earth as a uniformly magnetized sphere.

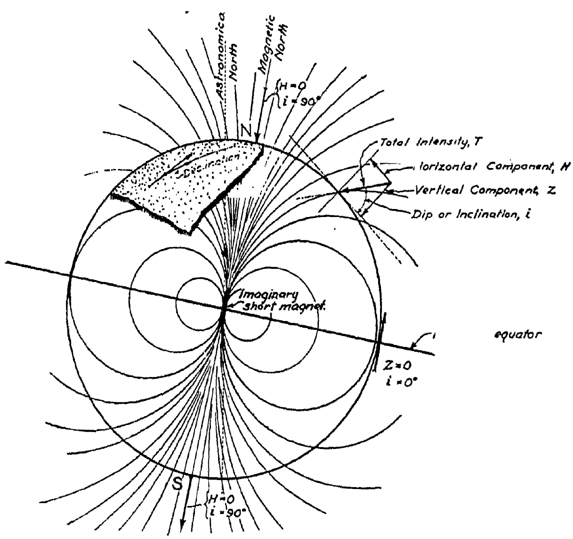

FIG. 1.—Schematic diagram illustrating magnetic field produced by an imaginary bar magnet located at the earth's center.

The magnetic field at any point P on the surface of the earth is completely described when its direction and intensity are specified. In practice, the direction is obtained by measuring the components of the field. This is illustrated for a point on the northern hemisphere in Figure 1. T is the resultant magnetic field or total intensity; H is the horizontal component; Z is the vertical component; and i is the dip or inclination (angle which the resultant field makes with the horizontal, i.e., angle between T

† J. H. Jeans, *The Mathematical Theory of Electricity and Magnetism* (Cambr. Univ. Press, 5th Ed.) p. 400.

and H). It is convenient in some cases to resolve the component H into a north-south component X and an east-west component Y.

The magnetic quantities usually obtained at an observation point are the declination d (angle between the magnetic meridian and the astronomical meridian), the dip i and the horizontal component of the earth's field H. These observed quantities are related to T, X, Y, and Z by the relations:

$$\left.\begin{array}{ll} T = H \sec i, & Y = H \sin d, \\ X = H \cos d, & Z = H \tan i. \end{array}\right\} \qquad (1)$$

The total or resultant intensity of the earth's magnetic field in the United States is of the order of 0.6 gauss.

The accuracy achieved in many practical explorations is 1/10,000 of the earth's field. The unit of field strength commonly employed in geomagnetic explorations is the *gamma* which is equal to one hundred thousandth (10^{-5}) of a gauss. Expressed in this unit, the total intensity of the earth's field would be:

$$0.6 \text{ gauss} = 0.6 \cdot 10^5 = 60{,}000 \text{ gammas}$$

The distribution of the earth's magnetism at distances far removed from the magnetic poles is mapped by means of lines showing locations where a magnetic property is constant. Lines of equal declination are called *isogonal* lines. (In general, isogonal maps have found little use in applied geophysics, although they have been used to some extent in the Lake Superior iron ore region.) *Isoclinal* lines are lines of equal inclination. *Isodynamic* lines refer to lines of equal intensity. On maps of national and international surveys such lines are "smoothed-out" because it would be impossible to show each minute variation in any particular area. These large scale maps chiefly portray regional characteristics. Exploratory geomagnetics on the other hand is concerned with specific local variations such as would be associated with changes in the immediate local subsurface structure; that is, exploratory geomagnetics is concerned with relatively "microscopic" variations in the regional magnetic properties.

Numerous magnetic observatories have been established by various agencies throughout the world. Large scale magnetic studies in the United States have been made chiefly by the Coast and Geodetic Survey. The need for magnetic information has extended to land surveying, marine and air navigation, and finally to the geophysicist. Magnetic observations along the coasts were begun shortly after establishment in 1843 of the Coast and Geodetic branch of the U. S. Department of Commerce. A systematic magnetic survey of the country which included occupation of at least one station in every county, the stations being situated about 30 to 40 miles apart, was initiated in 1899.† This work was completed in 1915.

† D. L. Hazard, *U. S. Magnetic Tables and Magnetic Charts for 1925* (U. S. Dept. of Commerce, Coast & Geodetic Survey).

Since the later date, reoccupation of selected stations has been made for the determination of the changes in the earth's magnetism.* Graphs showing declination, vertical and horizontal intensity, and dip in the United States may be obtained from the United States Coast and Geodetic Survey.

Although these large scale maps give very little information regarding the relatively small areas investigated by the geophysicist, they provide absolute values of the magnetic elements (dip, horizontal intensity, etc.) to which to "tie" the relative results of the detailed surveys. †

Time Variations in the Earth's Magnetic Field

Data obtained in regional or local surveys must be corrected for certain time variations. The secular variation, i.e., the systematic change of the earth's magnetism over long periods of time, is of slight importance in applied geophysics. The diurnal variation, however, which appears to be a terrestrial phenomenon varying chiefly with the position of the sun above the horizon, is ordinarily of such magnitude that it must be taken into account in nearly all applications of geomagnetics. Also, variations produced by magnetic storms, which are violent, occasional disturbances, lasting hours or days in some cases, must be taken account of, because they may change the total earth's field by as much as 1%.

Law of Force.—It is found experimentally that magnetic poles always occur in pairs. Thus a bar magnet has a positive pole at one end which is equal in strength to a negative pole at its other end. It is possible, however, to state a law of attraction between "isolated" magnetic poles which is analagous to the law of attraction between electric poles (or charges). In the magnetic case, the "isolated" poles occur, for example, at the ends of two long bar magnets, the other poles of the two bar magnets being sufficiently far away that their effects may be neglected (Figure 2). The law of force between the two poles of strength m_1 and m_2 is:

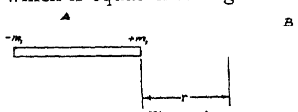

FIG. 2.—Sketch illustrating concept of "isolated" poles appearing in the inverse square law (Equation 2).

(2)

where C is a constant which depends on the medium and r is the distance between the two poles.

A unit pole is a pole of such strength that at a distance of one centimeter from a like pole the repulsion between them is one dyne. In a unit magnetic field, i.e. in a field of 1 gauss, a unit positive (N) pole is acted on by a force having a magnitude of one dyne and a direction parallel to that of the field.

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* Time variations in the earth's magnetic field are discussed in the following section.

† D. L. Hazard, "The Relation of the Magnetic Work of the U. S. Coast and Geodetic Survey to Geophysical Prospecting Methods," *Terr. Mag.* No. 3, pp. 130, 131, 1938.

Properties of a Bar Magnet.—Permanent bar magnets constitute an important part of practically all geomagnetic instruments. It will be of value, therefore, to describe briefly certain well known properties of these magnets.

Magnetic Field of a Bar Magnet

The magnetic field surrounding a magnet is a vector field, the properties of which are usually described with reference to the magnetic lines of force. The assumed number of lines per unit area at any point is taken as being proportional to the strength of the field at that point. The lines of force surrounding a bar magnet are illustrated in Figure 3 which shows the positions assumed by iron filings when sprinkled over a sheet of cardboard beneath which lies a bar magnet. This figure indicates also the distortion of the lines of force caused by the presence of a piece of iron in the field.

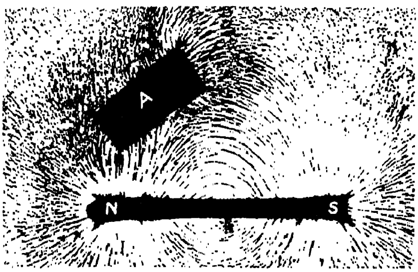

FIG. 3.—Photograph of iron filings illustrating the lines of force due to a bar magnet N-S and the distortion produced by a piece of iron A.

The magnitude of the field at any point due to a bar magnet is readily calculated by using Equation 2. †

Case 1. Field at P situated on a line passing through the poles of the bar magnet (Figure 4A).

Let r be the distance of P from the center of the magnet and let the length and pole strength of the magnet be $2l$ and m respectively. From the definition of field strength and Equation 2:

$$\text{Field at } P \text{ due to } N = \frac{m}{(r \cdot \cdot l)^2}$$

$$\text{Field at } P \text{ due to } S = - \; m \text{ [illegible]}$$

Since the two are in the same line,

$$\text{Resultant Field} = \frac{m}{\text{[illegible]}} - \text{[illegible]} \quad 4mlr$$

For a magnet of such length that $l^2 << r^2$,

$$\text{Resultant Field} = \frac{4ml}{\text{[illegible]}} = \frac{2M}{\text{[illegible]}} \tag{3}$$

where M is the *magnetic moment* of the bar magnet and is equal to the pole strength m times the distance $2l$ between the poles.

† See, for example, S. G. Starling, *Electricity and Magnetism* (Longmans, Green and Co., Ltd., 5th Ed. 1929) pp. 3-20.

Case 2. P on a line bisecting the magnet at right angles (Figure 4B).

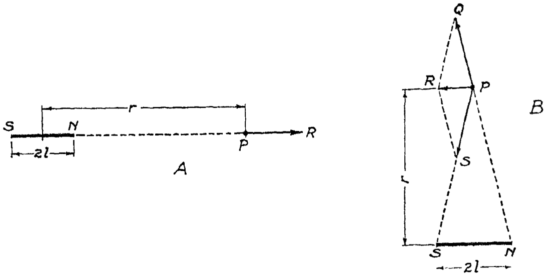

Fig. 4—Field at an external point P due to a bar magnet SN.

From geometry, $PN = \sqrt{r^2 + l^2}$; hence

$$\text{Field } PQ \text{ due to } N = \frac{m}{r^2 + l^2}$$

$$\text{Field } PS \text{ due to } S = \frac{m}{r^2 + l^2}$$

The resultant field is PR and this may be obtained by noting that from the geometry of the figure,

$$\frac{PR}{NS} = \frac{PQ}{PN}$$

Hence,

Resultant Field = m $2l$ $2ml$ M

If, as in case 1, $l^2 << r^2$,

Resultant Field = M (4)

Turning Couple Experienced by Magnet in Uniform Field

If a magnet is suspended freely at its center of gravity, it will align itself in the direction of the resultant magnetic field (normal earth's field plus any anomalous field). The couple tending to align the bar magnet may be computed by referring to Figure 5. SN represents a magnetized needle of pole strength m and magnetic moment M; the needle is suspended at a and is free to turn in the plane of the paper. H represents the component of the resultant magnetic field in that plane. The couple tending to rotate the needle about its center of gravity a is due to the two forces acting on the poles: namely, $+Hm$ on the N pole and $-Hm$

on the S pole. The magnitude of this couple is equal to the product of either force multiplied by the perpendicular distance, PN, between them; i.e.,

$$\text{couple} = (Hm)\,\overline{PN} = (Hm)\,\overline{NS}\sin\theta$$

but

$$= M$$

Hence,

$$\text{couple} = HM\sin\theta \tag{5}$$

The couple is zero when $\sin\theta$ is zero, i.e., when the magnet is parallel to the magnetic field H; it is a maximum when $\sin\theta = 1$, i.e., when the magnet is at right angles to the field.

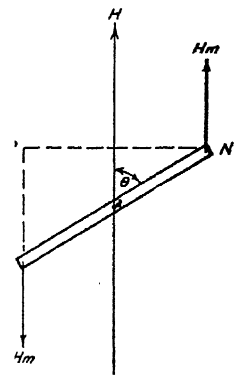

Fig. 5.—Sketch illustrating ouple experienced by a bar agnet NS placed in a uniorm magnetic filed H.

Magnetic Properties of Materials.—All materials may be classified as *diamagnetic, paramagnetic,* or *non-magnetic* depending on whether they repulse, attract, or are neutral to the magnetic lines of force. A non-magnetic or neutral material does not distort the external magnetic field, and the magnetic flux (number of lines of force per unit area) inside is the same as that outside the material. (Figure 6A.) A para- or ferromagnetic material tends to attract the lines of force. (Figure 6B.) A diamagnetic material situated in a uniform magnetic field, such as the earth's field, distorts the lines of force away from it. (Figure 6C.) In the case of para- and diamagnetic materials, the modification of the original field H produced by the presence of the material is slight; for ferromag-

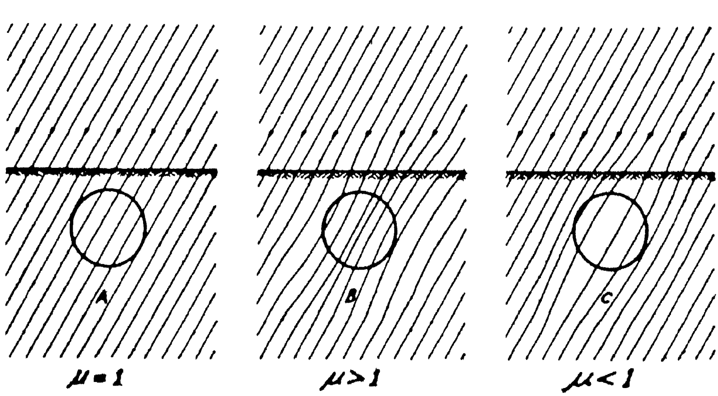

Fig. 6.—Distortion of earth's field due to various materials; A.—non-magnetic material; B.—para- or ferromagnetic material; C.—diamagnetic material.

netic materials, the distortion of the original field near the magnet is very pronounced.

The magnetic *permeability* of a material is defined as the ratio of the magnetic flux inside the material to the flux in vacuum for a given applied field. Numerically, the permeability, μ, is equal to the reciprocal of the constant C which appears in the inverse square law, Equation 2. That is,

$$F = C$$

μ is essentially unity for air and all other so-called non-magnetic materials. For iron, nickel, cobalt, and other magnetic minerals, the permeability is greater than 1 and depends not only on the material but also on the magnitude of the external field.*

For most rocks and ores, μ has a small value and a related quantity, the magnetic *susceptibility,* is used in describing the magnetic properties of these materials. The magnetic susceptibility, which is usually denoted by the letter k, is defined as the ratio of the *intensity of magnetization*, or the magnetic moment per unit volume, to the magnetizing force. That is,

$$k = \frac{I}{H} \tag{6}$$

where I is the magnetic moment per unit volume and H the magnetizing force or field strength. Due to the fact that I depends on H, k is a constant which is essentially independent of H, except in the case of highly magnetic materials such as metallic iron, magnetite, pyrrhotite, etc.

k has an especially interesting physical significance for magnetic material in the form of a bar. When magnetic material is placed in a magnetic field, it becomes magnetized by induction and exhibits the properties of a magnet. In particular, if a bar of magnetic material is placed with its axis parallel to a uniform field H, the susceptibility is given by the expression,

where σ is the pole strength per unit area induced on the two end surfaces of the bar.

It may be proved from elementary considerations of magnetic theory that the susceptibility is related to the permeability by the expression †

$$= 1 + \tag{8}$$

An additional magnetic property which is of considerable practical importance in laboratory determinations of the magnetic susceptibility is

* According to Coulomb's law, the force of attraction between unlike charges decreases with an increase in the permeability of the medium in which the charges occur. This apparently paradoxical result is due to polarization effects produced by the charges.

† S. G. Starling, *loc. cit.*, pp. 266-8.

the *residual magnetism* or *retentivity*. The residual magnetism refers to the magnetism retained by a magnetic material after it has been removed from an inducing field.

GEOMAGNETIC ANOMALIES

The outer crust of the earth comprises a magnetically heterogeneous assemblage of rocks which exists in the earth's magnetic field. Certain rocks exhibit the properties of magnets (induced or permanent) and superimpose their own magnetic fields on that of the earth. The superimposed fields are termed major, continental, regional, and local anomalies, depending on the scale of the geologic irregularities which produce them.

The major and continental magnetic anomalies may be defined as variations of the earth's magnetic field associated with major and continental irregularities (inhomogeneities of composition and/or structural distortions). Continental anomalies are negative in Europe and positive in North and South America. Major anomalies may show pronounced trends, such as a positive major trend paralleling the Rockies and the Andes and a negative major trend paralleling the Alpine chains. †

The regional anomalies cover smaller areas, one of the best known being in the Province of Kursk, Russia. Regional anomalies indicate the tectonics and the stratigraphic and the petrographic character of the upper portion of the rock zone. A basin that has strongly magnetic beds, such as Illinois, is positive, and a basin that does not have strongly magnetic beds, such as West Texas, is negative. Uplifts that have a thick section of Ordovician limestones, such as the Ozarks, are negative because the flanks have magnetic shales and sands above the limestones. The Nemaha ridge is positive because the granite is sufficiently close to the surface to outweigh stratigraphic influences. ‡

Local anomalies, as the name implies, extend over relatively small areas. The magnetic intensity in these areas may occasionally reach a high value; for example, near Juneau, Alaska, a local magnetic pole produces an anomaly of sufficient magnitude to cause a dip needle to stand vertical.

Intensity of Magnetization of Rocks and Formations.—The intensity of magnetization of rocks and formations, and hence the magnetic anomalies which they produce, is determined principally by the nature of the constituent materials and the percentage relations among these materials and to a minor extent by the geologic history. The importance of the first factor derives from the following consideration. The intensity of magnetization depends on the susceptibility of the rocks and formations;

† W. P. Jenny, Abstract of paper delivered before the American Association of Petroleum Geologists, *Oil and Gas Journal*, April 11, 1940.

‡ W. P. Jenny, *loc. cit.*

the effective susceptibility, in turn, depends chiefly on the relative amount and the distribution of magnetic material, notably magnetite, contained in the formations. The importance of the geologic history will be evident from a consideration of the forces affecting magnetization *during* the geologic life of the formations.*

For the most part, the forces which have been active in changing the magnetization are thermal, chemical and mechanical in nature, and are associated with erosion, tectonic movements, metamorphic processes and igneous intrusions. Also, disintegration and decomposition alter the magnetization of formations. Finally, the magnetization may be affected by lightning.

In general the permeability of magnetic materials increases continually with increase of temperature until a point known as the "temperature of recalescence" is reached.† After passing this temperature, μ decreases very rapidly, and within a few degrees of the recalescence temperature, ferromagnetic materials appear to lose their magnetic properties completely. The temperature at which magnetic materials become non-magnetic is known as the *Curie point*. The Curie points for some ferromagnetic minerals are: magnetite ‡ 515° C., pyrrhotite ‡ 300° C., nickel § 310° C., iron § 690° C. to 870° C. These values for the Curie point have an important bearing on the question of the depth in the earth's crust at which magnetization of rocks can exist. It is estimated that due to increase of temperature with depth, permanent magnetization of rocks disappears below about 60,000 feet. ††

Some metamorphic processes which take place in the zone of flowage change hematite to magnetite. This increases the potential magnetization, because the susceptibility of magnetite is much greater than that of hematite. The chemical and mechanical processes which take place in the zone of fracture have differing effects. *Disintegration* (a mechanical process) may result in concentration of magnetite, as in placers. *Decomposition* (a chemical process) invariably results in demagnetization.

A dissemination of magnetite particles produced by disintegration will cause a decrease in the intensity of magnetization. This is attested by Figure 7 which shows the variation with per cent voids.

* From a theoretical viewpoint, it appears to be unnecessary to assume permanent magnetization of rocks, inasmuch as the rocks never get out of the earth's field. However, it is entirely plausible to consider that plutonic rocks could have become permanently magnetized, with an intensity of magnetization corresponding to the momentary value of the earth's field in which they occurred, at that period of their cooling when their temperature was less than the Curie point. (R. Ambronn, *Elements of Geophysics* (McGraw-Hill Book Co., Inc., New York, 1928) p. 95.)

† J. H. Jeans, *loc. cit.*, p. 412.
‡ R. Ambronn, *loc. cit.*, p. 95.
§ S. G. Starling, *loc. cit.*, p. 287.
†† See also J. A. Fleming, *Terrestrial Magnetism and Electricity* (McGraw-Hill, 1939) p. 321.

A change in position or orientation of dikes, lava flows, etc., may produce anomalous magnetization. Such effects, if present, are often masked by the abnormal polarization effects of steep topography. Rocks of high retentivity quite often retain their original magnetization. If changed in position without excessively high temperatures, their polarity will produce a local anomaly.

Lightning may affect the local magnetization of exposed rocks. The large current intensities in lightning discharges (of the order of 20,000 amperes) give rise to relatively strong magnetic fields. Hence, lightning discharges may offer an explanation for the irregular rock magnetization sometimes observed on exposed hill tops.

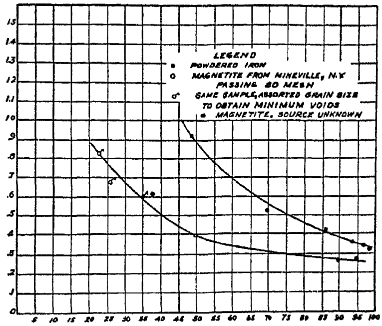

Fig. 7.—Variation of susceptibility of magnetite and iron powders with per cent voids in a field of 0.64 gauss. (L. B. Slichter, *A.I.M.E. Geophysical Prospecting,* 1929, p. 247.)

Numerical Data on Magnetic Susceptibilities.—The usefulness of geomagnetic methods in obtaining information regarding subsurface conditions depends chiefly on the relative magnetic susceptibilities of the materials constituting the outermost portion of the earth's crust. Because in general rocks are mixtures of numerous mineral components, the effective susceptibilities depend on those of the individual constituents, and on the percentage of each constituent.

As a general rule, reliable magnetic anomalies cannot be measured by the usual field techniques of exploration geomagnetics unless the magnitude of the *difference* between the susceptibility of the anomalous geologic feature and the susceptibility of the surrounding media is equal to or greater than plus or minus 0.00015. If the susceptibility difference is positive, the anomalous subsurface feature usually possesses some ferromagnetic material. The *amount* of ferromagnetic materials which must be present is small due to the fact that a majority of the compounds of iron have very high susceptibilities. (See Figure 7.) The most familiar geologic feature possessing a *negative difference* with respect to surrounding media is a salt dome.

The magnetic susceptibilities of various materials which occur in the outer portion of the earth's crust are shown in Table 5.† Other values appear elsewhere in the literature.‡

† N. H. Stern, "A Background for the Application of Geomagnetics to Exploration," *A.I.M.E. Geophysical Prospecting,* 1929, pp. 315-342.

‡ L. B. Slichter, *A.I.M.E. Geophysical Prospecting,* 1929, p. 343.

F. Stutzer, W. Gross and K. Borneman: "Ueber magnetische Eigenschaften der Zinkblende und einiger anderer Mineralien," *Metall und Erz* (1918) *15,* 1, quoted by L. B. Slichter, *A.I.M.E. Geophysical Prospecting,* 1929, p. 241.

TABLE 5.

SUSCEPTIBILITIES OF VARIOUS ELEMENTS AND MINERALS

Ferromagnetic	*Susceptibility* $\times 10^6$
Ilmenite	4,400
Iron	80,000
Magnetite	32,122

Paramagnetic	*Susceptibility* $\times 10^6$
Albandite	177.28
Bieberite	68.00
Brookite	0.262
Chalcanthite	14.30
Cuprite	4.38
Hausmannite	318.07
Hematite (Amor)	107.12
Hematite (Xtal)	426.00
Limonite	57.00
Manganosite	349.44
Melanterite	80.00
Morenosite	18.00
Octahedrite	0.257
Platinum	26.00
Pyrite	120.00
Pyrolusite	131.22
Rutile	0.28

Diamagnetic	*Susceptibility* $\times 10^6$
Anhydrite	1.12
Arsenic	1.70
Berzelianite	1.01
Bismuth	14.00
Bromyrite	1.53
Calcite	1.00
Cerargyrite	1.55
Chalcocite	0.78
Copper	0.80
Cotunite	1.31
Covellite	0.74
Diamond	1.80
Epsomite	0.63
Fluorite	2.00
Graphite	8.00
Iodyrite	1.66
Kalinite	1.00
Lead	1.30
Marble	0.75
Niter (KNO_3)	0.72
Niter ($NaNO_3$)	0.70
Quartz	1.20
Rock Salt	0.82
Sassolite	0.89
Silver	1.50
Sulfur	0.85
Sylvite	0.91
Villiaumite	1.12
Zincite	1.85

LABORATORY METHODS FOR DETERMINING SUSCEPTIBILITIES

Several investigators have devised various laboratory methods for determining rock susceptibilities or permeabilities. † It will suffice here to indicate the operating principles of one method. ‡ Two identical solenoids are mounted symmetrically on either side of one magnet of an astatic magnetic system. (Figure 8.) The solenoids are series-connected, near poles in opposition, and the system is carefully balanced by adjustment of the potentiometer until the magnetometer deflection is independent of the current flowing through the solenoids.

The test specimen S is now inserted in one solenoid. The specimen may be ground to size or else pulverized and placed in the glass specimen tube. When current is passed through the coils, the different permeability of the specimen upsets the magnetic balance of the solenoids and produces a deflection of the astatic magnetometer system. This magnetometer deflection is reduced to zero by adjusting the potentiometer. The magnetic moment may be calculated from the change in potentiometer setting or ratio of current in the two coils and the constants of the apparatus. Knowing the magnetic moment of the specimen, the intensity of magnetization I may be calculated from the relation: $I =$ magnetic moment per unit volume.

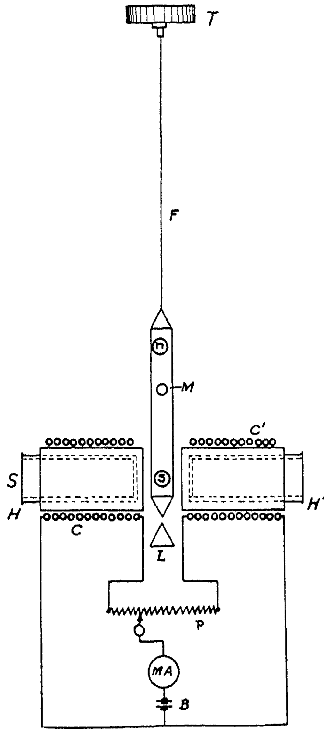

Fig. 8.—Method for determining magnetic susceptibility. C and C', two identical solenoids with like poles opposing; M, mirror; n and s, astatic magnetic system; T, torque head; F, suspension fibre; H and H', specimen holders, usually glass, in one of which is placed solid or pulverized specimen; L, leveling points; P, potentiometer; MA, milliampere meter; B, battery.

METHODS AND INSTRUMENTS FOR GEOMAGNETIC INVESTIGATIONS

These may be divided into two classes: (a) instruments and methods for determining the absolute values of the magnetic elements of the earth's field over extended areas, and (b) instruments and methods for determining the relative values of the magnetic elements over limited areas.

Methods and Instruments Usually Employed for Surveys of Extended Areas.—The magnetic elements *usually* measured are the declination, the horizontal intensity, and the dip.* In addition, complete investigations at a station may include observations on the total intensity and the vertical intensity.

† R. Ambronn, *loc. cit.*, p. 92.
W. M. Barrett, "A Method for Determining Magnetic Susceptibility of Core Samples *A.I.M.E. Geophysical Prospecting*, 1932, pp. 216-233.
D. M. Collingwood, *A.A.P.G. Bull.*, *14*, Part 2, pp. 1187-1192, 1930.
C. A. Heiland, *A.I.M.E. Geophysical Prospecting*, 1932, pp. 234-236.
‡ See also, Bozorth, *Review of Scientific Instruments*, May, 1925.
* Compare p. 56.

Earth inductors are used for accurate measurements of the dip and for measurements of the intensity (horizontal, vertical, or total). The dip circle or dipping needle is used chiefly to measure the dip or inclination of the earth's field at any point. In addition, it is sometimes used for total intensity measurements. The mean pocket chronometer is used for determinations of the geographic meridian.

In addition to the earth inductor and mean pocket chronometer, which are usually standard equipment in extended surveys, various other devices have been developed for determining the magnetic elements. These include the sine galvanometer (horizontal intensity), the compass-variometer (horizontal intensity), magnetic gradiometer (magnetic gradient), iron induction instruments (total intensity, vertical intensity, direction of the magnetic meridian).

Determination of the Magnetic Declination

The determination of the magnetic declination comprises two operations: the determination of the astronomical meridian and the determination of the magnetic meridian. In one convenient method, the sun's altitude is determined at or near apparent noon by using a small theodolite equipped for azimuth observations.† The horizontal circle of the theodolite is read and a distant mark is observed through the telescope; this operation gives the approximate azimuth of the mark. With the longitude of the place of observation and the equation of time known, the direction of true north can be computed.

The magnetic meridian may be determined by a *declinator* which consists of a magnetic needle, provided with a mirror and free to rotate in a horizontal plane, and a telescope mounted on a transit base. In some declinators, the magnetic needle or compass is suspended by a torsion thread; in others, it is supported by means of a jewel suspension. The determination of the magnetic meridian consists in observing the position of a reflected image on the horizontal transit circle.*

Determination of the Horizontal Intensity

Method of Oscillations and Deflections: The Magnetometer.—The absolute value of the horizontal intensity can be determined by a combination of two operations called "oscillations" and "deflections" respectively.

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† D. L. Hazard, "Directions for Magnetic Measurements," United States Dept. of Commerce, *Coast and Geodetic Survey Serial No. 166*, pp. 62-73.

* The *relative* declination of two consecutive stations may be determined as follows. The observer first measures the angle between the magnetic meridian and the forward sight at the first station. He then proceeds to the second station and measures the angle between the magnetic meridian and the backward sight (on the first station). If the magnetic meridians at the two stations are parallel, the sum of these angles will equal 180°. If the sum is not equal to 180°, the difference between the sum of the observed angles and 180° is a measure of the difference in declination at the two stations.

Oscillations

A magnet free to rotate about a vertical axis through its center of gravity comes to rest with its magnetic axis in the plane of the magnetic meridian. If the magnet is displaced out of that plane and then released, it will oscillate in a horizontal plane under the influence of the horizontal component of the earth's field, precisely as a pendulum oscillates in a vertical plane under the influence of the force of gravity. Since the magnet executes a simple harmonic motion, its period is:

$$2\pi \sqrt{-\frac{\text{displacement}}{\text{acceleration}}} \tag{9}$$

The displacement θ at any instant is equal to the angle made by the magnetic axis of the magnet with the magnetic meridian. The acceleration at any instant is equal to the couple produced by the horizontal component of the earth's field divided by the moment of inertia I of the magnet about the axis of rotation. From Equation 5 the couple acting on the magnet is $MH \sin \theta$ where M is the magnetic moment of the suspended magnet, H is the horizontal intensity, and θ is the displacement. Hence, the acceleration is equal to $MH \sin \theta / I$. Also, for simple harmonic motion, the displacement and acceleration have opposite signs; hence, Equation 9 becomes:

$$T = 2\pi \sqrt{\frac{\theta}{MH \sin \theta / I}}$$

For small displacements, $\sin \theta$ is very nearly equal to θ; hence

$$T = 2\pi \sqrt{I/MH} \qquad \text{(approx.)} \tag{10}$$

or

$$MH = 4 \frac{\pi^2 I}{T^2} \qquad \text{(approx.)} \tag{11}$$

T may be determined by observing the time for several oscillations and calculating the mean value. The moment of inertia I is either calculated or determined experimentally. Hence, a determination of the time of oscillation T yields the value of the quotient MH directly.

It is of interest to note that relative values of the horizontal intensity H may be obtained by determining T for the same magnet at several stations.

Deflections

If the suspended magnet described in the last section is at rest and another magnet ("deflecting magnet") is brought close to it, the suspended magnet will be deflected out of the magnetic meridian until the restoring couple due to the horizontal intensity exactly balances the deflecting couple. The displacement of the suspended magnet produced by the deflecting magnet depends on the relative orientation of the two magnets. Generally, one of the four relative orientations shown in Figure 9 is employed. The suspended magnet of moment M is assumed to make an angle θ with the

direction of the horizontal intensity H. The vector F represents the field at M due to the deflector of magnetic moment M_d.

The orientation of the suspended magnet when it comes to rest is determined by the condition that the couple due to the deflector be equal

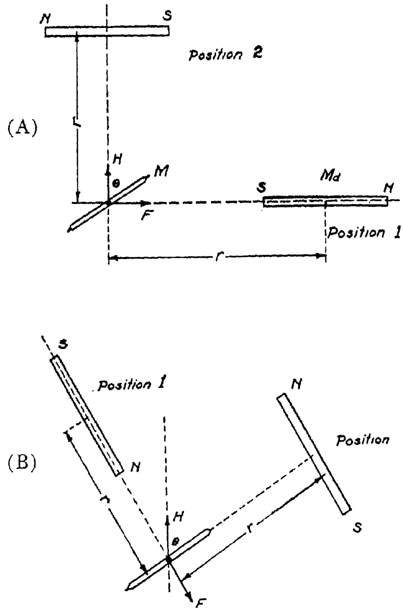

Fig. 9.
(A) Tangent Method (Gauss positions).
(B) Sine Method (Lamont positions).

written in the form:

$$\sin\theta = FM \sin (90$$

or

$$\sin\theta = F\cos\theta$$

On substituting appropriate values for F from Equations 3 and 4, the following relations are obtained:

Tangent Method

Position 1
$$H\sin\theta = \frac{2M_d}{r^3}\cos\theta$$

or
$$H/M_d = \frac{2}{r^3\tan\theta} \text{ (approximately)} \qquad (12)$$

Position 2
$$H/M_d = \frac{1}{r^3\tan\theta} \text{ (approximately)} \qquad (12a)$$

Sine Method

Position 1 $$H \sin \theta = \frac{2M_d}{r^3} \sin (\pi/2) =$$

or $$H/M_d = \frac{2}{r^3 \sin \theta} \text{ (approximately)}$$

Position 2 $$H/M_d = \frac{1}{r^3 \sin \theta} \text{ (approximately)} \qquad (13a)$$

It will be noticed that the magnetic moment of the suspended magnet M does not appear in Equations 12 or 13. Hence, the two operations required for determining the absolute value of the horizontal intensity at any point are: (1) The *deflecting magnet* M_d is suspended by a torsion wire and its period of oscillation T is measured. This yields the value of the *product* M_dH, because from Equation 11,

$$M_dH = \frac{\cdot}{T^2} \qquad (14)$$

(2) The deflecting magnet M_d and a suspended magnet M are mounted in one of the relative orientations shown in Figure 9, and the deflection θ is measured. This yields the value of the *quotient* H/M_d. In particular, if the tangent position 1 is utilized, the quotient is given by Equation 12. r, the separation of the magnets, can be measured. Equations 12 and 14, therefore, constitute two equations in two unknowns: viz., M and H; hence, the absolute value of H is obtained by solving the two equations simultaneously.

The instrument used for carrying out the two operations of deflections and oscillations is called a magnetometer.* This instrument consists essentially of a small magnet which is suspended by a practically torsionless fiber in a non-magnetic case and two horizontal arms which are attached to the base of the case. One arm carries a cradle which supports the scale and the other arm carries a cradle which supports the auxiliary magnet during the deflection experiments.**

Sine Galvanometers.—The sine galvanometer is an observatory instrument used for precision determinations of the absolute value of the hori-

* It should be pointed out that various types of field instruments which are used for deflection measurements only are also called magnetometers.

** A detailed description of the technique of carrying out oscillation and deflection experiments will be found in Starling, *loc. cit.* pp. 26-31.

zontal intensity. The horizontal intensity is determined by the deflection method described above. The deflecting field in this case is produced by an electric current flowing in a coil of known dimensions. Sine galvanometers for the determination of the horizontal intensity are described in detail by Barnett † and by Hazard. ‡ Figure 10 shows a photograph of a sine galvanometer designed and constructed by the Carnegie Institution of Washington for determining the magnetic horizontal intensity. The standard cells and electrical measuring appurtenances used when making observations with this sine galvanometer are mounted in a heat insulated cabinet.

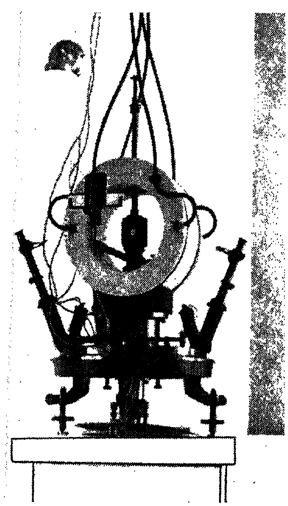

Fig. 10.—Carnegie Institution of Washington sine galvanometer for determining magnetic horizontal intensity. (Courtesy of Carnegie Institution of Washington.)

Compass-Variometers.—The Carnegie Institution of Washington has developed several compass-variometers capable of measuring relative values of the horizontal intensity. §

One type of compass variometer which has been used for measurement aboard ships is shown in Figure 11. Two disc-shaped magnets of equal magnetic moment are suspended independently one above the other. The distance between the two disc magnets is regulated by a graduated micrometer screw. A fine quartz-rod pointer, or index, is attached to each of the magnet supports. The pointer of the upper magnet is in the vertical plane through the magnetic axis of the upper magnet (a diameter of the disc), and the pointer of the lower magnet is in a vertical plane making an angle ψ with the magnetic axis of the lower magnet. (In general, the angle ψ is made 60°.)

On looking down on the instrument through a lens one sees the quartz pointer of the lower support and an image of the pointer of the upper support reflected from a mirror which is mounted centrally with respect to the magnet system. The angle ψ between the two pointers is read off a

† S. J. Barnett, *Carnegie Institution of Washington, Publication 175*, pp. 373-394, Dec. 1921.

‡ D. L. Hazard, *loc. cit.*, p. 38.

§ L. A. Bauer, W. J. Peters, and J. A. Fleming, "The Compass Variometer," *Carnegie Institution of Washington, Publication 175*, Vol. V, pp. 339-357.

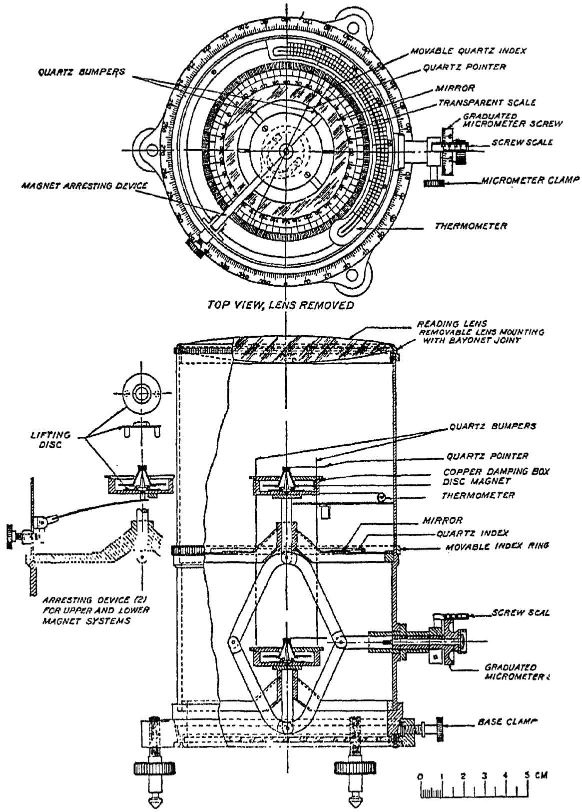

FIG. 11.—Horizontal and vertical sections of a C.I.W. compass-variometer (Bauer, Peters, and Fleming, Carnegie Institution of Washington, Publication 175, Vol. V.).

graduated circle which is photographed on a glass plate mounted approximately in the plane of the reflecting mirror.

It may be shown that changes in the horizontal intensity are related to changes in the angle ψ by the equation

$$\Delta H = - \quad DM \quad \sin$$

where

ΔH = change in horizontal intensity
D = instrumental constant
M = magnetic moment of either magnet
e = vertical distance between magnets
ψ = angle between two imaginary vertical planes passing through the magnetic axes of the two magnets

The usual method of measuring variations in H at different stations consists in keeping the distance e fixed and observing the changes in ψ. $\Delta\psi$ at any station will depend on the initial (base station) value of ψ and on the anomaly ΔH at the station. For a given variometer and given base station value of ψ, the value of $\Delta\psi$ at any station will depend only on the anomaly ΔH at that station.

It is evident that, for a given value of the magnetic moment M of the magnets, the sensitivity of the instrument depends on the value of ψ and of e. For $\psi = 60°$, $e = 6.5$ cm., the sensitivity (ΔH for $\Delta\psi = 1°$) is of the order of 75 gammas.

Inclination or Dip Determinations

Dip Circle.—The dip angle which the total intensity makes with the horizontal intensity is usually determined at magnetic stations by means of an earth inductor; occasionally, however, a dip circle is employed. The dip circle comprises a magnetized needle, which is supported so that it is free to swing in a vertical plane, and a circle graduated in degrees. An axle passes through the center of gravity of the magnetized needle and terminates in small pivots which rest on pivot bearings. The angle of dip may be read directly off the vertical graduated circle which is concentric with the axis of rotation of the needle, provided this axis is perpendicular to the plane on the magnetic meridian. If the needle does not swing in the plane of the magnetic meridian, measurements must be made for two orientations of the instrument at right angles to each other. From these two readings, i_a and $i_{(90-a)}$, the true dip angle i can be calculated by the formula

$$\text{ctn}^2 i_a + \text{ctn}^2 i_{(90-a)} = \text{ctn}^2 i$$

Earth Inductors.—The earth inductor may be used to determine the inclination of the earth's magnetic field, any component of the earth's magnetic field, or the direction of the magnetic meridian.*

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* For a detailed description of the theory of the standard earth inductor used for inclination measurements see N. E. Dorsey, "The Theory of the Earth Inductor as an Inclinometer." *Terr. Mag.* 18 (1) pp. 1-37, 1937

Various designs have been suggested. The operating principle of all of them, however, is essentially similar to that of a small dynamo. When a coil of wire is rotated in a magnetic field so as to cut magnetic lines of force, an E.M.F. is induced in the coil. The magnitude of the induced E.M.F. depends on the number of magnetic lines of force which are cut by the coil and may be expressed by an equation of the form:

$$E = 2NHA$$

where

$E =$ the average E.M.F. induced in the coil on rotating it through one-half turn.

$N =$ the number of turns of wire in the coil.

$H =$ component of the earth's magnetic field parallel to the axis of rotation of the coil.

$A =$ area of the coil.

$\tau =$ time for one-half turn.

The induced E.M.F. creates a flow of current when the circuit is closed. The magnitude of this current is determined by the induced potential and the impedance of the coil and auxiliary circuit.

Evidently, when the plane of the coil is parallel to the magnetic field, no lines of magnetic force are cut, and the induced E.M.F. and current are zero. Hence, to determine the magnetic inclination with an earth inductor, it is only necessary to measure the angle of inclination of the brush assembly system making contact with the commutator of the coil when no current flows as the coil is rotated.

The inductor is usually mounted on a modified gimbal support so that the axis of rotation of the coil may be placed in any direction in space. The magnitude of the earth's total field, or any component of the total field, may be computed from the magnitude of the current induced in the coil and the constants of the instrument. The direction of the total field, or one of its components, may be ascertained by noting the inclination of the axis of rotation for zero current.

A photograph of the Wild pattern earth inductor employed by the United States Coast and Geodetic Survey is shown in Figure 12. The instrument comprises essentially a coil of copper wire wound on a cylindrical frame, a flexible shafting which is connected to the cylindrical frame, a commutator, a ring, and a graduated vertical circle which is rigidly attached to the ring. The cylindrical frame is supported

by an axis (coil axis) that passes through a central diameter and rests in bearings in the ring. The axis of the ring (inclination axis) is perpendicular to the axis of the coil and is supported in a horizontal position on bearings in uprights attached to the alidade. The graduated vertical circle is parallel to the axis of the coil and concentric with the inclination axis.

The azimuth of the coil axis may be adjusted by turning the ring about the inclination axis, and the azimuth may be read by means of the

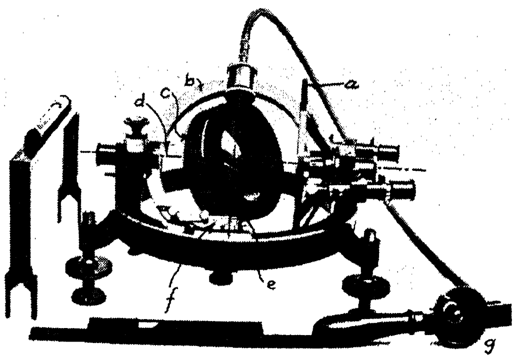

FIG. 12.—Wild pattern earth inductor. *a*, vertical circle; *b*, ring; *c*, coil; *d*, inclination axis; *e*, coil axis (extended); *f*, brush assembly and commutator; *g*, hand cranked gear drive. (Courtesy of the United States Coast and Geodetic Survey.)

graduated vertical circle and comparator or microscope shown in the right hand portion of the figure. The coil may be rotated about its axis by means of the flexible shafting, which connects one end of the axis with a gear, and a hand crank.

The commutator consists of two brass half rings which surround the lower end of the axis of the coil. The half rings are connected to the ends of the wire of the coil but are insulated from the coil axis and from each other. The commutator brushes, one on each side of the axis of the coil, are attached to the large ring but are well insulated from it. They are so placed that commutation occurs when the plane of the coil is parallel to the inclination axis. The alternating current produced in the rotating coil by the induced E.M.F. is conveyed to the half rings of the commutator, taken off by the brushes as a direct current, and carried by the attached leads to the galvanometer.

The operation of the instrument for dip determinations consists in placing the axis of the coil in the plane of the magnetic meridian and al-

tering the inclination of the coil axis (by turning the ring) until a position is found where the galvanometer indicator stays at zero when the coil is rotated. In this position, the axis of the coil is parallel to the resultant or total field of the earth, and the angle of inclination of the coil axis as measured on the vertical circle is the dip.

If only the *direction* of the earth's field is to be measured, any type of sensitive null-point galvanometer (string, loop, astatic, etc.) may be employed. If the *strength* or *intensity* of the field is to be determined, a ballistic galvanometer is usually employed. (If intensity measurements are based on the potential compensation principle, however, null-point galvanometers are again usable.)

Another type of *portable* earth indicator and galvanometer has been designed and constructed by the Carnegie Institution of Washington. The galvanometer is so designed that the arm carrying the telescope and scale can be folded and carried in the earth-inductor carrying case. † Peep sights and a compass attachment are sometimes provided for setting the earth inductor in the magnetic meridian. If the magnetic declination and the azimuth of a reference mark are known, the meridian setting may be made with the peep sights alone. Ordinarily, however, it is necessary to use the compass attachment.

Special Devices for Determining the Magnetic Gradient, Intensity, and Meridian

Magnetic Gradiometer.—The magnetic gradient is measured by comparing the induced electromotive forces in two similar coils spaced apart with parallel axes of rotation and rotatable at the same speed.‡ If the magnetic field intensities perpendicular to the axes of rotation are different at the two coils, the relative magnitudes of the electromotive forces induced in the coils will depend on the relative magnitudes of the field intensities. The two coils of the gradiometer are mounted in a frame and are free to rotate about axes at right angles to the length of the frame. The ratio of the E.M.F.'s induced in the two coils is measured by connecting the coils to opposite arms of a Wheatstone bridge circuit. A sensitive galvanometer is employed for indicating the null-point.*

The principles embodied in instruments of this type open up many interesting applications and will become of more commercial importance

† Hazard, *loc. cit.*, pp. 80-93.

‡ J. Roman and T. C. Sermon, "A Magnetic Gradiometer," *A.I.M.E. Geophysical Prospecting*, 1934, pp. 373-388.

* For a derivation of the magnetic gradients as a function of the ratio of the induced E.M.F.'s, see Roman and Sermon, *loc. cit.*, pp. 381-384.

as instrumental improvements are made. Instruments of this type practically eliminate the effects of diurnal and other magnetic variations for which corrections must be made when absolute or relative magnetic anomalies are observed.

Magnetic Balance for Intensity Measurements.—Various instruments have been proposed for accurate measurement of the earth's field. The various modifications of the dynamic type appear to be the most successful in elimination of fragile bearings and other friction supports that limit sensitivity and give difficulty in field use. A majority of the dynamic instruments utilize an electric means for accurate neutralization of the earth's field and for measuring the current required for neutralization.† Knowing this current and the constants of the instrument, the strength of the earth's field can be computed by use of fundamental formulas.

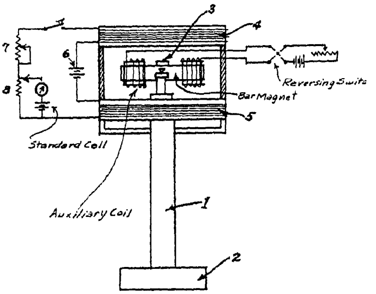

FIG. 13.—Balance for measurements of vertical component of earth's magnetic field. 1, support; 2, base; 3, mirror of optical system; 4, and 5, Helmholtz coils; 6, energizing battery; 7, rheostat; 8, potentiometer. (After Vacquier, U. S. Patent 2,151,627.)

An improved modification of this type of instrument is shown on page 90. In that design, all sliding contacts and brushes have been eliminated, the only moving part being a sturdy metal rotor.

Vacquier Balance for Absolute Measurements of the Vertical Intensity ‡

The operating principle of this instrument consists in neutralizing the vertical component of the earth's field by an opposing magnetic field created by an electrical current. The apparatus (Figure 13) comprises a magnet which is conveniently though not necessarily shaped as a bar and is supported for rotation about an axis very slightly above its center of gravity by means of knife-edges. The latter are made of quartz or other suitable material and rest on level quartz plates. (The plates are mounted on a standard which has a base.) In use, the axes of the knife-edges are aligned horizontally in a direction parallel to the magnetic meridian, so that only the vertical component will affect the magnet. If the magnet were not magnetized, it would assume a horizontal position (as shown) because of the small separation between the axis of rotation and the center of gravity. Under the influence of the earth's field, however, the magnet experiences a turning couple, the magnitude of which depends on the vertical component of the earth's field at that station. The angular deflection of the bar magnet is determined with the aid of a conventional optical system which consists of a small plane mirror affixed to the magnet and a lamp, lens, scale, etc.

The magnetic field for neutralizing the vertical intensity is obtained by means of a Helmholtz coil arrangement. The electrical energizing and measuring circuit com-

† E. A. Johnson, "A Primary Standard for Measuring the Earth's Magnetic Vector," Jour. *Terrestrial Magnetism and Atmospheric Electricity*, Mar. 1939, Vol. 44, No. 1, pp. 29-42.
‡ Victor V. Vacquier, "Apparatus for and Method of Measuring the Terrestrial Magnetic Field," U. S. Patent 2,151,627. Issued Mar. 21, 1939.

prises the Helmholtz coils, a battery, a rheostat, and a potentiometer. The applied field strength is altered by adjusting the resistance so as to bring the magnet to a standard position. Measurements are made, therefore, on the null or balancing principle.

The auxiliary electrical circuit shown in the right hand portion of Figure 13 provides a reversible magnetic field for reversing the direction of the induced magnetism of the bar magnet. This field is used to compensate the effect produced by a possible displacement of the center of gravity of the bar magnet due to warping, temperature effects, dust on the knife-edges, etc. The balance obtained by adjusting the current in the Helmholtz coils is the true balance if the scale deflection remains constant on reversing the induced magnetism of the bar magnet by means of the auxiliary magnetic field.

It is evident that the intensity of the applied field can be determined absolutely provided the instrumental constants are known.

Iron Induction Instruments.—The effects observed with iron-induction instruments are produced by magnetization induced in iron bars by the earth's field. The iron-induction instruments described below are used for determining the vertical component of the earth's field and the magnetic meridian.

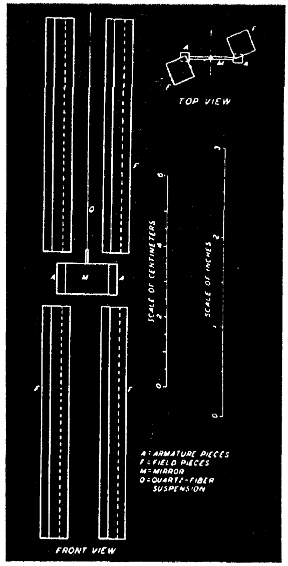

Fig. 14.—Schematic diagram of induction variometer. (McNish, *Rev. Sci. Ins.,* Sept. 1936, p. 338.)

Induction Variometer

The design of a recent type of induction variometer employed by the Carnegie Institution of Washington for vertical intensity measurements is illustrated schematically in Figure 14.† Four staves milled from a perminvar bar serve as field pieces.* These field pieces are mounted with their long axes vertical; hence, the induced magnetization in the direction of the long axes depends on the vertical component of the earth's field only. Two smaller pieces, milled from the same bar, serve as armature pieces. The latter are mounted symmetrically on two sides of a frame of brass to which a mirror is attached, the assembly being suspended by a quartz fibre. Variations of the magnetic fields associated with the induced magnetization of the field-pieces cause a rotation of the armature, and, as usual with suspension instruments, this rotation is restrained by the torsion of

† A. G. McNish, "An Induction-Variometer to Measure Magnetic Anomalies," *Rev. Sci. Ins.,* Vol. 7, 1936, pp. 336-338.

* Perminvar, which is an alloy of 45 per cent nickel, 25 per cent cobalt and 30 per cent iron, shows an extremely small hysteresis loss. A comparison of the magnetic properties of perminvar and other less highly magnetic alloys is given by G. W. Elamn, "Magnetic Alloys of Iron, Nickel and Cobalt," *Electrical Engineering,* Dec. 1, 1935, pp. 1292-1299.

the suspension fibre. The angular deflection, which depends on the vertical component of the earth's field, is measured with the aid of a telescope by observing the deflection of a beam of light reflected from the mirror attached to the armature. The variometer is calibrated by producing a known change of field with a Helmholtz coil arrangement.

Micromagnetometer

The *micromagnetometer*† originally was used to determine the magnetic meridian. Its operation depends on the induction produced by the earth's field in bars made of a magnetic alloy of high permeability. The bars are mounted end to end on a horizontal platform with a small air gap between them. A fine wire, which is free to vibrate, is stretched vertically between the two bars and energized with a 100 cycle alternating current. For most positions of the bars, the fine wire will vibrate under the action of the magnetic field produced by the induced magnetization of the magnetic bars. If, however, the bars are accurately perpendicular to the earth's field, the induction is zero and the wire will cease to vibrate. It is reported that the instrument will detect directional changes of less than 0.01 degrees.

The Magnetron as a Prospecting Instrument

A *magnetron* is a diode or thermionic tube having a straight axial cathode surrounded by a cylindrical anode. ‡ Its proposed use as a magnetic prospecting instrument derives from the fact that in the presence of a magnetic field the electrons do not travel radially from the cathode to the anode. Instead, they spiral around the cathode in circular paths, and after a critical magnetic field intensity is reached, the electrons will return to the cathode without reaching the anode. At this field strength, the plate current will drop abruptly.

The procedure in operating the instrument consists in decreasing the plate voltage on the diode until a voltage is reached at which the current falls off rapidly. The plate voltage at which this occurs is related to the critical field strength H by the relation

$$H = \frac{\quad}{r}$$

where r is the radius of the anode. In practice, a compensation procedure is used, wherein the field to be measured is nullified by a known field produced by a Helmholtz coil arrangement.

The magnetron is affected only by the component of the earth's field which is parallel to its axis. Hence, the instrument theoretically may be used to measure any component of the earth's field by suitable orientation. The sensitivity of the tube may be increased through regeneration by passing the plate current through an additional solenoid. Interesting experimental results have been obtained by using magnetic alloy field pieces to increase the effective magnetic field.

† F. Rieber, "A New Micromagnetometer," *A.I.M.E. Geophysical Prospecting*, 1929, pp. 409-415.
‡ A. W. Hull, *Phys. Rev.* 22 (Second Series), 1923, pp. 279-292
M. Rossiger, *Zeits. für Physik*, 43, 1927, pp. 480-488
M. Rossiger, *Zeits. für Instrumentenkunde*, 49, 1929, pp. 105-113.

Conventional Instruments and Methods for Explorations of Local Magnetic Anomalies.—Magnetic instruments used in explorations for local magnetic anomalies may be classified into four illustrative groups corresponding to their principles of operation: (1) dip needles, (2) deflection instruments, (3) magnetic torsion balances, and (4) magnetic field balances.

Dip Needles

Swedish Mining Compass.—This compass was probably the first "dip needle" employed in magnetic prospecting. At present, it is used widely in regions where the magnetic anomalies are very large and hence easily detected. Well-known regions of this type are the Lake Superior region in America and the Swedish iron ore districts. The Swedish mining compass consists essentially of a pillar which is mounted on the bottom of a cylindrical glass case and supports a needle point pivot.† The needle is free to move in a vertical and a horizontal plane. The rotation in the horizontal plane permits automatic alignment with the magnetic meridian. A counterweight is provided to compensate the effect due to the vertical intensity.

Because this mining compass is affected by the horizontal as well as the vertical intensity, its action is somewhat involved. In practice, the instrument is used for qualitative observations, and H and Z are not measured separately. Despite this limitation, it has been used extensively in prospecting and for preliminary magnetic surveying.

The modern dip needles commonly employed for reconnaissance surveys are more accurate than the Swedish mining compass.

Modern Dip Needles.—The modern dip needle comprises an elongated magnet suspended at its geometric center by fixed bearings in such fashion that it is free to rotate in a single plane.‡ One end of the elongated magnet is provided with a counterweight. Thus the point of suspension of the dip needle does not coincide with its center of gravity, and this design permits the dip needle to be utilized for relative intensity measurements.* If it is assumed that the instrument is orientated in the plane of the magnetic meridian, the needle will be acted on by two couples: viz., a couple produced by the vertical component of the earth's field and a couple produced by the force of gravity acting at the counterweight. The needle, therefore, rotates about a horizontal axis until the torque due

† Eugene Haanel, *On the Location and Examination of Magnetic Ore Deposits by Magnetometric Measurements* (Ottawa, 1904), pp. 65-66.

‡ N. H. Stearn, "The Dip Needle as a Geological Instrument," *A.I.M.E. Geophysical Prospecting*, 1929, pp. 345-363.

* If the suspension point coincided with the center of gravity, the instrument would be a *dip circle*, i.e., a device for measuring the *inclination* of the earth's field.

to gravity just balances the torque produced by the vertical intensity. (Compare also p. 86.)

The dip needle does not give the absolute value of the vertical intensity for several reasons, the most important of these being that the strength of the magnetic poles of the elongated magnet, and hence the effective magnetic torque, varies with the position of the magnet.† The sensitivity of the dip needle depends on several factors: such as the magnetization of the needle, the amount of friction in the bearings, and the adjustment of the normal position of rest with respect to the direction of the earth's field.

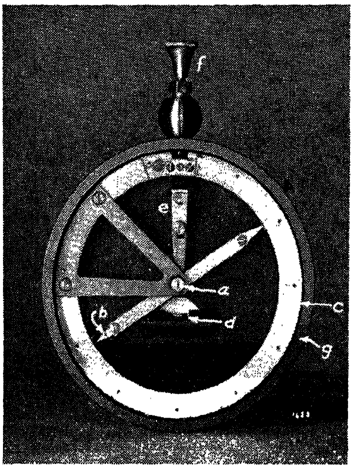

FIG. 15. — Dip Needle — Lake Superior Model; *a*, bearing support screw; *b*, counterweight; *c*, graduated arc; *d*, level; *e*, release finger; *f*, release button; *g*, non-magnetic case. (Courtesy of W. and L. E. Gurley.)

A photograph of a dip needle of recent design is shown in Figure 15.‡

The arc is graduated in degrees through practically three quadrants. The horizontal position is established by means of a spirit level. The latter is located just under the needle support; hence, it is convenient to check the position of the bubble while noting the swing of the needle.

The needle is made of an alloy of magnetic steel and is supported by a sturdy triangular mount in which sapphire jeweled bearings are carefully aligned and centered. A "permanent" or stable balance of the needle is secured by brass plugs which screw into the needle near each point. Changing the balance is accomplished by filing both plugs. The needle release assembly or arresting device is designed to provide for instant and uniform release of the needle.

The counterweight is a brass rivet mounted at one of the tapered ends of the needle. (The instrument can be made to serve as a dip circle, i.e., an inclination-measuring device, by removing the counterweight.)

The sensitiveness of the instrument usually is sufficient to permit checking quarter-degree readings. The counterweight is made of a non-magnetic material and is mounted so that its distance from the pivot axis may be varied.

All ordinary dip needles are relatively insensitive except for large anomalies. Stearn estimates that a one per cent variation in the intensity of the earth's field produces a deflection of the dip needle of from 1°

† Compare also C. O. Swanson, "The Dip Needle as a Magnetometer," *Geophysics*, Vol. 1, Jan. 1936, pp. 55 to 63.
‡ W. and L. E. Gurley, Bulletin No. 300-B (Troy, New York).

to 2°; hence, in regions where the average value of the earth's field is 0.6 gauss, the sensitivity of the dip needle is 1° to 2° per 600 gammas.

Dip needles have been applied successfully in the following problems: † location of faults in magnetic formations, mapping of the areal distribution of lavas and intrusives, tracing igneous contacts, dikes, ore lodes, etc.

Deflection Instruments

In general, deflection magnetometers are designed to measure the horizontal component of the earth's field. Some deflection instruments, notably the Thalen-Tiberg magnetometer, may be used to measure both the horizontal and the vertical intensities.

Kohlrausch Magnetometer.—The Kohlrausch magnetometer consists essentially of a circular, glass-covered case which houses a pivoted

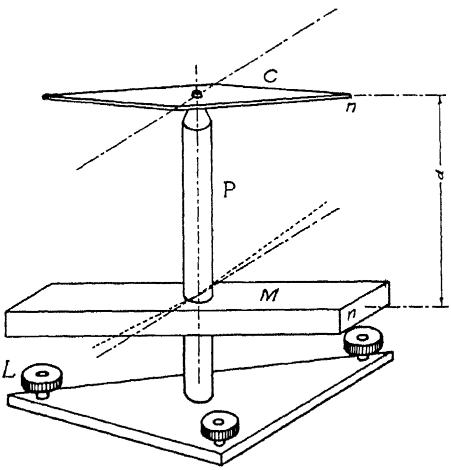

FIG. 16.—Kohlrausch magnetometer. *C*, pivoted magnetic needle; *P*, supporting pillar; *M*, bar magnet; *L*, leveling screws; *d*, sensitivity adjustment.

horizontal swinging magnetic needle. ‡ The case is supported by a vertical rod which is coaxial with the pivot of the swinging needle. (Figure 16.) An auxiliary bar magnet is also supported by the vertical rod. The distance *d* between the bar magnet and the needle is adjustable, thereby controlling the sensitivity. The bar magnet may be rotated either clockwise or counterclockwise, the rotation to either side being limited by stops. The vertical rod is supported on a three-legged base which is provided with screws and a small bubble level so that the rod can be orientated in a vertical position.

The magnetometer is first set up at a base station with the needle and magnet orientated in the direction of the magnetic meridian. The bar

† Stearn, *loc. cit.*, p. 358.
‡ A. S. Eve and D. A. Keys, *Applied Geophysics* (Cambr. Univ. Press, 1938), pp. 31-35.

magnet is rotated until the compass needle is deflected 90° and one of the adjustable stops is clamped at this position to provide a reference mark. Next, the bar magnet is rotated in the opposite direction until the compass needle is again deflected through 90°, and the second adjustable stop is clamped at this position. The supplement θ of the mean value of the angle of rotation of the magnet is computed. The horizontal component H_0 of the earth's field at the base station and the horizontal component F of the field produced by the bar magnet are related by the equation

$$H_0 = F \cos \theta \tag{15}$$

The instrument is now moved to a new (field) station and leveled as before. If the horizontal intensity H at the field station is less than H_0 a rotation of the bar magnet to its stop will deflect the compass needle through an angle of magnitude $90° + \phi$. The components of H and F perpendicular to the plane of the resultant field are equal; that is,

$$H \cos \phi = F \cos (\theta + \phi) \tag{16}$$

On replacing F by $H_0/\cos \theta$ (Equation 15),

$$H = H_0(1 - \tan \theta \tan \phi) \tag{17}$$

Thus, if H_0 and θ are known for the base station, the relative variation in the horizontal intensity can be calculated by observing ϕ.

The angle ϕ in an ore-free district will vary only by one or two degrees; the variations over a magnetite deposit, however, may be considerable.

Electromagnetic Deflection Magnetometer.—A deflection magnetometer of an improved type is shown in Figure 17.† The principle of operation is similar to that of the Kohlrausch magnetometer except that the magnetic field utilized for deflecting the magnetic needle is created by current flowing in a coil. A coil of insulated copper wire is placed so that its plane is vertical and passes through the axis of rotation of the needle. The instrument is oriented so the coil lies in the magnetic meridian. Current from two dry cells in the case of the instrument is then passed through the coil and the current recorded to cause a given deflection (usually 60 degrees) of the magnetic system. The current is then reversed and the milliamperes recorded to give an equal deflection in the opposite direction. From an average of the two current readings and the constants of the coil, the field strength at each station may be calculated. The sensitivity of the instrument is controlled by a compound multipoint switch connected to different taps on the coil and shunts on the potentiometer.

For many mining and other investigations where large anomalies are obtained, the magnetic system is the conventional Brunton compass. More accurate work is done by removing the Brunton compass and substituting a fine ribbon-suspended magnetic system, with optical magnification. With the Brunton compass, a sensitivity of 150 gammas per scale division of

† See also J. J. Jakosky, "The Application of Geophysics to Mining," *The Mining Journal*, April 15, 1930.

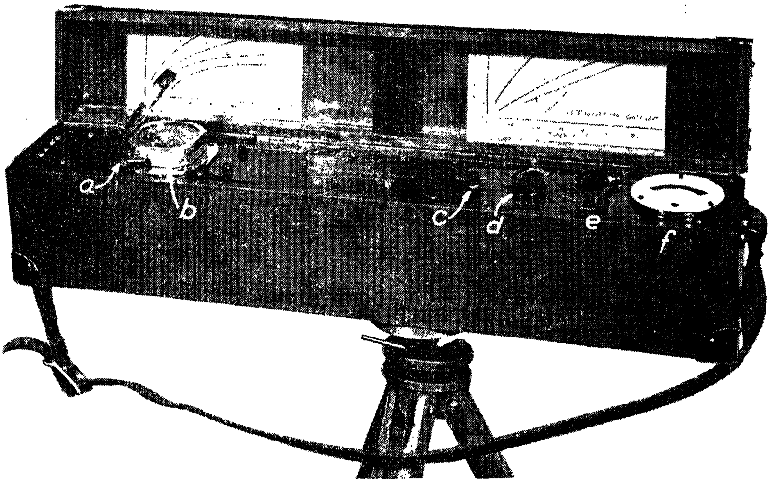

FIG. 17.—Electromagnetic deflection magnetometer. *a*, mounting for Brunton or torsion magnetometer; *b*, Brunton compass; *c*, reversing switch; *d*, current control rheostat; *e*, range switch controlling taps or coil and shunts on potentiometer; *f*, shielded galvanometer. (Courtesy International Geophysics, Inc.)

the milliammeter is obtainable when using a small magnifier for observing the compass. With the suspended magnetic system a sensitivity of about five gammas per scale division may be obtained; the usual working range is about ten to thirty gammas per scale division.

Thalen-Tiberg Magnetometer.—This instrument may be used to measure the horizontal intensity and the vertical intensity. In the former case, it functions as a deflection magnetometer and in the latter case as a magnetic balance. Essentially, the instrument consists of a magnetometer which is rotatable about a horizontal axis † and may be placed either in a horizontal or vertical plane. The magnetic needle system comprises a magnetized needle which carries a sliding weight and is pivoted at its center between two hardened steel bearings to permit free rotational movement. The needle can be adjusted to balance horizontally by altering the position of the sliding weight. A deflecting magnet of known magnetic moment is supported at any desired distance from the pivot by a fixed graduated horizontal arm. The instrument is supported on a light tripod, and means are provided for accurate leveling. The sensitivity of the instrument can be varied by altering the position of the center of gravity of the magnet system with respect to the axis of suspension.

The declination can be measured by orienting the instrument in the geographical meridian. For field intensity measurements, the operation of the instrument is essentially as follows. The deflecting magnet is attached to the horizontal arm; this arm is placed at right angles to the magnetic meridian and the magnetic needle is re-

† The description given here follows Broughton Edge and Laby, *Geophysical Prospecting* (Cambr. Univ. Press, 1931), p. 182.

leased. The needle deflects to an azimuth θ where the turning moment due to the magnet counterbalances that due to the earth's field. The deflection θ, the magnetic moment M_d of the deflecting magnet, and the distance r to the pivot can all be measured; hence, the horizontal component H of the earth's field can be calculated from the tangent formula, $H = 2M_d/r^3 \tan\theta$ (approx.). The horizontal intensity can also be calculated from the sine formula, $H = 2M_d/r^3 \sin\theta$ (approx.). The procedure in this case is to rotate the instrument about its vertical axis until the deflecting magnet and magnetized needle are mutually perpendicular and to observe the deflection θ of the needle as it returns to the meridian when the deflecting magnet is removed. Both the tangent and the sine methods yield the *total* horizontal field. The value of the magnetic *anomaly* is obtained by subtracting the normal field at the chosen base station from the field measured.

To measure anomalies in vertical intensity, the magnetometer is placed ertical plane at right angles to the magnetic meridian and is balanced so that it is orizontal in the normal earth's field. If the instrument is then set up at successive [illegible] of the angles made by the magnetized [illegible] to the vertical intensity [illegible] at these stations.

The Thalen-Tiberg magnetometer is most useful in regions where the vertical intensity anomalies exceed 20 gammas.

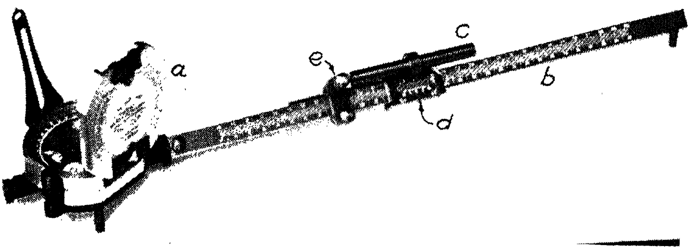

FIG. 18.—Wilson magnetometer attachment. *a*, Brunton compass; *b*, graduated arm; *c*, calibrated magnet; *d*, vernier scale; *e*, vernier adjustment knob with rack and pinion. (Courtesy of Wm. Ainsworth and Sons.)

Wilson Magnetometer Attachment; Dahlblom Pocket Magnetometer.—Various other deflection instruments, e.g., the Wilson magnetometer attachment for the Brunton pocket transit and the Dahlblom pocket magnetometer have been used in field investigations.

The Wilson magnetometer attachment (Figure 18) comprises an auxiliary attachable arm which is graduated in millimeter divisions and carries a magnet holder with a vernier. The magnet holder may be moved along the auxiliary arm by means of an adjustable rack and pinion device.†

The theory of the instrument depends on the deflection principles developed on page 68. That is, the needle rotates under the action of two torques, due to the earth's field and the deflecting magnet, respectively, and comes

† Wm. Ainsworth and Sons, Denver, Colorado, *The Wilson Magnetometer Attachment for Use with the Brunton Pocket Transit.*

to rest at an orientation for which these couples are equal and opposite. By an application of the principles already developed, it may be proved that when the needle is at rest in the 30 degree position, the horizontal intensity at that location is given by the expression $H = 4M/r^3$ (approx.) where M is the magnetic moment of the deflecting magnet and r the distance read on the auxiliary arm, i.e., the distance between the center of the needle and the center of the magnet.

The operation of the instrument consists in moving the magnet carrier (with magnet in place) by means of the rack and pinion device until the north end of the Brunton needle is exactly at the 30 degree mark and then reading the distance r on the auxiliary magnet arm with the aid of the vernier. (This value of r is then substituted into the relation $H = 4M/r^3$.)

The advantages of the instrument are: its cheapness, portability, and the fact that it may be attached to the Brunton compass which is a part of the equipment of nearly every geologist and mining engineer. The instrument gives satisfactory results in the class of work for which it is designed: viz., anomalies of 200 gammas or more.

In another type of magnetometer based on the compass principle, the opposing torque is supplied by the torsion of a helical spring instead of the field due to an auxiliary magnet. One of the earliest instruments of this type, the Dahlblom pocket magnetometer, comprises a compass mounted in a circular case, the latter being free to rotate about a horizontal axis. Because the magnetic needle system can swing in both a horizontal and a vertical plane, the magnetometer may be used to measure either the horizontal or the vertical intensity.† A magnetometer which is similar to the Dahlblom instrument, except that it measures the vertical intensity only, has also been proposed.‡

Magnetic Torsion Balances

A magnetic torsion balance is a modified gravitational torsion balance in which the suspended weight is replaced by a magnet. The first torsion balance for measuring magnetic and gravity *gradients* was developed by Eötvös.§ Later Berroth†† developed an instrument for determining vertical intensity gradients. A disadvantage of instruments of this type is that to obtain accurate results it is necessary to determine magnetic and gravity gradients separately. This is accomplished in Berroth's torsion balance by employing a double balance; one beam is constructed with a magnetic system and is used for determining the combined magnetic and gravity effect and the other beam (non-magnetic mass) is used for measuring the gravity effect alone.

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† E. Haanel, *On the Location and Examination of Magnetic Ore Deposits by Magnetometric Measurements* (Ottawa, 1904), pp. 99-106.

‡ C. A. Heiland, *Physics*, Vol. 3, pp. 18-22, July, 1932.

§ R. von Eötvös, *Ann. der Physik und Chemie* (new series), Vol. 59, 1896, pp. 373-383.

†† Berroth, *Zeit. für Geophysik*, 9, 1933, pp. 355-368.

Magnetic Field Balances

The Hotchkiss Superdip.—The Hotchkiss superdip magnetometer combines in one instrument the functions of a dip needle and a magnetometer. Essentially, the superdip is an improved dip needle with adjustable sensitivity limited only by friction and other mechanical difficulties. The operating principles may best be explained by comparing the ordinary dip needle with the Hotchkiss Superdip. †

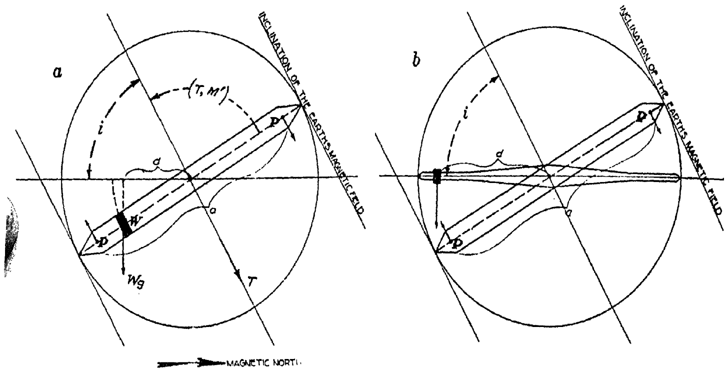


FIG. 19.—Sketches illustrating working principles of the ordinary dip needle (*a*) and the Superdip (*b*). (After Stearn, *A.I.M.E. Geophysical Prospecting*, 1932.)

The action of the ordinary dip needle in measuring the intensity of the earth's field derives from the effect of the counterweight with which the needle is provided. (Figure 19.) When the instrument is oriented in the plane of the magnetic meridian, the counterweight serves to balance the magnetized needle at some fixed angle to the direction of the earth's resultant field. That is, the magnet is acted on by two opposing couples and comes to rest in an azimuth where the couples are equal in magnitude. The magnetic couple (equal to the product of the magnetic moment M' of the needle times the sine of the angle between M' and T) tends to rotate the needle in the direction of the earth's field, while the gravitational couple (equal to the product of the weight of the counterweight times its perpendicular distance from the center of gravity of the needle) opposes this rotation.

The equilibrium condition for a dip needle is given by the equation:

$$Wgd = TM' \sin (T,M') \tag{18}$$

or

sin

† N. H. Stearn, "Practical Geomagnetic Exploration with the Hotchkiss Superdip," *A.I.M.E. Geophysical Prospecting* (1932), pp. 169-197.

where

Wgd = couple due to the counterweight.

T = total earth's field.

M' = magnetic moment of needle.

(T,M') = angle between T and M'.

The characteristic feature of the Hotchkiss Superdip is that d and $\sin (T, M')$ are made to vary proportionately in such manner that d is a maximum when $\sin (T, M')$ is 1. This could be achieved in an ordinary needle only for the unusual case of inclination equal to 90°. It is accom-

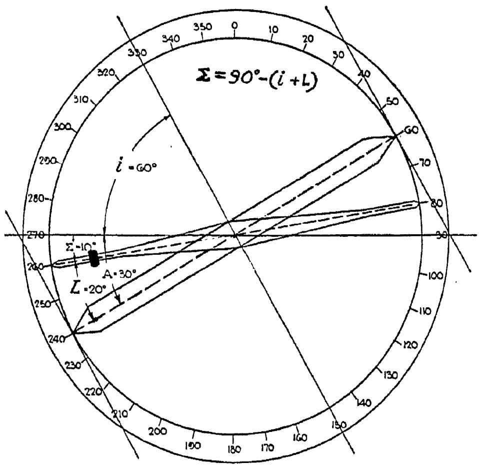


FIG. 20.—Setting of Hotchkiss Superdip. (After Stearn, *A.I.M.E. Geophysical Prospecting*, 1932.)

plished in the Hotchkiss Superdip by an ingenious arrangement employing a duplex needle assembly which consists of an adjustable counterarm (provided with a counterweight) frictionally pivoted with the magnetized needle. The theoretical sensitivity of the Superdip is determined almost entirely by the angle between the counterarm and the needle. (Figure 20.)

In practice, the limiting angle is established by the inclination of the earth's field. For example, if the inclination i of the earth's field is 60° the limiting angle A between the counterarm and the magnet would be 30° for maximum sensitivity, because in this position the counterarm is horizontal when the magnet is at right angles to the earth's field. Practically, however, maximum sensitivity is seldom desirable in field work, and the angle L which is actually set off is smaller than the limiting angle. The difference is called the sensitivity or Σ angle ($\Sigma = 90° - i - L$) and is adjustable to any desired degree of sensitivity.

A photographic view of the Hotchkiss Superdip is shown in Figure 21. The dust-tight cylindrical case is made of brass or aluminum and is mounted vertically. It is about 6 inches in diameter and 1½ inches thick and has a removable glass face. The swinging assembly which is mounted in the case consists of the magnet, counterarm, counterweight, and pivot, as well as a release device, scale circle, thermometer, and a mechanical adjustment finger. The instrument is mounted on a sturdy tripod which is equipped with quadrature leveling screws, two levels (at right angles), and vernier azimuth adjustment.

The magnetic system is a thin bar magnet made of tungsten-cobalt steel. The moving system swings in a vertical plane and is mounted on a pivot. The pivot is located at the center of gravity of the magnet and rolls on a pair of level jeweled knife-edges. The counterarm, which is attached to the same pivot, may be adjusted to any desired angular position with the axis of the magnet. The counterweight is placed on one end of the counterarm and is adjusted by varying its distance from the pivot.

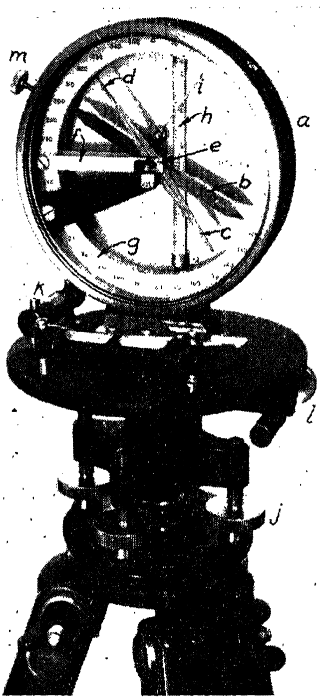


FIG. 21.—The Hotchkiss Superdip Magnetometer. *a*, non-magnetic case; *b*, magnet; *c*, counterarm; *d*, counterweight; *e*, pivot; *f*, release; *g*, scale circle; *h*, thermometer; *i*, adjustment finger; *j*, quadrature leveling screws; *k*, levels; *l*, azimuth adjustment; *m*, release screw.

The release device comprises two levers operated by a thumb screw, and is so designed that the swinging assembly may be raised from the horizontal knife-edges and clamped against two centering pivot guides. The scale circle is graduated in degrees of arc so that the position of the magnet can be measured. The thermometer is mounted in the case to provide data for temperature corrections. A light wire arm, operated from the back of the case, moves the swinging assembly to its zero position. Orientation of the instrument in the field of the magnetic meridian is accomplished by means of an auxiliary compass which is placed on the tripod table. After proper orientation, the compass is removed and the Superdip fastened to the tripod table.

Figure 22 is a graph of the practical working sensitivity of a typical superdip. The curve marked $\Sigma = 0°$ is of interest. Theoretically it should coincide with the zero abscissa; its departure therefrom indicates the amount of the departure from infinite sensitivity introduced by the mechanical

features of the instrument, including friction. Although the sensitivity curves for the various values of Σ are curved rather than straight, a simple linear coefficient of sensitivity is sufficiently accurate—e.g., 12½ gammas per scale division for a Σ of 1°.

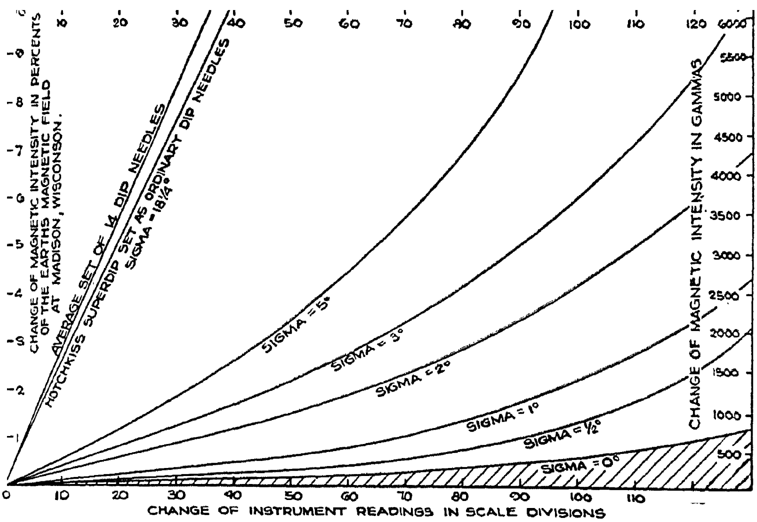


FIG. 22.—Sensitivity of Hotchkiss Superdip. (Stearn, *A.I.M.E. Geophysical Prospecting*, 1932.)

Preliminary Adjustment of Superdip Magnetometer in the Field

(1) The instrument is placed in the magnetic meridian by means of the orientation compass.

(2) The counterarm is set parallel to the magnet, and the angle of inclination for the area is measured by using a small auxiliary scale.

(3) The counterarm is set at the desired sensitivity (sigma value).

(4) The counterweight is adjusted until the position of rest of the assembly is approximately at right angles to the magnetic inclination.

Each station reading of a Hotchkiss Superdip magnetometer involves the following steps:

(1) Set up the tripod and level the mounting plate.

(2) Place the compass on the mounting plate. Orient the plate in its proper relation to the earth's magnetic meridian, and clamp it.

(3) Remove the compass to a safe distance (20 feet or more) and mount the magnetometer.

(4) Move the north pole of the magnet to the zero position on the circular scale.

(5) Carefully lower the swinging assembly to the agate edges and note the end of the swing. (The reading taken is the position of the north pole of the magnet relative to the circular scale at the end of the swing.)

(6) Clamp the swinging assembly and record the reading, temperature, time, and location of the station.

Electromagnetometer.—This type of instrument is a modification of the earth-inductor type and is gaining in favor for the field technique where both the horizontal and vertical components are measured. In this design the only moving part is a metal, non-magnetic ring inductively coupled to the balance indicating circuit. (Figures 23 and 24.) The three chief advantages of the design are: (1) almost any desired sensitivity

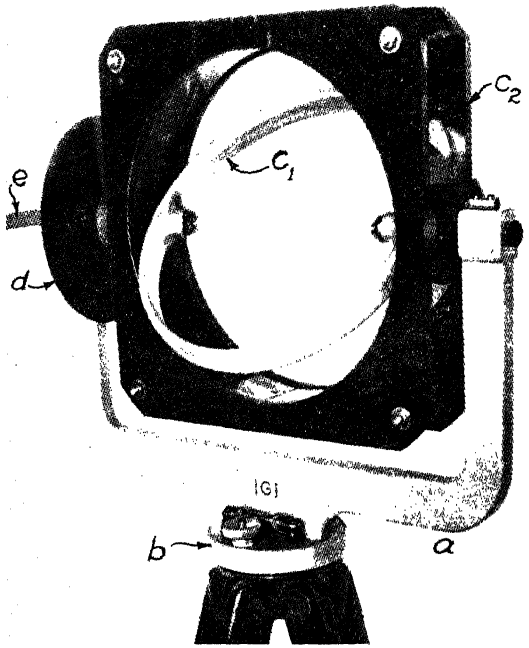


FIG. 23.—Improved electromagnetometer for measuring any desired component of the magnetic field. *a*, non-magnetic support; *b*, tripod head and orientation plate; c_2, neutralizing coil; c_1, nc magnetic rotor coil; *d*, dip circle and vernier; *e*, flexible remc drive shaft. The neutralizing coil is positioned vertically wh measuring the horizontal component and horizontally when meas ing the vertical component. Detecting coil not shown. (Courte of International Geophysics, Inc.)

may be obtained by use of standard potentiometers or other measuring instruments; (2) any desired component of the earth's field may be measured by proper orientation of the neutralizing coil; (3) the instrument has a rugged design and may be constructed to be practically independent of temperature variations; and (4) the instrument is ideal for rapid field use as both the horizontal and vertical components may be readily measured. The balance utilizes a neutralizing coil C_2 to provide a magnetic field for opposing the component of the earth's field which it

is desired to measure. At right angles to the neutralizing coil is a detecting coil. A four-pole double-throw switch allows the right angle components to be measured rapidly.

A single turn metal ring or coil C_1 rotates in the differential field. Unless the earth's field is exactly neutralized, a current will be induced in the rotating ring and this current will induce by transformer action an alternating current flow in the detecting coil circuit. By means of transformer T_1, this alternating component is fed to a one or two stage amplifier. The output of the amplifier is passed through a rectifier, and then to a galvanometer for accurate detection of the null point.

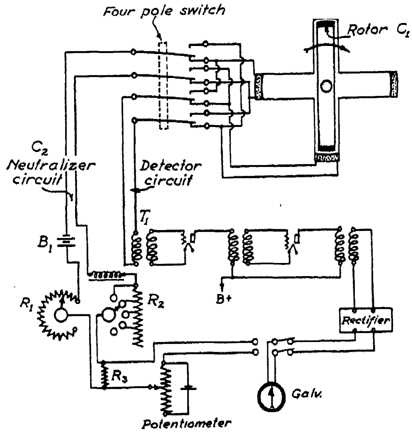


FIG. 24.—Wiring diagram for electromagnetometer. (Courtesy of International Geophysics, Inc.)

The current for neutralization is measured by determining the potential drop across a standard resistor R_3. By means of a double-pole double-throw key switch, the same galvanometer may be utilized in the potentiometer and the rectified A.C. circuit.

The method of measurement consists in orienting the neutralizing coil at right angles to the component of the earth's field to be measured, and then adjusting the current by resistors R_1 and R_2 until that component is exactly neutralized. This condition will obtain only when the induced current is zero. Hence, the earth's field strength may be calculated from the value of the current required for neutralization and the known constants of the instrument.

The field measurements utilize the following steps:

(1) Orientation of the axis of rotation of the rotor at right angles to the magnetic meridian (by compass or minimum signal).

(2) Rotation of the ring C_1 at any frequency, which need not be constant, by turning the gear box crank.

(3) Adjustment of the neutralizing current by means of rheostats R_1 for fine, and R_2 for coarse adjustment until no deflection is obtained in the galvanometer. Rotation of the ring may now be stopped.

(4) The galvanometer is now switched to the potentiometer circuit and the voltage drop across R_3 measured.

(5) Strength of the earth's field component may be computed as a function of the current and the constants of the coil.

Schmidt Magnetic Field Balance.—Two types of the Schmidt field balance are available; one measures the vertical component of the earth's field and the other the horizontal component. In both instruments the mass distribution of the balance system is so arranged that the force exerted by gravity holds the magnetic system when properly oriented for use in such a position that it is acted upon chiefly by the desired component (vertical or horizontal) which is to be measured.†

The operating principle of the vertical magnetometer, i.e., the instrument which measures the vertical component of the earth's field, is indicated schematically in Figure 25. A magnetic needle provided with counterweights, W_1, W_2, W_3, is mounted so as to be free to rotate about an axis A perpendicular to the magnetic meridian. The needle is acted on by two opposing torques: namely, a torque due to the force of gravity and a torque due to the vertical component of the earth's magnetic field. Because the horizontal component of the earth's field passes through the axis of rotation A, the horizontal component will exert a zero torque on the system, while the vertical magnetic component exerts its maximum torque. The instrument therefore mechanically differentiates between the horizontal component and the vertical component.

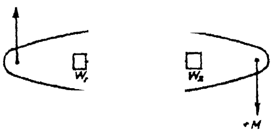


FIG. 25.—Diagram illustrating method of balancing torque due to vertical intensity against torque due to gravity.

The operating principle of the horizontal magnetometer is illustrated schematically in Figure 26. The magnetic system is mounted in a vertical position with its longitudinal geometric axis parallel to the magnetic meridian so that the needle is affected principally by the horizontal component of the earth's field. The application of the instrument for relative determinations of the horizontal intensity is discussed by Joyce.‡

An interior view of a horizontal component field balance is given in Figure 27.

Because both instruments are essentially similar only one, viz., the vertical balance, will be described in detail. The vertical balance is chosen because of its generally greater applicability, especially in the field of oil-structure and mining work. The description given here is limited to the fundamental principles of operation and a brief review of field manipulation. For more detailed instructions, the reader is referred to the manufacturers or to the excellent manual of Joyce.

FIG. 26. — Diagram illustrating method of balancing torque due to horizontal intensity against torque due to gravity.

† J. Wallace Joyce, *Manual on Geophysical Prospecting with the Magnetometer*, U. S. Bureau of Mines Publication (Printed by the American Askania Corporation, Houston, Texas, 1937).

‡ J. Wallace Joyce, *loc. cit.*

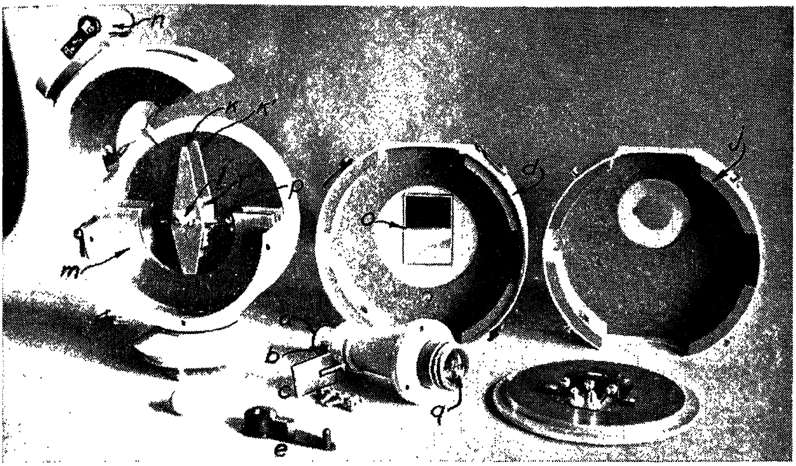


FIG. 27.—Interior view of Askania horizontal component field balance. *a*, gauss eyepiece; *b*, light window; *c*, mirror; *d*, instrument case; *e*, clamp lever; [illegible], cork thermal insulation; *k* and *k'*, magnetic system; *l*, quartz knife-edge; *m*, copper damping plates; *n*, level bubbles; [illegible], door in case with mirror attached for reading thermometers; *p*, counterbalances; *q*, objective lens; *r*, mirror.

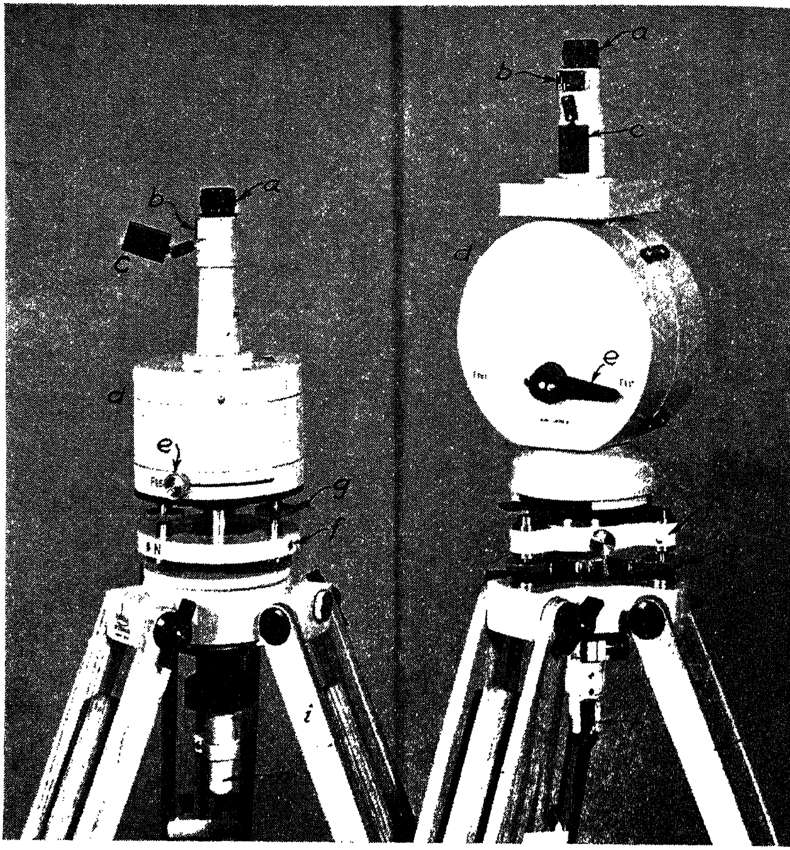
Exterior views of late models of the Schmidt-Askania type magnetometers are shown in Figure 28.

The magnet system consists of two gold-plated, magnetized bars of tungsten or cobalt steel fastened together by a cube-shaped, structural member made of aluminum.† A view of two types of vertical and horizontal component magnetic systems is shown in Figure 29. A quartz knife-edge is fastened to the block at right angles to the long axes of the magnets. The quartz knife-edge rests on two semi-cylindrical quartz bearings and supports the moving system. An arresting system is provided to lift the quartz knife-edges from the cylindrical quartz bearings while the instrument is being moved. This arrangement prevents "chipping" or other injury to the edges and also provides a means for resetting the system in the same place on the bearings. For transportation of the instrument, the magnet system must be clamped in the arrested position. Oscillations of the magnet system are dampened by eddy currents induced in copper plates (removable) placed near the poles of the magnets.

In order to provide for temperature corrections, two thermometers (having ranges of $-15°$ to $24°$ C. and $17°$ to $55°$ C.) are fastened inside the case. The magnet system and thermometers are enclosed in a cork-lined, aluminum casing to minimize rapid temperature changes.

Small displacements of the magnet system from the horizontal are directly proportional to the vertical intensity. They are measured by means of a telescope with a gauss eyepiece using the optical arrangement shown

† Erwin Roux, "Magnetic Balance for the Measurement of Intensities," U. S. Patent 1,976,636, Oct. 9, 1934.



A B

FIG. 28.— A, standard Askania magnetometer for measuring vertical or horizontal magnetic intensity; B, simplified Askania vertical magnetometer. *a*, gauss eyepiece; *b*, light window; *c*, mirror; *d*, instrument case; *e*, clamp lever; *f*, tripod head; *g*, leveling screws; *h*, auxiliary magnet holder; *i*, tripod legs.

in Figure 30. Sky light is reflected from a mirror, passes through a ground glass window in the side of the telescope and falls on a reflecting plate which deflects it downwardly so that it illuminates a scale. The scale is etched on a transparent glass plate situated at the focus point of the objective lens. (The latter is located vertically above the mirror attached to the magnet system.) The light transmitted through the scale plate passes through the objective lens and is reflected back by the plane mirror fastened to the magnet system. Thus, two scales are seen superimposed in the eyepiece of the telescope; one is the direct and the other the reflected image. The difference in position of corresponding marks on these scales is a measure of the amount of the deflection of the magnet system.

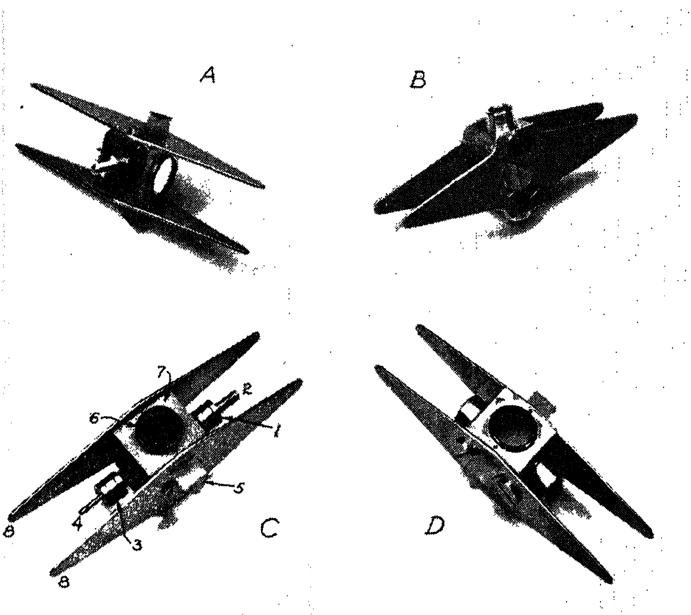


FIG. 29.—Magnetic systems of Askania magnetometers.
A—New-type, horizontal component, temperature-compensated system.
B—Uncompensated, horizontal component system.
C—New type vertical component, temperature-compensated system.
D—Uncompensated, vertical component system.
1—Temperature compensation weight.
2—Temperature compensation spindle (aluminum).
3—Latitude adjustment weight.
4—Latitude adjustment spindle (invar steel).
5—Quartz knife-edge.
6—Mirror.
7—Aluminum frame.
8—Magnets.

A vertical brass screw weight suspended on the under side of the magnet system affords a means of varying the sensitivity of the instrument. For ordinary subsurface structural investigations, the magnetic system of the vertical balance is usually adjusted for a sensitivity of about 10 to 30 gammas per scale division. The scale is divided into 120 divisions; hence, the intensity range of the instrument is approximately 1200 to 3600 gammas. In areas of large magnetic disturbances, the sensitivity is decreased, with a resultant increase in total range of the instrument.

As shown in Figure 31, the magnetometer is clamped on a special adjustable wooden tripod, the graduated rotational head of which is leveled by means of a center bubble and three leveling screws. Fastened to the under side of the tripod head is an extension tube holder for an auxiliary magnet. A thumb screw clamps the auxiliary magnet at any desired distance from the magnetometer. (The distance may be read directly on a graduated scale etched on the magnet holder.)

Accessories consist of a compass which can be fitted into the tripod head for determination of the magnetic meridian, auxiliary magnets of various strengths, and small tools for adjusting and cleaning the instrument. Canvas cases are provided for carrying the magnetometer and the tripod.

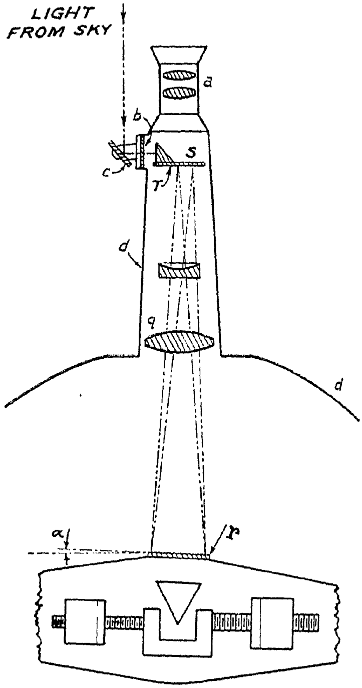
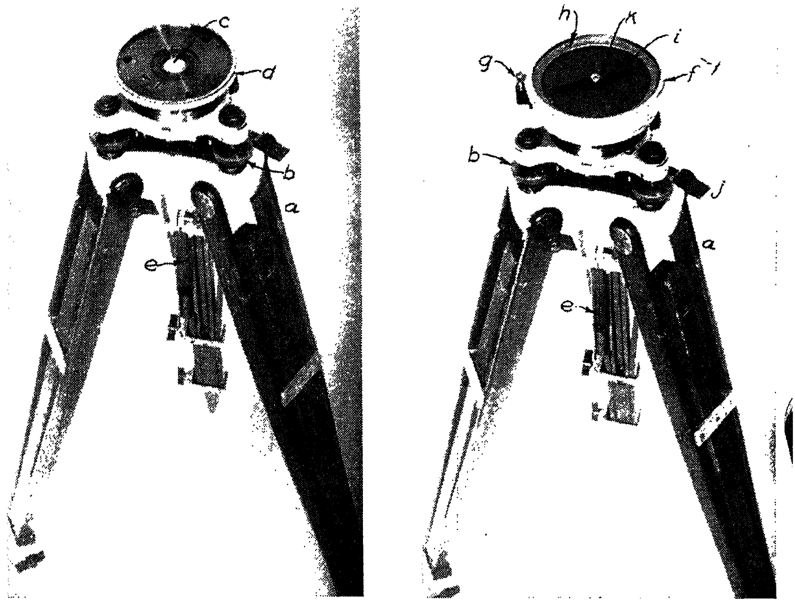


FIG. 30.—Optical system of the Schmidt field balance. *a*, gauss eyepiece; *b*, ground glass window; *c*, reflecting mirror; *d*, case; *q*, objective lens; *r*, mirror on magnet system; *s*, scale; *T*, transparent
α, angle of deflection.

Factors Affecting Magnetometer Readings

Orientation.—Orientation of the magnetometer must be such that the horizontal axis of rotation of the instrument is perpendicular to the magnetic meridian for the vertical magnetometer, or parallel to the magnetic meridian for the horizontal instrument. Orientation of the instrument is accomplished indirectly as one of the steps in the initial set-up of the tripod. The tripod head consists of a turntable, adjustable auxiliary magnet holder, turntable stop-ring, turntable bearing, and leveling screws. The tripod head is also provided with three lugs to which are fastened three wooden tripod legs. The top of the turntable (Figure 31A) contains guide holes so that either the compass or the magnetometer may be placed in the same relative position on the turntable at a series of stations. In



A B

FIG. 31.—Tripod leveling and orientation. *a*, tripod; *b*, level screws; *c*, level bubble; *d*, turntable; *e*, extension tube for auxiliary magnet; *f*, compass; *g*, compass needle clamp screw; *h*, graduated arc of 360°; *i*, compass needle; *j*, leg clamps; *k*, jewel support.

the center of the turntable is the level bubble used in preliminary leveling of the tripod head for initial orientation. The stop-ring is so adjusted that the turntable may be turned exactly 180° without reading the scale or compass, thereby facilitating check readings.

The turntable is oriented by placing the compass on it. (See Figure 31B.) Orientation of the instrument must be correct to within one-half degree of arc. This requires care and is one of the sources of errors militating against accuracy. Misalignment of the vertical instrument causes an error which is chiefly due to two factors: (a) A small portion of the horizontal magnetic component of the earth's field will act on the magnet system, and (b) the scale constant of the instrument will be altered. Misalignment of the horizontal instrument introduces a small portion of the vertical component which decreases the true reading. However, the sensitivity of the horizontal instrument is not affected by misalignment of a few degrees.

Leveling.—Initial leveling of the tripod head is accomplished by the level bubble. This leveling is sufficiently accurate for orientation of the turntable by the compass. After the orientation step is completed, the compass is removed from the turntable and the magnetometer is placed on it in the proper orientation for that station. The instrument is now

accurately leveled by adjusting the three leveling screws on the tripod head and observing the two levels fastened to the magnetometer itself. The leveling should be accurate to within a half-division of the level bubbles (15 seconds of arc); under these conditions, errors due to incorrect leveling will be less than 0.1 of a scale division. If the leveling is changed by one division, due to clamping, unclamping, or rotation of the turntable, the instrument should be releveled and the reading repeated.

One prevalent cause of shifting of the instrument is due to improper placing of the tripod when first set up at the station. The tripod legs should make an angle of about 30° with the vertical and should be firmly pressed into the ground when initially set up. Apparent shifting of the instrument may be due to unequal expansion of the level glasses and supports when in the direct rays of the sun. The operator should stand between the instrument and the sun or employ an umbrella. Instruments at base stations should be shielded from direct sunlight and protected from wind and rain by suitable shelter.

Temperature Effects.—The three chief effects on a magnetic system due to changes in temperature are: (a) changes in magnetic moment (decreasing moment with increasing temperatures); (b) unequal expansions of the component parts of the moving system with a resultant change in scale constant, variations in optical constants, and changes in gravitational moment accompanied by changes in scale value; (c) thermal and elastic lags of the component parts of the system.

It is the chief function of the heavy cork insulation of the case of the instrument to minimize temperature variations—particularly, rapid temperature fluctuations. Slow changes in temperature may be compensated by using a calibration curve which shows the correction for various changes in temperature. This curve is usually plotted with the temperature correction in gammas as ordinate and the change in temperature ($t—t_0$) in degrees Centigrade or Fahrenheit as abscissa. It is usually convenient to assume some mean temperature for the area under investigation and apply the appropriate correction factor for each reading. (As stated previously, thermometers are provided in the instrument so that the temperature may be read at the time of each magnetic reading.)

Certain instruments of recent design incorporate temperature compensation,† and the effects of moderate temperature variations are usually negligible with such equipment. In the temperature-compensated system, two threaded rods are employed; the rod on the north side of the axis is made of aluminum (see Figure 29) and carries the temperature-adjustment weight. The rod on the south side of the axis is made of invar steel and carries the latitude-adjustment weight. The thermal coefficient of the invar rod (carrying the latitude weight) is very small; hence, changes in the position of the latitude-adjustment weight with variations in tempera-

† Joyce, *loc. cit.*, p. 42.
C. A. Heiland and W. E. Pugh, "Theory and Experiments Concerning a New Compensated Magnetometer System," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 483, 1932.

ture are extremely slight. Furthermore, the slight change in position of the latitude weight is largely compensated by the change in the position of the center of gravity of the aluminum rod.

Temperature changes greatly affect the scale readings when employing auxiliary magnets. This is caused by the following factors: (a) change in magnetic moment of the auxiliary magnet and (b) change in length of the magnet support which is fastened to the base of the turntable. For this reason, accurate measurements in an area must never be attempted with auxiliary magnets. Oftentimes, auxiliary magnets are employed to bring the scale of the instrument within view. This procedure is permissible where large magnetic anomalies are to be measured. In regions of small anomalies, the instrument must be opened and proper adjustment made by rotating the latitude-adjustment weight until a suitable position of the scale is observed.

Latitude and Longitude Corrections

As stated previously, the normal values of the components of the earth's magnetic field vary over the surface of the earth. The vertical component of the earth's field varies from approximately *minus* 67,400 gammas at the south magnetic pole through zero at the magnetic equator to about *plus* 63,500 gammas at the north magnetic pole. The horizontal component has a maximum value of about 39,000 gammas at the equator and decreases to zero at each of the two poles. This variation of the magnetic field strength is usually unimportant where studies are confined to traverses of only a few miles length. In larger surveys, or when tying-in one survey with another at a different location, latitude corrections average from 10 to 12 gammas per mile for the vertical component and from 5 to 8 gammas per mile for the horizontal component; the longitude corrections average 2 to 3 gammas per mile.

Due to the irregular distribution of the isodynamic lines along the earth's surface, the earth's normal field strength cannot be calculated theoretically from the latitude and longitude position, but must be obtained from maps of isodynamic lines.

Latitude Corrections.—Calling d the distance measured perpendicular to the geographical latitude between two successive isodynamic lines differing in intensity by ΔZ or by ΔH, as the case may be, the corrections are: vertical component $= \Delta\gamma_z = \Delta Z/d$; horizontal component $= \Delta\gamma_h = \Delta H/d$. The vertical correction is to be subtracted from readings north of the base station in areas north of the magnetic equator or south of the base station in areas south of the magnetic equator. (The rule regarding signs derives from the fact that the magnetic intensity increases positively toward the north magnetic pole and negatively toward the south magnetic pole.) The horizontal correction is added to readings north of the base station in areas north of the magnetic equator or south of the base station in areas south of the magnetic equator. (This sign rule obtains because the horizontal intensity decreases toward the two poles.)

Usually it is convenient to employ a latitude correction curve, wherein distances measured perpendicular to the geographical latitude are plotted as abscissa and the calculated correction from the base station as ordinate.

Longitude Corrections.—Vertical component $= \Delta\gamma_z = \Delta Z/d$; horizontal component $= \Delta\gamma_h = \Delta H/d$, where d is the distance measured perpendicular to the geographical longitude between two adjacent isodynamic lines differing in intensity by ΔZ or ΔH, as the case may be.

The longitude correction curves are plotted in the same manner as the latitude curves. The algebraic signs of the corrections depend on the magnetic declination in the area. For west declination areas, the corrections are added for all stations east of the base station. For east declination areas, the corrections are subtracted for all stations east of the base station. These corrections are generally quite small and often may be neglected in ordinary work. Latitude and longitude corrections can be combined into a single correction by using the magnetic north and south rather than the geographical north and south as the direction of measurement from the base station. Also, corrections for latitude and longitude are oftentimes included in the "regional gradient" in final interpretation. When the latter procedure is employed, the regional gradient is drawn in by inspection of the magnetic profiles, and anomalies are measured using the regional gradient as a reference line. This is illustrated in connection with Figure 68.

Preliminary Adjustment of Schmidt Field Balance

Latitude Adjustment.—This adjustment is accomplished by rotating the lateral screws until the zero point of the reflected scale is approximately at the middle of the scale. In regions where the magnetic gradient is small, no other adjustment will be required to keep the scale in the field of vision. However, in regions where igneous rocks outcrop at or near the surface (e.g., in many mining regions), the magnetic anomalies are often large and show rapid variations. Under these conditions, it is often necessary to use auxiliary magnets below the magnet system of the instrument to keep the scale in the field of vision. (The auxiliary magnets are so mounted that their effect on the magnetic system of the balance opposes that of the geologic anomaly.)

Conditions of the Knife-Edge.—The sensitivity of the instrument depends in large measure on the condition of the knife-edge. The operator must take every precaution, therefore, to protect the fragile quartz-edges on which the system pivots. The magnetic system must always be clamped, except during the time when a reading is being made. If the knife-edge is liberated with special care and always with a slow movement, it should last as long as the instrument.

Special precautions must be taken to protect the instrument from dust and lint when the case is open for adjusting the magnet system. The instrument should never be opened in windy or dusty places. A minute particle of grit between the knife-edge and the bearings will cause erratic readings and changes in sensitivity. To remove such foreign material, or oil films, the bearing surfaces should be wiped carefully, just prior to closing the instrument case, with a linen cloth dampened with pure ether or chloroform.

Determination of the Scale Value.—Before conducting a magnetic survey, it is desirable to determine the scale value of the instrument. The differential magnetic intensity per scale division, generally expressed in gammas per scale division, is obtained by observing the deflection of the magnetic system produced by a magnet of known

magnetic moment placed in one of two positions and applying the required mathematical formulas involving the magnetic constants and the distances. (Compare pp. 69 and 70.)

Instead of using a magnet of known magnetic moment, the scale value may also be determined with the aid of a Helmholtz coil arrangement. This device consists of two identical coaxial coils of equal radius placed at a distance apart approximately equal to this radius. It may be shown† that the current in the coils produces a magnetic field along the common axis which is substantially constant over a considerable distance near the midpoint of the axis, because the decrease in field intensity of one coil is offset by the increase in field intensity of the other coil. The field intensity is a function of the current and the characteristics of the coils, and may be expressed by the relationship

$$F = IC = I$$

where

F = field intensity, in gammas

C = coil constant

D = diameter of coils, centimeters

I = current, in milliamperes

n = number of turns in each coil

E = mean distance between coils, centimeters

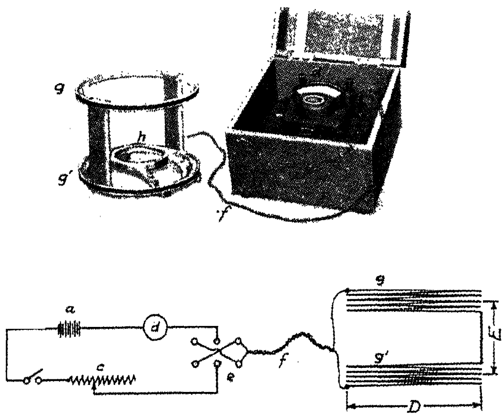


Fig. 32.—Helmholtz coil and diagram of connections. *a*, battery; *b*, switch; *c*, rheostat; *d*, milliammeter; *e*, reversing switch; *f*, twisted flexible connecting cord; *g* and *g'*, identical coils; *h*, coil holder bracket.

The diagram of connections is shown in Figure 32. Usually the coil constant C is supplied by the manufacturers. The coils are approximately 35 centimeters in diameter,

† See for example, S. G. Starling, *Electricity and Magnetism* (Longmans, Green and Co., 5th Ed.), p. 57.

J. W. Joyce, *loc. cit.*

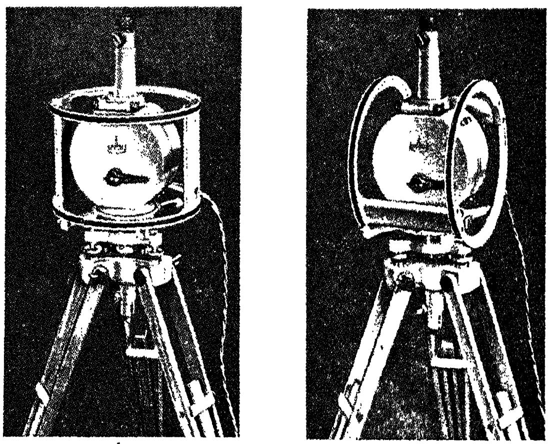
and are spaced approximately 20 centimeters apart. A mounting bracket is fastened to the tripod turntable, and the coil system is placed on the bracket (Figure 33), with the coils in a horizontal position when calibrating a vertical component magnetometer and in a vertical position with their axes parallel to the magnet system for calibration of a horizontal component magnetometer. The scale reading of the magnetometer may be brought to any desired value either by using the latitude adjustment screw or the auxiliary magnet. If the auxiliary magnet is employed, caution must be taken to avoid appreciable temperature changes during the calibration.

When calibrating a vertical magnetometer it is carefully leveled and oriented with the coils in position (Figure 33A). The scale reading and temperature is recorded when no current is flowing through the coils. The switch is then closed and the scale reading recorded with the value of current flowing through the coils. The direction of current flow is then reversed and the scale reading again recorded with the value of current. The circuit is opened and the zero or initial position of the scale checked. The scale value of the magnetometer may now be calculated by the relationship:

$$S = \frac{CI}{(S_0 - S_1) + (S_2 - S_0)} \text{ or } S = \frac{CI}{S_2 - S_1}$$

where S = scale value in gammas at the temperature of calibration, S_0 = scale reading for zero current, S_1 and S_2 = scale reading with current in first one and then the other direction.

If the values of $(S_0 - S_1)$ and $(S_2 - S_0)$ differ by more than a few per cent, the condition of the knife-edges should be carefully checked. A series of observations should be made with different values of current, when the north pole of the magnetometer is pointing east and then west. Any variation in deflection will be shown by irregularities in the calibration curves.



A B

FIG. 33.—*A*, vertical component magnetometer, and *B*, horizontal component magnetometer with calibration coil in position. (Courtesy American Askania Corp.)

In calibration of a horizontal component magnetometer, the instrument is oriented in the magnetic meridian. As before, the values of initial and final scale readings, current, and temperature are recorded. The values are substituted in the preceding formulas and a series of tests conducted from which calibration curves may be drawn.

It will be apparent that the Helmholtz coils also may be used for determining the magnetic moment of the auxiliary magnet.

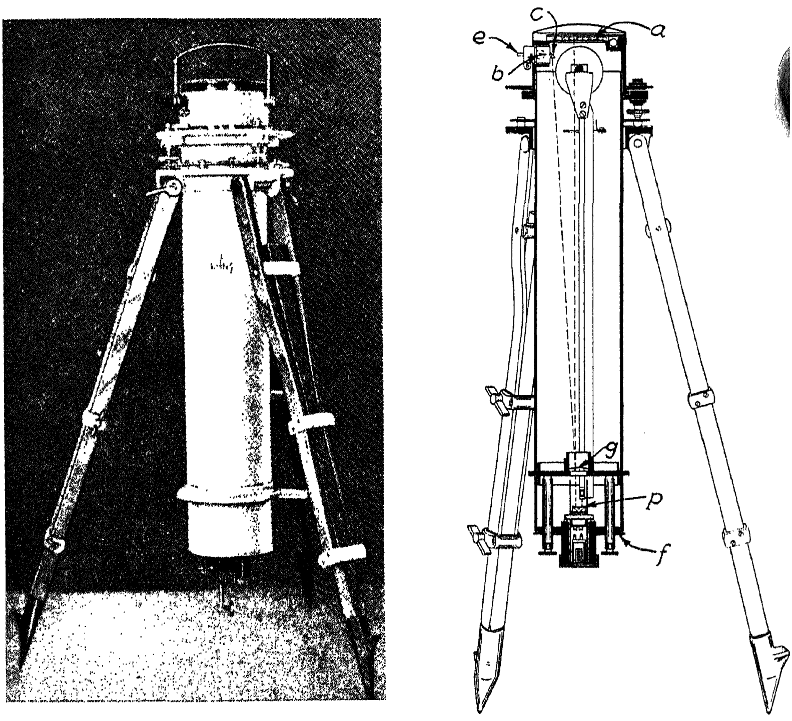
The optimum scale sensitivity depends on local conditions. However, for ordinary structural geological work, the sensitivity should be adjusted to about 10 to 30 gammas per scale division by moving the vertical screw under the magnet system. The nearer this mass is placed to the axis of rotation, the higher is the sensitivity of the instrument.

Determination of the Magnetic Moments of the Auxiliary Magnets.—The auxiliary magnets used in the field may be calibrated by means of the graduated tube magnet holder which extends from beneath the tripod head. A standard magnet with the moment M_{st} of the same length as the auxiliary magnet, is placed in the holder at an arbitrary distance and the deflection $(S_1—S_2)_{st}$ is observed. (S_1 is the reading when the north pole of the deflected system is in the north, and S_2 is the reading when it is in the south.)

After the standard magnet has been removed, the auxiliary magnet with the moment M_x is placed at the same distance and the deflection $(S_1—S_2)_x$ is noted. The moment of the auxiliary magnet is given by the expression

$$M_x = M_{st} \; (S_1 - S_2)_{st}$$

The magnetic moments of the auxiliary magnets may also be determined with a Helmholtz coil arrangement. This is conveniently accomplished by noting the scale deflection produced by the auxiliary magnet at a given distance and position from the magnetometer as compared to the deflection produced by a given current flow in the Helmholtz coils.



(A) (B)

FIG. —Askania continuous recording magnetometer. *A*, exterior view; *B*, interior view. *a*, plate holder; *b*, flash-light bulb; *c*, prism; *e*, clock drive; *f*, support; *g*, lens; *p*, prism.

Continuous Recording Magnetometers.—Practically all types of continuous recorders employ photographic recording on sensitized plates or paper. One type of instrument utilizing photographic recording on plates is illustrated in Figure 34. The magnetometer and the recording apparatus are both mounted in a cylindrical housing, which is supported on a tripod. The instrument is leveled and oriented in the same manner as the standard field balance. The recording mechanism is located in the upper part of the cylindrical housing and comprises: plate holder, flashlight bulb, prism, moving plate holder, clock-drive mechanism. A removable brass plate carries the magnetic system which is similar to that of the standard magnetometer except that a prism is employed for deflecting the light beam instead of a mirror.

The light from the bulb passes through a diffusing glass and a diaphragm with a point aperture. The image of the aperture is projected on the photographic plate by means of the objective lens, the prism on the moving magnetic system, and the fixed mirror. The fixed mirror receives a portion of the light beam and reflects a continuously moving light spot in a plane perpendicular to the plane of vibration of the magnetic system. Like the fixed mirror which records the base-line, the counter-mirror is fixed to a pivot rotated from a clockwork by a long arm carrying a toothed segment. After the lapse of a given time interval, the toothed segment is thrown out of gear and returns to its starting position.

The photographic plates used with the magnetometer shown in Figure 34 are 45 mm. by 60 mm. and are read by means of a graduated plate. One millimeter of record equals 10 minutes time along the long axis of the plate and approximately 10 gammas along the field strength scale. (The latter is at right angles to the time axis or direction of movement of the plate.) A record obtained with a continuous recording instrument exhibits the minute variations which cannot be obtained with any type of intermittent reading method.

Field Technique in Surveys Employing Magnetic Balances.—The field balances described above measure a differential value or anomaly. In a great majority of problems these relative values are sufficient because, by proper interpretation, the observed anomalies can be translated into their geological terms. In special cases where the absolute value of the magnetic intensity is required, it may be obtained either by comparison of anomalies at several stations with the known intensity at one or more stations or by mathematical computations utilizing the constants of the magnet system and the auxiliary magnets used in calibration.

The magnetic element measured in most geomagnetic surveys is the vertical intensity.† The field technique for carrying out a survey consists in setting up a systematic gridwork of stations at each of which readings are taken, corrected, and reduced to true values of the vertical magnetic

† M. C. Alexanian (trans. by W. Ayvazoglou), "Practical Rules for the Use of the Magnetometer in Geophysical Prospecting" (U. S. Bur. of Mines, I. C. 6527, 1931).

anomaly. These various stations are consecutively occupied during the progress of the survey. The closeness of the net of stations depends on the special purpose of the survey, the detail and the accuracy required, and the magnitude of the magnetic gradient as influenced by the geologic phenomena being examined. In general, the same factors will also determine the necessity of applying the various corrections.

In subsurface structural investigations, the stations are usually located at distances varying from ¼ to 1 mile apart in detailed work and from 1 to 3 miles apart in reconnaissance work. The stations are preferably located along traverse lines on roads. For mining and other detailed shallow structural work, the stations may be placed less than 100 feet apart. Generally, the distance between stations should not exceed about one-third to one-half of (a) the smallest horizontal dimension or (b) the depth of the geologic feature being explored. Closer spacing than this distance is preferable to avoid errors due to local irregularities, even though the cost of the work will be increased almost proportionally.

During the progress of a magnetic survey, the readings at the various stations are "tied in" to readings made at the same time at one or more "base stations."

Stray Magnetic Material

During the conduct of a magnetic survey, the operator must always be on the alert to note the presence of any "tramp" iron which may cause erroneous anomalies. Iron or steel structures, pipe lines, railroads, direct current power lines (including street car lines with ground return circuits), junk piles, etc., are possible sources of error and a careful record of observed surface conditions should be kept in order that the readings made in the neighborhood of such disturbances may be properly evaluated in the final interpretation of the magnetic data.

The operator should also avoid personal articles which may be magnetic. Errors are oftentimes caused by wrist watches, steel belt buckles (usually plated with the more expensive metals), iron arch supports in shoes or boots, pocket-knives, Brunton compasses, cameras, and other personal articles containing steel or iron.

Corrections for Time Variations of the Earth's Field

The irregular variations in the strength of the earth's magnetic field necessitate corrections for each of the various stations occupied during the course of a survey. These corrections are always the negative of the magnetic variations; e.g., if the magnetic intensity in the area *increases* at a certain time, the amount of increase will be *subtracted* from the magnetic reading made at that particular time. In general, the annual and secular variations are of interest in geomagnetic explorations only when surveys are conducted over a considerable period of time or when a lapse of several months occurs in extending or checking a given survey. The diurnal variations, however, are of considerable importance.

Corrections for magnetic variations during the progress of a survey are made by recording the variations of the magnetic intensity at one or more base stations. The ideal procedure is to employ a continuous recording magnetometer at the base station. This ideal procedure is approximated by setting up one instrument at a fixed station and taking intermittent readings. In many surveys where a separate instrument cannot be utilized for these fixed or base station measurements, results of lower (but often adequate) accuracy are obtained by readings made every hour or two at a central base station.

Intermittent Readings: Diurnal Variation.—In practice, intermittent readings of the diurnal variation frequently are made by setting up a magnetometer at a base station in the area under investigation and having an observer read the magnetic field strength at this station at intervals of 15 to 20 minutes. The readings should be started about an hour prior to the other field work and extend for a similar period of time beyond the other field measurements in order to establish general trends. Any variations in the field instruments are immediately apparent when check readings are taken at the base station. The two instruments must, of course, be mounted at a sufficiently large separation to insure that the mutual attraction between the two magnet systems is negligible.

If an extra instrument is not available for use as a base station magnetometer, usable results may be obtained by "checking-in" at the base station a number of times during the day. In this procedure, the operator returns to the base station at regular intervals and takes readings with

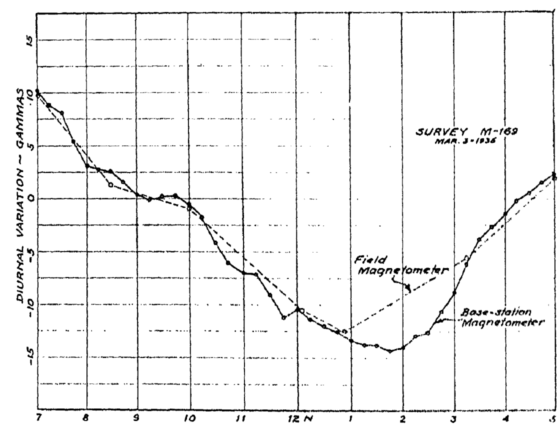


FIG. 35.—Graph showing diurnal variation as given by field and base station magnetometers.

his field magnetometer. These readings, when corrected for temperature changes, give a fair approximation of the diurnal variation. In this method, any change in instrument calibration will introduce an error which cannot be differentiated from diurnal variations.

An alternative field technique involves checking back on a previously occupied station rather than the base station. Occupy base station *A*, then stations, 1, 2, 3, 4 and back to *A*. Thus, corrected values for stations 1, 2, 3, and 4 are available for making any one of them usable as a check station. The survey might then proceed with stations 5, 6, 7, 8 and back to 4, etc. This saves time in difficult country.

Figure 35 shows corresponding curves obtained with a base station magnetometer and a field magnetometer "checked-in" at intervals of about two hours. The results are in fairly good agreement, with the exception of the time period between 1 P. M. and 3:15 P. M. Here the error would amount to about 4.5 gammas which may be sufficient to mask or distort a minor magnetic feature.

The graph (Figure 35) also shows that the diurnal variation fluctuated from +10 gammas at 7 A. M. to −14 gammas at 1:45 P. M., i.e., the variation had an amplitude of 24 gammas for that particular station and date. The maximum magnetic variation due to structure in this area was about 10 to 15 gammas, i.e., approximately $\frac{2}{5}$ to $\frac{3}{5}$ the diurnal variation. It is evident, therefore, that in this case the diurnal variation had to be measured with the same accuracy as the anomalies.

In certain types of mining investigations where the magnetic anomalies are very large, the diurnal variation may oftentimes be neglected. Generally speaking, however, it is poor policy and false economy to omit the diurnal corrections.

Corrections for Auxiliary Magnets

In regions where the anomalies are large and precise measurements are not required, auxiliary magnets may be employed to extend the scale value of the instrument. (Compare p. 100.) The preferred procedure is as follows: Observe the scale deflection at the station where the end of the scale has been almost reached and record the reading. Choose an auxiliary magnet to bring the scale reading to the desired value and again record the reading. Then go back to one or two prior stations and repeat the procedure. The average difference in the two sets of readings at each of the stations is the auxiliary magnet correction in scale divisions. This correction will then be added or subtracted, as the case may be, for all readings employing the auxiliary magnet. A similar procedure will be employed if it becomes necessary to change the auxiliary magnets.

If precise work involving an accuracy of 5 gammas or better is necessary, auxiliary magnets should not be employed for extending the scale of the instrument. At such times, the only procedure is to open the case of the instrument and adjust the latitude screw until the desired scale reading is obtained. The sensitivity adjustment should not be changed unless the work clearly shows the need for a change in sensitivity. After shifting the scale reading, repeat readings should be made at one or two prior stations to effect a proper tie-in of data.

Recording of Data

It is preferable to employ a standard printed form for all field work. A recommended form (Figure 36) allows the operator to carry out the computations conveniently.* Good technique requires that a written record be made at each reading.

Outline of Procedure at Each Station

The steps outlined below illustrate the procedure at each station when using a vertical component Schmidt field balance after the instrument has been removed from the box and has acquired the temperature of the air.

(1) Remove all iron objects from the observer and immediate vicinity of the instrument; set auxiliary magnets and other magnetic materials that are in the carrying box at a distance of 30 to 50 feet from the station.

(2) Set up and level the tripod. Carefully orient the tripod head in the magnetic meridian with the aid of the compass; turn the head 90° and clamp it. (With the tripod head oriented in this direction, the magnetometer will set in the head at right angles to the magnetic meridian.) Remove the compass to a safe distance from the tripod.

(3) Clamp the magnetometer on the turntable. Level the instrument accurately by means of the level screws.

(4) Release the magnetic system by means of arresting device and note whether the reflected scale remains in the field of vision. If the instrument has been properly adjusted for latitude, the magnetic anomaly will be well within the scale reading for practically all structural investigations. If it disappears because of *large anomalies* employ an auxiliary magnet to bring it into the field of vision, noting which magnet is used and its position and distance.

(5) Read and note the position of the magnet system, revolve the instrument through 180° and again read the position of the magnet system.

(6) Repeat these readings two or more times to obtain representative readings. (If instrumental trouble is not encountered, careful manipulation will give readings which check within one or two tenths of a scale division.)

(7) Record these separate readings in scale divisions of deflection.
case, and progress to the next station.

(8) Read and note the temperature and time.

(9) Clamp the magnet system, remove instrument from the tripod and place in

Treatment of Data Obtained at Each Station.—In computing the results of a vertical intensity magnetometer survey, the "average observed reading" at each station is obtained by averaging the various readings obtained in the two azimuths at each station and applying the following corrections:

(1) Temperature correction.
(2) Correction for auxiliary magnet if such has been used.
(3) Base station correction for instrumental changes.
(4) Base station or observatory diurnal variation correction.
(5) Latitude and longitude corrections.

In making these corrections the base station is usually assumed to have no magnetic anomaly; hence, its observed reading is the true reading at an arbitrary temperature, usually 0°C or 20°C.

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* It should be understood that the magnetometer survey forms are not exactly alike in all cases, but are modified so as to be appropriate for the instrument used and the magnetic conditions existing in the area under investigation.

FIGURE 36

MAGNETOMETER SURVEY

Observer: Date: Area: Instrument No. Sensitivity: 28.6 γ per scale division.

Station	Location	Time	Temp.	Aux. Magnet	Readings	Average	Total Average	Corrections T.	Corrections A. Mag.	Corrections Base	Corrections T. plus Base	Total Average plus B. & T. Correction	Minus Base	Times Scale Value	Long. and Latit. Correct.	Corrected Reading	Cor. Reading plus Assum. Base	Remarks
Base		8:45	+4.1 +5.1	—	E. 5.2, 5.1, 5.1 W. 5.4, 5.6, 5.5	5.1 5.5	5.3	+0.7	—	—	+0.7	6.0	—				Assumed 1000 gammas Base Value	
1	N.Smith's farm See map	10:10	—.7 —1.0	—	E. 16.0, 15.9 W. 14.9, 14.9	16.0 14.9	15.5	—0.1	—	—0.2	—0.3	15.2	9.3	+266	—138	+128	1128	
2	3 mi. E. 2 mi. S.	11:20	—1.1 —1.3	—	E. 16.8, 17.2 W. 19.5, 19.4	17.0 19.5	18.2	—0.2	—	—0.2	—0.4	17.8	11.8	+338	—131	+207	1207	
3	1 mi. S.	11:40	—1.5 —1.7	—	E. 17.7, 17.8 W. 17.6, 17.6	17.7 17.6	17.7	—0.2	—	—0.3	—0.5	17.2	11.2	+320	—120	+200	1200	
4	4 mi. E. of St. 1	12:30	—0.6 —1.1	—	E. 18.5, 18.9 W. 19.1, 19.3	18.7 19.2	19.0	—0.1	—	—0.4	—0.5	18.5	12.5	+359	—159	+200	1200	
5	2 mi. E.	1:05	—2.1 —2.5	—	E. 14.7, 14.5 W. 14.1, 14.1	14.6 14.1	14.4	—0.3	—	—0.5	—0.8	13.6	7.6	+218	—168	+50	1050	
Base		3:45	+9.0 +8.4	—	E. 5.3, 5.5 W. 5.7, 5.9	5.4 5.8	5.6	+1.2	—	—	—	6.8	—					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	

EXPLANATION OF HEADINGS

No. of Column

1 Base location is given in detail. Location of stations is given relative to preceding station.

2 Time of day is read to the nearest 5 minutes.

3 Temperature is read on the thermometer attached to the magnetometer.

4 If auxiliary magnet is used, its number and its deviation are given. Also rod distances are given.

5 A few deviations east and west by deflection are usually taken at each station.

6 Separate averages for eastern and western deviations are computed.

7 Total average is one-half of the sum of the eastern and western averages.

8 Temperature correction from assumed 0.0 degree is expressed in scale divisions.

No. of Column

9 Correction due to the introduction of auxiliary magnet is given in scale divisions.

10 Base correction for diurnal magnetic variation at the base station, in scale divisions.

11 The sum of the temperature and base corrections in scale divisions.

12 Total average plus base and temperature corrections in scale divisions.

13 The difference between the station values and the base as given in column No. 12.

14 Scale values as given in the column 13 multiplied by the sensitivity of the instrument.

15 Corrections due to deviation in latitude and longitude from the base station, in gammas.

16 Difference between columns 14 and 15 in gammas.

17 Corrected reading plus 1000 gammas assumed as a base station value.

The temperature correction for each station is the increase or decrease in temperature above or below an arbitrary temperature (0° C. or 20° C.) multiplied by the temperature correction factor. (Compare p. 98.) The auxiliary magnet correction depends on the amount which the use of the auxiliary magnet decreased or increased the reading. Corrections for instrumental behavior and diurnal variation are obtained by comparing field station readings with base station readings at corresponding times. The latitude and longitude corrections usually are applied only in detailed oil-structure work where large areas are mapped.

When these corrections are all figured in terms of scale divisions and applied to the average observed readings, the result is the corrected vertical anomaly in scale divisions. This value multiplied by the sensitivity of the instrument gives the anomaly in gammas.

Plotting Magnetic Data.—The corrected data of a magnetic survey may be presented in a number of ways. Usually, however, the results are shown as: (1) magnetic profiles along traverse lines so chosen that the magnetic profiles may be correlated with known geological drill-hole or other data or (2) isanomalic contour maps. In the first type of presentation, the data are usually plotted with the anomaly in gammas (or scale divisions) as ordinate and the traverse distance in feet or miles as abscissa. The second type of presentation (isanomalic contour map) is similar to the conventional topographic map; that is, the magnetic datum at each station is indicated on a plan view of the area at a point corresponding to the location of the station, and isanomalic magnetic contours are drawn by inspection.* The small closed contours representing curves of greatest magnetic strength are usually shaded outwardly, and are termed magnetic "highs." The contours representing curves of lowest field strength are usually shaded inwardly, and are termed magnetic "lows."

Illustrations of magnetic anomaly profiles and isanomalic contour maps are given in the section dealing with results of surveys

"Normal" Values of the Earth's Intensity Components

Interpretation of magnetic anomaly curves is sometimes facilitated by knowing the "normal" intensity components in the area under investigation.

A precise definition of the normal intensities cannot be given. The procedure for arriving at the normal values for a given area is an approximation process. Consideration must be given the shapes of the anticipated geological features and their relative magnetic effects. In regions where large anomalies exist, the normal values are the intensity components measured at a station removed from the anomalous area.

---

* In many surveys the method of least squares is applied before drawing final isanomalic contours. Barton † and Roman ‡ describe the application of this method to magnetic prospecting.

† D. C. Barton, "Control and Adjustment of Surveys with the Magnetometer or the Torsion Balance," *Bull. A.A.P.G.*, Vol. 13, 1929, pp. 1163-1183.

‡ I. Roman, "Least Squares in Practical Geophysics," *A.I.M.E. Geophysical Prospecting*, 1932, pp. 460-506.

Summary

Preliminary

In Laboratory

Determining temperature coefficient of instrument
Determining magnetic moments of auxiliary magnets

In Field

Adjusting for latitude
Adjusting for sensitivity
Checking scale value to determine gammas per scale division
Determining location of stations to allow best correlation with geology
Establishing base stations
Preliminary traverses: one traverse in a N-S direction to obtain maximum anomalies due to polarization

Field Work

Setting up tripod at station
Leveling tripod
Orienting tripod head in magnetic meridian
Leveling instrument
Obtaining scale readings
Recording

Station number or location
Scale readings
Temperature of instrument
Time of observation
Local conditions which may affect the reading

Corrections

Base reading to datum plane (for survey employing more than one instrument)

Latitude

Average for U. S.: 10-12 gammas per mile for vertical component
5-8 gammas per mile for horizontal component
When movement proceeds to the North

Subtracted for vertical component
Added for horizontal component

Longitude

Average for U. S.: 2 to 3 gammas per mile
When movement proceeds to the East

Subtracted for east declination areas
Added for west declination areas

Base station corrections
 Diurnal and other variations in earth's field
 Instrument variations
Temperature corrections
Correction for deflection due to use of auxiliary magnets

Accuracy to Which the Work is Conducted
 Depends upon anomalies in the region

Accuracy of Data Depends on
 Errors in scale reading: 2 to 4 gammas
 Temperature effects: 1 to 3 gammas
 Diurnal variations: 2 to 5 gammas
 Regional variation

Final Interpretation Includes
 Checking scale value
 Making corrections and converting readings to gammas
 Correcting for regional gradient if necessary
 Plotting of magnetic data
 Correlating with areal and subsurface geology
 Correlating with other geophysical work

THEORETICAL ANALYSIS OF MAGNETIC DATA

As stated previously, magnetic anomalies are produced chiefly by differences in the magnetic permeability of the rocks and formations comprising the earth's crust. In addition to the variation in permeability, remanent magnetism, such as is possessed by lodestone and a few other materials occurring in nature, may contribute to the anomalies. The problem of interpretation consists in inferring the position (or attitude), depth, configuration, and general character of the subsurface body or structure from the magnetic anomaly observed at the surface of the earth.

The usual treatment of magnetic data is almost exclusively empirical. Deductions and inferences are drawn in a qualitative manner from the sizes and configurations of the magnetic anomalies. (Compare p. 135.) However, the empirical treatment of magnetic data is based in part on a knowledge of the theoretical anomalies produced by certain inhomogeneities. Furthermore, a theoretical type of analysis is applicable in certain cases. Hence, it is advantageous to consider the theoretical procedure for inferring the subsurface structure from magnetic data obtained at the surface.

The theoretical procedure for deducing subsurface structure from magnetic data is: (1) to assume (a) a geologically plausible configuration of the subsurface formations and (b) probable values of the permeabilities (or susceptibilities); (2) to compute the magnetic effects which the assumed configuration (and permeabilities) would produce at the surface;

(3) to compare the theoretical and observed results; (4) to modify the assumptions until a satisfactory agreement is obtained between the observed and theoretical data.

Three theoretical methods will be considered. The first method is used when the subsurface body has a small cross section relative to its length. The procedure followed in this method is to set up expressions for anomalies characteristic of a single pole, a vertical magnetic dipole and an inclined dipole, and to compare the theoretical results with the observed anomalies. This method is described in section A below.

The second method is used for calculating the magnetic anomalies produced by magnetized strata. The theoretical technique for obtaining the magnetic anomalies of magnetized strata may be subdivided into two separate methods. In one, use is made of a functional relation between

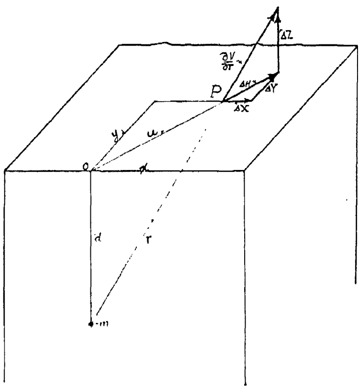

FIG. 37.—Field at P due to S pole of strength m located at a depth d.

the magnetic and gravitational effects of magnetic layers in order to obtain the unknown theoretical magnetic anomalies from known theoretical gravitational anomalies. In the other, it is postulated that the magnetic effects produced at the surface of the earth by the volume distribution of magnetization throughout the layers are approximately the same as those produced by surface distributions of magnetic charge located on the upper and lower faces and on vertical contacts of the layers. This method is described in section B below.

The third method is used to calculate the magnetic effects produced by uniformly magnetized geologic bodies having the shape of spheres, ellipsoids of revolution, etc. In the case of a uniformly magnetized sphere, the theoretical magnetic effects at the surface of the earth may be obtained by replacing the sphere by a bar magnet having a magnetic moment equivalent to that of the sphere. This method is described in section C below.

A. Anomalies Produced by Certain Ore Bodies and Igneous Intrusions.—Certain ore bodies and igneous intrusions which occur in the form

1. *Field Due to a Magnetic Pole*

tial. The potential at any point due to a mag be defined as the work done in carrying a unit infinity to the point in question. That is, the potential V may be by the equation:

$$\int_{\infty}^{r} \frac{m}{r^2}\, dr \tag{19}$$

The components of the magnetic field (Figure 37) at a point (x, y) due to a pole of strength m located at a depth d are:

$$\partial V$$

$$\partial V \tag{20}$$

The horizontal component of the field due to the pole m is:

$$\Delta H \quad \frac{\partial V}{\partial u} \quad mu \tag{21}$$

The actual shape of the curve obtained on plotting ΔH or ΔZ depends on the depth to the top of the ore body and the pole strength. Figure 38 shows the profiles in the x-direction $(y = 0)$ when $d = -1$ and $m = -2$. For this case, Equations 20 and 21 for the vertical and horizontal intensity anomalies become

$$\Delta Z = md - \frac{md}{} = \frac{2}{} \tag{20a}$$

and

$$\Delta H = \Delta X = \frac{}{r^3} = - \quad 2x \tag{21a}$$

An examination of the expression for ΔZ reveals that ΔZ is symmetrical with respect to the z-axis, has a maximum for $x = o$, i.e., directly over the pole and approaches the x-axis asymptotically. The expression (21a) for ΔH shows that ΔH is an odd function of x. (ΔH for negative values

of x is equal to $-\Delta H$ for the corresponding positive values of x.) ΔH vanishes for $x = o$ and approaches the x-axis asymptotically for large positive or negative values of x.*

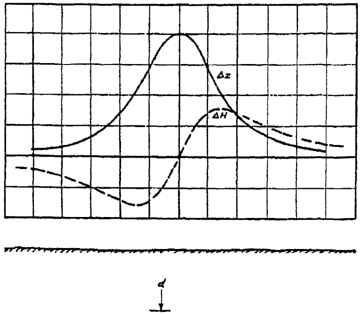

FIG. 38.—Vertical and horizontal intensity anomalies produced by an ore body equivalent in its magnetic effects to a single *s* pole of strength *m* located at a depth *d*.

Two "depth rules" are readily deduced from Equations 20 and 21.† (1) The horizontal distance u from a point directly over the pole to a point where ΔZ equals $\frac{1}{2}\Delta Z_{max}$ is approximately equal to $\frac{3}{4}$ the depth of the pole. (Tiberg depth rule.) (2) At a horizontal distance u equal to d, $\Delta Z = \frac{1}{3}\Delta Z_{max}$ (approximately). That is, the distance from the origin to a point at which ΔZ is approximately equal to $\frac{1}{3}\Delta Z_{max}$ is equal to the depth of the pole. (Haanel depth rule.)

An alternative method for obtaining an approximate value of thickness of the overburden is to draw the vector diagram of the total anomalies near the point of maximum vertical anomaly.‡

The anomalies in the horizontal and vertical intensities are obtained subtracting the normal values of the horizontal and vertical intensities in the area under investigation from the observed values. (Compare p. 110.) Corresponding values of the horizontal and vertical anomalies are then combined vectorially in order to obtain resultant vectors having the direction of the *total anomaly*. Since the resultant vector at any point is tangent to the line of force at that point, the resultant vectors near the point of maximum vertical intensity will intersect very nearly at the pole of the ore body. Figure 39 shows a diagram for the resultant magnetic anomaly due to an ore body. In this case the vectors indicate that the pole is about 150 to 175 feet below the surface. This "depth" is of course an approximate value, because vector diagrams indicate a point near the center of attraction of the upper pole of the body and do not give the thickness of the overburden, which will always be less than that deduced from the diagram.

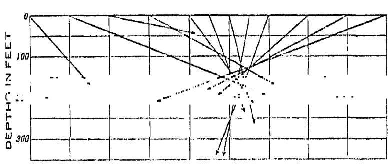

FIG. 39.—Vectors of resultant magnetic anomalies due to ore body. (Eve and Keys, *Canadian Memoir* 170, p. 39.)

* To draw the curves given in Figure 38, it is sufficient to assume four or five values of x and compute ΔZ and ΔH from Equations 20a and 21a.

† Compare C. A. Heiland, *A.I.M.E. Geophysical Prospecting*, 1932, p. 213.

‡ A. S. Eve and D. A. Keys, "Studies of Geophysical Methods," 1930, Department of Mines (Canada), *Memoir* 170, pp. 36-37.

2. *Field Due to a Magnetic Dipole*†

If the depth extent of the subsurface magnetic feature (ore body, etc.) is too small to permit neglecting the magnetic effect of the pole on the deep end of the ore body, the total effect may be approximated by assuming that the ore body is equivalent to a magnetic dipole. (Figure 40.)

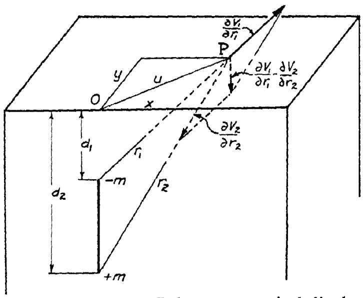

FIG. 40.—Field at P due to a vertical dipole.

The magnetic anomalies produced by a dipole are calculated by adding the effects of a south and a north pole vectorially. The application of Equations 20 and 21 to the poles $-m$ and $+m$ yields:

$$\Delta Z = -m\left(\frac{d_1}{r_1^3} - \frac{d_2}{r_2^3}\right) \tag{22}$$

$$\Delta H = -mu\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right) \tag{23}$$

Sketches of the horizontal and vertical intensity anomalies produced by a vertical dipole are shown in Figure 41. The profiles in the x-direction are drawn for the case: $m = 2$, $d_1 = -1$, and $d_2 = -3$; that is

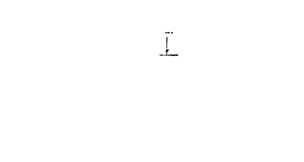

FIG. 41.—Vertical and horizontal intensity anomalies for an ore body which is magnetically equivalent to a vertical dipole.

$$\frac{1}{(9+x^2)^{3/2}}\Bigg]$$

An approximate depth rule for an ore body which is equivalent in its magnetic effects to a vertical dipole was given by Thalen in 1879. The rule may be stated as follows: The depth d_1 to the top of the ore body is equal approximately to 0.7 of the horizontal distance u from a point directly over the dipole to a point where $\Delta Z = 0$.

† H. Haalck, *Die Magnetischen Verfahren der Angewandten Geophysik* (Gebrüder Bornträger), Berlin, 1929.

A. Nippoldt, *Verwertung Magnetischer Messungen* (Springer), Berlin, 1930.

C. A. Heiland, Chapter on Magnetic Prospecting, pp. 136-139, *Terrestrial Magnetism and Electricity*, edited by J. A. Fleming (McGraw-Hill), 1939.

If the dipole is inclined (Figure 42), Equations 22 and 23 must be modified to include the cosine of the dip of the ore body and its length.

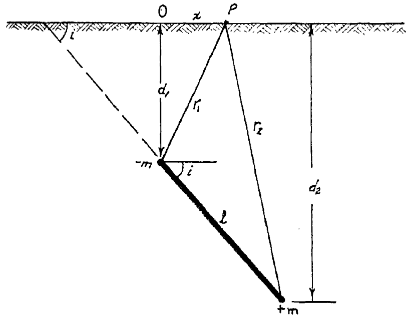

Fig. 42.—Sketch illustrating geometric relations of dip of ore body and vectors r_1 and r_2 to the point of observation P on an x traverse.

It may be shown from an analysis similar to that given for the vertical dipole that the expressions for the vertical and horizontal intensity anomalies along the x-axis are:

$$= - m$$

$$\Delta H = - m \left[\frac{x}{r_1^3} - \frac{x - l \cos i}{r_2^3} \right] \tag{25}$$

where d_1 is the depth to the top of the ore body; and $(d_1 + l \sin i) = d_2$ is the depth to the bottom of the ore body; r_1 and r_2 are the distances from

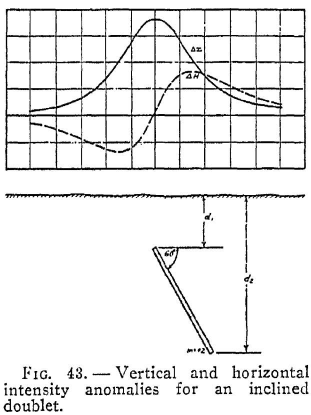

Fig. 43. — Vertical and horizontal intensity anomalies for an inclined doublet.

the point of observation to top and bottom respectively; l is the length and i the dip of the ore body; x is the distance from the origin to the point of observation.

Profiles of the horizontal and vertical intensity anomalies in the x-direction are shown in Figure 43. The curves were plotted for the case: $d_1 = -1$, $d_2 = -3$, $m = 2$, $i = 60°$, and $l = 2.305$; that is,

$$\Delta Z = +2 \left\{ \frac{1}{(x^2+1)^{3/2}} - \frac{3}{[9+(x-1.15)^2]^{3/2}} \right\} \tag{24a}$$

and

$$: -2 \cdot \frac{x}{\quad} - \frac{x-1.15}{\quad} \Big)$$

3. *Anomalies Due to Long Narrow Dikes*

If the dike is approximately vertical, it is equivalent in its magnetic effects to a vertical magnetized "sheet" of finite thickness. Also, if the depth extent is sufficiently great that the effects due to the induced pole strength at the deep end may be neglected, the magnetic anomalies produced at the surface are essentially the same as would be produced by a linear distribution of magnetic charge. In particular, it may be shown that the maximum value of the vertical intensity anomaly is:†

$$\Delta Z_{\max} = \frac{2mb}{d} \tag{26}$$

where m is the magnetic pole strength per unit area, b the width of the dike, and d the depth to the effective linear distribution of magnetic charge. (d is, of course, somewhat greater than the distance from the surface to the top of the dike.)* The anomaly at a point P located at a distance x from the point of maximum anomaly along the line at right angles to the strike is:

$$\frac{2mbd}{x^2+d^2} \tag{27}$$

If we choose the point P such that $\Delta Z = \frac{1}{2}\Delta Z_{\max}$, and combine the last two equations, we obtain

$$\frac{2mbd}{x^2+d^2} = \frac{mb}{d}$$

or

$$d = x$$

Thus the thickness of the overburden is equal approximately to the distance along the line perpendicular to the strike of the dike at which ΔZ is equal to $\frac{1}{2}\Delta Z_{\max}$.

† The method outlined here is described by D. A. Keys, "Determining Depth of Magnetic Ore Bodies," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 830, p. 5.

* Equation 26 which gives the maximum value of the vertical anomaly is essentially a particular application of the expression for the field due to a linear distribution of magnetic charge. (Compare p. 115.)

An alternative method for determining the approximate thickness of the overburden over a dike makes use of vector diagrams, as illustrated in connection with Figure 38.

B. Theoretical Anomalies Produced by Magnetic Strata.—Two methods may be employed to calculate the magnetic effects of two-dimensional bodies such as strata. In both methods it is assumed that the difference in susceptibilities between the anomalous body and the adjacent formation is such that *demagnetization effects** are negligible.

Method Utilizing Functional Relation Between Magnetic and Gravitational Anomalies

As a consequence of a theorem due to Poisson, it may be shown that the following relationships obtain between the magnetic and gravitational anomalies produced by two-dimensional bodies: †

$$\Delta H = \frac{\Delta k}{G\Delta\sigma}\left(H \frac{\partial^2 U}{\partial x^2} \sin\alpha + Z \frac{{}^2 U}{\partial x \partial z} \right) \qquad (29)$$

$$\Delta k \qquad H \frac{\partial^2 U}{\partial x \partial z} \sin\alpha - .$$

where

$\Delta k =$ susceptibility difference of the geologic body relative to the adjacent rock.
$G =$ gravitational constant ($6.68 \cdot 10^{-8}$ c. g. s.)
$\Delta\sigma =$ density difference.
$U =$ gravitational potential.
$\alpha =$ angle between geographical north and the west direction of strike.
$H =$ horizontal component of earth's field.
$Z =$ vertical component of earth's field.

The applicability of the ΔH and ΔZ formulas derives from the fact that the derivatives of the gravitational potential $\left(\frac{\partial^2 U}{\partial x^2} \quad - \right)$ have been evaluated for numerous, two-dimensional geologic bodies. (Compare chapter on gravimetric methods.)

Method Utilizing Equivalence of Volume and Surface Distributions of Magnetic Charge

This method consists in calculating the magnetic effects of a uniformly magnetized formation in terms of its effective surface distribution of magnetic charge. This method postulates that the magnetic effects at the

* A discussion of demagnetization effects is given on p. 132 *et seq.*

† R. von Eötvös, "Bestimmung der Gradienten der Schwerekraft und ihrer Niveauflachen mit Hilfe der Drehwage," *XV. Allgemeine Konferenz der Intern. Erdmessung,* Budapest, 1906. See also W. P. Jenny, "Experimental Interpretation of Magnetic and Gravimetric Anomalies," *Terr. Mag.* 40 (No. 1) March 1935, pp. 71-78.

surface due to a volume distribution of magnetization throughout a stratum are approximately the same as those produced by surface distributions of magnetic charge located on the upper and lower faces of the stratum.

Before deriving the anomalies produced by two-dimensional geologic bodies, it will be convenient to describe: (a) the magnetic fields produced by linear distributions of charge and (b) the general expressions for the magnetic potential and magnetic field produced by surface distributions of charge.

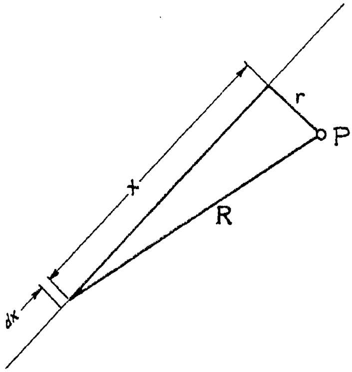

FIG. 44.—Sketch illustrating geometric relations between a linear distribution of magnetic charge in the direction x and an external point P.

Magnetic Fields Produced by Linear Distributions of Charge.—The magnetic field of force in the direction R due to element dx (Figure 44) is:

$$m\,dx$$

where m equals the pole strength per unit length. The force dF is the vector sum of the components of dF_r in the direction normal to x and dF_x in the direction parallel to x.

But mr

Hence, the component in the direction r due to the whole line is

$$= \int_{-\infty}^{\infty} \frac{mr}{R^3}\,dx = 2\int_{0}^{\infty} \frac{mr}{R^3}\,dx = 2mr\int_{0}^{\infty} \frac{dx}{(x^2+r^2)^{3/2}}$$

Let

$$x = r\tan\theta$$
$$dx = r\sec^2\theta\,d\theta$$
$$x^2 + r^2 = r^2\sec^2\theta$$

then

$$F_r = \frac{2\,m\,r}{r^2}\int_{0}^{\pi/2}\cos\theta\,d\theta = \frac{2m}{r} \tag{30}$$

The component in the direction x is

$$F_x = \int \frac{mx}{D^3}\,dx = 0 \text{ *}$$

Hence, the resultant field for an infinite linear distribution of magnetic poles has the direction r and a magnitude $2m/r$.

The potential due to the linear distribution of charge is given by the

i

Field Due to Two Parallel Linear Distributions of Equal Strength and Opposite Polarity (Figure 45)

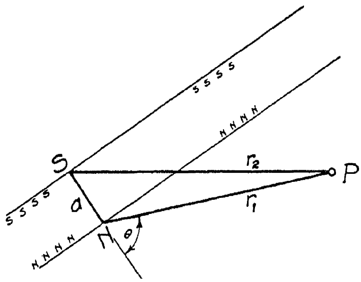

FIG. 45.—Sketch illustrating geometric relations between two parallel linear distributions and an external point P.

If the separation a of the two parallel lines is small compared to r, then the potential $V = V_1 + V_2 = 2m \log r_1 - 2m \log r_2$

$$= -2m \log \frac{r_2}{r_1} = -2m \log \left(\frac{r_1 + a\cos\theta}{\quad}\right)$$

$$= -2m \log \qquad a\cos\theta$$

* To evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{mx}{R^3}dx = \int_{-\infty}^{\infty} \frac{mx\,dx}{(\quad}$

set

$$x = r\tan\theta$$

This yields

$$F_x = m\int_{-\pi/2}^{\pi/2} \frac{\quad}{r^3\sec^3\theta} \qquad \int_{-\pi/2}^{\pi/2}$$

** It will be noticed that this expression for V makes V infinite at r equal to infinity. However, we are chiefly interested in the magnetic field, i.e., the derivative of the potential at a finite value of r, and this derivative always has a finite value.

where m is the pole strength per unit length. Since $a \cos\theta$ is small when a is small compared with r_1, the power series expansion of the logarithm gives $V = -2m \frac{a\cos\theta}{}$ + terms of the order

That is,

$$' = -2m\frac{a\cos\theta}{r_1}\text{(approx.)} \tag{32}$$

General Expressions for the Magnetic Potential and Field Produced by Surface Distributions of Charge.—The magnetic potential produced by a uniformly magnetized body at any external point P is given by the equation†

$$V = \quad \cos a \quad dS$$

where r is the distance between P and the element of surface dS and α is the angle between the magnetization I and the outward normal at dS. For the cases to be considered, the magnetization is normal to the surface, i.e., $\alpha = 0$; hence,

$$V = \tag{33}$$

The magnetic field at P is obtained by taking the derivative of V. Differentiation of Equation 33 with respect to r yields: *

$$= \iint \frac{IdS}{r^2} \tag{34}$$

Equations 30 to 34 are the fundamental equations utilized in deriving the magnetic effects produced by two-dimensional geologic bodies.

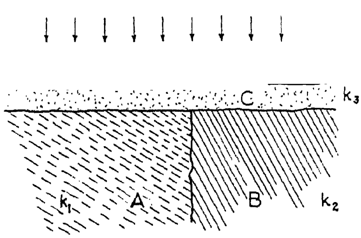

FIG. 46.—Buried contact of two thick layers.

Magnetic Anomalies Due to a Buried Contact of Two Very Thick Horizontal Layers Having Different Susceptibilities.—Assume that the materials A and B (Figure 46) are separated by a vertical plane, i.e., have a vertical contact, and that d is the thickness of layer C, k_1 susceptibility of layer A, k_2 susceptibility of layer B, k_3 susceptibility of layer C. On the upper surface of C there is a uniform distribution of magnetism of surface density $I_3 = k_3 Z$ units of pole per unit area.** This is south

† J. H. Jeans, *Electricity and Magnetism*, 5th Edition, p. 374.

* In carrying out the differentiation with respect to r, the quantity IdS is held constant.

** Note that Z is the vertical component of the earth's field.

polarity or magnetism if C is paramagnetic and north polarity if C is diamagnetic. On the lower surface of C the surface density of magnetism is $-I_3$. On the upper surface of A the density of magnetism is $I_1 = k_1 Z$ and along the upper surface of B it is $I_2 = k_2 Z$. This condition is equivalent to the following: Density I_3 on upper surface of C, density $I_1 - I_3$ on plane CA, density $I_2 - I_3$ on plane CB.*

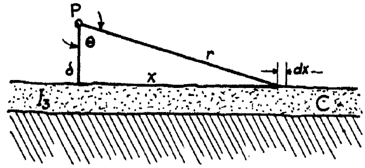

FIG. 47.—Sketch showing quantities which enter into the calculation of the effect of upper surface of C.

1. *Effect at P of Upper Surface of C (Figure 47).*** The magnetic field in the direction of r is computed by substituting $dx\,dy$ for dS in Equation 34; that is,

$$\Delta Z_r = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{I_3\,dx dy}{r^2}$$

If the height of the measuring instrument above the surface of the ground is designated by δ, the vertical component ΔZ_1 may be written

$$= \int_{-\infty}^{\infty}\int$$

$$= I_3\,\delta \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \frac{dx}{[(x - x_1)^2} \right]$$

To integrate the quantity within the brackets, set $(y - y_1)^2 + \delta^2 = a^2$ and introduce a variable ϕ defined by the relation $x = a \tan \phi$. Thus

$$\int \frac{dx}{x_1)^2 + (y} \qquad \int_{-\infty}^{\infty} \frac{}{[(x}$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{a^2} \cos\phi\, d\phi = \frac{}{a^2} = \frac{}{(y}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{}{[(x} \qquad I_3\,\delta \qquad \delta \int_{-\infty}^{\infty} \frac{2}{(y - y_1} \qquad dy$$

$$= \left[\frac{2I_3\,\delta}{\delta} \right.$$

That is, the vertical component ΔZ_1 due to the magnetic density on the upper surface of C is

* This is an approximate treatment which does not hold strictly at the boundary AC and BC.

** For simplicity, the illustration shows a vertical section through a point P (x_1, y_1, δ)

BC and AC).—The effects of sur-
as

the field *BC* is:

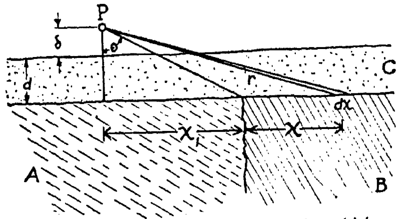

FIG. 48.—Sketch showing quantities which enter into the calculation of the effect of contact *BC*.

$$\Delta Z_2 = \int_0^{\infty}\int_{-\infty}^{\infty} \frac{(I_2 - I_3)\ (d+\delta)}{[(y-y_1)^2 + (x-x_1)^2 + (d+\delta)^2]^{3/2}}\, dx dy$$

In this case, it is simpler to integrate first with respect to y.

$$\frac{dy}{[(y-y_1)^2 + (x-x_1)^2 + (d+\delta)^2]^{3/2}}$$

Hence

$$\Delta Z_2 = 2(I_2 -$$

$$\Delta Z_2 =$$

The effect due to surface *AC* at *P* is obtained in similar fashion by integrating between x and ∞; this gives:

$$\left(\frac{\pi}{2} - \tan^{-1}\frac{x_1}{\delta + d}\right)$$

Hence, the total vertical anomaly (vertical component of the field due to induced magnetism) is:

$$\Delta Z = \Delta Z_1 + \Delta Z_2 \qquad 2(I_1 - I_3)\left(\frac{\pi}{2} - \tan \quad \frac{x_1}{\delta + (}\right.$$

$$\cdot\, 2(I_2 - I_3)\left(\frac{\pi}{2} + \tan^{-1}\frac{x_1}{\delta + d}\right)$$

or

$$Z = 2I_1 \qquad \frac{\pi}{2} + \tan^{-} \qquad (35)$$

It is evident that ΔZ does not depend on I_3 but does depend on the layer thickness d. If the overlying formation consisted of a number of layers having different values of k, the effects would still cancel out. Figure 49 illustrates the vertical anomaly profiles for the case that I_2 is much greater than I_1.†

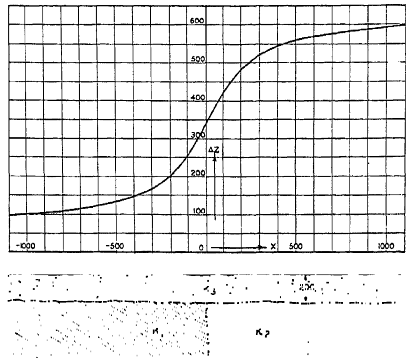

FIG. 49.—Anomaly due to buried contact of two thick layers. (Equation 35.)

Anomaly Due to Horizontal Component of Induction

If, as is generally the case, the earth's field is not vertical, the phenomenon would be the same as though two fields were present, i.e., a vertical field equal to the vertical component Z and a horizontal field equal to the horizontal component H. The calculations just made may be used to give the effect of the vertical field. The horizontal component, however, would produce an additional distribution of magnetism along the plane AB. (The effects produced by such a distribution will be considered in a later section.)

Thin Layer Terminating at a Distance d Below the Surface.—In this case there are two kinds of effects to be taken into account: the effect due to the end of the layer and the effect due to magnetization at right angles to the plane of the layer. (Figure 50.) We shall consider the end effect first. If I_1 is the intensity of magnetization parallel to the plane of the paper and normal to the end of the layer and t the thickness of the layer, the pole strength per unit length of the end of the layer is tI_1. If the layer is thin, the end effect is the same as that produced by a linear distribution of magnetic poles oriented at right angles to the plane of the paper. The potential due to such a distribution is given by Equation 31; that is,

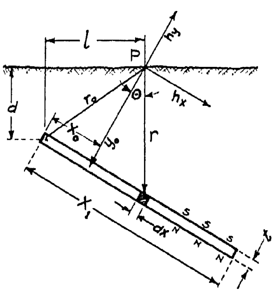

FIG. 50. Sketch showing quantities which enter into the calculation of the potential due to magnetization at right angles to the surface of the layer.

$$V_1 = 2tI_1 \log r_0 = 2tI_1 \log (d^2 + l^2)^{1/2}$$

The vertical component of the anomaly at P is:

$$\frac{\partial V_1}{\partial d} = \frac{2tI_1 d}{d^2 + l^2} \qquad (36)^*$$

† See also L. B. Slichter, *A.I.M.E. Geophysical Prospecting*, 1929, p. 240.

* The field is expressed as the positive derivative of V in order to avoid carrying negative signs. Actually, the sign of the anomaly depends on the sign of I and this in turn depends on the orientation of the magnetic layer with respect to the earth's field.

The horizontal component at P is

$$\frac{\partial V_1}{\partial l} = \frac{2tI_1 l}{d^2 + l^2} \tag{36a}$$

The second effect (due to magnetization of the layer by the field at right angles to its surface) may be obtained as follows: Let I_2 and $-I_2$ be the densities of magnetic charge on the two surfaces of the layer. The potential at the point P due to a magnetized strip which is perpendicular to the plane of the paper and has a thickness t is $2I_2 t \cos\theta/r$. (Compare Equation 32.) The potential V_2 due to the magnetized *surface* may be obtained by integrating the potential due to the magnetized strip with respect to x; that is,

$$V_2 = \int_0^{x_1} \frac{2I_2 t \cos\theta}{r} dx = 2I_2 t \int_0^{x_1} \frac{y_0\, dx}{y_0^2 + (x - x_0)^2}$$

$$= 2I_2 t \left[\tan^{-1}\frac{x_0}{y_0} - \tan^{-1}\frac{x_0 - x_1}{y_0} \right]$$

The x and the y components of the magnetic field at P are:

$$\ldots {}^2 + x_0^2 \qquad y_0^2$$

and

SPECIAL CASE

Suppose that one end of the layer is very far removed from the earth's surface; that is, suppose that x_1 is very large. For this case, the equation given above for the potential due to cross magnetization reduces to

The x and y components of the anomaly due to the cross magnetization for this case are:

$$\frac{\quad}{\partial y_0} = -2I_2 t \frac{\quad}{y_0^2 + x_0^2}$$

and

$$\frac{\quad}{\partial x_0} = 2I_2 t \quad y_0^2 + x$$

* This equation is obtained by differentiating V_2 with respect to y_0 under the integral sign and then integrating the resultant expression with respect to x_0 between the limits x_0 and x_1-x_0. In carrying out the integration use is made of the formula

$$\int \frac{2y_0^2\, dx}{[y_0^2 + (x - x_0)^2]^2} = \frac{x - x_0}{y_0^2 + (x - x_0)^2} + \int \frac{dx}{y_0^2 + (x -}$$

The vertical component of the anomaly is $A_v = (y \text{ component}) \cos\theta$—($x$ component) $\sin\theta = h_y \cos\theta - h_x \sin\theta$ and the horizontal component is $A_h = h_y \sin\theta + h_x \cos\theta$. (Compare Figure 50.) Thus, for the cross magnetization:

$$A_v = \cdot \quad \cdot \cos\theta -$$

But

$$y_0 \sin\theta + x_0 \cos\theta = l \quad \text{and} \quad x_0^2 + y_0^2 = r_0^2 = d^2 + l^2$$

$$\therefore A_{v_2} = -2I_2 t \frac{,}{d^2 + l^2} \tag{37}$$

Similarly

$$_{h_2} = \frac{\partial V_2}{\partial y_0} \sin\theta + \frac{\partial V_2}{\partial x_0} \cos\theta = 2I_2 t \left(- \frac{x_0 \sin\theta + y_0 \cos}{y_0^2 + x_0^2} \right.$$

But

$$y_0 \cos\theta - x_0 \sin\theta =$$

$$\therefore A_{h_2} = \frac{2I_2 td}{d^2 + l^2} \tag{37a}$$

The vertical and horizontal components of the anomaly due to the end effect have already been computed. In the new notation, the anomalies are

$$= \frac{\partial V_1}{\partial d} = \frac{2tI_1 d}{d^2 + l^2} \tag{36}$$

and

$$A_{h_1} = \cdot \quad \partial l \quad = \quad d^2 + l^2 \tag{36a}$$

A comparison of Equation 36 (end effect) and Equation 37 (cross magnetization effect) shows that the horizontal component due to the end effect has the same form as the vertical component due to cross magnetization. Also, the horizontal component due to cross magnetization has the same form as the vertical component due to the end effect.

The relative magnitudes of the two effects depend on the orientation of the layer with respect to the magnetic field of the earth. For example, if the surface of the layer were perpendicular to the total magnetic field of the earth, the end effect would be zero; on the other hand, if the sur-

face of the layer were parallel to the total magnetic field, the end effect alone would be present. In the general case, both effects are present, and the magnitude of each depends on the components of the magnetic field. The end effect depends on I_1 (which is equal to the susceptibility k of the layer times the component of the magnetic field normal to the end of the layer), and the cross-magnetization effect depends upon I_2 (which is equal to k times the component of the magnetic field normal to the layer).*

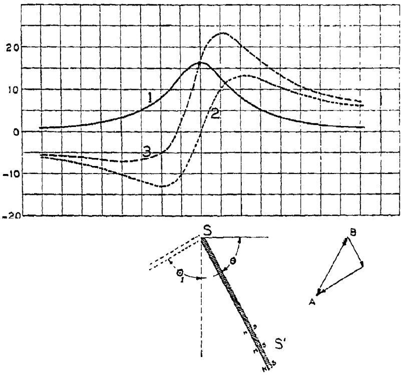

FIG. 51.—Anomalies due to a thin layer terminated by a horizontal plane. Curve 1 is the vertical component of the end effect (Equation 36); curve 2 is the vertical component due to magnetization of the layer by the field at right angles to its surface (Equation 37), and curve 3 is the resultant vertical anomaly.

The vertical anomalies due to the two effects are plotted in Figure 51 for the case that the layer makes an angle of about 32° with the vertical component of the earth's magnetic field. It should be emphasized that the end effect can be either positive or negative. That is, the sign, as well as the magnitude of this effect, depends upon the orientation of the layer. For example, if the layer in Figure 51 were horizontal ($\theta = \pi/2$), the direction of the vertical component due to the end effect would be opposite to that of the ertical component of the earth's field; that is, the end effect would be gative. Thus, in some cases, *the presence of a magnetic low may indi-e that a layer of strongly ferromagnetic material lies below the surface.* general, the end of the layer does not lie directly under the maximum point of the magnetic anomaly, and the depth to the layer can be estimated only if its orientation relative to the earth's resultant field is taken into account.

Curves 1, 2, and 3 in Figure 51 also represent the horizontal anomalies. Curve 1 is the horizontal anomaly due to cross magnetization, curve 2 is the horizontal anomaly due to the end effect, and curve 3 is the total horizontal anomaly.

Effects Due to a Thick Layer.—Consider the effect produced at a point P (x_0y_0) by a magnetized strip which is perpendicular to the plane of the paper. (Figure 52.) The magnetized strip corresponds essentially to a linear distribution. Hence, the potential at an external point due to a magnetized strip of thickness dx is given by Equation 31; that is, $dV =$

* It is assumed here that the susceptibility of the surrounding medium is negligible.

$2I \log r\, dx$. The potential due to the entire surface between $x = x_1$ $x = x_2$ is

$$V = 2\int_{x_1}^{x_2} I \log r\, dx = 2\int [I \log \{y_0^2 + (x - . \qquad dx$$

The x and y components of the magnetic force due to the surface layer at the point $(x_0 y_0)$ are

$$\frac{\partial V}{\partial x_0}\Big) = 2\int I \frac{\partial \log r}{\partial x_0}\, dx = 2I\int \frac{(x - x_0)\, dx}{}$$

$$I \{\log [y_0^2 + (x_2 - x_0)^2]^{1/2} - \log [y_0^2 + (x_1 \qquad (38)$$

$$2\int_{x_1}^{x_2} I \frac{\partial \log r}{\partial y_0}\, dx = 2I\int_{x_1}^{x_2} \frac{y_0\, dx}{y_0^2 + (x - x_0)^2}$$

$$2I\left[\tan^{-1}\frac{(x_2 - x_0)}{y_0} - \tan^{-1}\frac{(x_1 - x_0)}{y_0}\right] \qquad (39)$$

The vertical and horizontal components due to the end of the thick layer are given by the end effect equations for a thin layer. The total magnetic anomaly produced by a thick layer may be calculated by adding the effects produced by its surfaces and its end.

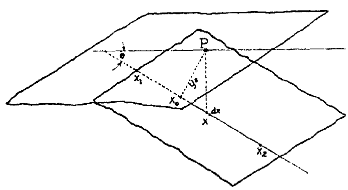

FIG. 52.—Sketch showing quantities which enter into the calculation of the magnetic potential produced by a thick layer at an external point P.

Special Cases

1. *Vertical Anomaly Near the End of a Thick Horizontal Layer.*—Assume that the upper surface of the layer is buried at a depth of 200 feet and that the layer is 1,000 feet thick. It will be convenient to consider separately the effects of the upper surface, the lower surface, and finally, the end of the layer. For a horizontal layer, the vertical anomalies produced by the surfaces are equal to the y components. Hence, the vertical component due to the upper surface of the layer is:

$$\frac{\partial V}{\partial y_0} \cdots - \tan^{-}$$

If it is assumed that one end of the layer is vertical and the other end is very far removed from the region in which anomalies are being measured, the first quantity in the bracket approaches $\pi/2$. The second quantity in the bracket may be simplified by setting x_1 equal to zero. When these substitutions have been made, the last equation becomes:

$$\frac{\partial V}{\partial y_0} \cdots \Big[\frac{\pi}{2} \cdots \frac{}{y_0} \Big]$$

where $y_0 = 200$ and $x_1 = 0$. Similarly the vertical anomaly due to the lower surface is

$$\frac{}{y_0}$$

where $y_0 = 1200$ and $x_1 = 0$. The vertical component due to the end is equal to the

horizontal component due to the surface. (Compare p. 127.) Thus, the vertical anomaly due to the end is

$$= I_2 [\log \quad {}_0 - x_2)^2\} - \log \{y_0$$

2. *Vertical Anomaly Due to a Thick Vertical Vein.*—The calculation of the vertical anomaly due to a thick vertical vein, the upper surface of which is located at a depth d below the surface, is essentially similar to that for a thick horizontal layer. First, the effect due to the upper surface of the layer is calculated. This may be assumed to be equivalent to the effect of a horizontal strip. Hence, the effect due to the surface of the layer is given by Equation 39; that is,

$$= 2I_1 \left[\tan^{-1} \frac{\quad}{y_0} - \tan^{-1} \frac{\quad}{y_0} \right]$$

The effect due to the side of the layer is essentially the same as that due to a strip placed at right angles to the surface. Hence, the vertical component due to the side is given by Equation 38; that is,

$$\frac{\partial V}{\partial x_0} = I_2 \{\log [y_0^2 + (x_2 - x_0)^2] - \log [y_0^2 + (x_1 - x_0)^2]\}$$

If this equation is used for estimating the effects of each side of the layer separately, the following difficulty is encountered: When one end of the layer is assumed to be far away, the vertical anomaly due to one side becomes very large and positive, while the vertical anomaly due to the other side of the layer also becomes very large but negative. For this reason it is necessary to add the effects due to the two sides before

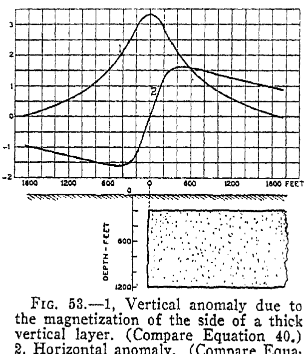

FIG. 53.—1, Vertical anomaly due to the magnetization of the side of a thick vertical layer. (Compare Equation 40.) 2. Horizontal anomaly. (Compare Equation 39.)

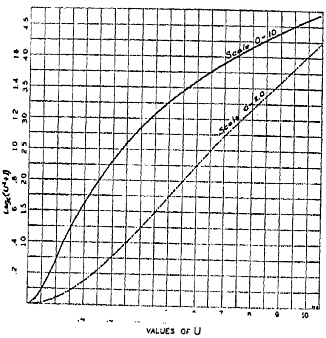

FIG. 54.—Auxiliary curve.

integrating Equation 38. A material simplification is achieved by assuming the other end of the layer removed to infinity as then the expression for the vertical anomaly due to the side reduces to

(40)

where $y_1 = y_0 +$ the width of the layer.

The vertical anomaly due to the side of the thick vein is plotted in Figure 53 for an assumed negative value of I_2. (To simplify calculations, auxiliary curves are plotted in Figure 54.)

Effects Due to Surface Irregularities. *(A Cliff.)*—Consider the vertical anomaly in the neighborhood of a vertical cliff. (Figure 55.) The vertical component of the field along a line AA' is the resultant of the effects due to surfaces A, C, and B. The effect of surface A will be approximately constant, as the instrument is usually relatively close to the ground and the effects due to the edge are not apparent until the distance from the edge becomes comparable with the height of the instrument above ground. Surfaces C and B may be considered as strips and their effects calculated from the formulas already given.

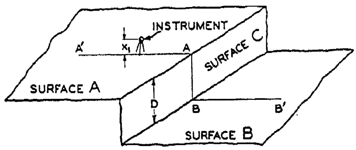

Fig. 55.—Surface irregularity. (A cliff.)

C. Magnetic Anomalies Produced by a Uniformly Magnetized Sphere.—The effect of induction in the earth's magnetic field for a sphere may be analyzed as follows: † Consider that the sphere has two volume densities of magnetic charge, $+\tau$ and $-\tau$, which coincide when there is no external field. (Figure 56.)‡ In the presence of a field, the sphere of positive charge is displaced relative to the sphere of negative charge in the direction of the field by a distance $00'$. As a consequence of this displacement the sphere becomes uniformly magnetized with an intensity of magnetization.

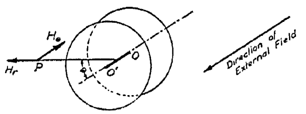

Fig. 56.—A uniformly magnetized sphere is equivalent to two spheres having equal and opposite densities of magnetic charge.

$$I = \tau \cdot \overline{00'} \tag{41}$$

It may be proved that the field at outside points due to a spherical distribution of magnetic charge is the same as that due to a single charge concentrated at the center of the sphere. Hence, the magnetic field at an external point P is the same as that produced by a dipole of magnetic moment:

$$M = 4/3\ (\pi R^3\, \tau \cdot \overline{00'}) = 4/3 \tag{42}$$

where R is the radius of the sphere.

† Compare Starling, *loc. cit.*, p. 142.
‡ Starling, *loc. cit.*, pp. 268, 269.

The field H' due to the magnetic moment M is shown by the broken lines of Figure 57. It is apparent that inside the sphere and in certain regions outside, the induced field H' opposes the earth's field (solid lines). The demagnetizing field H' is proportional to the moment of the magnetic dipole to which it is due, and this in turn is proportional to the intensity of magnetization. Hence, $H' = -NI$, where N is a constant. The "demagnetizing factor" N depends on the geometrical form of the magnetized body. It may be shown that for a sphere $N = 4/3\pi$. Therefore, if the total intensity of the earth's field is denoted by T, the effective field inside the sphere may be written: $T - 4/3\pi I$.

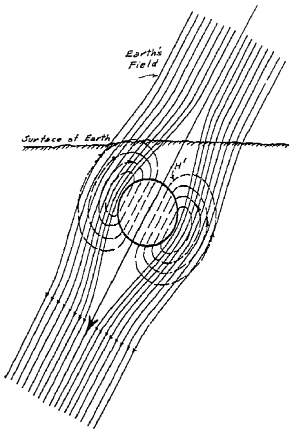

FIG. 57. — Diagram illustrating demagnetization effect: The field H' due to a uniformly magnetized sphere opposes the earth's field.

The intensity of magnetization is proportional to the effective field inside the sphere; that is,

$$kT \tag{43}$$

where k is the susceptibility of the paramagnetic material constituting the sphere.

Equation 43 corresponds to the case that the susceptibility of the medium in which the sphere is imbedded is zero. If the susceptibility k_0 of the surrounding medium is not zero, the last equation becomes

$$I = \frac{(k - k_0)\, T}{4/3\pi\,(k - k_0)}$$

and

$$M = \frac{}{1 + 4/3\pi\,(k - k_0)} \tag{44}$$

On introducing a new variable c defined by the equation

$$c \equiv \frac{4/3\,\pi R^3\,(k - k_0)}{4/3\pi\,(k - k_0)} \tag{45}$$

the expression for the magnetic moment M becomes

$$M = cT \tag{46}$$

The potential ΔV at an external point P due to the uniformly magnetized sphere is

$$\Delta V = \frac{M \cos\theta}{r^2} \tag{47}$$

where M is the magnetic moment of the sphere, r is the distance between the center of the sphere and the point P, and θ is the angle formed by r

and θ. (Figure 58.)† The components of the magnetic field in the direction of increasing r and θ are

$$2M\cos\theta$$

(48)

$$\frac{M\sin\theta}{r^3}$$

respectively.

The horizontal and vertical components of the magnetic anomaly at a point P on the surface of the earth may be obtained by resolving the magnetic moment M of the sphere into the components M_x and M_z. (Figure 58a.)

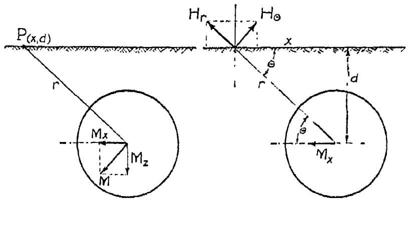

FIG. 58.
a. Sketch illustrating method for resolving magnetic moment M into components M_x and M_z.
b. Sketch illustrating method for determining horizontal and vertical magnetic anomalies due to magnetic moment M_x.

The components of the field in the directions of increasing r and θ due to M_x are

$$(\Delta H_r)_x = 2M_x\cos\theta \quad 2M_x x$$

and

$$_x \sin \qquad r^4$$

The net x component $(\Delta H)_x$ of these fields is

$$2M_x x^2 \qquad M_x d^2$$

and the net z component $(\Delta Z)_x$ is

$$2M_x xd + \qquad 3M_x xd$$

In a similar manner it may be shown that the net x component $(\Delta H)_z$ of the fields due to M_z is

and the net z component $(\Delta Z)_z$ is

† Compare Starling, *loc. cit.*, p. 15.

The total x component ΔH due to M_x and M_z is

$$r^5 \qquad r^5 \qquad r^5 \tag{49}$$

and the total z component ΔZ due to M_x and M_z is

$$2M_zd^2 \tag{50}$$

Equations 49 and 50 may be simplified by expressing the magnetic moments M_x and M_z in terms of the horizontal and vertical components H and Z of the earth's normal field. That is,

$$M_x = cH = cZ$$

and

$$M_z = cZ = cH$$

On substituting these values into Equations 49 and 50 and simplifying, one obtains †

$$= \frac{cH}{r^5}\left(2x^2 - d^2 - 3xd\,\frac{H}{Z}\right) \tag{51}$$

$$Z = \frac{cZ}{r^5}\left(x^2 - 2d^2 + 3xd\,\frac{H}{Z}\right) \tag{52}$$

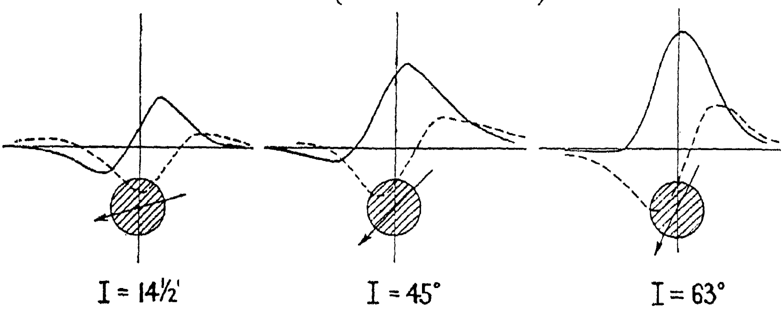

FIG. 59.—Sketches illustrating the anomalies produced by a uniformly magnetized sphere in regions of different inclination. (After Haalck, *Die Magnetischen Verfahren der Angewandten Geophysik.*)

Equations 51 and 52 show that the anomalies ΔH and ΔZ depend on the normal values H and Z. That is, the anomalies produced by a uniformly magnetized sphere depend on the normal value of the inclination of the earth's field in the region in which the subsurface sphere is located. The ΔH and ΔZ profiles for three values of the inclination are shown in Figure 59. ‡

† Compare H. Haalck, "Die Magnetischen Methoden der Angewandten Geophysik," pp. 330-332. *Handbuch der Experimentalphysik,* edited by W. Wien and F. Harms. (Akademische Verlagsgesellschaft M.B.H., Leipzig 1930.) Vol. 25, part III.

‡ H. Haalck, *Die Magnetischen Verfahren der Angewandten Geophysik* (Gebrüder Bornträger, Berlin, 1927).

EMPIRICAL METHODS OF INTERPRETATION: CORRELATION WITH KNOWN GEOLOGY

In the practical application of magnetic measurements to the solution of subsurface problems of economic geology, theoretical calculations can be carried out only for relatively simple bodies, e.g., two-dimensional bodies, spheres, etc. However, the usual configurations of geologic bodies rarely produce the simple magnetic effects equivalent to those produced by simple geometric forms. The interpreter, therefore, must resort to other aids besides direct theory when solving his field problems. The three most effective aids are: first, and most important, adequate geologic or subsurface control; second, magnetic studies over known geologic conditions in the immediate area; and third, small scale model experiments.

The usual interpretative technique is almost exclusively empirical. Deductions and inferences are drawn qualitatively from the sizes, shapes, and configurations of the magnetic anomalies and are coordinated with the known regional and local geology.

In many problems where prior subsurface development work has been done, it is advantageous to carry out magnetic studies over known subsurface features or producing fields and utilize the results for interpretative control. For instance, in magnetic studies of the Sparta-Wilcox trend in central Louisiana, control studies were conducted over the Eola and Cheneyville structures, and the control used as a guide in interpretation. Similar studies have been employed in mapping the extension of the Kettleman structure in California. The magnetic results obtained over the Hobbs' field in Lea County, New Mexico (see Figure 65) have been used as interpretative control for numerous studies made in that area. The rapidity and low cost of magnetic work allows this type of interpretative control to be economically feasible. The combined reconnaissance and detail work in this type of study seldom cost more than 1½ to 2½ cents per acre.

Model Experiments.—Model experiments are of value in determining the relationship between the directions of the magnetic vectors in space and the shape of a magnetic structure. The operating principle is to construct small-scale models having relative dimensions and geometric disposition similar to those assumed for the anomalous geologic bodies.

Experiments on model ore bodies have been reported by Hotchkiss,† Keys,‡ and Jenny.§

Hotchkiss investigated the magnetic effects as a function of the dip and strike of model formations. One of the experimental arrangements consisted of a drawing board with a sheet of tin suspended by a wood frame from its side. The sheet was positioned at various distances from

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† W. O. Hotchkiss, *Mineral Land Classification*, Wisconsin Geological and Natural History Survey Bulletin, No. 44, Madison, 1915, Ch. IV, "Magnetic Observations," pp. 112-126.

‡ D. A. Keys, *A.I.M.E. Geophysical Prospecting*, 1932, pp. 205-208.

§ W. P. Jenny, "Experimental Interpretation of Magnetic and Gravimetric Anomalies," *Terr. Mag.*, 40 (1), 1935, p. 72.

the board and at different angles of dip. The magnetic declination was ed with a compass at various positions along "traverses" parallel sides of a wood T square. The sheet could also be oriented at different strike angles with the magnetic field.

The Keys' experiment was a laboratory investigation of the magnetic effects of a "thin magnetized dike of limited length." † A. S. Eve, ‡ in collaboration with Keys and several other investigators, had carried out a survey over a portion of the Falconbridge ore body in which two sets of measurements were made with an Askania magnetometer, one set being made on the ground and the other on elevated platforms. The purpose of the survey was to determine the lower depth of the ore body from the differences in the anomalies at the two levels (ground and p survey did eld an accurate value of the lower depth, experiment correctness of the method of interpretation used in the survey.

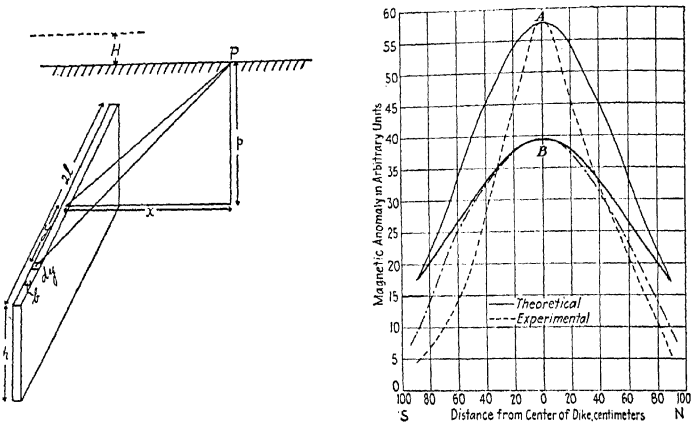

FIG. 60.
(a) Sketch of narrow magnetized dike of limited dimensions.
(b) Experimental and theoretical vertical magnetic anomalies at two different levels over a model magnetic dike. (Keys, *A.I.M.E. Geophysical Prospecting*, 1932, p. 206.)

A sheet of mild steel, which was approximately one-half inch thick, four feet high, and ten feet long, was supported vertically on its edge with its length perpendicular to the magnetic meridian. To magnetize the sheet ten turns of insulated copper wire were wound lengthwise around its center and a current of one ampere passed through the wire in a direction to make the upper edge a south pole. An Askania vertical variometer was

† D. A. Keys, *loc. cit.*

‡ A. S. Eve, "A Magnetic Method of Estimating the Height of Some Buried Magnetic Bodies," *A.I.M.E. Geophysical Prospecting*, 1932, pp. 200-205.

mounted on a platform so that the vertical distance between the needle of the instrument and the top of the steel plate was 88 cm. Measurements were made of the vertical anomalies at various points along a line parallel to the magnetic meridian through the center of the plate. A second set of measurements was then made along the same "traverse" with the needle 111 cm. above the top of the steel plate.

The results of the laboratory investigations are shown in Figure 60. In plotting these curves, the experimental readings were all multiplied by an arbitrary factor to facilitate the comparison of the theoretical and experimental results.

The "height" h of the dike, i.e., the vertical distance between the poles, is related to the ratio r of the readings over the center of the dike at two levels separated by a vertical distance H by the formula

$$1 \qquad\qquad 1$$

$$(p \qquad \overline{\cdot\, h)^2} \cdot \qquad (p + 2h$$

where $p =$ the depth to the top of the dike, and $l =$ one-half the length of the dike.

In addition to investigations of model ore bodies, laboratory experiments have been carried out to determine the depth and geometric configuration of regional subsurface structural features. Figure 61 shows a schematic section of an apparatus devised by Jenny† which permits quantitative interpretation of vertical magnetic anomalies. Vertical bar magnets *1* are connected with counterweights *4* by means of ribbons *2* which pass over pulleys *3*. The displacements of the dip needles *5, 6, 7* produced by the magnets when they are all at the same level are compensated by counterweights (not shown) which are attached to the needles and by screws *8*. To determine depths, the bar magnets are displaced by means of the counterweights *4* so as to correspond to the assumed structure. For example, the positions of the bar magnets shown in Figure 61 would correspond to an anticline. Contours of equal depth may be obtained by studying several cross sections.

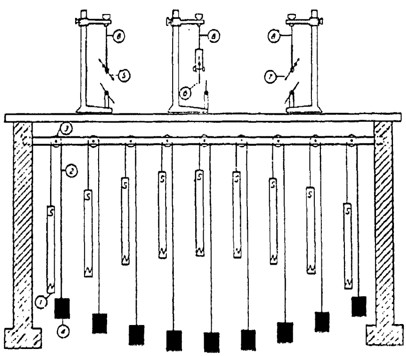

FIG. 61.—Schematic diagram of apparatus for experimental interpretation of magnetic and gravimetric anomalies. (Jenny, *Terr. Mag.* 40 (1), 1935, p. 72.)

† W. P. Jenny, *loc. cit.*, p. 74.

The most flexible model arrangement for magnetic studies comprises a mixture of coarse iron filings and clean white sand placed in a large wooden tank, preferably about 8 feet in diameter and 2 feet high. The tank should be provided with a cover in order to exclude moisture and minimize rusting of the iron. The orientation of the tank with respect to the magnetic field can be varied by mounting it on a turntable. The latter should be placed on a wooden track with rollers so that it can be readily moved. The magnetic observations are made with the regular horizontal and vertical component magnetometers. The latter are mounted on a rigid platform above the movable tank. The "depth" of the magnetic body is simulated by varying the height of the magnetometers above the tank. Any desired shape and configuration of the body may be simulated by proper shaping of the sand-iron mixture.

The iron filings are preferably of the coarse type obtained from milling operations. Those employed by the author were obtained by milling a stack of laminated iron sheets across grain. (The iron sheets came from the core of an old power transformer.) The milling was done on a bias to give short cuttings. A mixture of sand and 10 to 30 per cent of filings, by weight, gives an easily handled material with sufficient magnetic permeability to allow sharp readings.

The movable tank arrangement with stationary magnetometer involves considerably more preparatory work than the use of a stationary tank or earth pit with a movable magnetometer. However, more accurate and rapid work may be done with the movable tank arrangement, because the magnetometer may be adjusted at the beginning of the experiment and not disturbed thereafter. Thus, the errors of misorientation, leveling, and other similar instrumental errors are avoided. The magnetometer is unclamped at the beginning of the experiment and readings are made thereafter without disturbing the instrument.

ILLUSTRATIONS OF GEOMAGNETIC SURVEYS

Regional Surveys.—Magnetic investigations of a deep-seated regional structure usually necessitate a careful interpretation based on comparisons of the observed and calculated (theoretical) values of the vertical and horizontal components along traverse lines. Figure 62 shows an investigation of this type across the Los Angeles Basin.† This basin is a broad syncline with an axial trend northwest to southeast and is about 75 miles long and 25 miles wide. The geologic section shown in the figure is based on geologic, drill, seismic, and gravity data. It is predicted that this structural basin reaches a depth of over 40,000 feet at its deepest point. The basement is granitic and is overlain with about 15,000 feet of schists and other non-crystalline metamorphic rocks, which are probably of Franciscan age. Overlying the metamorphosed rocks is a section of about

† L. F. Uhrig and S. Schafer, "Observed and Calculated Values of the Magnetic Intensity over a Major Geologic Structure," *Gerland's Beiträge zur Geophysik*, vol. 49, pp. 129-139, 1937.

25,000 feet of soft sandstones and shales of marine origin, chiefly of Tertiary age. Shallow alluvial deposits form the surface covering. High angle faults of considerable displacement trend in a general northwest and southeast direction.

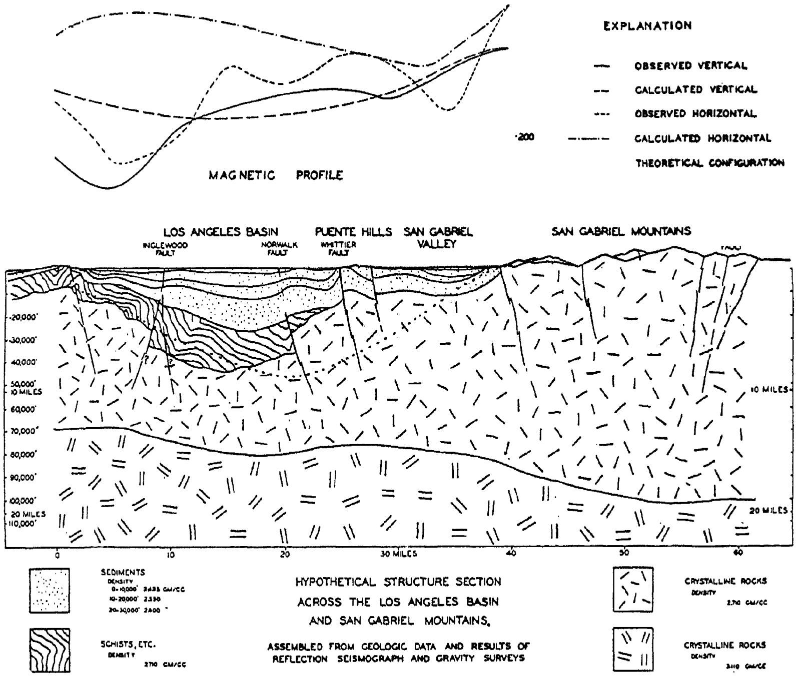

FIG. 62.—Illustration of regional survey in the Los Angeles Basin. (After Uhrig and Schafer, *Gerland's Beiträge zur Geophysik.*)

The magnetic investigations were conducted along a 45 mile traverse and comprised: (a) vertical component measurements at intervals of ¼ to ½ mile and (b) horizontal component measurements at intervals of from 3 to 4 miles.

The magnetic work failed to show the positions of the large faults which traverse the basin parallel to its major axis. At the shallower depths, the materials on both sides of the faults have about the same permeability; hence, they show no definite anomalies at their contacts. The computations for the calculated vertical and horizontal components were made by assuming that the contact between the overlying materials and the granite is an inclined smooth surface. The air, sediments, and metamorphics were given a weighted susceptibility of about 40×10^{-6} c.g.s. units while the granite was assigned a value of 1012×10^{-6} units.† It was assumed also that the

† J. L. Soske, Unpublished Doctor's Thesis, Calif. Inst. of Tech., Pasadena, California.

field strength was 50,000 gammas and that the direction of the field made an angle of 30° with the vertical. The general agreement between the theoretical calculated configuration and the assumed configuration (as based on drill hole and geologic control, supplemented by seismic and gravity work) is fairly typical of the results which may be expected in a complex problem of this type.

Mapping Contacts by Magnetic Methods.—The contact between two materials of different magnetic permeabilities is readily disclosed by magnetic studies provided their difference in permeability is sufficient to produce a measurable anomaly. An application of magnetic work in contact mapping is shown in Figure 63. The high magnetic permeability of

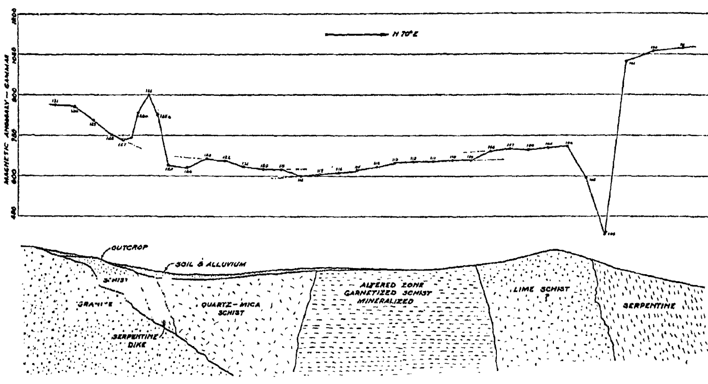

FIG. 63.—Magnetic profile over contacts of lime schist and serpentines.

the serpentine gives rise to the large anomaly at the contact between the lime schist and the serpentine at the right end of the traverse and the short abrupt anomaly at the left end of the traverse. Due to the relatively small difference in permeability between the quartz mica schist and the altered zone of garnetized schist, and between the altered zone and the lime schist, there are no appreciable magnetic anomalies or change in trend produced at their gradational contacts. The effect of the high permeability of the granite basement rocks is shown by the gradual rise in the magnetic trend starting at about station 118 and extending to the left of the figure.

The profile also shows the magnetic high produced by an intrusive serpentine dike (at the left of the traverse). (Direct location of the dike could not be made by surface geology because of the covering of 50 feet of soil and alluvium fill.)

Magnetic Anomalies Over Steeply Dipping Structures.—In regions containing steeply dipping structures, the crest of the deeper sedimentary structure is usually over the crest of the uplift in the basement rocks. The magnetic anomaly is caused by the high magnetic permeability of the igneous basement rock. The overlying sediments usually have a relatively minor effect on the anomaly.

The peak of the magnetic vertical intensity often is shifted laterally due to the inclination of the earth's field. The amount of shift depends upon many factors, including depth to the basement rocks, relative permeabilities, inclination of the earth's field, etc. This effect is illustrated schematically in Figure 64.

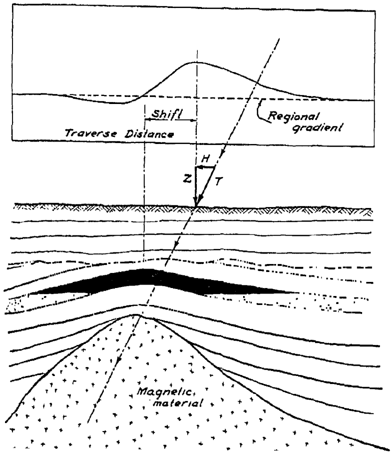

FIG. 64.—Diagram illustrating displacement of the peak of the magnetic anomaly with reference to the crest of the structure.

An example of this phenomenon is furnished by the Hobbs' oil field in Lea County, New Mexico. The field was originally discovered as the result of magnetic work in 1926. The discovery well (1928) was located in the center of the magnetic high, which fortunately was sufficiently "on structure" to give a small producer. † Figure 65 shows the magnetic contours and the final structure contours based on well logs obtained as the field was developed.‡ In this case there is a shift of about eight thousand feet between the center of the structural high and the center of the magnetic high produced by the basement rock. The field includes approximately 6,000 acres and 143 producing wells. Two oil producing zones exist: the Bowers sand at a depth of 3,170 to 3,225 feet; and the white lime, a white or crystalline lime of Permian age and occurring at a depth of about 4,000 to 4,200 feet. The sedimentary series has a magnetic permeability of approximately one, and the basement granite has a relatively high magnetic permeability. The topography is flat, and the surface has a southeast slope of about 10 feet to the mile. The surface formation is caliche, and there are no outcrops.

Mapping Faults and Flows by Magnetic Methods.—The magnetic method may sometimes be employed to map the locations of faults in sedimentary rocks. Ordinarily the magnetic location of faults is contingent upon: (1) concentration of magnetic mineralization along the fault or, (2) displacement by the fault of the subsurface body which exerts the controlling influence on the magnetic field at the surface. In the latter

† C. B. Carpenter and H. B. Hill, "Petroleum Engineering Report, Big Spring and other Fields in West Texas and Southeastern New Mexico," Dept. of Interior, R. I. 3316, Nov. 1936.

‡ A somewhat similar figure is shown in a trade pamphlet entitled "Mapping Geologic Structure with the Magnetometric Methods," published by William M. Barrett, Inc., Shreveport, Louisiana.

case, the field at the surface may be associated with a displacement of a relatively weak magnetic stratum or with a displacement of the relatively high magnetic basement complex.

The magnetic effect or anomaly at the surface is chiefly a function of the difference in depths to the controlling magnetic body on adjacent sides

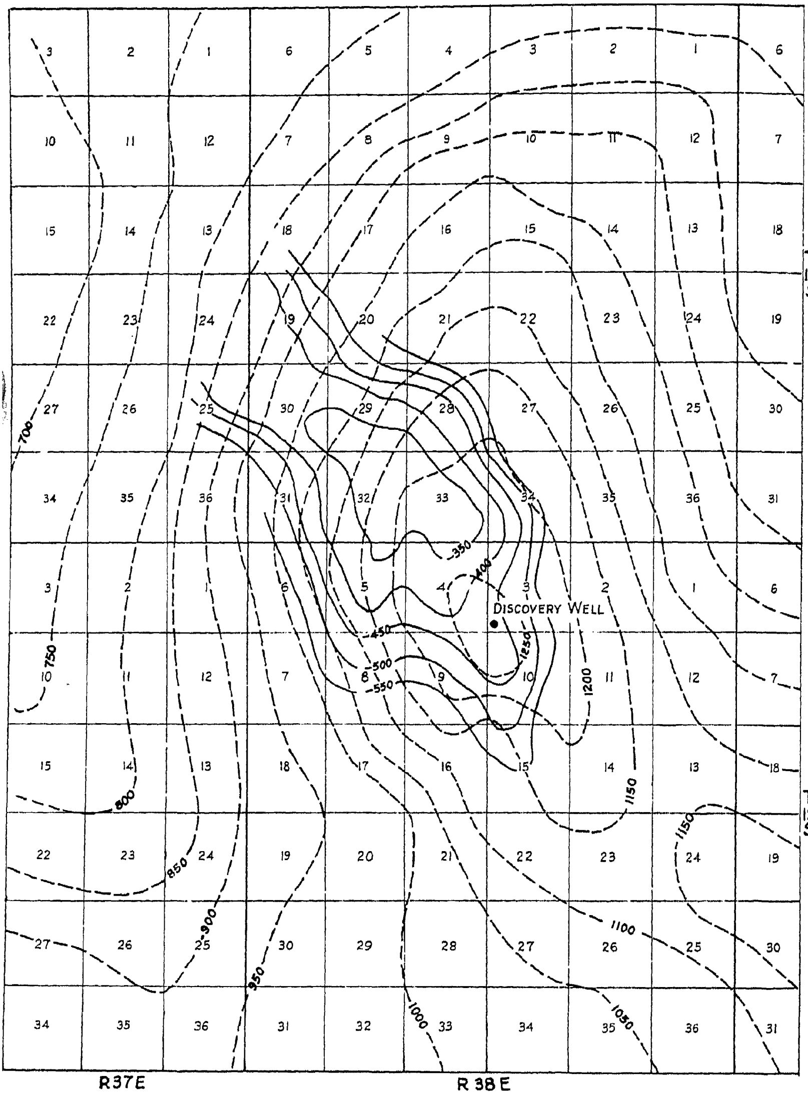

FIG. 65.—Magnetic contours and structure contours of Hobbs' Oil Field, Lea County, New Mexico. Structural contours (solid lines) after R. S. Christie, *A.I.M.E. Petroleum Tech.* 1932; magnetic contours (dotted lines) after H. T. Morley, Stanolind Oil and Gas Company.

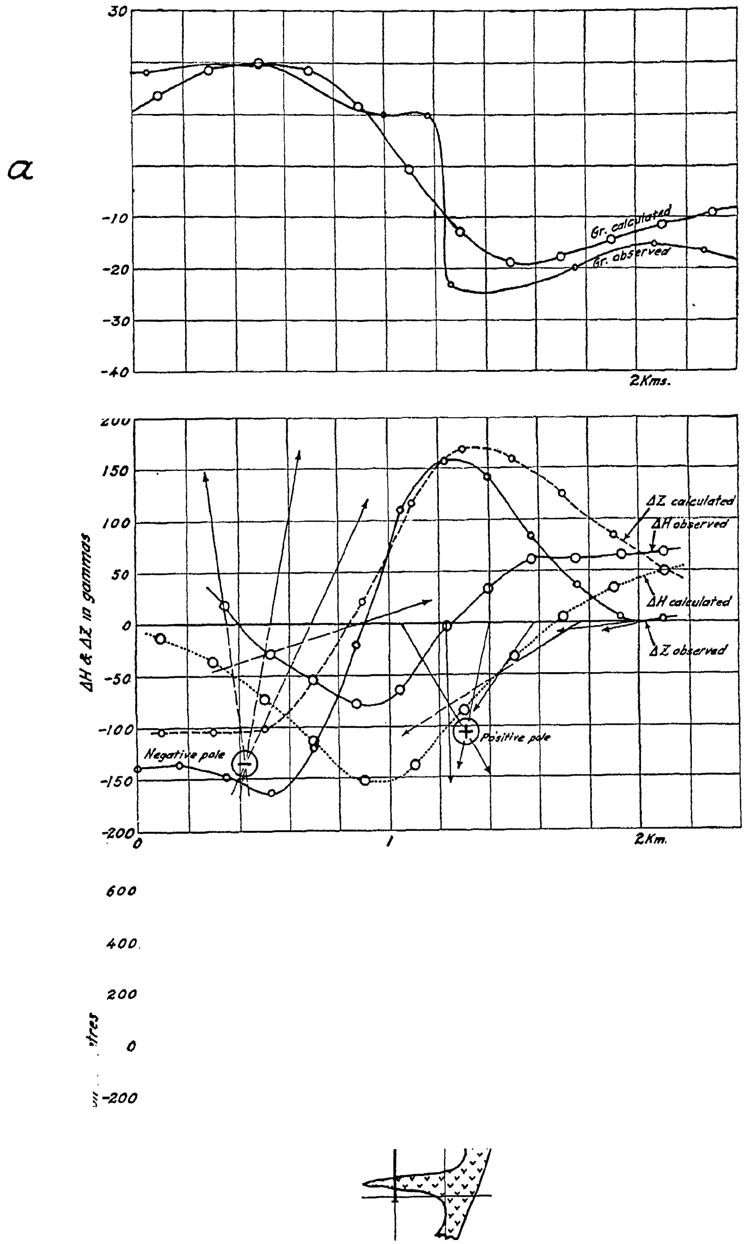

FIG. 66.—Magnetic contours over irregularly-shaped laccolithic mass. (*a*) observed and (*b*) calculated anomalies over intrusive mass of predicted shape shown in (*c*). (Malamphy, *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 696.)

of the fault; i.e., the higher magnetic intensity occurs over the upthrow side of the fault. The plan location of the magnetic anomaly with reference to the fault is subject to the same considerations regarding displacement discussed in the preceding example. Ordinarily, however, this displacement is slight, due to the usually moderate depth to the controlling magnetic feature.

An example of magnetic work over an irregularly-shaped laccolithic mass at moderate depth (about 1500 feet) is given by Malamphy. † The results obtained from the magnetic and gravitational studies are shown in Figure 66. The magnetic inclination in this area is only 20° and has a reversed polarity with respect to that existing in the northern hemisphere.

The gravity gradients were calculated by the method described by Barton. ‡ The theoretical and observed gravity gradient profiles are substantially in agreement.

The theoretical and observed magnetic profiles do not agree very well. According to Malamphy, the observed and theoretical magnetic and gravity gradient data can all be consistent only if the laccolithic mass possesses "magnetic properties in excess of those that normally would be induced by the earth's magnetic field alone."

The gravitational and magnetic anomalies are due chiefly to the igneous intrusions of the diabase, and to a minor extent, to the crystalline basement rocks.

Magnetic Anomalies Produced by Upper Magnetic Beds of Variable Thickness.—The magnetic anomalies produced by magnetic sedimentary beds or lava flows which overlie a substantially non-magnetic or diamagnetic basement depend on the depth to the contact with the underlying rocks. Thus, if the magnetic beds overlie

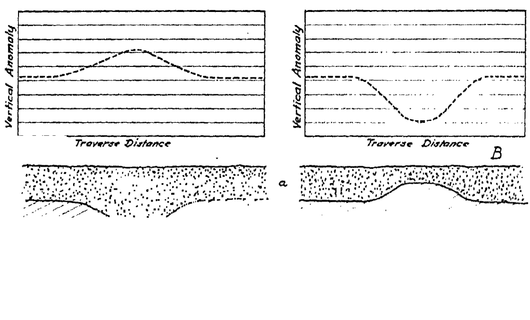

FIG. 67.—*A*. magnetic high associated with increased thickness of shallow magnetic beds, and *B*, magnetic low associated with decreased thickness of shallow magnetic beds.

† Mark C. Malamphy, "Geophysical-Geological Study of the Sao Pedro Area, Brazil," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 696, 1936.

‡ D. C. Barton, "Eötvös Torsion Balance Method of Mapping Geologic Structure," *A.I.M.E. Geophysical Prospecting*, 1929, p. 429.

a synclinal or old erosional low feature having a relatively low permeability, a magnetic high may be produced, as illustrated in Figure 67A, due to the greater thickness of the magnetic material over the deeper contact. When the upper magnetic beds overlie an anticlinal feature of relatively low permeability, as in B, a magnetic low may result due to the lesser thickness of magnetic material over the basement high. (A supplemental geophysical method (such as electrical or seismic) capable of indicating variations of thickness in the upper layer should be employed in problems of this type. Interpretation would then be based upon both types of data.)

Placer Deposits.—In a majority of placers the best gold values in the pay streaks will be found immediately above bedrock. In other placers the best values occur on a false bedrock ("hardpan" or conglomerate) which overlies the bedrock. The magnetic methods are especially valuable in reconnaissance work in location of placer deposits. Because the gold is found adjacent to the bedrock or false bedrock, it is important to know the depths to the bedrock and the thickness of the fill material. This may be accomplished by electrical or shallow refraction seismic methods.

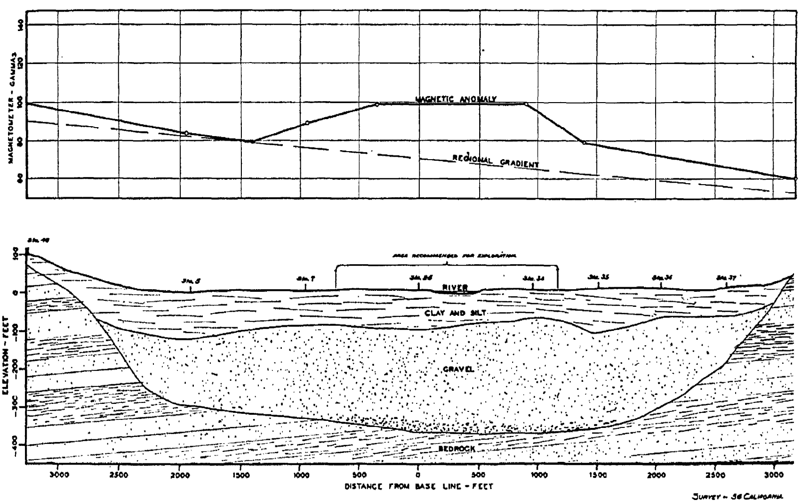

FIG. 68.—Magnetic traverse across a simple placer concentration in an old stream bed, Trinity County, California. (Jakosky, *Arizona Mining Journal*, Aug. 1931.)

Figure 68 shows a typical cross-section from a geophysical survey conducted in Trinity County, California. By means of the combined magnetic and electrical surveys the complete outline of the old stream bed was obtained, together with the thickness of the lower gravels and the overlying clay and sand fill. With this information the necessary depth of test pits and drill holes was easily determined.

Another simple magnetic survey is illustrated by the contour results of

a survey of a placer property in Pinal County, Ari:
The general regional gradient extended in a northwest-southeast direc..
The concentrations of placer materials is well shown by the magne

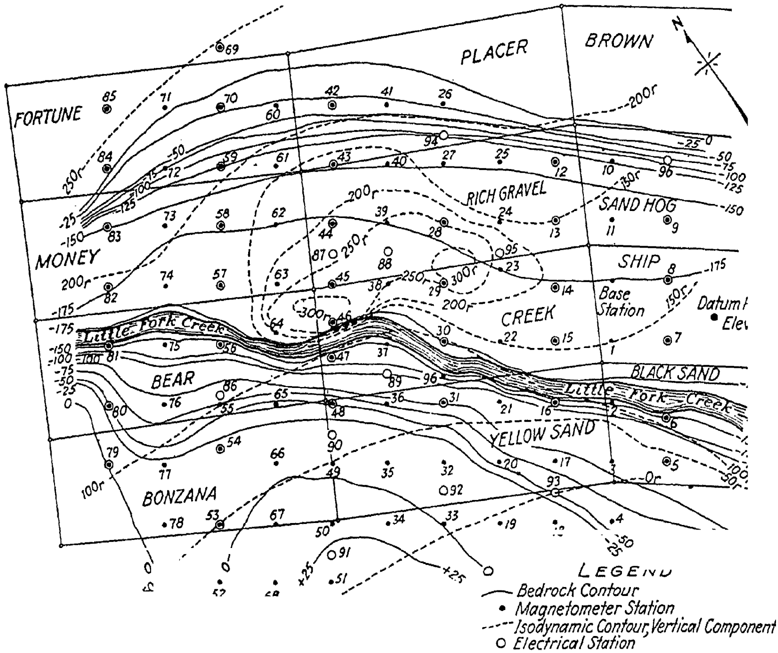

FIG. 69.—Magnetic and geoelectrical contours on a gold placer property in Pinal County, Arizona. (Magnetic contours dotted curves and geoelectrical bedrock contours solid curves.) (Jakosky and Wilson, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 515.)

highs.* The magnetic conditions in this particular area are almost ideal for application and interpretation of magnetometer studies.

Ilmenite, Pyrrhotite and Nickel Ore Deposits.—Ilmenite deposits have been located by magnetic methods by Gillson ‡ and Keys. § Also, certain nickel-ore bodies which contain pyrrhotite have been located by magnetic investigations. ††

† J. J. Jakosky and C. H. Wilson, "Geophysical Studies in Placer and Water-Supply Problems," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 515, 1933.

* In the application of magnetic measurements to placer gold location, the interpretation must be made from the viewpoint that the magnetic methods are not locating the gold itself, but only the placer magnetic materials with which the gold is associated.

‡ J. L. Gillson, "Genesis of the Ilmenite Deposits of St. Urbain County, Charlevoix, Quebec," *Economic Geology*, Vol. 27, Sept.-Oct. 1932, pp. 554-577.

§ F. W. Lee, "Results of Some Magnetic Measurements on Dikes . . . in the Sudbury Dist. Ontario, Canada," U. S. Bureau of Mines, Tech. Paper No. 510, 1932.

†† Lee, *loc. cit.*

L. B. Slichter, "Certain Aspects of Mag. Surveying," *A.I.M.E. Geophysical Prospecting*, 1929, pp. 238-260.

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CHAPTER IV

GRAVITATIONAL METHODS

Prospecting by gravitational methods is the technique of measuring the differences or deviations of the gravitational field at the earth's surface and utilizing the data thus obtained to predict the subsurface structure. Gravitational methods are analogous to magnetic methods in that quantitative investigations are made of a natural field of force. The magnetic and gravitational methods are also analogous in their fundamental relationships, as will be seen during the development of the theory.

The physical property of the subsurface materials which produces the significant or diagnostic gravitational anomalies is *density*. It is necessary that the effects caused by the changes in density of the subsurface materials be of sufficient magnitude to manifest themselves over near-surface, topographic, and regional effects.

FUNDAMENTAL PRINCIPLES AND PHENOMENA

The force of gravity is the familiar force commonly called weight. Numerous investigations have attested that every particle of matter in the universe attracts every other particle according to a universal law formulated by Isaac Newton. This law states that if two objects or masses m_1 and m_2 are separated by a distance r—where r is much greater than the maximum dimension of either object—the masses attract each other with a force, acting along the line joining their centers, that is directly proportional to the product of the masses and inversely proportional to the square of the distance between them. In symbols,

$$F = G \frac{}{r^2} \qquad (1)$$

Experiments show that G is a universal constant which is independent of the material of the masses and of their states. In all common units G is very small; thus, when the mass is measured in grams, the distance in centimeters, and the force in dynes, G is equal to 6.68×10^{-8}.

The expression "force of gravity" or "gravity" as used in geophysics usually applies to the gravitational attraction between the earth and a *unit* mass located on or near the earth's surface. In this case, r is not

large compared to the dimensions of the earth and the expression for the gravity becomes:

$$\text{Force per unit mass} = g = G \int \frac{dm}{r^2} \qquad (2)$$

where *dm* is an element of mass of the earth located at a distance *r* from

face.

to Newton's s

Force

A force per unit mass, therefore, is equal
gravity may be expressed in units of force per
or in units of acceleration, e.g., cm./sec².

An approximate average value of gravity may be obtained readily from Equation 2. For purposes of calculation, it will be assumed that the earth is a uniform, stationary sphere of mass M. The uniform stationary sphere is equivalent in its gravitational effects to a "particle" which has a mass M and is located at the center of the sphere. Hence, Equation 2 may be written

where R is the radius of the earth. The mass of the earth is approximately $5.99 \cdot 10^{21}$ metric tons or $5.99 \cdot 10^{27}$ grams. The value of R at sea level at the equator is approximately $6.38 \cdot 10^6$ meters or $6.38 \cdot 10^8$ centimeters. Hence

$$g = \frac{6.68 \cdot 10^{-8} \cdot 5.99 \cdot 10^{27}}{(6.38)^2 \cdot 10^{16}} \cong 980 \text{ cm/sec}^2 \cong 980 \text{ dynes/gram}$$

The simplified calculation given above assumes that the earth is a homogeneous, stationary, isolated sphere. Such a sphere would exert a radial force on objects at its surface and the force per unit mass would be the same at all points on the surface. Actually, the earth is neither homogeneous, stationary, isolated, nor spherical. The result is that the force of gravity per unit mass varies, both in direction and in magnitude, from place to place and from time to time. Furthermore, data on gravitational anomalies furnish information about the deviation of the earth from a homogeneous, stationary, isolated sphere. Interpretation of the field measurements made in gravity surveying is based upon these variations or gravitational anomalies.*

Factors Causing Variations in Gravity.—The earth's deviation from a sphere derives from two factors: (a) the topographical features, i.e., the local phenomena of valleys, hills and mountains; and (b) the oblate spheroid shape resulting from its rotation. This second factor introduces a variation in the force of gravity which is a function of latitude. This variation and the effect of the centrifugal force of rotation are usually

* At present, as far as geophysical prospecting is concerned, the fact that the earth is not isolated is of little importance. The attractions of the sun and the moon and the effects of the tides caused by them introduce changes in gravity which are smaller than the errors in observations.

lumped together and called a "latitude, or north-south correction."* From a knowledge of the mean shape of the earth and its speed of rotation about its axis, the latitude correction may be calculated mathematically; the correction amounts to about 1/194 x $\sin^2$ (latitude).

Another variation in gravity is caused by the differences in density of the materials of the earth. This phenomenon furnishes the physical basis of the gravitational methods of prospecting. Thus, a structure that is denser than the average material of the earth's crust in its neighborhood exerts a greater force on a unit mass than would have been exerted if the structure had been absent. The magnitude of this gravitational anomaly is dependent upon the density, geometric configuration, depth, and location of the structure with reference to the point at which the effect of the structure is measured.

Before outlining gravitational methods of prospecting in detail, it will be instructive: (1) to define certain concepts which are used in describing the earth's gravitational field and (2) to consider the relation between gravitational prospecting and the science of geodesy.

Gravitational Field of the Earth.—As a consequence of the inverse square law (Equation 1), there exists a physical quantity described as the gravitational potential which is analogous to the magnetic potential. The *gravitational potential* at a point is defined as the work required to move a unit mass from that point to a point infinitely distant. Gravitational potential is thus a scalar quantity.**

A *level surface* is a gravitational equipotential surface. It is a surface such that no work is done against gravity when a mass is moved between two points on it. (A level surface is not necessarily a horizontal surface.) Because the potential is constant on a level or equipotential surface it is impossible for equipotential surfaces corresponding to different values of potential to intersect, for the two surfaces would have the same values of the potential at the points of intersection and hence the same values at all other points on the surfaces.

A level or equipotential surface has the following very important property: The force of gravity is perpendicular to the surface at every point.*** Hence, if a curve is drawn which is normal at each point to the gravitational equipotential surface through that point, the force of gravity will be parallel to the curve. Such a curve is called a *vertical line*. Ordinarily, over distances which are not too large, the vertical line will be practically straight.

* The rotation of the earth about its axis causes particles on its surface to be subject to a centrifugal force. One component of the centrifugal force acts in an opposite direction to the force of gravity, and increases from zero at the poles to about 1/289 of the force of gravity at the equator.

** An alternative definition of potential may be stated as follows: The gravitational potential at any point is that quantity whose rate of change in the direction of gravity is the force of gravity at that point.

*** The mean surface of a body of water such as an ocean is a portion of a gravitational equipotential surface.

Gravitational Prospecting and the Science of Geodesy. — The science of geodesy deals with measurements of large areas of the earth's surface by triangulation and astronomical observations. One department of geodesy comprises the study of the figure of the earth. This branch investigates deviations from homogeneity of segments of the earth's crust which have a large areal extent and thickness.† The main problem in gravitational prospecting, on the other hand, concerns the deviation from homogeneity of relatively small portions of the near-surface crust of the earth. However, the methods of geodesy are instructive because similar principles of measurement are applied.

In geodetic terminology, the term *sea level* applies to the equipotential surface that most nearly coincides with the average level of the ocean. This surface is called the *geoid*. The determination of the shape of the geoid involves a combination of surveying, astronomy, and gravitational methods.

Geodetic latitude and longitude determinations at a point involve a determination of the direction of the plumb line at that point. The direction of the plumb line may be greatly affected by the presence of large bodies of abnormal density near the point of measurement. If the body is denser than the surrounding material, it will exert an unduly large force of attraction on the plumb bob and cause the plumb line to deflect toward it. Differences in latitude or longitude due to this effect are usually only a few seconds of arc. Although these differences generally do not exceed ten seconds, except in mountainous areas, they are occasionally quite large. For example, between the north and south coasts of Puerto Rico the anomalous deflection of the plumb line produced by local inhomogeneities if not taken into account would lead to an error in distance of about 1 part in 50. This example is sufficient to show that the effects of local inhomogeneities may be great enough to cause considerable difficulty in geodetic measurements.

Isostasy

If the shape of a mountain or valley or any other topographical feature and the densities of the materials composing the feature and surrounding it are known, it is possible to compute the effect of the topographical feature on the gravitational field. It is found, however, that for large features, the effects so computed are generally much greater than the measured effect. The discrepancy in most cases is far in excess of the probable variations due to inaccuracy of data or calculations. This fact forms the experimental basis of the hypothesis of *isostasy* or *isostatic compensation*.‡

According to this hypothesis the apparent excess of matter represented by a hill or the apparent deficiency of matter represented by a valley or an ocean basin is compensated by underlying materials. Thus, beneath each hill there is somewhere sufficient

† National Research Council, *Physics of the Earth-II: The Figure of the Earth*, Washington 1931.

‡ William Bowie, *Isostasy* (E. P. Dutton Co., 1927); "Isostatic Investigations, etc." *U. S. Coast and Geodetic Spec. Pub.* Dept. of Commerce, 99, Serial 246, 1924.

material of lower density to compensate for the hill so that in reality there is little, if any, real excess of matter. Quite generally, the hypothesis of isostatic compensation postulates that the amount of material standing upon a unit area will be the same regardless of whether it is under highlands or lowlands, continents or ocean depths. The unit area, of course, cannot be taken indefinitely small; also, the state of isostasy is not perfect. In general, a circle 100 miles in radius is large enough to serve as a unit area, and frequently a much smaller circle may be used.

Two theories have been advanced as an explanation of the phenomenon of isostasy. One of these, published by J. H. Pratt in 1856, holds that there is a definite depth of compensation. The material constituting the outermost layers of the earth's crust is assumed to be less dense than the material below these layers. As a result, the total mass standing on any unit area is substantially the same. This hypothesis lends itself more readily to computation than the subsequently described theory. Topographical computations give 60 miles as an average depth of compensation.

Another theory published by Sir George Biddell Airy in 1855 is somewhat more in accord with geological theories. According to the Airy theory, blocks of the earth's crust are floating in a relatively dense plastic material, usually called magma or sima. The excess of mass corresponding to high blocks is compensated for by a displacement of the denser plastic magma or sima. Thus, blocks of crust containing excess mass produce a greater displacement of the magma than blocks containing a "normal" amount of mass. Calculations on this theory give an average depth of about 30 miles for the lighter crusts. This depth is, of course, less under the oceans and more under the continents and highlands.

Isostatic Method of Determining the Figure of the Earth.—In one method of determining the figure of the earth, the deflections of the plumb line are used to determine the ellipsoid which best fits a relatively small region. If it is desired to make the region representative of the earth as a whole, it is necessary to correct the deflections for the visible surface topography and its isostatic compensation. This isostatic method was applied by Hayford to observations extending over the United States, and the figure of the earth deduced by him was adopted in 1924 by the International Geodetic and Geophysical Union as the best available figure of the earth as a whole. The ellipsoid thus determined is known as the International Ellipsoid of Reference. The semi-major axis (or equatorial radius) equals 6,378,388 meters and the ellipticity is equal to 1/297. The mass of the ellipsoid, assuming a mean density of 5.527, is 5.988×10^{21} metric tons. Later measurements of the deflection of the plumb line in Europe gave isostatic results which are substantially in agreement with Hayford's. Observations on the value of gravity discussed in the next section in general support the conclusions regarding isostasy.

"Normal" Variation of Acceleration Due to Gravity.—If it is assumed that the earth is an ellipsoid of revolution revolving about an axis of symmetry with a constant angular velocity and that the surface of the ellipsoid is a gravitational equipotential surface, it can be shown that the value of gravity at any point on the surface is given by the expression:

$$b - b\ \mathrm{si}$$

where g_ϕ is the value of gravity at sea level in geographic latitude ϕ, g_E is the value of gravity at the equator, and a and b are constants which depend upon the gravity at the equator, the angular velocity of rotation, and the departure of the shape of the earth from a true ellipsoid. The constant b is small and may be taken as 0.000007. This leaves two coefficients g_E and a. A commonly accepted value of g_E is 978.039 cm./sec.^{2}

and a commonly accepted value of a is 0.005294. On substituting these values into Equation 3, one obtains

$$g_{\phi} = 978.039\,(1 + 0.005294 \sin^2 \phi - 0.000007 \sin^2 2\phi) \qquad (3a)^*$$

In general, it is not possible to make gravitational measurements on the surface of the ellipsoid corresponding to sea level. If the height of the point of observation above the level surface is h, the value of gravity is obtained by adding a correction proportional to h to the observed value of gravity. The factor by which h must be multiplied amounts to a variation of gravity of about seven one hundred millionth of the total value of gravity for each foot of elevation. This correction for h may be the "free air correction" or the Bouguer correction, as described later.

Stokes showed as early as 1849 that it was possible by gravitational observations to determine not only the flattening of the terrestrial ellipsoid, but also the deviation of the geoid from this assumed ellipsoid. When the suggestion was first made, it seemed to be of speculative interest only because it required that gravity be observed at intervals over the entire globe including the sea. No practical method existed for observing the value of gravity at sea at that time. An interesting method for carrying out observations at sea was devised later by Hecker. In this method the boiling point of water and the height of the mercury barometer are determined simultaneously. Because the atmospheric pressure governs both the boiling point of water and the height of a column of liquid of given density, the boiling point and fluid height data may be used to determine the density of the mercury which is proportional to the value of gravity. Recently, a much more accurate method has been developed by Vening Meinesz. By an arrangement of three pendulums the effect of the motion of the vessel is decreased. By using this apparatus in submarines submerged deeply enough to avoid most of the surface motion, it has been possible to make gravity determinations on the open ocean.†

If there were no irregularities on the surface of the earth, the water would stand at a uniform depth over the whole earth. The plumb line at all places would be normal to the spheroid and the value of gravity would be the same for all points having the same latitude, and would increase uniformly north and south of the equator. Actually, the surface of the earth is not a regular mathematical surface but is quite irregular. However, it is found that the surfaces of the oceans and the imaginary sea levels continued under land areas approximate the surface of the spheroid which would exist if there were no irregularities. For any large area the plumb line will, on an average, be normal to the spheroid and the values of gravity after applying a correction for the elevation of the station above sea level will be very nearly the same.

Geodesists have made searching investigations to determine the causes

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* This formula is sometimes referred to as the Bowie Formula, Number 2.

† F. A. Vening Meinesz, "Projet d'un nouvel appareil pendulaire," *Bulletin Geodesique* Nr. 5, 1925; "Theory and Practice of Pendulum Observations at Sea," *Publict Netherl. Geod. Comm.* 1929.

W. Heiskanen. *Handbuch der Geophysik* (Gutenberg) Vol. I, p. 765-780.

of the deviation from normal of the plumb line and the variation of gravity along parallels of latitude. It was readily realized that the land masses above sea level and the deficiencies in density of water in ocean basins as compared with equal volumes of surface rock would cause irregularities. But even after corrections are applied for the effects of these excesses and deficiencies of mass, there are anomalies. The causes of these irregularities must lie in the outer portion of the earth because stations only short distances apart are affected differently. Consequently the causes of these effects must lie at depths comparable to the distances between the stations. In other words, there must be abnormal densities of materials in the outer portion of the earth. These abnormal densities at relatively shallow depths are the variables of interest in gravitational surveying.

Geodesists have made computations on the theory that at some depth below sea level the pressure for a unit area is the same, i.e., does not depend on the density of the rocks near the earth's surface. The computations have confirmed this assumption of isostasy to a remarkable degree. Thus, it has been shown that at least a greater portion of the abnormal densities occurs in the outer 60 miles of the earth. By these isostatic computations the anomalies in the goedetic data have been reduced to from 1/7 to 1/10 of what they would have been if there were no such compensation as the theory of isostasy postulates. These small residual anomalies are believed to be caused by rapid changes in density of the materials at a shallow depth and therefore are properly the object of investigations in applied science, because the variations in density producing these anomalies are caused by the juxtaposition of the different rocks close enough to the surface to interest the geophysicist and geologist.

DENSITIES OF MATERIALS

The applicability of the gravitational methods depends on the existence of detectable *differences* in density between a subsurface body and the surrounding medium. The density in grams per cubic centimeter of some of the materials found in the earth's crust are given in Table 5. The table affords only a general survey. Densities of materials in a particular region are preferably obtained from measurements made on samples taken from that region.

In a majority of petroliferous areas the density increases with depth. The increase in the Gulf Coast of the United States is 0.07 per thousand feet to depths of about 8,000 feet. † When applied to petroleum exploration, a decrease in density is found: (1) if a massive salt series is present at considerable depth; (2) if a large thickness of diatomaceous shale is present at considerable depth; and (3) if a thick section of limestone or anhydrite overlies a thick section of sands, shales, clay marls or other poorly consolidated materials.

† D. C. Barton, "Gravitational Methods of Prospecting," *The Science of Petroleum* (Oxford Univ. Press, 1938), Vol. I, p. 374.

TABLE 6

DENSITIES OF VARIOUS CONSTITUENTS OF THE EARTH'S CRUST*

Rock Materials	*Density (grams per cubic centimeter)*
Acid (massive) lava	2.4-2.7
Anhydrite	2.8
Argillite (clay slate)	2.8-2.9
Basalt	2.7-3.3
Beaumont formation, Pleistocene sands and clays, Texas-Louisiana Coast	1.7-2.2
Chalk	1.8-2.6
Diabase	2.9
Diatomaceous shale, San Joaquin Valley, California	0.9-1.1
Diorite	2.7-3.0
Dolerite	2.5-3.1
Dolomite	2.4-2.9
Ferruginous rocks, such as those of Lake Superior iron ore series. Magnetic ore deposits, such as those of the iron deposits of northern Sweden, and those of the sulphide Sudbury district in Ontario	3.0-5.0
Gabbro and certain basic rocks	2.7-3.0
Gneisses and schists	2.4-3.0
Granite and other "acidic" rocks	2.4-3.0
Greenstone	2.9-3.0
Lava (basaltic)	2.8-3.0
Limestone	2.1-2.6
Marl	2.25-2.6
Peridotites and allied rocks	3.0-3.3
Pitchblende	8.0-9.7
Rock salt	2.1-2.4
Sandstones	2.0-2.6
Serpentine	2.4-2.8
Soil and alluvium	1.2-2.0
Tertiary sands and clays, (a) at shallow depth	2.0-2.2
(b) at 7,000 ft. Gulf Coast of Texas and Louisiana	2.5

Mineral Substances	*Density (grams per cubic centimeter)*
Arsenopyrite	6.0-6.2
Barite	4.3-4.6
Bauxite	2.0-3.0
Bornite	4.9-5.2
Calamine	4.1-4.5
Calcite	2.6-2.8
Cassiterite	6.8-7.0
Chalcopyrite	5.7
Coal (hard)	1.4-1.8
Coal (soft)	1.2-1.5
Corundum	3.9-4.0

data given here represent averages of typical values found in the literature sical prospecting.

TABLE 6 (Continued)

Mineral Substances	*Density (grams per cubic centimeter)*
Cuprite	5.7-6.2
Feldspar	2.5-2.6
Fluorite	3.1-3.2
Galena	7.5-7.8
Graphite	2.2-2.3
Gypsum	2.2-2.3
Hematite	4.5-4.9
Hematite (specular)	5.1-5.3
Hornblende	2.6-3.4
Limonite	3.4-4.0
Magnetite	5.0-5.4
Manganese ore (red and black)	3.9-4.1
Marble	2.5-2.9
Marcasite	4.9-5.0
Molybdenite	4.4-4.8
Petroleum	0.7-1.0
Pyrite	4.9-5.2
Pyrrhotite	4.5-4.64
Quartz	2.6-2.8
Sea water	1.0-1.05
Silica	2.7-2.7
Water	1.0
Zincblende	4.0

Elements	*Density (grams per cubic centimeter)*
Iron	7.7-7.9
Lead	11.2-11.4
Sulphur	1.9-2.1

PROPERTIES OF THE GRAVITATIONAL FIELD MEASURED IN PROSPECTING

The properties of the gravitational field which are usually measured in gravitational surveys are: (1) the relative gravity, (2) the gravity gradient, and (3) the curvature quantity or horizontal directive tendency (H.D.T.). *Relative gravity* is the value of gravity at any station in reference to the known or assumed absolute value of gravity at some base station. Relative gravity is measured directly by pendulums and by gravimeters. The *gravity gradient* is defined in gravitational exploration work as the rate of horizontal variation of gravity, i.e., the rate of change of gravity per unit horizontal distance. The gravity gradient is measured directly by torsion balances and by gradiometers. The *curvature quantity* is equal to the product of g times $(1/\rho_1 - 1/\rho_2)$ where ρ_1 and ρ_2 are the minimum and maximum values of the radii of curvature of the equipotential surface through the

point of measurement. The curvature quantity is measured by the torsion balance.

Units of Acceleration, Gradient, Curvature.—The unit of acceleration due to gravity is the centimeter per second per second and is commonly called a *gal.* Thus, the acceleration at the earth's surface is approximately 980 gals. Most local structures have an attraction which produces a variation in the acceleration of considerably less than one gal. For convenience, therefore, the one thousandth part of a gal, i.e., the *milligal* is the usual field unit.

A force which gives an acceleration of one gal to a mass of one gram is a dyne, and it is therefore common in the literature to use *dyne,* instead of dyne per gram, and *millidyne,* instead of millidyne per gram, as units in describing gravitational anomalies, although this usage is confusing.

The unit most commonly employed in describing the gravity gradient is the Eötvös. An Eötvös unit, frequently abbreviated to E, equals $1 \cdot 10^{-9}$ dynes (per gram) per horizontal centimeter.

The curvature quantity, $g(1/\rho_1 - 1/\rho_2)$ has the same units as the gravity gradient, viz., dynes (per gram) per centimeter, and is usually expressed in Eötvös units.*

PROSPECTING METHODS AND INSTRUMENTS

Present day gravitational methods of prospecting may be classified according to the three types of instruments in most general use. The methods will be described as gravitational prospecting: (a) with the pendulum; (b) with the gravimeter and (c) with the torsion balance.

From the standpoint of instrument design, gravitational prospecting represents achievement of the highest quality. In order to be of value for oil exploration, the instruments must measure extremely minute variations of the gravity and of the gravity gradient. For example, instruments for determining variations of gravity must be capable of measuring about *one ten-millionth of the total acceleration due to gravity.* To accomplish this, considerable instrumental refinement was necessary, and the success of gravitational surveying has been due largely to the development of suitable apparatus. †

GRAVITATIONAL EXPLORATION WITH THE PENDULUM

Absolute Gravity Measurements.—A simple pendulum consists of a small, heavy mass suspended by a theoretically massless, perfectly flexible, string of unvarying length. Obviously, such an object does not exist but it offers a simple theoretical basis for the discussion of more complicated pendulums. As shown in Figure 70, the downward force on the bob is *mg,*

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* The quantity usually plotted on gravitational contour maps, however, is the "differential curvature" $(1/\rho_1 - 1/\rho_2)$ and this quantity is usually expressed in units of 10^{-12} cm^{-1}.

† M. K. Hubbert and Frank A. Melton, "Gravity Anomalies and Petroleum Exploration by the Gravitational Pendulum," *A.A.P.G.*, Vol. 12, No. 9, 1928, pp. 889-898.

Clarence H. Swick, "Modern Methods of Measuring the Intensity of Gravity," Dept. of Commerce, Ser. No. 150, Special Publication No. 69, 1921, U. S. Coast and Geodetic Survey.

where g is the acceleration due to gravity and m is the mass of the bob. The component of this force acting perpendicular to the string is $mg \sin i$, where i is the angle between the string and the vertical. The acceleration of the mass is $l \frac{d^2i}{dt^2}$ where l is the length of the string. Hence, from Newton's second law of motion,

$$ml \frac{d^2i}{dt^2} = -mg \sin i \qquad \text{or} \qquad \sin i = 0$$

The solution of this equation is a periodic function with period T, where T is the time for the pendulum to swing from A to A' and back to

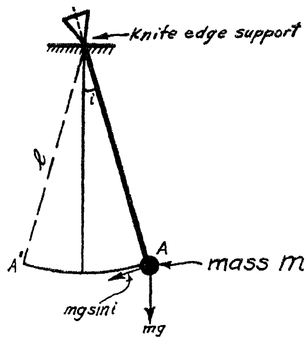


FIG. 70.—Simple pendulum. (mg is the force of gravity acting vertically downward. The force tending to move the mass m along the arc AA' is $mg \sin i$.)

A—assuming that A represents the highest point of the path. T is a function of l, g, and the azimuthal angle from which the pendulum was initially set into oscillation. It is given by the formula:

$$g \qquad \sin^2 \frac{i_0}{2} + (1.3/2.4)^2 \sin^4 \frac{i_0}{2} \quad] \qquad (4)$$

where i_0 is the angle made by the string with the vertical at the start of the oscillations. For small amplitudes the period formula becomes*

$$(5)$$

Compound Pendulum: Reversible Type

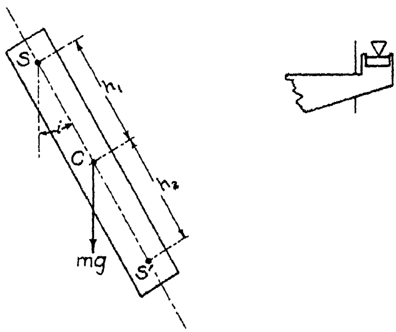
A compound pendulum is a rigid body of any shape suspended by a horizontal axis and swinging through a small angle with negligible friction. Figure 71a shows a vertical section through the center of mass C and

* In some of the literature the half period (time to swing from A to A') is referred to rather than the time for a complete oscillation.

perpendicular to the axis of suspension S. The gravitational moment tending to rotate the pendulum about S is $mg \sin i$. The acceleration is $\frac{d^2i}{dt^2}$. Hence, from Newton's second law of motion

$$I_s \frac{d^2i}{dt^2} = - \quad \sin i$$

where I_s is the moment of inertia about the axis through S and m is the mass of the pendulum. This equation of motion for a compound pendulum



a b

FIG. 71.—Schematic diagrams of reversible type, compound pendulum.

is similar to that for a simple pendulum. Hence, for small values of i the solution is a periodic function with period T_1, where

$$2\pi \sqrt{\quad I_s}$$

A comparison of this expression with the corresponding expression for a simple pendulum (Equation 5) shows that the compound pendulum is equivalent to a simple pendulum of length l_1 where $l_1 = \frac{I_s}{mh_1}$. Hence, a compound pendulum may be used to determine the absolute value of g provided the equivalent length l_1 can be evaluated.

To determine the equivalent length l_1, use is made of the fact that to every point of suspension S there corresponds another point S', known as the center of oscillation, which has the property that the period of vibration about it is the same as the period of vibration about S. The proof that there is such a center of oscillation is readily shown by using a well known relation between the moment of inertia I_c about an axis through the center of mass C and the moment of inertia I_s about a parallel axis through S. That is,

$$I_s = I_c + \quad \text{or} \quad I_s = mk^2 + mh_1^2$$

where k is the radius of gyration about the center of mass. If one substitutes the latter value for I_s into the expression for the period one obtains

$$T_1 = 2\pi \qquad \text{or} \qquad T_1 = 2\pi \left(\frac{k^2}{h_1} + h_1\right)\frac{1}{g}$$

Hence, the length l_1 of the equivalent simple pendulum may be written in the form

$$l_1 = \frac{k^2}{h_1} + h_1$$

In the same manner it may be shown that the period of vibration about a parallel axis through another point such as S' on a line through S_c extended is

$$\frac{1}{g}$$

where h_2 is the distance between parallel axes through S' and C. The length l_2 of the equivalent simple pendulum for this case is

$$l_2 = \frac{k^2}{h_2} + h_2$$

For $l_1 = l_2$ the periods of vibration about the two axes S and S' will be the same. This condition may be written in the form

$$l = l_1 = l_2 = \frac{k^2}{h_1} + h_1 = \frac{k^2}{h_2} + h_2$$

On elimination of k, this condition becomes

$$l = h_1 + h_2$$

which is the relation sought.

The principle of the practical method advanced by Kater for determining the distance l between a point of suspension S and the corresponding center of oscillation S' will be evident from Figure 71b. The procedure consists simply in adjusting the two weights A and B until the period of vibration is the same when the pendulum is caused to vibrate first about S and then about S'. (An alternative design applicable in certain cases employs only one movable weight mounted between S and S'.) When the periods about S and S' have been made equal, the distance between the two parallel axes of suspension through S and S' is equal to the length of the equivalent simple pendulum. This distance can be measured with considerable accuracy and from it and the measured period of the pendulum, the value of g can be computed. Thus, the Kater pendulum can be used for absolute determinations of g in terms of measurable distances and times.

Accuracy of Measurements

The accuracy of absolute gravity measurements with the reversible pendulum depends chiefly upon three factors: (1) measurement of length, (2) accuracy with which the center of gravity is placed on the line joining the two axes and the parallelism of the two axes, and (3) degree of perfection with which friction is eliminated. In use, the accuracy is chiefly dependent upon the accuracy of time measurement. The most accurate measurements of the absolute value of g are believed to be those of Kühnen and Furtwängler at Potsdam in 1898-1904 which had a probable error of three parts in a million.† Such accuracy of measurement cannot be achieved in field measurements and recourse generally is had to relative gravity measurements.

Relative Gravity Measurements.—If a pendulum has the period T_1 at a base station where g has the value (measured or assumed) of g_1, the corresponding value of g at a second station where the period of the pendulum is T_2 may be obtained from Equation 4. That is,

$$\frac{T_1^2}{T_2^2} \tag{6}$$

From which

$$g_2 - g_1 = g_1 \frac{T_1^2 - T_2^2}{T_2^2} \tag{6a}$$

When relative values of the gravity are desired, it is not necessary to determine the absolute value of g_1 with an accuracy greater than about 1/10 of 1% for average structural investigations.

The application of Equation 6 or 6a to data obtained with field pendulums gives accurate results only when the assumptions implicit in the derivation of Equation 4 are adequately approximated and when all of the quantities in Equation 4, except T and g, are the same at the two stations or accurate corrections have been made for any changes.

It is obvious that the amplitude factor must be taken into account. Unfortunately, there are many other less obvious factors. One of these is that the quantity l, the equivalent length of the pendulum, must not vary. To accomplish this, the pendulums are made of an alloy with a low coefficient of thermal expansion, e.g., invar. If the pendulum were swung in air, the resistance of the air to the motion of the pendulum would add a force that does not enter into the simple formula (4). Moreover, the buoyant effect of the air would change the mass of the pendulum. For this reason the pendulum is swung in an evacuated case and a correction applied for the residual air pressure.

In deriving Equation 4 it is assumed that the pendulum rotates about a linear axis. In practice, this is approximated by supporting the pen-

† *Handbuch der Geophysik*, Vol. 1, p. 753

dulum by a hard knife-edge on an agate plate. (However, even the best knife-edges are not lines.) The derivation of the period formula (Equation 4) also assumes that the point of support is stationary. Obviously, it is impossible to swing a large mass without transmitting some of the motion to the case—particularly if the equipment is light enough to be portable. To minimize this effect, it is customary to employ two or more pendulums of equal periods swinging in opposition so that the total effect on the case balances out.

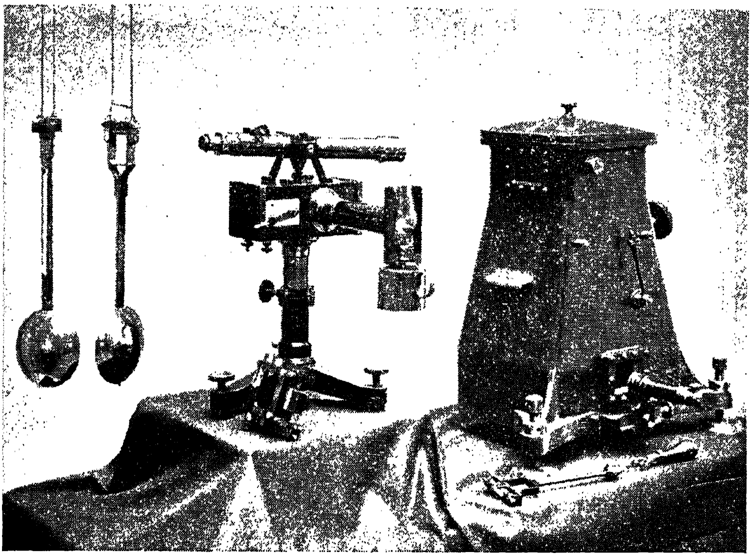


FIG. 72.—Pendulum prospecting equipment typical of the late 1920's. Left to right: Two invar pendulums; light source for photographic recording; pendulum case. (Courtesy of L. F. Athy.)

Apparatus

Relative gravity measurements are usually effected with the aid of a so-called invariable type compound pendulum. The pendulum as used in geophysical applications consists of two or more bobs or masses, supported by knife-edges and having lengths such that they swing in synchronism.* The bobs are released simultaneously so that their horizontal components of reaction on the support are of opposite phase and cancel each other. This minimizes the tendency of the system to sway upon its support.

Early Pendulum Prospecting Equipment.—Figure 72 shows pendulum prospecting equipment which was typical in the late 1920's. The two pendulums shown at the left of the figure are made of invar.

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* A pendulum comprising two bobs is also described as a 2-pendulum apparatus; a pendulum comprising three bobs as a 3-pendulum apparatus, etc.

Von Sterneck-Askania 4-Pendulum

illustra

n base

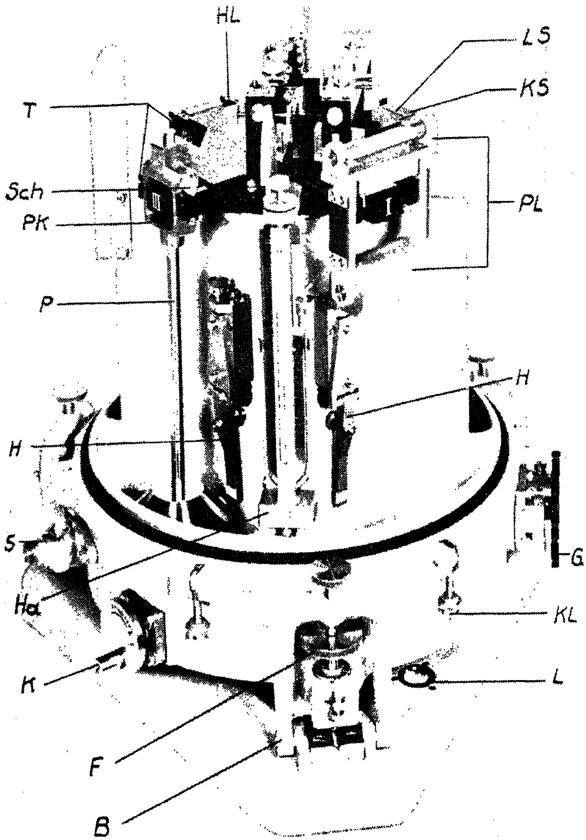


FIG. 73.—Base plate with pendulum tripod. (Courtesy of the Askania Corporation.)

F—level screw
B—holding stirrup for level screw
KL—swing bolt for hood
S—evacuation petcock
KS—headpiece of pendulum tripod
LS—agate bearing in head piece
P—pendulum
PK—pendulum head
Sch—pendulum wedge
PL—pendulum level
K—crank for releasing and clamping
HL—bearings in the arresting device
G—amplitude limiting device
H—adjusting lever for the amplitude limiting device
Ha—holder with thermometer and barometer

pendulum hood. The base frame fits into the tripod which also accomodates the pendulum and the hood.

The tripod contains all the equipment necessary for clamping; setting the pendulum into oscillation; adjusting the amplitude, etc. On the tripod

is mounted an optical bridge which reflects the light coming from the point of observation via the four pendulum mirrors.

The pendulum tripod is supported by 3 level screws which rest on the base frame and are used for leveling and locking the apparatus. A disk is provided for starting the oscillation of one, two, or all four of the pendulums. A crank facilitates the releasing and clamping of the pendulums. The pendulums are supported by agate bearings and each pendulum swings in a separate chamber. (Figure 73.) The pendulum bridge consists of 6 prisms, each of which can be turned and tilted for adjusting purposes.

The pendulums are made of invar material and are gold-plated for protection against corrosion. (Figure 74.) The wedge is made of quartz and is mounted in the pendulum head.

The metal mirror which is located in the pendulum head is ground in such a way that errors due to displacements of the plane of the mirror are minimized. The pendulum hood which covers the tripod and all its accessories is so mounted that a close air-tight fit is assured. Two windows are provided to read the thermometer and barometer and to permit the light ray to travel from the point of observation to the optical bridge and return.

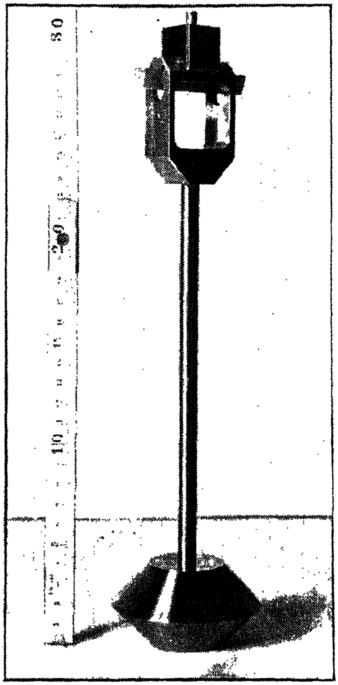


FIG. 74.—Von Sterneck type invariable pendulum. (Courtesy of the Askania Corporation.)

Two levels, having a sensitivity of 10 sec. per division, are mounted so that they can be conveniently lowered to the agate bearings, the exact level position of which they control. Two heavy handles are provided for carrying the instrument.

During transport, the holder for the thermometer and barometer is carried in a box. Suitable means are provided so that the barometer will not be damaged while in transport, even if the mercury should be subject to considerable movement.

GENERAL OUTLINE OF PROCEDURE AT EACH STATION FOR PENDULUM OBSERVATIONS

1. Selecting a Station

To obtain a high accuracy of the pendulum observation (about 1 milligal) it is necessary to select a place free from vibrations, preferably with a hard rock or cement base, and not subject to large temperature fluctuations. Sufficient space must be provided for the setting up of the coincidence apparatus. (See p. 167.)

2. Setting-up and Leveling of the Askania Equipment

After the proper spot for the station has been located, the base frame is set on the ground, oriented, leveled and grouted with plaster-of-Paris in order to furnish a firm connection between base and ground. The pendulum apparatus is then mounted on the

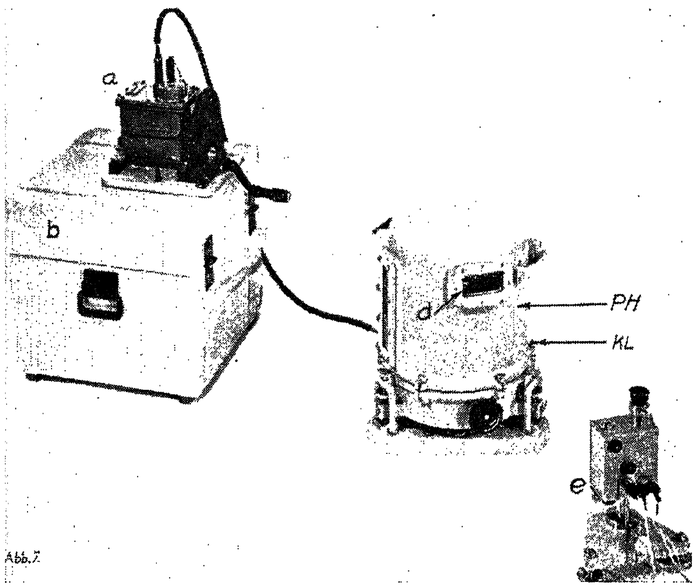


FIG. 75.—Askania 4-pendulum apparatus ready for operation.

a—oil pump
b—instrument box
PH—pendulum housing
d—observation window
e—observation equipment
KL—locking screw

frame with the window facing the observation equipment. The level screws *F* and the pendulum levels *PL* (Figure 73) permit accurate leveling of the plane of the bearings. (Such leveling is necessary to prevent the pendulums from slipping off the bearings during oscillations, which last several hours.)

3. Precautions to be Followed in Suspending the Pendulums

In order to suspend the pendulums freely, it is first necessary to move the selector disk and the oscillation knob to the fixed stop. The operator should wear gloves when handling the pendulums in order to minimize temperature variations due to handling and deposition of oily films.

4. Control of the Light Path and of the Amplitudes

Before placing the hood over the apparatus, the observation equipment must be adjusted and the light path checked. Also, the magnitude of the oscillations is controlled and corrected, if necessary. The proper operation of the selector disk and oscillation knob is checked as well as the proper fitting of the hood.

The pendulum hood is fastened to the tripod base by means of 6 swing bolts. The equipment is then ready for evacuation of the air in the pendulum chamber.

5. *Evacuating the Apparatus*

For convenience, the oil pump is mounted on the shipping box (Figure 75) and the hose is fastened to the apparatus. The manufacturers recommend reducing the pressure to about 5 mm. of mercury and then admitting sufficient air to raise the pressure to about 10 mm. of mercury. By following this procedure, the danger of dew formation is practically eliminated.

6. *Adjustment of the Selector Disk: Oscillating the Pendulums*

The pendulums are lowered so that the wedge touches the bearings. (This should only be done when the oscillation knob touches the fixed stop. Also, the selector disk should be operated only when the knob is in this position.)

The various marked positions of the knob permit oscillating one, two, or all four of the pendulums at any desired amplitude.

Tieing-In Observations at a Series of Stations

The field procedure for determining the relative gravity by pendulum methods is based on the use of two or more pendulums. The beats of the pendulum at the base station are transmitted by a portable radio transmitter to the field station where they are recorded photographically on a strip of film. The same strip also records the beats of the pendulum at the field station.

If the times of swing (half periods) of the two pendulums are exactly the same, the marks on the film, which correspond to the beats of the two pendulums, will coincide or lie in a straight line.† If the times of swing are not exactly the same, the marks will be displaced with respect to one another. After a certain number of vibrations of the base station pendulum, the marks due to the two pendulums will again coincide. The number of oscillations of the base pendulum N can be read off the photographic film directly. During this interval, the field station pendulum will have made $(N+1)$ or $(N-1)$ vibrations, depending on whether it is faster or slower than the base pendulum. Corresponding to these two cases, the time of swing of the field pendulum is $N/(N+1)$ or $N/(N-1)$ times as great as that of the base pendulum.

If the field and base pendulums are exactly similar, the relative value of g at the field station referred to that at the base station may be obtained directly from Equation 6, which for convenience may be written in the form $\frac{g_f}{g_b} = \frac{T_b^2}{T_f^2}$ where the subscripts f and b apply to the values of gravity and time at the field and base stations respectively. If the two pendulums are not precisely similar, a modified form of this equation must be used.

† Compare R. Ambronn, *Elements of Geophysics* (McGraw-Hill, 1928).

Reduction of Observations

The relative gravity values obtained with gravimetric instruments nearly always include effects of appreciable magnitude which are not related to the geologic anomalies under consideration. For example, the relative gravity values obtained with pendulums and gravimeters are the sum of effects due to: † latitude, elevation above sea level, layers of sediments between sea level and the level of the station, large topographic features, and structural irregularities of density. Hence, appropriate corrections must be applied so that the "reduced" data correspond chiefly to effects due to the geologic inhomogeneities under investigation.*

Limitations of Commercial Pendulums

About two hours' time is required for making a station reading with the pendulum. This time requirement is greater than that usually necessary to carry out a gravimeter observation. Furthermore, according to Barton‡ the best commercially purchasable pendulums have a probable error of ± 0.5 milligals. It is this large probable error which limits the use of pendulums in geophysical prospecting to surveys of regional features or abnormally large anomalies. The gravitational anomalies characteristic of structures associated with oil deposits may be less than 1.5 milligals. Hence, measurements of the relative value of g in oil exploration work require a more sensitive type of instrument than the pendulum. The more sensitive instruments commonly employed to determine the relative value of g are: (a) gravity meters, or gravimeters, for direct determination of the relative value of g and (b) the torsion balance and, to a smaller extent, the gradiometer for indirect determinations.

GRAVITATIONAL EXPLORATION WITH THE GRAVIMETER

Direct determinations of the relative gravity with a gravimeter or gravity meter consist in "weighing" the same object with very great precision at several stations. The weight of an object at any location on the earth's surface is equal to the force of attraction exerted by the earth on the object. That is, the weight is equal to the product of the mass m of the object, which remains the same at all locations, and the acceleration g due to gravity. Hence, the weight of a constant mass m at any station is affected by the nature of the subsurface materials. For example, it is larger at stations where the subsurface material is relatively dense. The

† D. C. Barton, "Gravitational Methods of Prospecting," *Science of Petroleum*, Vol. I, pp. 370-371.

* The corrections to be applied to relative gravity data are discussed in the section on gravitational exploration with the gravimeter, p. 185.

‡ D. C. Barton, *loc. cit.*, p. 369.

observed changes in weight are very small, being of the order of *one part in ten thousand,* or one part in ten million of the total value of gravity.

Gravimeters.—Prior to 1936, gravimeters were used to a very limited extent in prospecting in the United States. Since that date, the application of gravimeters in oil prospecting has been increasing markedly as shown by the comparative data plotted in Figure 76.†

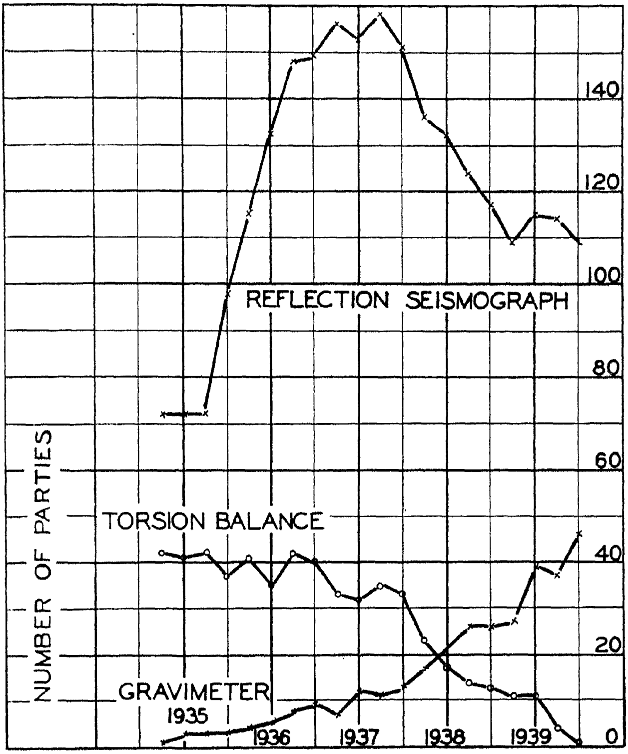


Fig. 76.—Plot showing field parties in oil exploration in states bordering on the Gulf of Mexico. (After Eckhardt, *The Explosives Engineer.*)

Classifications of Gravimeters

The gravimeters used in geophysical prospecting may be classified into two types: static and astatic instruments. The simplest type of static gravimeter comprises a heavy mass M rigidly attached to an elastic spring. Some form of amplification system is employed, often optical, whereby slight displacements of the mass may be measured. The operative technique may be summarized briefly as follows. The equilibrium extension,

† E. A. Eckhardt, *The Explosives Engineer,* April, 1940, p. 104.

or compression, of the spring supporting the weight is determined at a base station, and the reading recorded. The meter is then moved to one of the field stations and the equilibrium extension, or compression, at this station is recorded. A comparison of the two readings, after making the necessary corrections, gives the relative gravity of the field station as compared to the base station.

Astatic gravimeters employ an auxiliary restoring force opposite in sign and approximately equal in magnitude to the elastic restoring force. In general, therefore, the mass M or bob of an astatic gravimeter is subject to three reactions: (1) the pull of gravity, (2) the elastic restoring force of a spring, and (3) an astatizing or labilizing force which acts in the same direction as gravity and has a magnitude nearly, but not quite, the same as the elastic restoring force of the spring.

For small displacements, the force due to gravity tending to displace the mass is $A(g-g_0)$ where g_0 is the gravity value at a base station; the elastic force is $-Bx$, where x is the displacement of the mass from the position occupied at the base station; and the astatizing force is Cx. Hence, the resulting force on the mass when it is in equilibrium is:

$$A\,(g-g_0)-(B-C)\,x=0$$

The displacement is:

$$=\frac{A}{\qquad}\,(g-g_0)$$

It is evident from the last relation that if C can be made approximately equal to B, the sensitivity, $A/(B-C)$, can be increased to any desired value. Naturally, this involves many instrumental difficulties.

It is interesting to notice that if Dd^2x/dt^2 is the force required to produce an acceleration of d^2x/dt^2, the equation for small motions, neglecting the effect of damping, is

which has the solution

$$x=\frac{A}{B-C}\,(g-g_0)+A_0\sin\,(2\pi\,t/T)$$

where

$$A_0 = \text{a constant which depends on initial conditions}$$

and

$$2\pi\sqrt{\frac{D}{B-C}} = \text{period of oscillation about the new equilibrium position.}$$

The last relation shows that the sensitivity is proportional to the square of the period; that is, $1/(B-C)$, and hence $A/(B-C)$, is proportional to T^2.

A second classification of gravimeter types depends upon the method of reading the change in gravity. In scale reading instruments, an indi-

cator of some kind moves across a graduated scale. In null reading instruments, an auxiliary force is applied to return the mass to a standard position and the amount of this force is measured on a scale based on prior calibration of the instrument.

Instrumental Problems in Gravimeter Construction

Many instrumental difficulties were encountered in the development of the gravimeter. The chief problem was measurement of the minute displacement of the mass produced by a change in the gravitational force of attraction. The measurement must be made with an accuracy of about one ten-millionth of an inch. Several systems have been developed which possess the required sensitivity. The most satisfactory method employs a compound spring system which is mechanically equivalent to a simple spring of very large extension. Compound spring systems increase the displacement some ten to one hundred times that of a practicable simple spring. A sensitive optical system, working in conjunction with the compound spring, constitutes the essential means for measuring the displacement to the required accuracy.

Another difficulty encountered in the gravimeter development was the variation in the displacement of the spring system due to changes in temperature. This variation is caused primarily by: (a) thermal expansion and (b) changes in the elasticity of the spring material. The first effect is large due to the high coefficients of expansion of the materials most commonly used for springs. For example, a steel spring may increase in length from one-half to two parts in ten thousand per degree centigrade rise in temperature. Thus exposure of a gravimeter having a steel spring to ordinary temperature variations would result in displacements due to temperature variations which would be very large in comparison to those due to gravity. Temperature effects have been minimized by a number of methods, chief of which may be mentioned: (1) compensation by use of bimetallic materials similar to those employed in the new temperature-compensated magnetic systems, (2) maintenance of constant temperature by use of electric thermostats, (3) utilization of spring materials having a very low temperature coefficient. Usually, all three of these methods are employed. The most effective control, however, is the use of a well insulated cabinet provided with accurate thermostatic control. Such equipment can maintain the temperature constant within one hundredth to one five-hundredth degrees centigrade.

The most troublesome difficulty to be overcome was the imperfect elasticity exhibited by all elastic materials. This imperfection appears in two distinct manners termed *creep* and *elastic-after-effect* or *elastic lag*. Creep is simply a gradual yielding of any "solid" material when under load. This effect is usually negligible in ordinary engineering applications but must be considered for precise work as it may in unseasoned common steel springs cause an increase in length of twenty parts in ten million per hour. Elastic

lag inhibits the rapid return of a spring to its original length after a displacement. The gradual return of a spring to its original condition depends on the amount of displacement and the length of time it has been displaced. Elastic lag, if present to an appreciable extent, impairs the accuracy because its effects are unpredictable.

The limitations of the gravimeter are due almost exclusively to the limitations of available spring materials. In addition to the preceding problems there are numerous secondary ones. The calibration of the instrument must not be affected by jars or vibration such as encountered in average field operations. Also, the damping of the elastic system should be as nearly aperiodic as possible, consistent with sensitivity. Unless this is done the system will be affected unduly by disturbances, such as might be caused by a passing automobile which sets the system into oscillations that persist for an extended period of time. Under these conditions, observation would be impossible if disturbances are frequent. Average vibration of the ground usually will not cause appreciable unrest of a properly designed elastic system; however, near the ocean, the pounding of the waves does produce a quite noticeable unrest of the elastic system.

Operating Principles of Various Gravimeters

The particular instruments described in the following pages illustrate representative types. The list is not intended to be exhaustive, but merely illustrative of the general trend of design.

Hartley Gravimeter.—The Hartley instrument, illustrated in Figure 77, is one of the simplest types of gravimeters.† In this instrument, displacements of the mass M due to variations in gravity are compensated by changing the pull of the spring S_2 so that the mirror D will show the reflected image of the lamp L at some fixed position. The main spring S_1 supports the mass M. The cross member AB is hinged at A with a flexible metal strip, or Galitzin hinge, which permits the end B to move up and down as M moves up and down. The movement of B causes the small mirror D which bridges the gap to tilt from side to side. Adjustment of the instrument is accomplished by returning the reflected image to a previously chosen reference line in the eyepiece E by turning the divided head H.

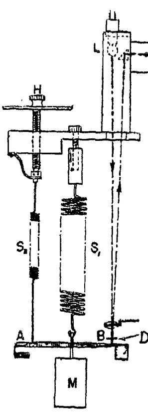


Fig. 77.—Diagrammatic sketch of Hartley type gravimeter. (Bryan, *Geophysics.*)

The Hartley meter employs no astatizing force, depending instead on mechanical and optical magnification of the displacement of the

† A. B. Bryan, "Gravimeter Design and Operation," *Geophysics*, Vol. II, No. 4, pp. 302-308.

beam for its sensitivity. A movement of the mass of 10^{-5} mm. results in a movement of 0.6 mm. at the eyepiece. The meter employs a thermostat to keep the temperature constant to within 0.01 °C. The instrument is highly damped and relatively insensitive to leveling. The mechanism is in an air tight container, with the clamps and screw H working through mercury seals.

Truman Gravimeter.—This instrument is illustrated in Figure 78. An approximately triangular framework A is hinged at N by flexible metal strips.† The spring S_1 supports the mass M which is attached to the outer end of the framework. A second spring S_2, which is attached to the lower end of the framework, pulls directly upward through the hinge line. This spring acts to increase the sensitivity of the instrument. A third very small spring (not shown in the sketch) is attached to the frame and works in parallel with S_1.

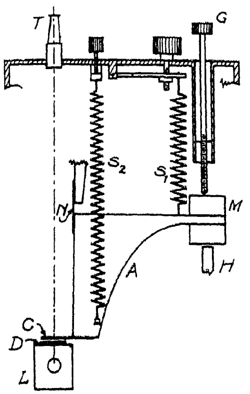


FIG. 78.—Diagrammatic sketch of Truman type gravimeter.

An optical system T is focused from above on a reference line C attached to the frame. Readings are made by observing the lateral motions of this reference line corresponding to vertical motions of M. A lamp L shining through a ground glass D illuminates the reference line or scale. Means are provided to damp the motion of the moving system electromagnetically and to clamp it.* Also, provision is made for the frame to carry a small rider of known weight which is used in calibrating the meter. A clamp screw G locks the system. The temperature of the case in which the meter is mounted is controlled accurately. The behavior of the commercial type meter is indicated by Table 7, which is a summary of data for five closed loops. Loop B, for example, involved a total of 20 miles of line, 6 gravity stations, a maximum gravity difference of 200 units between the highest and the lowest station, and an error in closing of 2.6 units. (The closure error, or the amount by which the algebraic sum of the gravity differences around a closed loop differs from zero, is a measure of the combined effects of: (1) uncompensated instrumental errors such as creep, elastic-after-effect, temperature changes, pressure variations, etc.; (2) inaccuracies in reading of the instrument; and (3) errors due to variations in calibration on scale value.)

† A. B. Bryan, *loc. cit.*

* The instrument usually is either undamped or else damped electromagnetically.

TABLE 7 †

SAMPLE GRAVITY DATA OBTAINED WITH TRUMAN GRAVIMETER

Loop	Total Miles	Total Stations	Max. Gravity Difference in 10^{-4} C.G.S. Units *	Closure Error 10^{-4} C.G.S. Units
A	18	6	208	2.3
B	20	6	200	2.6
C	33	30	208	1.9
D	50	31	514	4.7
E	22	16	270	1.7

Hoyt Gravimeter. ‡—A schematic vertical section of this instrument is shown in Figure 79. The gravimeter consists primarily of a helical spring formed of steel or of other suitable elastic material. The helical spring is suspended at one end from a fixed support through an adjustable hanger 72 which allows the spring to assume freely a vertical position. The other end of the spring 64 is joined to another helical spring by means of a post 82. The post is provided with a collar 83 which carries

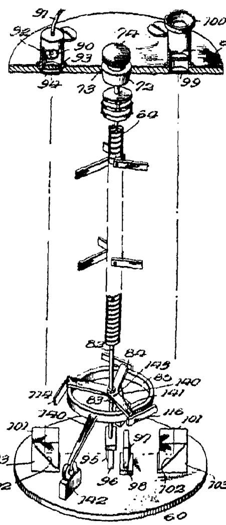


FIG. 79.—Hoyt gravimeter—schematic vertical section. (Hoyt, U. S. Patent 2,131,737.)

† Bryan, *Geophysics, loc. cit.*

* The unit employed is 0.1 millidyne or approximately 1/10,000,000 of the total value of gravity.

‡ Archer Hoyt, "Gravimeter," U. S. Patent 2,131,737, issued Oct. 4, 1938. (Assignor to the Gulf Research and Development Company.)

a spider 84, which in turn carries a narrow-rimmed annular weight 85 concentric with the axis of the helix. The spring and weight combination hangs freely. The weight of the annulus 85 changes in accordance with the force of gravity at the location where the apparatus is set up. This change in weight produces a greater or less pull on the spring, as the case may be, and this causes an angular deflection of the lower end of the spring.

The angular deflection is observed with the aid of the lens 96 attached to the lower portion of the post 82 and additional optical apparatus including a source of light, scale and two totally reflecting prisms.

It is claimed that this instrument is capable of measuring the force of gravity with an accuracy of one ten-millionth of the total value of gravity.

Thyssen Gravimeter.†—The principle of this meter will be evident from a consideration of Figure 80. Mass M, which is made of platinum and weighs approximately 20 grams, is fastened to one end of the quartz beam B. This beam is supported by a knife-edge K. At the other end of the beam is a spring S (of relatively small weight compared to M) fastened at its lower end to a micrometer screw J. The spring is enclosed within a closely fitted metal tube which in turn is enclosed in an insulating medium. Vertically above the knife edge there is supported an adjustable astatizing weight A. By suitable adjustment of this weight the astatizing force can be made to approach the spring force. Ordinarily, the astatizing mass is adjusted to give a period of from 6 to 10 seconds. The end of the mass carries a small scale which is read by means of a microscope and prism P—an arrangement which gives an optical magnification of 60. The meter has very little damping. The spring material and construction, and the material and length of the tube surrounding the spring, are so chosen that the temperature effect is very small. In normal operation the entire meter is enclosed in a thermally insulated box, but usually without thermostat control. The instrument is very sensitive to level; consequently, in practice, two nearly identical units are employed

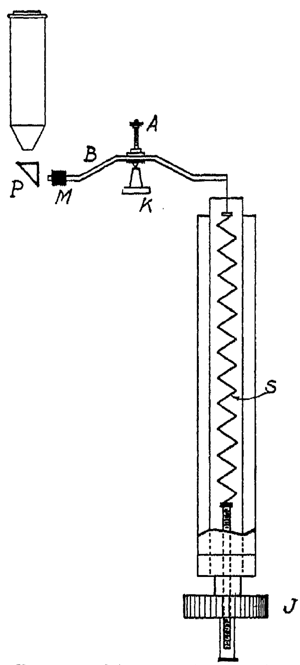


FIG. 80. — Diagrammatic sketch of Thyssen gravimeter. (*Zeitschrift für Geophysik.*)

† A. Schleusener, "Messungen mit Transportablen Statischen Schweremessern," *Zeitschrift für Geophysik,* Vol. 9, 1934, p. 369.

St. v. Thyssen, "Ueber die Wirkungsweise von einigen Federgravimetern," *Zeitschrift für Geophysik,* Vol. 15, 1939, p. 121.

with opposite orientation, i.e., with masses mounted at approximately the same level but on opposite sides of the knife-edges. This expedient is employed in several other meters which are sensitive to level.

"Zero Length" Spring Gravimeter.—Another commercial gravimeter is a modification of a vertical seismograph employing a "zero length" spring.† Figure 81 shows a diagram of the essential part of the meter. The spring S is so wound that its elongation is equal to the distance between the points where it is attached; that is, if one defines the initial length as the actual physical length minus the elongation, this type of spring has zero initial length.

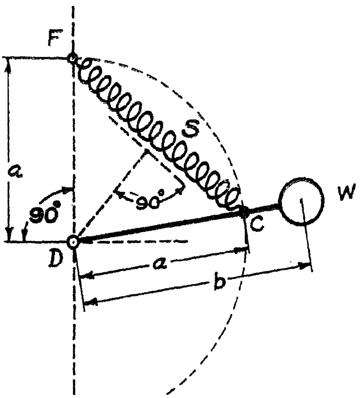


FIG. 81.—Diagrammatic sketch of spring gravimeter. (After La Coste, *Physics*.)

The weight W is fixed to the arm DC which is pivoted at D. The spring S is attached to the arm at the point C and to the frame at the point F vertically above the pivot D. The distances a and b are about 45 cm. and the weight W about 2.5 kg.

The period of the spring can be given almost any desired value by choosing a spring material which has an appropriate constant. For periods of about one minute the system tends to become unstable and hence very sensitive to variations in the torque exerted by the weight.

The meter is damped and employs an additional spring in order to return the mass to its standard position making the instrument a null reading type. The instrument is compensated for small changes in temperature.

Wright Gravimeter.‡—The Wright meter (Figure 82) comprises primarily two helical elastic springs which taper from opposite ends toward a common center portion to which is attached a relatively short aluminum rod. The rod carries a weight at its outer end and a mirror at or adjacent its inner end on the axis of the two helical springs. The helical springs are attached at their outer ends to a frame 12 which is rotatably mounted. The frame, together with the springs, may be rotated by means of a shaft extension terminating in a knurled head 14. Clamps are provided for preventing serious vibrations of the springs during transportation. Also, to minimize temperature variations within the case, and hence changes in the elastic constants of the spring, the case is thermally insulated by use of several outer casings.

† L. J. B. La Coste, Jr., "A New Type of Long Period Vertical Seismograph." *Physics*, Vol. 5, 1934, p. 178.

‡ F. E. Wright, U. S. Patent 1,579,273.

The operating procedure may be summarized as follows. At a given location (base station) the frame and springs are rotated by means of the knurled head 14 until the aluminum rod and weight W are in an approximately horizontal position (standard reference position). The standard position of rest is determined by means of an autocollimating telescope sighted on the mirror M. The reference angle is read off the graduated circle 15 attached to the axis of rotation of the frame 12. The difference in the readings of the graduated circle for the two horizontal positions of the aluminum arm is a measure of the elastic deformation set up in the springs by the mechanical torque exerted by this arm in response to the gravity pull on weight W. The instrument is then moved to another location (field station) and the procedure repeated. The relative difference in the readings of the horizontal positions at the field station as referred to that at the base station is a measure of the relative gravity at the field station.

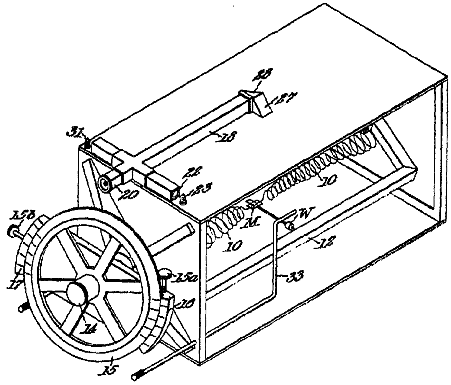


FIG. 82.—View of the Wright gravimeter. (U. S. Patent 1,579,273.) 10, helical elastic springs; W, mass; M, mirror; 12, supporting frame of helical springs; 14, adjustment knob; 15, graduated circle; 16 and 17, vernier scales; 18, autocollinating telescope; 23, light; 27, totally reflecting prism.

Boliden Gravimeter. †—This instrument is a static type wherein the determination of the relative gravity is based on a change in capacity produced by a vertical displacement of a spring-supported plate.

A schematic section of the gravimeter is shown in Figure 83. The movable body is suspended by the plate springs F which are attached to the cross piece of the support E. The top of the movable member forms one plate of a parallel plate condenser, the other plate of which is fixed to a thick steel disc A by an electric insulator B. The contact C connects the condenser with the oscillatory circuit of an ultramicrometer coupling which registers minute variations in capacity between the two plates. The function of the lower plates (the returning plates) is to provide a means for returning the movable body to the zero (base

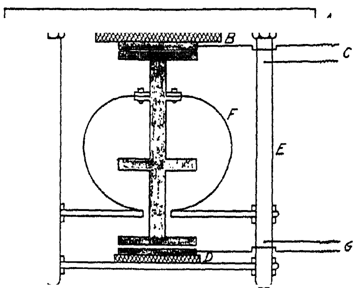


FIG. 83.—Schematic drawing of Boliden gravimeter. (After Hedstrom, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953, p. 13.)

† H. Hedstrom, "A New Gravimeter for Ore Prospecting," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953, pp. 12-15 (Feb., 1938).

station) position. This is accomplished by applying a potential to one of the plates thus causing the movable plate to be displaced due to the electrostatic forces produced by the potential difference between the plates.

The ultramicrometer coupling is designed so that variations in the anode current caused by changes in the capacity between the two plates are registered on a milliammeter. To accomplish this, the plates are connected to the grid circuit of an oscillating tube. By choosing the instrument constants such that the plate separation of the condenser is 0.002 cm. and the coupling sensitivity 10 ma. per cm., a gravimeter sensitivity of 0.01 milligal may be obtained. An instrumental sensitivity between 0.01 and 0.05 milligal is claimed.

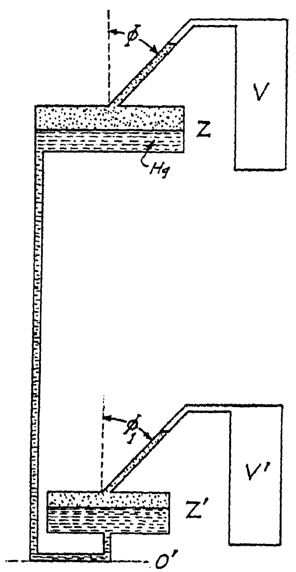


FIG. 84.—Diagran gravimeter. (*Zeitschrift für Geophysik.*)

Haalck Gravimeter. †—The Haalck meter is a static meter using a sealed volume of gas as the elastic medium. A schematic representation of the meter is shown in Figure 84. The long vertical tube and the lower portions of the two containers Z and Z' are filled with mercury. The upper portion of each of the containers Z and Z' and a portion of the tubes above them are filled with a light fluid, such as toluol. When the value of gravity varies, the density of the mercury column varies correspondingly. Also, the volumes of gas in the two chambers V and V' vary in such manner that the total net gas pressure on the ends of the fluid column is equal to the weight of the fluid. By making the ratio of the area of the containers Z and Z' to the area of the capillary tubes containing the toluol very large, the displacement of the liquid in the capillary tubes can be made many times the displacement of the mercury in the containers so that a high sensitivity can be obtained by readings of the toluol menisci. Furthermore, if the proper choice of volumes is made the temperature effect can be somewhat decreased, but at best, the meter is quite sensitive to temperature and is usually used in an ice bath. The instrument is highly damped and has been used with considerable success for coarse work on ship board.

Astatic Hydraulic Gravimeter. ‡—This instrument is an astatic type which utilizes the flow of liquid as an astatizing means. The meter comprises a specially shaped container connected to a vertical spring and a fluid system whereby upon lengthening of the spring fluid flows into the mass thus increasing its weight and further lengthening the spring. A schematic view of one form of the meter is shown in Figure 85. The tube system is completely filled with the fluid and the containers 4 and 13 are partially filled. Stops 8 and 9 on the scale 10 define the limits within which the balance can move.

† H. Haalck, "Ein Statischer Schwerkraftsmesser," *Zeitschrift für Geophysik*, Vol. 7, 1931, p. 95; Vol. 8, 1932, pp. 17 and 197; Vol. 9, 1933, pp. 81 and 285.

‡ H. M. Evjen, "Gravimeter," U. S. Patent 2,117,471, issued May 17, 1938. (Assignor to Shell Development Company.)

The operation of the instrument may be summarized as follows. The gravimeter is leveled by means of the leveling screws so that the liquid in the container or pan

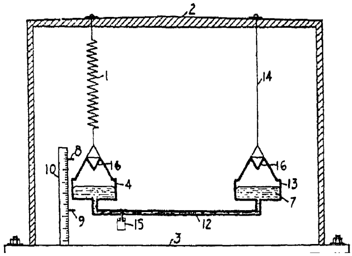
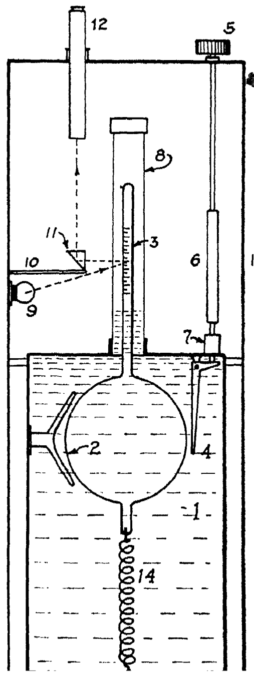


FIG. 85.—Schematic view of the astatic hydraulic gravimeter. (Evjen, U. S. Patent 2.117.471.)

4 is at the same level as in the container 13. Any increase in the value of the force of gravity acting on the mass of the pan 4 will cause the pan to move downward, thereby stretching the spring 1. The liquid in the pan 4, however, will tend to remain at the same level as in the container 13, so that a certain amount of liquid will flow into the pan 4 from 13 through the tube system 12. The resulting increase in the quantity of liquid in pan 4 will cause a further stretching of the spring 1 and accentuate the downward movement of 4. A decrease in the value of gravity will cause a reverse process to take place. It is claimed that the instrument is capable of high precision provided it is leveled with very great accuracy.

Hydrometer-Type Gravimeter.—A gravimeter of simple mechanical design is shown in Figure 86. The hydraulic buoyant force of a liquid on a cylinder is balanced against the downward pull of a long spring. The greater the attraction of gravity, the greater is the buoyant force. High sensitivity is obtained by use of a long spring, the weight of which is made relatively negligible by use of a fluid of appropriate density. The instrument is highly damped and aperiodic. Precise leveling is not necessary. Thermal effects are minimized by so constructing the float that its temperature coefficient produces effects opposite to those of the liquid. Barometric changes are minimized by sealing the container hermetically. A mechanical clamping device is employed for locking the float while the instrument is being moved. A cork insulated case protects the instrument against rapid temperature variations. Slow temperature changes of the liquid, although of small magnitude, are corrected for by prior calibration. The optical system comprises a source of light focused on a small scale carried by the float, and read by a magnifying eyepiece with index reference line. The temperature of the liquid is shown by a small thermometer fastened to the liquid container. The thermometer is illuminated by the lamp, and read through a small window in the case of the instrument.



type float; 3, scale; **4, clamp; 5, clamp** screw; 6, thermal insulation; 7, stuffing box; 8, glass tube; 9, lamp; 10, light reflector; 11, scale prism; 12, magnifying eyepiece; 13, thermally insulated housing; 14, spring.

Mott-Smith Gravimeter. †—This instrument is an astatic meter employing an auxiliary quartz fiber as a labilizing spring to affect a further movement of the "weight" after it is initially moved by gravity.

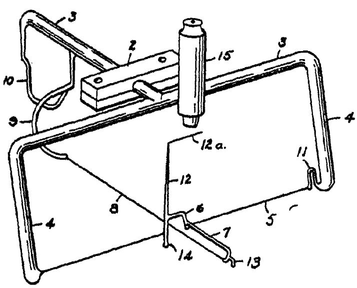


FIG. 87.—Perspective view of the Mott-Smith gravimeter. (Mott-Smith, U. S. Patent 2,130,648.)

The component parts of the meter are shown schematically in Figure 87. A T-shaped frame 3 having arms 4 is rigidly attached to a casing 1 by means of a clamp 2. A torsion fiber 5, which is about 1½ inches long and about 0.002 inches in diameter, carries a weight arm 7, which, in turn, is connected to the labilizing fiber 8. The latter is connected to the T-frame 3 through two springs 9 and 10. The weight arm 7 is fixed and normally extends in a substantially horizontal position from the torsion fiber 5.

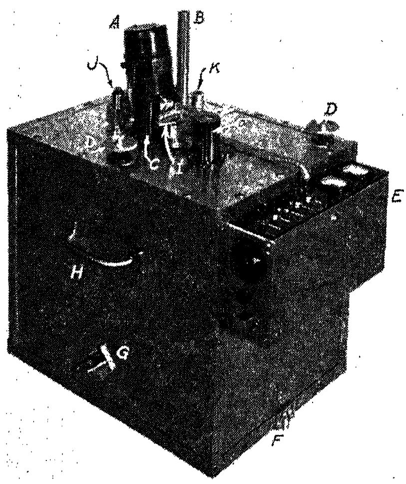


FIG. 88.—Exterior view of Mott-Smith gravity meter. (Courtesy of Mott-Smith Corporation.)

A—Bath stirring motor
B—Thermometer housing
C—Observing microscope housing
D—Leveling screws
E—Electric control panel for thermostat
F—Foot mounting bracket
G—Truck mounting bracket
H—Carrying handle
I—Reading light
J—Thermometer reading microscope
K—Thermometer reading light

The center of gravity of the suspended system, which includes the arm 7, its extension 13, pointer 12, and counterweight 14, is placed as nearly

† L. Mott-Smith, "Torsion Gravimeter," U. S. Patent 2,130,648, issued Sept. 20, 1938.

as possible at the same level as the torsion fiber 5 by adjusting the counterweight 14. The substantially-horizontal equilibrium position of the weight arm 7 is adjusted by adding or removing quartz by fusion, or by bending the extension 13.

Operation of the instrument is as follows: The weight arm is adjusted at a base station until it is in a substantially horizontal position and the position of the pointer arm 12a attached to the pointer 12 is observed with the aid of the microscope 15. In this position, the auxiliary fiber 8 passes through the center of gravity of the suspended system and hence exerts no torque on it. The instrument is then moved to another station. If the acceleration due to gravity is greater at this station, the weight arm 7 will

TABLE 8

CLOSURE DATA, ILLUSTRATING ACCURACY OF MOTT-SMITH GRAVIMETER †

Total Gravity Difference (millidynes)	Number of Differences in Loop	Error of Closure (millidynes)
17.07	4	0.19
15.20	3	0.26
14.90	4	0.11
15.65	5	0.03
6.05	4	0.05
8.67	3	0.23
6.48	3	0.16
8.16	4	0.04

be pulled downwardly against the resistance of the torsion fiber 5; at the same time the fiber 8 exerts a torque on the suspended system tending to displace it still further. The total displacement of the pointer arm 12a is then measured by means of the microscope 15.

An exterior view of the Mott-Smith gravity meter is shown in Figure 88. The instrument is capable of measuring the force of gravity with an accuracy of 0.1 millidyne or better. To achieve this accuracy, it is necessary to observe various precautions. The casing must be air tight and contain a drying agent to keep the air inside at a constant density. (If a drying agent, such as calcium chloride, is not included, corrections must be made for changes in barometric pressure and relative humidity.) Also, it is necessary that the temperature within the case be kept constant to within about 0.001° C. The latter requirement is fulfilled by immersing the casing and level (not shown) in a water bath whose temperature is controlled by a mercury-toluene thermostat element.

The accuracy of the instrument is indicated by Table 8 which is a summary of closure data for eight loops. The first column gives the total numerical change in gravity around the loop, i.e., the sum of the differ-

† Mott-Smith, *Geophysics*, Jan. 1937, p. 29.

ences added without regard to sign; the second column gives the number of observed differences which were added together to form the loop; the third column gives the error of closure. The probable error of closure was 0.11 millidyne, the average error 0.13 and the largest error 0.26 millidyne.

Electric Gauge Type Gravimeters.—Electric gauge type gravimeters offer several advantages. The ratio of the displacement of the indicator to the displacement under measurement can be made as high as 100,000 without introducing frictional effects, such as hysteresis or sticking. Also, the gauge permits measurements of very high accuracy and allows remote reading or recording.

The electric gauging device may be: (1) capacity type, (2) saturation type, (3) eddy current type, or (4) bridge type. †

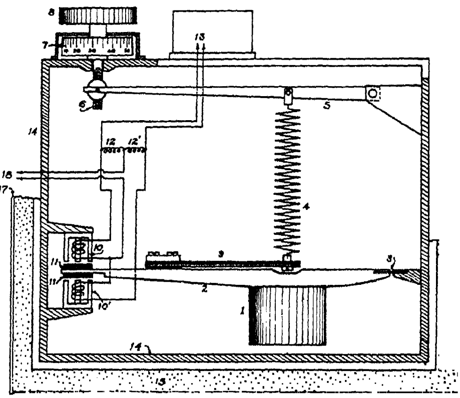


FIG. 89.—Electric gauge, bridge-type gravimeter. 1, mass; 2, suspension lever; 3, spring hinge; 4, suspension spring; 5, spring lever; 6, adjustment screw; 7, adjustment scale; 8, adjustment knob; 9, bimetallic temperature compensation lever; 10 and 10′, electric gauge heads, differential type; 11 and 11′, armatures; 12 and 12′, reactor coils; 13, alternating current vacuum tube voltmeter; 14, cast aluminum case; 15, thermal insulation; 17, polished duraluminum case; 18, leads to alternating current supply. (Courtesy International Geophysics, Inc.)

The first method is illustrated by the Boliden gravimeter previously described. Because the total capacity is usually small, measurements are generally made at high frequency, which is most conveniently supplied by a small vacuum tube oscillator. In general, electric gauging methods involving measurements of changes in capacity are not as applicable as other methods such as the bridge type for the usual gauging applications. The saturation method is usually desirable where unusually low contact pressure on the part being gauged is desired. The eddy current method of measurement is best known commercially as a means for measuring the thickness of non-magnetic metallic sheets, particularly thin foils. Eddy currents are induced in the sheet and the magnitude of these currents, which is dependent on the thickness of the sheet, is measured by the effect on the inducing field. The bridge-type gauge has several

† C. M. Hathaway and E. S. Lee, "The Electric Gauge," *Mechanical Engineering*, Sept. 1937.

advantages. The bridge circuit is simple, comprising a small number of simple parts. Sufficient sensitivity and output can generally be obtained without auxiliary amplifiers.

An electric gauge, bridge-type gravimeter is indicated schematically in Figure 89. The displacement of the mass 1 causes a displacement of a suspension lever 2 to the end of which is attached a light arm composed of a magnetic material. The motion of the magnetic arm between the pole pieces of the electromagnet varies the current through the reactors 12 and 12′, thereby producing a deflection of the voltmeter 13.

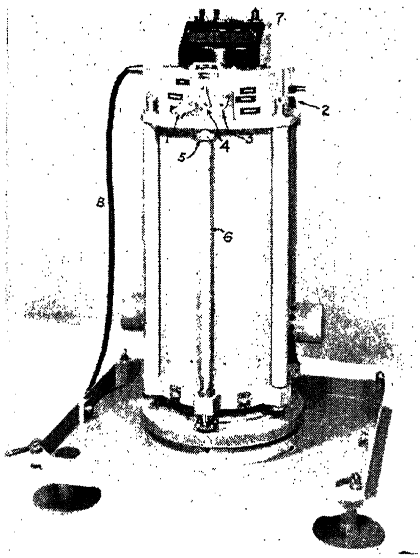


FIG. 90.—Exterior view of the Graf-Askania mechanical-electrical gravity meter. 1, arresting clamp; 2, scale range adjustment; 3, calibration weight; 4, spring adjustment; 5, leveling screw knob; 6, leveling screw shaft; 7, microammeter; 8, cable to storage battery. (Courtesy of the Askania Corporation.)

In another electric gauge-type gravimeter, determinations of the relative gravity are based on electrical measurements which show the change in length of a vertical spring to the lower end of which is attached a mass M.† Measurements of the relative gravity are made on a microammeter (without amplifier) or on a reading drum. The sensitivity of the instrument can be modified within a fairly wide range; for field investigations, 3 to 6 scale divisions of the microammeter are made to correspond to 1 millidyne. A double thermostat is provided which, however, is used only when the apparatus is moved between two observation stations far apart from each other. No temperature control is necessary for local investigations.

In one American modification of this method, the electric measuring device utilizes a change in impedance as a measure of the movement of the mass. In this system, a

† A. Graf, "Ein neuer statischer Schweremesser zur Messung und Registrierung lokaler und Zeitlicher Schwereänderungen," *Zeit. für Geophysik,* No. 14, Vol. 5/6, 1938, pp. 153-172.

medium frequency alternating current is passed through a differential coil system, and any movement of *M* causes a change in impedance, with resultant unbalance of an alternating current, bridge measuring circuit.

An exterior view of the Askania meter is shown in Figure 90. The spring system and the electrical measuring device are contained inside a thick-walled, air-tight housing. All of the levers are so connected that they can be operated from outside the housing. The levers include: the arresting lever for the mass (left side of Figure 90), a drum for changing the measuring range (right side of Figure 90), a device for supporting a standard weight, and an arrangement for moving the spring. The last two devices are employed only occasionally and are not usually necessary for taking measurements. Levels with 60 second graduations are mounted inside the case and can be observed from above through a glass window. The leveling screws on the lower part of the instrument can be adjusted from the top of the instrument so that it is possible to view the inside graduated levels while making adjustments. Inside the main housing there is a triple-walled soft metal casing which holds a thermostat. The thermostat requires 20 to 25 watts for a temperature rise of 10°. The current source for operating the gravity meter is a 12-volt storage battery.

Calibration of Gravimeters

A gravimeter must be calibrated so that the change in the acceleration of gravity can be determined from the measured displacement of the mass. This may be done by several methods. One method, which is applicable for practically all types of gravity meters, consists in measuring the variation of gravity as a function of height or elevation. Such calibration measurements may be carried out conveniently in any tall building.*

Less simple is the calibration of an instrument by observations made at stations whose gravity differences have previously been measured accurately with pendulums or torsion balance.

Certain meters, for example, the Thyssen instrument described on page 175, may be calibrated conveniently by adding small weights. The sensitivity, or number of milligrams required to produce one scale deflection is obtained directly from a graph of scale divisions versus load (in milligrams). Certain other meters of proper design may be calibrated by the "tilting method." Tilting the instrument decreases the effective value of gravity by a factor equal to the cosine of the angle of tilt. Hence, an indication of the sensitivity may be obtained by observing the scale deflections for various angles of tilt.

In all cases, it is desirable to provide a means by which calibration can be carried out in the field because changes in sensitivity may occur during the course of time. (Compare p. 172.) For instance, when dealing with a delicately balanced elastic system, small changes in geometry caused by relative motion of supports, or by readjustment to compensate for creep or large changes of gravity, may cause bothersome changes in calibration.

* See, for example, "Gulf gravimeter tests in Empire State Building, Washington Monument, and Pittsburgh Cathedral of Learning," Report in National Research Council, Transactions of American Geophysical Union, Vol. I, Washington, July, 1938.

Elevation Correction for Relative Gravity Data

The elevation correction takes into account changes in gravity caused by changes in elevation of stations. The acceleration of gravity decreases approximately six to eight hundredths of a milligal per foot of increase of elevation. Accordingly, except in an exceptionally level region, the observed gravity values will vary considerably from station to station due to changes of elevation. Relative gravity determinations, therefore, are usually reduced to the level of the base stations of the particular survey. Two methods are used: In the "free air" method, the layer between the datum level and the level of the field station is assumed to have no gravitational effect; i.e., it is assumed to consist of free air. In the Bouguer method, this layer is assumed to have a gravitational effect equivalent to that of a layer of rock of some definite density.

The free air correction is equal to $-2hg/R$ where h is the height of the place of measurement above sea level in meters and R is the radius of the earth at the latitude of the place of measurement.* The quantity $2g/R$ is approximately constant for all latitudes and is equal to 0.0003086. in English units, therefore, the free air correction is $-0.10h$ milligal, where h is the elevation difference in feet.

The Bouguer correction is $0.10h\ (1-3d/2)$ where d is the ratio of the density of the assumed rock to the mean density of the earth.

The proper elevation correction to be used in any region may be calculated from the Bouguer formula if a sufficiently good estimate can be made of the density of surface materials. It is also possible to find the elevation correction experimentally by obtaining gravity values at a series of closely spaced stations along a line chosen to include considerable changes of elevation; an appropriate value of the correction is chosen by trial and error to eliminate the effect of elevation difference in the corrected gravity values.

Having established the proper constant, the observed gravity values are all corrected to the elevation of the main base by adding or subtracting the proper amount from each.

Latitude Correction for Relative Gravity Data

The latitude correction is introduced to eliminate from the final gravity map the change of gravity which normally occurs due to a change of geographical latitude. This change amounts roughly to an increase of one milligal per mile of northerly displacement. To make the correction, curves are drawn from the standard Bowie formula giving the normal value of

* For a derivation of this equation see *A.A.P.G.* Vol. 12, No. 9, p. 889.

gravity as a function of the latitude. (Equation 3a.) The latitude of the main base and that of each field station are obtained from the map. For each station the gravity difference caused by difference in latitude is read off the curve and subtracted from the observed gravity differences.

Topographic Irregularities and Relative Gravity

The effect of topographic irregularities on the relative gravity is very small and may often be neglected. In areas of rugged relief, however, certain corrections pertaining to topographic irregularities are usually made. Hammar † gives a comprehensive study of the method of terrain corrections for gravimeter surveys.

Accuracy of Data *

The value of a gravity map depends largely on its accuracy. Various types of tests have been made to determine this accuracy. Aside from mistakes such as misreading the scale or error of computation, the error arises from three principal causes. The first is erratic fluctuations in the instrument reading from such causes as temperature fluctuation, elastic-after-effect, effect of jars, and the like. The second is inaccuracy of the calibration curve. The third is errors in making the latitude or elevation corrections due to inaccuracy of surveying or use of an incorrect elevation correction constant.

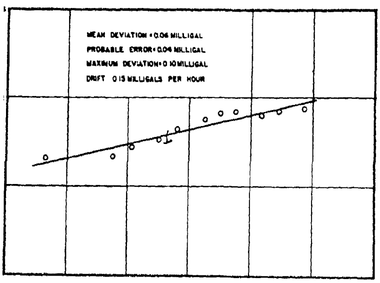


FIG. 91.—Plot of repeated observations at the base station. (L. M. and F. W. Mott-Smith, *The Petroleum Engineer.*)

The magnitude of the first type of error may readily be determined by noting the deviations of a succession of observations at the same station. In making such a test it is evidently necessary to simulate closely actual field conditions. The procedure, accordingly, is to occupy a station and make an observation in the manner customarily employed in the normal field work. The instrument is then transported in its truck a distance of one mile and returned to the same station. Here a second standard observation is made. A series of eleven such observations plotted as a drift curve are shown in Figure 91. ‡ These data were treated as follows for the

† Sigmund Hammar, "Terrain Corrections for Gravimeter Stations," *Geophysics,* Vol. IV, No. 3, July, 1939.

* The method of determining accuracy has been supplied largely by L. M. and F. W. Mott-Smith in a personal communication.

‡ L. M. and F. W. Mott-Smith, "Advancements in the Use of the Gravimeter in Oil Exploration," *The Petroleum Engineer,* July, 1939, pp. 85-97.

computation of the probable error of a single occupation of a station. The best straight line is first passed through the observed values. This may be readily done by a trial and error method or by the usual least square procedure. The deviations of the observed values from the drift line are then computed, and finally the probable error is obtained by the standard procedure of multiplying the square root of the average square deviation by the factor 0.67. In this test the mean deviation is found to be 0.06 milligal and the probable error of a single observation 0.04 milligal. The largest deviation from the drift line occurred at the second trial and is 0.10 milligal. This accuracy is somewhat better than necessary for obtaining useful gravity maps.

The error caused by inaccuracy of calibration can partially be tested by determining the error of closure of closed loops of gravity differences. If the scale readings are converted into gravity values by means of an incorrect calibration curve, which might be the case if the sensitivity of the instrument changed during the course of a survey, errors of closure will appear. There is, however, also the possibility that an initially incorrect calibration might give proper closure, and yet all the measurements might be too large or too small by a constant percentage. This possibility may be detected in several ways. One method is to make observations at different known elevations in a building. (Compare p. 184.) Another is to check the gravimeter against gravity differences known from pendulum observations. Gravimeters are considered satisfactory if such tests indicate calibration errors of less than two per cent.

Errors caused by incorrect latitude and elevation corrections can be reduced to practically negligible proportions by surveying with sufficient accuracy and by taking care to use the proper elevation correction constant. It is practicable to establish the latitude of each station to within about one-twentieth of a mile. Thus the error in the latitude correction is about five hundredths of a milligal. It is also perfectly feasible to require closure of level line loops to within one foot, so that the error from incorrect elevation is not greater than about seven-hundredths of a milligal.

Field Procedure.—The gravimeter is usually transported in a light closed panel truck. While making a measurement the instrument must rest on a firm base. For this purpose it may be removed from the truck and set up on the ground, but a better method, used by several organizations, is to support the gravimeter on a sturdy tripod arrangement inside the truck. When a measurement is to be made, the extension support is lowered to the ground through a hole in the floor of the truck. Figure 92 shows the instrument on such a tripod ready for a measurement. This method is advantageous in that the meter, and the operator, may be protected from inclement weather.

The general field procedure* with direct gravity instruments of the pendulum or gravimeter type is similar to that with a magnetometer. The initial step for a field survey is the lay-out of a set of station locations in the area to be explored. The location of stations is done by a field surveying party. Each station is marked with a flag or stake for easy discovery and

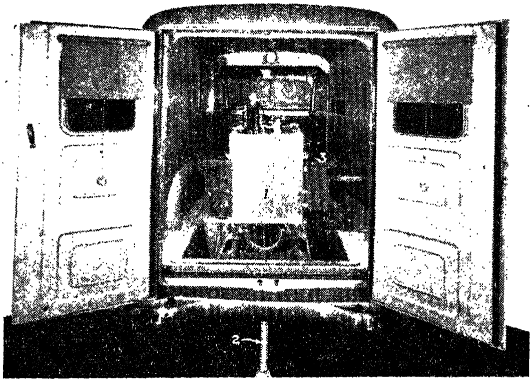


FIG. 92.—(1) Gravimeter on tripod in truck, with (2) extension leg resting on ground, (3) battery box for thermostat and motor battery. (L. M. and F. W. Mott-Smith, *The Petroleum Engineer.*)

preferably located along roads or trails. The stations may be arranged in checkerboard fashion, or placed along traverse lines, etc., depending upon the nature of the problem. Each station is plotted on a field map or recorded by proper survey notes. For most reconnaissance work one station per mile is sufficient. In areas where more detail is needed as many as ten stations to the mile may be required. Occasionally when making a detailed survey in areas where abrupt changes are present, a closer spacing may be advisable. The optimum spacing of stations is governed by local conditions. The surveyors run relative elevations on all of the stations, and if necessary plot topographic contours. The survey work proceeds simultaneously with the gravity survey, with the surveyors working sufficiently ahead of the instrument party to prevent delays in finding stations.

The instrument is first set up at a chosen base station and adjusted for the area. A measurement is then taken, followed after a short interval by a second observation to guard against mistakes and to allow averaging of results to increase the accuracy. This procedure is then repeated at successive stations. After five to ten stations have been occupied the instrument is returned to the base station for a check observation. Usually the next

* Personal communication from L. M. and F. W. Mott-Smith. Compare L. M. and F. W. Mott-Smith, *The Petroleum Engineer,* July, 1939. A major portion of this section is essentially a description of the general technique employed by the Mott-Smith Corporation and summarizes the important aspects of tested field procedure.

step is to make a subsidiary base station out of one of the new stations conveniently located for further expansion of the survey. This is accomplished by taking a second or repeat observation at the desired new base station. Each base station is simply a field station whose gravity value relative to another base has been made more certain by being occupied or measured at least twice. The survey then proceeds in the same manner from the new base, check readings being now taken at this base. In this way the area is gradually covered with a series of bases and ordinary stations. The use of subsidiary base stations avoids long trips back to the main base for check readings.

During the field work, a computer at the field office calculates the prior day's work and builds up the gravity anomaly map. The usual process of computation is as follows: (a) The instrument scale readings are first converted into milligals by using a calibration curve; (b) the drift curve is then obtained by plotting the value of gravity against the time at which the reading was taken. Figure 93 shows a drift curve representing the work of an average day. Analysis of the drift curves gives considerable information on the progress of the work during day. It also serves as a partial check on instrument operation and is used to compute the differences in the gravity between the several stations.

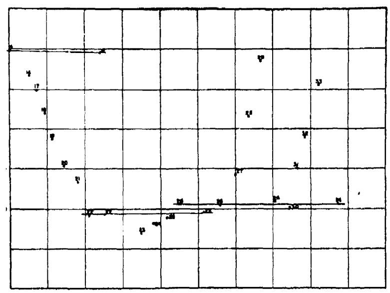


FIG. 93.—Drift curve for consecutive reading of stations. (L. M. and F. W. Mott-Smith, *The Petroleum Engineer.*)

On this particular day the operator first occupied base number 4 at 8:00 A.M. and made an observation. He then occupied stations 16 to 22 and returned to the initial base number 4 for a check reading at 10:20 A.M. If the instrument were perfect, he would have obtained exactly the same reading. Actually, due to slight instrumental inaccuracy or drift in the two hour and twenty minute interval, a reading was obtained which was 0.05 milligal less than the first reading. The instrument was then returned to station number 22, thus securing a second observation at this station and establishing it as a new base. Four new stations were then observed and a check reading at the new base taken. Station 26 was then established as a base and finally seven new stations were observed, with two check readings at base 26. It will be noted that on this particular day the instrument was set up 25 times, 18 new stations were determined, and two new bases established.

The difference in the gravity between the various bases and stations is taken directly from the drift curve. Small discrepancies between the various observations at a base are allowed for by drawing a mean straight line

through the points. This procedure involves the assumption that the instrument drifted at a constant rate between observations. In usual field work the drift is fairly constant, although erratic results occur occasionally due to peculiarities of gravity instruments when subjected to rough handling, etc. However, experience has shown that this method is sufficiently accurate for practical purposes. The gravity difference in milligals between each station and the base to which it is referred is taken from the plot by measuring vertically between the drift lines.

Knowing the difference in gravity between each station and one of the bases, and the differences between bases, the gravity difference between

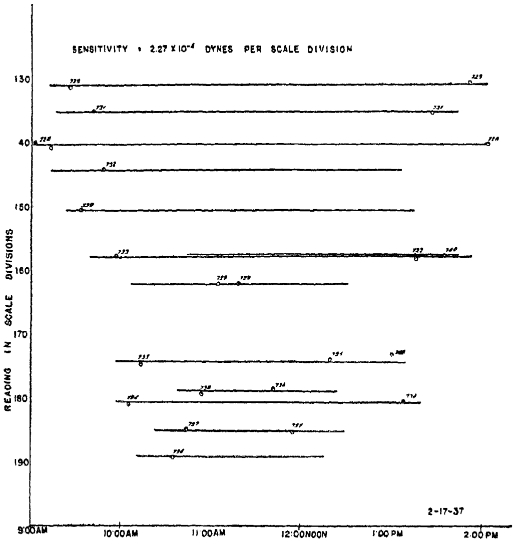


FIG. 94.—Drift curves for duplicate reading at each station, showing readings taken at 13 stations. (Bryan, *Geophysics.*)

all stations and bases and the main base can at once be obtained by simple arithmetic. At the start of the survey the main base in each area is given an arbitrary gravity value, preferably chosen great enough so that none of the station values are negative to avoid the inconvenience of carrying negative numbers. In all gravimeter work the actual numerical value of the acceleration of gravity is not obtained, because the instrument is capable only of giving the difference in gravity between two stations. Usually, however, this is not a disadvantage because interpretation is based upon the variations or anomalies in gravity exactly as is done in the interpretation of magnetic anomalies. If absolute gravity values are desired, the gravimeter survey must be supplemented with a pendulum gravity measurement at one or more of the base stations.

An alternative procedure, which is not as rapid as that outlined above, consists in occupying each station at least twice during the day. When this

procedure is used, the drift curve plot is similar to that shown in Figure 94.† For the case shown, the drift lines are practically horizontal, indicating that on that particular day the drift was very small and gratifyingly uniform. The gravity anomalies or differences between the various stations are obtained, as before, from the vertical interval on the plot.

After the gravity measurements at each station have been obtained, they should be subjected to two corrections: (a) elevation and (b) latitude.

The elevation correction has been previously described (p. 185) and amounts to approximately 0.06 to 0.08 milligals per foot increase in elevation. The value of the elevation correction is dependent upon the relative densities of the subsurface materials, and usually can be calculated with sufficient accuracy when the densities of the near surface and deeper materials are known. The differences in elevation are taken from the survey and topographic notes.

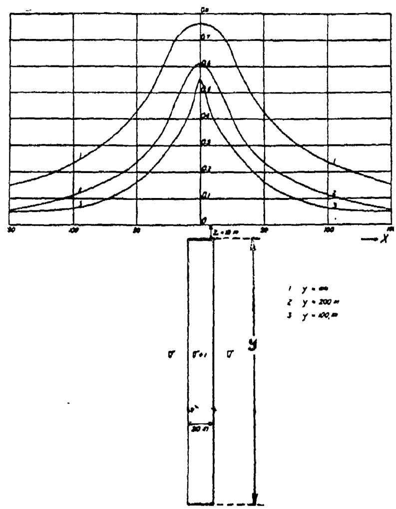


FIG. 95.—Gravity anomalies over a vertical body. (Hedstrom, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953.)

When the survey covers an area of considerable geographical latitude it is advisable to correct for variation of gravity with latitude before final interpretation is made. The correction is approximately one milligal per mile displacement away from the equator (p. 185). This correction is most conveniently made by plotting a latitude correction curve covering the latitude range of the area, and applying the necessary correction to each station. Latitude corrections are seldom necessary in mining work due to the small areal extent of the work. In petroleum exploration these corrections are usually desirable, especially when surveys along traverse lines extend an appreciable distance in a north-south direction or where an extended grid-work of stations is employed.

Interpretation of Relative Gravity Data (Contours and Profiles.)— The values of the relative gravity are corrected for elevation and latitude effects and then are plotted as isanomalic contours or as profiles. The isanomalic contours plotted in direct gravity work are lines of equal value of gravity anomaly referred to some point in the area, usually the main base

† A. B. Bryan, "Gravimeter Design and Operation," *Geophysics*, October, 1934, Vol. 2, No. 4, pp. 301-308.

station, where the anomaly is assigned some arbitrary value. The contour interval generally used is one-half milligal.*

Anomalies Over Assumed Bodies.—Theoretical gravity anomalies over hypothetical bodies of rectangular cross section are shown in Figures 95, 96, and 97.† In all three cases it is assumed that the body has a density

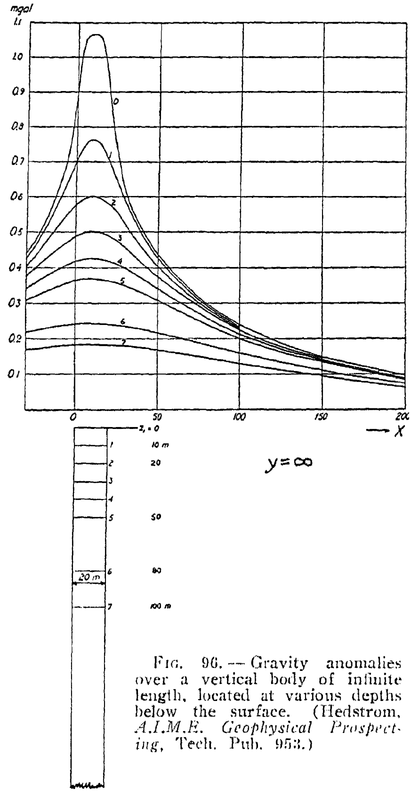


FIG. 96.—Gravity anomalies over a vertical body of infinite length, located at various depths below the surface. (Hedstrom, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953.)

one unit greater than that of the surrounding medium; that is, denoting the density of the medium by σ, that of the ore body is $\sigma + 1$. A density difference other than one unit would alter the amplitude but not the shape of the curves. In the case of a negative density difference, e.g., minus one unit, the anomalies will be inverted. Figure 95 shows the theoretical gravity anomalies over a vertical body whose length is ∞, 200 and 100 meters respectively. Figure 96 shows the theoretical gravity anomalies corresponding to a vertical body of "infinite" length, located at various

* An illustration of isanomalic contours is given in the section entitled "Examples of Gravimetric Surveys."

† H. Hedstrom, "A New Gravimeter for Ore Prospecting," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953, 1938.

depths below the surface. Figure 97 illustrates the theoretical anomalies corresponding to various dips for ore bodies of "infinite" length.

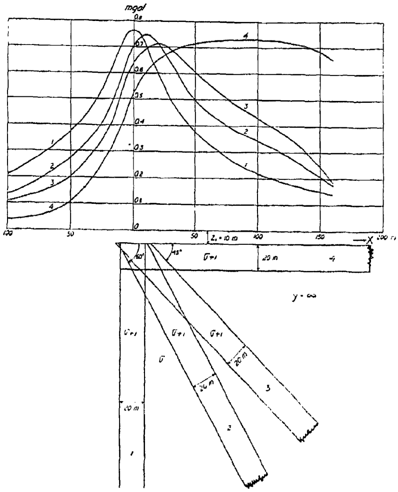


FIG. 97.—Gravity anomalies over bodies of different dips. (Heiland, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953.)

Figure 98 shows the gravity anomalies produced by step blocks located at various depths below the surface. The curves are drawn for the case that the density of the structure is greater than that of the surrounding medium.

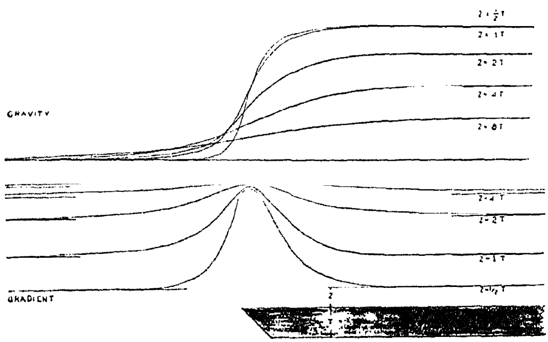


FIG. 98.—Relative gravity profiles for an infinite step block of thickness T, located at depths equal to $T/2$, T, $2T$, $4T$ and $8T$ below the surface. (Barton, *The Science of Petroleum*, Vol. I.)

Examples of Gravimeter Surveys.—The gravity anomaly over a large ore deposit in the Skellefte district is shown in Figure 99. The sulfide ore deposit, which had been previously disclosed by geoelectrical prospecting, consists of three parallel lenticular veins, dipping steeply to the north. One vein has a thickness of about 10 m. and the other two a thick-

ness of some 3 to 4 m. The uncorrected (primary) curve shows an effect of the contact with the schist. The reduced curve, which includes a correction for the effect of the contact on the assumption that the difference in

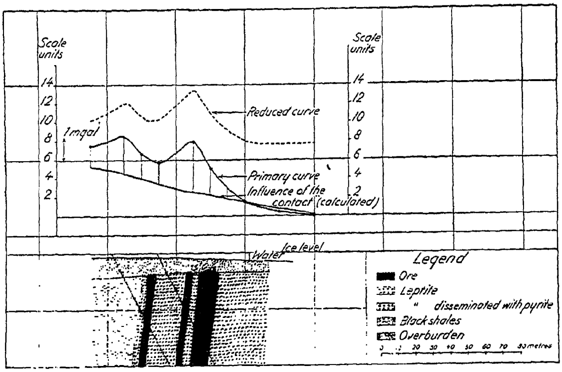


FIG. 99.—Gravity anomaly over a pyrite deposit located under a lake in the Skellefte district, Sweden. (Hedstrom, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 953.)

density between the schist and leptite is 0.2, shows indications of two ore zones.*

Petroleum Surveys †

Figure 100 shows an example of a recent survey conducted with the Mott-Smith gravimeter in the vicinity of Houston, Texas. The survey was made in nine working days, included 169 stations, and covered an area of approximately 100 square miles.

The area covered during the survey includes the Pierce Junction and the Mykawa oil fields. The Pierce Junction field is a piercement type dome having a substantially vertical salt plug approximately one mile in diameter. The cap rock overlying the plug is about 250 feet in thickness, and this in turn is covered with about 700 feet of recent material. The top of the salt is approximately 950 feet below the surface. The salt plug has a lower density than the surrounding rock, and its presence causes a gravity low or minimum. The gravity picture is complicated by the presence of a regional change of gravity, with the values increasing toward the south against the latitude gradient. Further complications are introduced by the neighboring gravity minimum due to the Mykawa field toward the southeast. The diagnostic gravity feature is a large so-called minimum nose, clearly indicated by several contours. This feature results when the gravity minimum caused by the salt plug superposed on the regional or lateral change of gravity. A study of the contours shows that the decrease in gravity due to the dome is about two milligals and its effect is noticeable

* Note that the gravimeter did not "resolve" or differentiate between the two southern veins.

† L. M. and F. W. Mott-Smith, personal communication; also, see "Advancements in the Use of the Gravimeter, in Oil Exploration," *The Petroleum Engineer*, July 1939, pp. 85-97.

at a distance of about three miles from the center. There is also a small positive gravity effect superposed on the large minimum, appearing as a maximum of four-tenths of a milligal near the center of the dome. This feature, no doubt, is caused by the shallow denser cap rock. The gravity

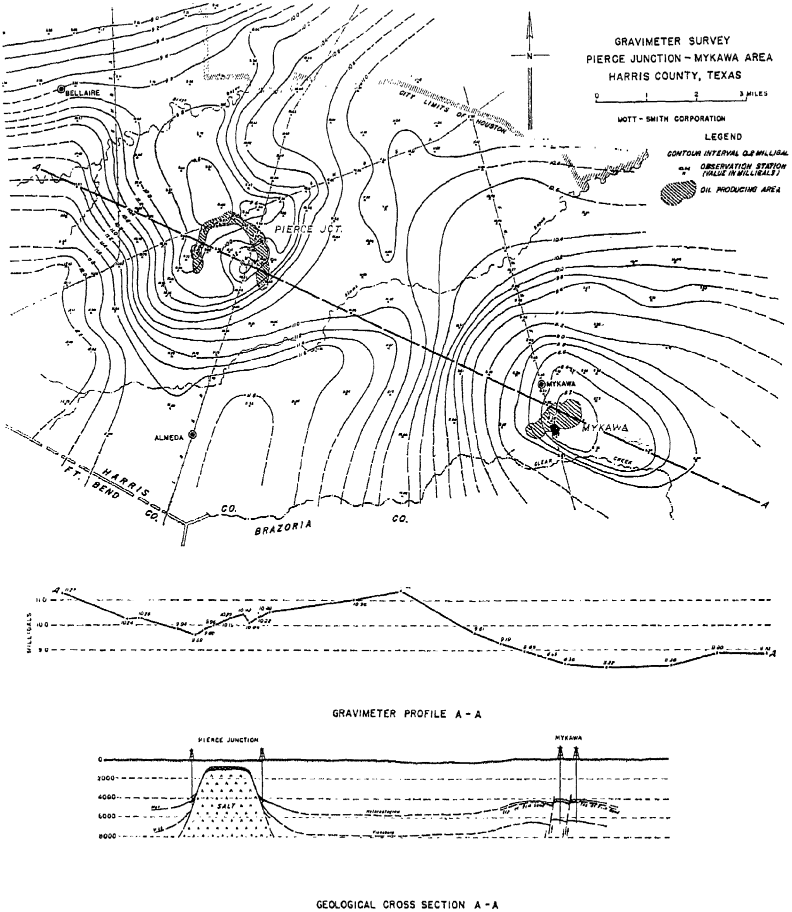


FIG. 100.—Pierce-Mykawa gravimeter survey. (L. M. and F. W. Mott-Smith, *Personal Communication.*)

maximum is somewhat displaced from the actual position of the cap rock as outlined by the producing area at the periphery of the dome. The displacement is principally caused by the regional gravity change and can be corrected for by applying corrections for regional change.* It can be seen, however, that even without correction, the gravity contours indicate the

* In using gravity work for detailing such structures, corrections for regional change are made whenever it is large enough to be important.

presence of the structure with sufficient accuracy for reconnaissance purposes.

The Mykawa field is on an uplift supposedly produced by a deeply buried salt mass. If this supposition is correct, the salt mass must lie at a depth greater than that of the deepest well which was drilled to 7,350 feet without hitting the salt. The Mykawa production comes from sands at depths of 4100 to 4900 feet.

The gravity minimum extends over a considerable area. In fact, it extends considerably beyond the limits of this survey in a southeasterly direction. The field is somewhat displaced from the center of the minimum, but such displacement could have been predicted by noting that the value of gravity drops toward the minimum much more rapidly on the northwest than on the southeast. This condition is produced by a salt mass of such vertical section that the center of gravity of the salt is not directly below its highest point. The center of the minimum is of course very nearly above the center of gravity, while the top of the uplifted beds is above the highest point. From a study of the exact shape of such a minimum it is possible to make a good estimate of the location most favorable for finding production.

GRAVITATIONAL EXPLORATION WITH THE TORSION BALANCE AND THE GRADIOMETER

The torsion balance measures the gravity gradient in a horizontal plane and the curvature quantity or horizontal directive tendency (H.D.T.) directly. (Unlike the gravimeter the torsion balance is not an instrument for measuring the component of gravity in the vertical direction.)

The torsion balance was introduced into the United States in 1922 and from that time until quite recently was very widely used in reconnaissance and detailed work in the American oil industry. At the peak of its popularity from 1928 to 1930 and from 1934 to 1936 inclusive, not less than 125 instruments have been engaged in exploration for oil each year.

Partial or total credit is given to the torsion balance for the discovery in the Gulf Coast area of seventy nine oil fields and salt domes up to the beginning of 1938.† The addition in oil reserves which should be credited entirely to the torsion balance probably amounts to 1,027,500,000 barrels. (The pendulum and gravimeter should be given credit in the same area for the discovery of 108,000,000 barrels of oil.)

The Eötvös Torsion Balance.—The values of gravity at neighboring points on the surface of the earth differ both in magnitude and direction. Thus, if two small masses at different elevations are supported at opposite ends of a light balance beam which is suspended from a torsion wire, the equipotential surfaces passing through the two small masses will not be parallel and the magnitudes of the force of gravity acting on the two

† V. G. Gabriel, "Probable Discovery Rates in the Gulf Coast Area," *Oil Weekly*, Vol. 95, No. 1, Sept. 11, 1939.

masses will not be the same. The non-parallelism of the two equipotential surfaces through the two small masses and the difference in the forces of gravity acting on the masses creates a rotational torque which acts on the suspended system.

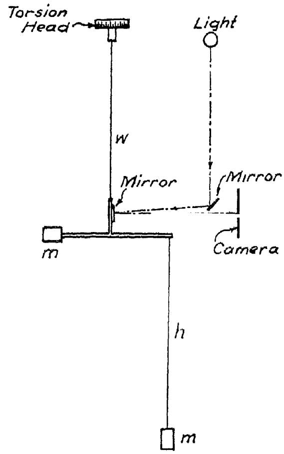


FIG. 101.—Schematic representation of Eötvös torsion balance.

A schematic representation of an Eötvös torsion balance is shown in Figure 101. W is a calibrated torsion wire of very small diameter on which is mounted a small mirror. An aluminum bar, suspended from the torsion wire and of negligible mass, has a small weight fastened to it on one side and a similar weight suspended from a fine wire on the other side. The angular deflection of the torsion wire is measured with the aid of a telescope by observing the shift of a scale image reflected by the mirror. The torque T required to rotate the swinging system (balance beam and torsion wire) through an angular deflection θ is

$$T = [illegible] \qquad (7)$$

The working equation of the torsion balance will be determined therefore by an expression for T in terms of the parameters measured at a particular station and the instrumental constants of the balance.

Derivation of the Basic Equation of the Torsion Balance

Let the origin of coordinates 0 of Figure 102 correspond to the intersection of the torsion wire and beam system of the torsion balance, and let U_0

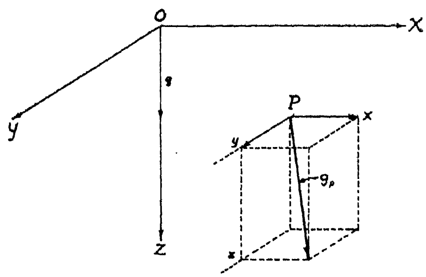


FIG. 102.—Resultant gravity at a point P located close to the origin of coordinates.

denote the gravitational potential at the origin. The z component of the force of gravity at the origin 0 is $\dfrac{\partial U_0}{\partial z}$.* Similarly, the x and y components

---

* This relation follows directly from the definition given on p. 151: viz., "the gravitational potential at any point is that quantity whose rate of change in the direction of gravity is the force of gravity at that point."

of the force of gravity at the origin 0 are $\frac{\partial U_0}{\partial x}$ and $\frac{\partial U_0}{\partial y}$ respectively.

The gravity components g_x, g_y, g_z at a point P which is close to 0 and has the coordinates $x\ y\ z$ may be obtained from a Taylor expansion in three variables.* That is,

$$\frac{\partial U_0}{\partial x}\Big)\, x + \frac{\partial}{\partial y}\left(\frac{\partial U_0}{\partial x}\right) y + \frac{\partial}{\partial z}\Big(\ \partial$$

. . . terms involving x, y and z to second and higher powers

$$g_y = \frac{\partial U_0}{\partial y} + \frac{\partial}{\partial x}\left(\frac{\partial U_0}{\partial y}\right) x + \frac{\partial}{\partial y}\Big(\ \partial y \tag{8}$$

Assuming that the balance beam comes to rest with the z axis (which passes through the center of gravity of the system as well as the origin 0) parallel to the direction of gravity at 0, †

$$\partial y \tag{9}$$

That is, the first terms of the g_x and the g_y expansions vanish. The first term of the g_z expansion does not vanish and will be denoted by g. That is,

$$\tag{10}$$

Also, the gravitational potential U_0 at the origin is approximately the same as the potential U at a point P close to the origin. On substituting U for U_0 into Equation 8 and eliminating the parenthesis signs, the expressions for the gravity components at the point P, to a first approximation, become:

$$g_x =$$

$$g_y\!: \quad \partial y \quad - \quad \partial x \partial y \quad x \quad \partial y \tag{11}$$

* Discussions of Taylor expansions in two and three variables are given in standard texts on calculus. See, for example, H. B. Fine, *Calculus*, p. 246 (Macmillan Co., 1927).

† R. von Eötvös, *Ann. der Physik*, Vol. 59, 1896, p. 356.

Not all of the coefficients appearing in the system of equations (11) are independent. Thus, because the gravitational potential is a continuous function

$$\partial^2 U \; - \; \partial^2 U \; . \; \partial^2 U \; - \; \partial^2 U \; . \; \partial^2 U \; - \; \partial^2 U \qquad (12)$$

On introducing the notation,

$$' \; \partial y^2$$

The system of equations (11) becomes

$$\begin{aligned} g_x &= U_{xx}x + U_{xy}y + \\ g_y &= U_{xy}x + U_{yy}y + \\ g_z &= g + U_{xz}x \; - \end{aligned} \qquad (14)$$

Consider now that the beam system of the balance is mounted so that the orientation of the beam with respect to the x axis is α. (Figure 103.)

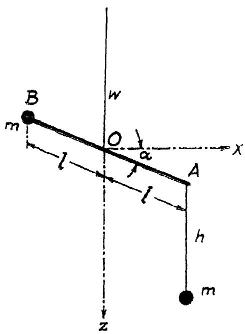


FIG. 103.—Eötvös torsion balance mounted with its beam in an azimuth α.

The gravity components g_x and g_y acting on each of the masses will exert torques on the suspended system (beam and torsion wire) tending to rotate it about the z axis. The magnitude of the torque acting at each mass m is:

$$(xg_y - yg_x)$$

For the lower mass, $x = l \cos \alpha$, $y = l \sin \alpha$, and $z = h$. Hence, on substituting for g_y and g_x their values from the system of equations (14), one obtains:

$$\begin{aligned} T_1 = {} & ml \cos \alpha \, (U_{xy} \, l \cos \alpha + U_{yy} \, l \sin \alpha + U_{yz} \, h) \\ & - ml \sin \alpha \, (U_{xx} \, l \cos \alpha + U_{xy} \, l \sin \alpha + U_{xz} \, h) \end{aligned}$$

or

$$\begin{aligned} T_1 = {} & ml^2 \, (U_{yy} - U_{xx}) \sin \alpha \cos \alpha + ml^2 \, U_{xy} \, (\cos^2 \alpha - \sin^2 \alpha) + mlhU_{yz} \cos \alpha \\ & - mlhU_{xz} \sin \alpha \end{aligned}$$

On making use of the trigonometric identities

$$\sin 2\,\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\,\alpha = \cos^2 \alpha - \sin^2 \alpha$$

the expression for T_1 becomes:

$$T_1 = \frac{ml^2}{2} U_\Delta \sin 2\,\alpha + ml^2 U_{xy} \cos 2\,\alpha + mlh U_{yz} \cos \alpha - mlh U_{xz} \sin \alpha$$

where

$$U_\Delta = U_{yy} - U_{xx} \tag{15}$$

For the upper mass, the coordinates are $x = -l \cos \alpha$, $y = -l \sin \alpha$, and $z = 0$. Hence,

$$T_2 = \frac{ml^2}{2} U_\Delta \sin 2\,\alpha + ml^2 U_{xy} \cos 2\,\alpha$$

By a similar procedure, the torque due to the mass of the beam itself may be shown to be:

$$T_3 = \frac{k}{2} U_\Delta \sin 2\,\alpha + k\, U_{xy} \cos 2\,\alpha$$

where k denotes the moment of inertia of the beam about the z axis.

The total twisting moment T acting on the torsion wire is equal to the sum of T_1, T_2, and T_3.

$$T = \frac{K}{2} U_\Delta \sin 2\,\alpha + K U_{xy} \cos 2\,\alpha + mlh\, U_{yz} \cos \alpha - mlh\, U_{xz} \sin \alpha \tag{16}$$

where K (the total moment of inertia of the beam system) equals $k + 2ml^2$.

The resultant torque given by Equation 16 will be resisted by the rigidity of the torsion wire, and the beam will come to rest when the two opposing torques balance each other. The condition of equilibrium in any azimuth α is given by Equation 7

where τ is the torsion coefficient and θ the angular deflection of the torsion wire from its zero position, i.e., the position it would take up if there were no twisting force on the beam.

In practice, θ is small and is measured by observing the displacement of a scale image reflected by the small mirror mounted on the torsion wire (telescope, mirror, and scale method). θ may be expressed, therefore, by the equation

$$\theta = \frac{n_\alpha - n_0}{2f} \tag{17}$$

where

$n_\alpha =$ scale reading for azimuth α

$n_0 =$ scale reading for torsion wire in "zero" or undeflected position

$f =$ distance between scale and mirror, expressed in scale divisions

On substituting Equations 17 and 16 into Equation 7 and rearranging terms, we obtain:

$$n_\alpha - n_0 = \frac{Kf}{\tau} U_\Delta \sin 2\alpha + 2\frac{Kf}{\tau} U_{xy} \cos 2\alpha + \frac{2fmhl}{} \quad U_{yz} \cos \alpha - U_{xz} \sin \alpha) \quad (18)$$

Equation 18 is the basic equation of the single beam Eötvös torsion balance.
To simplify the last equation let

$$(19)$$

Then

$$n_\alpha - n_0 = \frac{P}{2}(U_\Delta \sin 2\alpha + 2U_{xy} \cos 2\alpha) + Q\,(U_{yz} \cos \alpha - U_{xz} \sin \alpha) \quad (20)$$

Equation 20 contains five unknown quantities: viz., n_0, U_Δ, U_{xy}, U_{yz}, and U_{xz}. To determine the five unknown quantities the balance would have to be set up in five azimuths (five different values of α) at each station. On substituting the data (n_α and α) corresponding to these settings into Equation 20, one would obtain five equations, one for each setting.

$$\begin{aligned} &= \frac{P}{2}(U_\Delta \sin 2\alpha_1 + 2\,U_{xy} \cos 2 \\ &\quad + Q\,(U_{zy} \cos \alpha_1 - U_{xz} \sin \alpha_1 \\ n_2 - n_0 &= \frac{P}{2}(U_\Delta \sin 2\alpha_2 + 2\,U_{xy} \cos 2\alpha_2) \\ &\quad Q \qquad \cos \alpha_2 - U_{xz} \sin \alpha_2) \end{aligned} \quad (20a)$$

etc.

These equations could then be written in the form †

$$(20b)$$

where S_1, S_2, ... S_5 denote "a function of."

* P and Q are instrumental constants which are determined by calibration measurements.

† See also *The Eötvös Torsion Balance* (L. Oertling, Ltd., London).

Equations 20b are called the "working equations" of the single beam torsion balance. They express the unknown quantities n_0, U_Δ, U_{xy}, etc., in terms of the observed deflections n_1, n_2, etc., of the torsion wire and the instrumental constants P and Q.

Working Equations of the Double Beam Torsion Balance.— Modern torsion balances are equipped with two suspension systems so arranged that when one beam is in an azimuth α, the other is in an azimuth $180° + \alpha$. This arrangement requires only three settings at each station and therefore decreases the time for an observation. The settings are usually such that the azimuths of the first beam are 0°, 120°, and 240°, respectively; the corresponding azimuths of the second beam are 180°, 300°, and 60°, respectively. The instrumental constants and scale readings of the other system will be denoted by P', Q', n_0', n_1', n_2', and n_3', where n_1' corresponds to $\alpha = 180°$, n_2' to $\alpha = 300°$, and n_3' to $\alpha = 60°$. The equations of the double-beam balance corresponding to (20a) are:†

$$\begin{aligned} -n_0 &= PU_{xy} + QU_{yz} \\ -n_0' &= P'U_{xy} + Q'U_{yz} \end{aligned} \qquad \begin{cases} \alpha = 0° \text{ on first beam} \\ \alpha = 180° \text{ on second beam} \end{cases}$$

$\underline{Q}$

$$n_2' - n_0' = -\frac{P'\sqrt{3}}{4} U_\Delta - \frac{P'}{2} U_{xy} + \frac{Q'}{2} U_{yz} + \frac{Q'\sqrt{3}}{2} U_{xz}$$

$$\begin{cases} \alpha = 120° \text{ on first beam} \\ \alpha = 300° \text{ on second beam} \end{cases} \qquad (21)$$

$$n_3 - n_0 = +\frac{P\sqrt{3}}{4} U_\Delta - \frac{P}{2} U_{xy} - \frac{Q}{2} U_{yz} - \frac{Q\sqrt{3}}{2} U_{xz}$$

$$n_3' - n_0' = +\frac{P'\sqrt{3}}{4} U_\Delta - \frac{P'}{2} U_{xy} + \frac{Q'}{2} U_{yz} - \frac{Q'\sqrt{3}}{2} U_{xz}$$

$$\begin{aligned} \alpha &= 240° \text{ on first beam} \\ \alpha &= 60° \text{ on second beam} \end{aligned}$$

Combining these equations yields

$$n_1 + n_2 + n_3 = 3n_0\,;\; n_1' + n_2' + n_3' = 3n_0' \qquad (22)$$

On introducing the notation $N_1 = n_1 - n_0$, etc., $N_1' = n_1' - n_0'$, etc., Equation 22 becomes:

$$+ N_2' + N_3' = 0 \qquad (23)$$

† *The Eötvös Torsion Balance.*

On solving Equations 21 simultaneously and making use of Equations 23, we obtain

$$R\sqrt{3}\left[\frac{P'}{P}(N_2 - \right. \qquad (24)$$

$$P$$

where $R = PQ' + QP'$.

The relations given in (24) are the "working equations" of the double-beam balance.

Relation of Parameters Measured by the Torsion Balance to the Gravity Gradient and the Horizontal Directive Tendency

Consider the surface defined by the equation†

$$\tfrac{1}{2}(Ax^2 + 2Hxy + By^2 + 2Gxz + 2Fyz + Cz^2) + gz = 0 \qquad (25)$$

This is a surface which passes through the origin and has the xy plane as its tangent plane at the origin. Equation 25 may be written in the form

$$V(xyz) = 0$$

That is,

$$\tfrac{1}{2}(Ax^2 + 2Hxy + By^2 + 2Gxz + 2Fyz + Cz^2) + gz = V(xyz) = 0 \qquad (26)$$

It may be proved that for certain definite values of the coefficients, $V(xyz) = 0$ is identical with $U(xyz) = 0$, where $U(xyz) = 0$ is the equation of the *level surface* passing through the origin 0.

From analytical geometry, the direction cosines of the perpendicular to a surface given by an equation of the form $V(xyz) = 0$ are $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, and $\frac{\partial V}{\partial z}$. Differentiation of the left hand member of Equation 26 yields:

$$= Ax \cdot \qquad Gz$$

$$\frac{\partial V}{\partial y} = Hx + By + Fz$$

† *The Eötvös Torsion Balance.*

But these expressions are identical with the expressions for

1), provided we set

(27)

Hence $V(xyz) = U(xyz)$ is the equation of the level surface for points close to the origin 0. That is,

$$U(xyz) = \tfrac{1}{2}\,(Ax^2 + 2Hxy + By^2 + 2Gxz + 2Fyz + Cz^2) + gz$$

or

$$U(xyz) = \tfrac{1}{2}\,(U_{xx}x^2 + 2U_{xy}xy + U_{yy}y^2 + 2U_{xz}xz + 2U_{yz}yz + U_{zz}z^2) + gz = 0 \tag{28}$$

An inspection of Equation 28 reveals that, except for the term gz, the function $U(xyz)$ involves terms x^2, xy, etc., all of the second degree. Hence, z is of the same order of small quantities as x^2, xy, and y^2, while the terms xz, yz, and z^2 are of a higher order of small quantities. Hence, the terms $2U_{xz}xz$, $2U_{yz}yz$, and $U_{zz}z^2$ may be neglected, and the equation of the level surface for points close to the origin 0 becomes:

$$U(xyz) = \tfrac{1}{2}\,(U_{xx}x^2 + 2U_{xy}xy + U_{yy}y^2) + gz = 0 \tag{29}$$

(to a first approximation)

Equation 29 may be used to determine the total gravity g_p at a point P close to the origin (Figure 102) as will be evident from the following considerations.

The total gravity g_p at a point P close to the origin is given by the relation:

Differentiation of Equation 29 gives

$$\frac{\partial U}{\partial x} = U_{xx}x + U_{xy}y$$

(to a first approximation)

(31)

Also, from Equation 28,

$$\frac{\partial U}{\partial z} = g + U_{xz}x + U_{yz}y \quad \text{(to a second approximation)}$$

Whence

$$g_p = g + U_{xz}x + U_{yz}y \quad \text{(to a second approximation)} \tag{32}$$

where g_p is the resultant force of gravity for points on the level surface near the origin 0.

In particular, the resultant force of gravity at a point in the xz plane distant one unit from 0 is

$$g_p = g + U_{xz}$$

Because the value of the resultant gravity at the origin is g and the value at a point in the xz plane one unit distant from the origin is $g + U_{xz}$, U_{xz} is the increase of the resultant gravity per unit distance along the x axis in the level surface. That is, U_{xz} is the *gradient of gravity in the level surface* in the direction of the x axis. Similarly, U_{yz} is the gradient of gravity in the level surface in the direction of the y axis.

The total gravity gradient in the horizontal plane has a magnitude $\sqrt{U_{xz}^2 + U_{yz}^2}$ and makes an angle β with the y axis, where β is given by the relation: $\tan \beta = \frac{U_{yz}}{U_{xz}}$.

The relation between the radii of curvature of the level surface and the parameters U_{yy}, U_{xx}, and U_{xy} may be obtained by considering the sections of the level surface formed by various planes through the origin. The equation of the section of the level surface intercepted by the plane zx is obtained by setting $y = 0$ in Equation 29. This yields

or

(33)

The radius of curvature ρ_x of this curve may be obtained by using the general formula for the radius of curvature of any curve: namely,

$$\rho = \frac{\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]^{3/2}}{\partial^2 z} \qquad (34)$$

On differentiating Equation 33 with respect to x, we obtain

$2g$

and

$$\frac{U_{xx}}{g} - \frac{\partial}{g}$$

On setting x equal to zero and substituting the resulting values of $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x^2}$ into the formula for the radius of curvature (Equation 34) we obtain the following expression for the radius of curvature of the curve at the origin 0.

$$\rho_x = -\frac{1}{\frac{U_{xx}}{g}} \tag{35}$$

Hence, $-\frac{U_{xx}}{g}$ is the reciprocal of the radius of curvature of the section of the level surface intercepted by the zx plane. Similarly, $-\frac{U_{yy}}{g}$ is the reciprocal of the radius of curvature ρ_y of the section intercepted by the zy plane. The parameter $(U_{yy} - U_{xx})$ measured by the torsion balance, therefore, is a function of the radii of curvature of the level surface and the gravity. That is,

$$U_{yy} - U_{xx} = g\left(\frac{1}{\rho_x} - \frac{1}{\rho_y}\right) \tag{36}$$

Next consider a section of the level surface intercepted by a plane which makes an angle λ with the xz plane. Let r denote the distance of any point P in this plane from the z axis; the x and y coordinates of P are $r\cos\lambda$ and $r\sin\lambda$ respectively. The section of the level surface by the plane making an angle λ with the xz plane is obtained by substituting these values of x and y into Equation 29. The equation of the curve bounding this section of the level surface is:

$$\tfrac{1}{2} r^2 (U_{xx}\cos^2\lambda + 2U_{xy}\sin\lambda\cos\lambda + U_{yy}\sin^2\lambda) + gz = 0$$

or

$$z = -\frac{r^2}{2g}(U_{xx}\cos^2\lambda + 2U_{xy}\sin\lambda\cos\lambda + U_{yy}\sin^2\lambda)$$

The radius of curvature ρ of this curve at the origin may be obtained in the manner previously described. That is, one forms the first and second partial derivatives of z with respect to r, sets r equal to zero, and substitutes the resulting expressions for $\frac{\partial z}{\partial r}$ and $\frac{\partial^2 z}{\partial r^2}$ into the formula (34) for the radius of curvature. On carrying out these operations, we obtain

$$\rho = -\frac{\quad}{U_{xx}\cos^2\lambda + 2U_{xy}\sin\lambda\cos\lambda + U_{yy}\sin^2\lambda}$$

or

$$-\frac{g}{\rho} = U_{xx}\cos^2\lambda + U_{xy}\sin 2\lambda + U_{yy}\sin^2\lambda \tag{37}$$

The left hand member of Equation 37 will be a maximum or a minimum for

$$\frac{d}{d\lambda}(U_{xx}\cos^2\lambda + U_{xy}\sin 2\lambda + U_{yy}\sin^2\lambda) = 0$$

or

$$\frac{2U_{xy}}{\text{[illegible]}} = \frac{2U_{xy}}{\text{[illegible]}} = -\tan 2\lambda \qquad (38)$$

Equation 38 is satisfied by two values of λ. One value of λ defines a section for which the radius of curvature is a minimum and the other a section for which the radius of curvature is a maximum. The planes of these sections will differ by 90° so that assuming that λ_1 or λ_2 occurs either in the first or fourth quadrant

$$\sin^2\lambda_2 = \cos^2\lambda_1\ ;\ \ \cos^2\lambda_2 = \sin^2\lambda_1$$

$$\sin\lambda_2\cos\lambda_2 = -\sin\lambda_1\cos\lambda_1$$

Therefore

$$J_{xx}\cos^2\lambda_1 + 2U_{xy}\sin\lambda_1\cos\lambda_1 + U_{yy}\sin^2$$

and

$$-g\left(\frac{1}{\rho_2}\right) = U_{xx}\cos^2\lambda_2 + 2U_{xy}\sin\lambda_2\cos\lambda_2 + U_{yy}\sin^2\lambda_2$$

$$= U_{xx}\sin^2\lambda_1 - 2U_{xy}\sin\lambda_1\cos\lambda_1 + U_{yy}\cos^2\lambda_1$$

Whence

$$g\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) = U_\Delta\,(\cos^2\lambda_1 - \sin^2\lambda_1) - 4U_{xy}\sin\lambda_1\cos\lambda_1$$

$$= U_\Delta\cos 2\lambda_1 - 2U_{xy}\sin 2\lambda_1$$

On making use of the relation $\tan 2\lambda_1 = -\dfrac{2U_{xy}}{U_\Delta}$, the last equation becomes

$$g\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) = U_\Delta\cos 2\lambda_1 + U_\Delta\tan 2\lambda_1\sin 2\lambda_1$$

$$= U_\Delta\,(\cos^2 2\lambda_1 + \sin^2 2\lambda_1)\sec 2\lambda_1$$

or

$$g\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) = U_\Delta\sec 2\lambda_1 \qquad (39)$$

Also

$$\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) = -\ \text{[illegible]}\sec\text{[illegible]} \qquad (40)$$

Evidently if λ_1 and U_Δ are known, the quantity $g\ (1/\rho_1 - 1/\rho_2)$ may be evaluated (Equation 39 or Equation 40). This quantity, which is determined from measurements of U_{xy} and U_Δ, is the *curvature quantity* or *horizontal directive tendency.* The quantity $(1/\rho_1 - 1/\rho_2)$ alone represents the deviation of the level surface from a spherical shape at 0.

The results obtained on the relations between the quantities entering into the fundamental torsion balance equation (20) and the parameters of the level surface passing through the origin may be summarized as follows:

(1) The equation of the level surface passing through points close to the origin is

$$U(xyz) = \tfrac{1}{2}\ (U_{xx}x^2 + 2U_{xy}xy + U_{yy}y^2) + gz = 0 \qquad (29$$

(to a first approximation)

where g is the resultant gravity at the origin.

(2) The gradient of gravity in the level surface in the direction of the x axis is U_{xz}, and in the direction of the y axis it is U_{yz}.

(3) The gravity gradient in the horizontal plane has a magnitude equal to $\sqrt{U_{xz}^2 + U_{yz}^2}$ and makes an angle β with the x axis, where β is defined by the relation: $\tan \beta = \dfrac{U_{yz}}{U_{xz}}$.

(4) The radii of curvature of the sections of the level surface made by the zx and zy planes are $\dfrac{-g}{U_{xx}}$ and $\dfrac{-g}{U_{yy}}$ respectively.

(5) The curvature quantity $g\ (1/\rho_1 - 1/\rho_2)$ of the level surface is given by $-\ U_\Delta \sec 2\lambda$ where λ is determined by the relation: $\tan 2\lambda = \dfrac{-2U_{xy}}{U_\Delta}$.

(6) The working equation of the torsion balance is

$$n_\alpha - n_0 = \frac{P}{2}\ (U_\Delta \sin 2\alpha + 2U_{xy} \cos 2\alpha) + Q\ (U_{yz} \cos \alpha - U_{xz} \sin \alpha) \qquad (20)$$

wherein the first term represents the *curvature moment* or *curvature torque* tending to deflect the torsion wire of the balance, and the second term the *gradient moment* or *gradient torque* acting on the torsion wire.

The Gradiometer.—The gradiometer is essentially a modified torsion balance so constructed that the torsion wire is acted on by a gradient torque only. The beam system of the gradiometer is shown schematically in Figure. 104. Three small masses

are mounted at equidistant intervals (0°, 120°, 240°). Two of the masses m_2 and m_3 are supported on a light aluminum ring and a third mass m_1 is supported by a light aluminum rod above the level of the others. The mass m_2 is very approximately equal to m_3, and the combined mass of m_1 and the rod to which it is attached is equal to m_2 or m_3. †

The derivation of the fundamental equation of the gradiometer is similar to the derivation of the fundamental equation of the torsion balance. The origin of coordinates is taken at the center of the beam system. The x, y, and z axes are directed toward the north, the east, and vertically downwards respectively. (Figure 104.) The direction of the arm of the gradiometer which carries the upper mass m_1 makes an angle α with the x direction. As in the case of the torsion balance, the gravity components at a point near the origin are

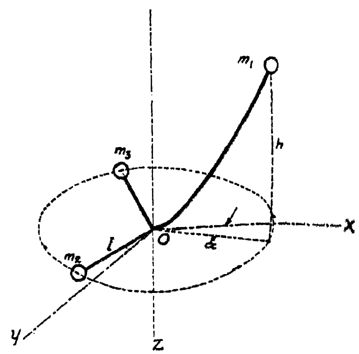


Fig. 104.—Diagrammatic sketch of the beam system of the gradiometer.

$$g_x = U_{xx}x + U_{xy}y + U_{xz}z$$
$$g_y = U_{xy}x + U_{yy}y + U_{yz}z \tag{14}$$

The g_x and g_y components of gravity acting on each mass produce a torque T about the z axis given by

$$T = (xg_y - yg_x)\, m \tag{41}$$

Substituting the values of g_y and g_x from the system of equations (14) yields

$$T = [x^2U_{xy} + xyU_{yy} + xzU_{yz} - yxU_{xx} - y^2U_{xy} - yzU_{xz}]\, m$$

or

$$T = [\,(x^2 - y^2)\,U_{xy} + xyU_\Delta + xzU_{yz} - yzU_{xz}]\, m \tag{42}$$

For the upper mass, $x = l\cos\alpha$, $y = l\sin\alpha$, and $z = -h$

For the lower masses, $x = l\cos(\alpha + 120°)$, $y = l\sin(\alpha + 120°)$, and $z = 0$

and $x = l\cos(\alpha + 240°)$, $y = l\sin(\alpha + 240°)$, and $z = 0$

respectively.

The value of the turning moment due to the circular beam and spokes is zero. Hence, the *total* torque acting at the three masses is:

$$T = m\,\{l^2\,[\cos^2\alpha - \sin^2\alpha]\; U_{xy} + l^2 \sin\alpha\cos\alpha\; U_\Delta - lh\cos\alpha\; U_{yz} + lh\sin\alpha\; U_{xz}\}$$
$$+\, m\,\{l^2\,[\cos^2(\alpha + 120°) - \sin^2(\alpha + 120°)]\; U_{xy} + l^2 \sin(\alpha + 120°)\cos(\alpha + 120°)\; U_\Delta\}$$
$$+\, m\,\{l^2\,[\cos^2(\alpha + 240°) - \sin^2(\alpha + 240°)]\; U_{xy} + l^2 \sin(\alpha + 240°)\cos(\alpha + 240°)\; U_\Delta\}$$

or

$$\begin{aligned} T &= ml^2\, U_{xy}\,[\cos 2\alpha + \cos(2\alpha + 240°) + \cos(2\alpha + 480°)] \\ &\quad + \tfrac{1}{2}\, ml^2\, U_\Delta\,[\sin 2\alpha + \sin(2\alpha + 240°) + \sin(2\alpha + 480°)] \\ &\quad - mhl\,[U_{yz}\cos\alpha - U_{xz}\sin\alpha] \end{aligned} \tag{43}$$

But the coefficients of U_{xy} and U_Δ are zero as is readily verified by expansion. Hence Equation 43 reduces to

$$T = -mhl\,(U_{yz}\cos\alpha - U_{xz}\sin\alpha) \tag{44}$$

† Compare also A. Broughton Edge and Laby, *Geophysical Prospecting* (Cambr. Univ. Press, 1931), p. 139, pp. 299-300.

Introducing the relation

$$T = \tau\theta$$

where θ is the angular deflection and τ the torsion constant of the torsion wire, Equation 44 may be written in the form:

$$\theta = -\frac{mhl}{\tau}\,(U_{yz}\cos\alpha - U_{xz}\sin\alpha) \tag{45}$$

Replacing θ by its value in terms of the scale readings n_a and n_0 and the distance f (expressed in scale divisions) between scale and mirror

$$\theta = \qquad 2f \tag{17}$$

Hence, the fundamental equation of the gradiometer becomes:

$$n_a - n_0 = -\frac{2fhml}{\tau}(U_{yz}\cos\alpha - U_{xz}\sin\alpha) \tag{46}$$

This equation states that the gradiometer deflection in any azimuth α is directly proportional to the horizontal gravity gradient $(U_{yz}\cos\alpha - U_{xz}\sin\alpha)$

Field Instruments.—In the United States use has been made chiefly of three types of Askania torsion balance and two types of Süss-Rybar torsion balance. The instruments are usually designed either for visual observation or photographic recording. The accuracy of the instruments is of the order of 1 to 3 Eötvös units. (1 Eötvös unit $= 10^{-9}$ cm./sec^2.)

To conserve space, the discussion of field instruments given here will be limited to a description of the Eötvös-Askania torsion balances.*

Large Torsion Balance

A vertical cross section of the large type instrument is shown in Figure 105. The torsion balance proper, with its two suspension systems, is mounted in the upper part of the instrument. The torsion wires are made of specially treated platinum-iridium and have a diameter of 0.04 mm. and an approximate length of 56 cm. Each torsion wire is held tightly by clamps. The upper end is fastened to a torsion head, and the lower end to a small vertical rod attached to a balance beam and carrying a small mirror for photographic recording of the position of rest or equilibrium of the beam. One end of each beam carries a gold weight of about 42 g.; the other end has a small hook from which a lead weight of the same mass as the gold weight is suspended by a thin brass wire some 60 cm. or so in length. To minimize temperature fluctuations, the suspension systems are enclosed in three airtight casings which are insulated from one another. The protective tubes containing the lower weights are unscrewed and packed separately for transport, as are also the weights and the brass

* For a description of the Rybar instrument the reader may refer to the catalogue of the Süss-Rybar manufacturing concern. A description of a gradiometer and the operating technique to be used with it may be obtained from L. Oertling, Ltd., London.

suspension wires. The torsion wires and the balance beams remain in the instrument during transport; the balance beams can be locked by turning the milled knobs on the short side of the casing.

A compass is mounted on the lid of the outer casing and is used for setting the upper part and the stops in the central portion of the instru-

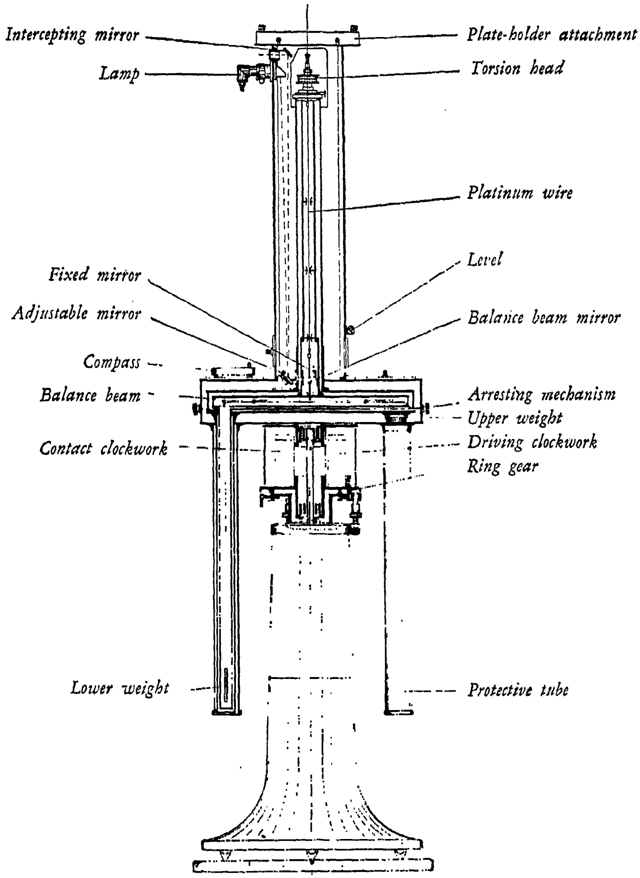


FIG. 105.—Large torsion balance (in section). (Courtesy of American Askania Corporation.)

ment in the magnetic meridian. The upper part of the instrument can be set into an exactly vertical position by means of two tubular levels. The torsion heads permit micrometric twisting, raising, lowering and lateral shifting of the torsion wires and hence of the balance beams.

The base of the instrument consists of two parts, placed one within the other. The central section, which contains the clockwork mechanism,

s screwed to the base. This section has three leveling screws for setting he instrument accurately in a vertical position. The lower part of the entral section contains a plug contact for supplying current to the contact nechanism from a 4- to 6-volt battery, the latter being located under the enter of the torsion balance. An azimuth ring is provided which has several adjustable stops for arresting the rotation of the instrument in different azimuths. Stops are provided for three, four, and five symmetrical positions of the torsion balance.

The contact mechanism is a precision clockwork which once an hour closes a contact and lights the electric lamps at the top of the instrument for making the photographic exposure. After the contact is opened, the upper part of the instrument is rotated to the next stop by means of a mechanical transmission gear between the contact clock and the driving mechanism.

The driving mechanism is mounted opposite the contact mechanism in the central portion of the instrument. A gear of the driving mechanism which meshes with the ring gear of the azimuth ring in the central portion of the instrument causes the upper section to rotate quietly and steadily until a lever in the driving mechanism runs against the next stop. The upper section of the instrument can be set free to rotate by using a control lever to disengage a friction clutch between the bevel pinion of the driving mechanism and the ring gear. The clockwork and driving mechanism are shielded from dust and damage by a cylindrical casing. The top plate of the central section carries a plug-in contact for electrical connection between the contact clock and the top of the instrument.

Current is supplied to the exposure lamps by a cable connected to the battery. The brightness of each lamp may be adjusted separately by use of rheostats. The light from the exposure lamps passes through a pinhole diaphragm, a prism, and then vertically downward to an adjustable mirror inclined at an angle of about 45° ; the beam of light is reflected horizontally from the inclined mirror, passes through an achromatic lens to the mirror on the stem of the balance beam, and is again reflected on to a photographic plate in the plate-holder attachment at the top of the balance. The plate holder is mounted in a slide on rollers and can be moved across a slit in the base of the plate-holder attachment at a speed of 3.5 mm. per

is also reflected to the photo

mirrors, one for each balance, which a

casing. One of these mirrors is attached to a bimetallic strip for recording the temperature. (A one-half mm. displacement of the spot of light represents a temperature variation of about 1°C.) The other mirror is fixed, and the spots of light reflected by it are used as fiducial marks in measuring the displacements of the spots of light reflected from the two balance mirrors. A copy of a photographic record obtained with this balance is shown in Figure 106. The start of the plate is at the left hand side of the

figure. The first two dots in columns I and II are made by a run which rotates the instrument until the first stop is reached. Then it is manually turned until the north direction coincides with this stop and the regular runs comprising three dots are recorded.

The time required for the beam to come to rest in any azimuth, i.e., the observation period, is approximately 60 minutes. Thus the time required to obtain data for one station, i.e., readings in 3 azimuths, is 3 hours. (Actually, it is common practice to have a repeated observation at the first azimuth to obtain better accuracy so that the total time per station is 4 hours.)

The overall height of the large torsion balance is 183 cm. (72 in.), and its weight when set up for measurement is about 130 pounds. An exterior view of the balance is shown in Figure 107.

The sensitivity of the balance is about 0.46 Eötvös units per 0.1 of a scale division of the graduated plate. The angular sensitivity is given by mhl/τ and the effective sensitivity by $\tau/2fmhl$. For the large torsion balance,

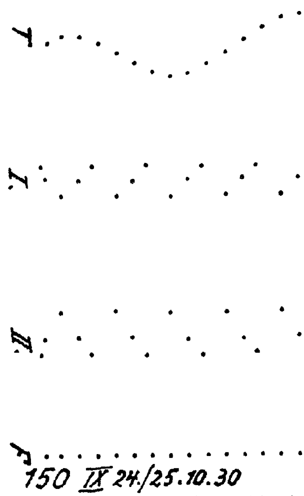


FIG. 106.—Record taken with the large torsion balance in 3 positions (repeated 4 times). (Courtesy of Askania Corporation.)

T—temperature points, i.e., spots of light reflected from mirror attached to thermometric device (bimetallic strip).

I—first balance beam.

II—second balance beam.

F—fixed points, i.e., spots of light reflected from fixed mirror.

$$\frac{mhl}{\tau} = 7.2 \cdot 10^4 \text{ cm. for each c. g. s. unit of gradient}$$

and

$$\frac{\tau}{2fhml} = 0.92 \cdot 10^{-9} \text{ c. g. s. units of gradient for each 0.01 cm. deflection}$$

as measured on the photographic plate.

The large torsion balance is especially useful: (a) in regions where the terrain corrections are large and (b) in explorations for ore deposits or other geologic bodies which produce a relatively small gradient anomaly.

Z-Beam Torsion Balance

In the Z-beam torsion balance (Figure 108) the end portions of each of the balance beams are bent so as to form right angles, and the weights are rigidly attached to the extremities of the vertical parts. With one weight above the balance beam and the

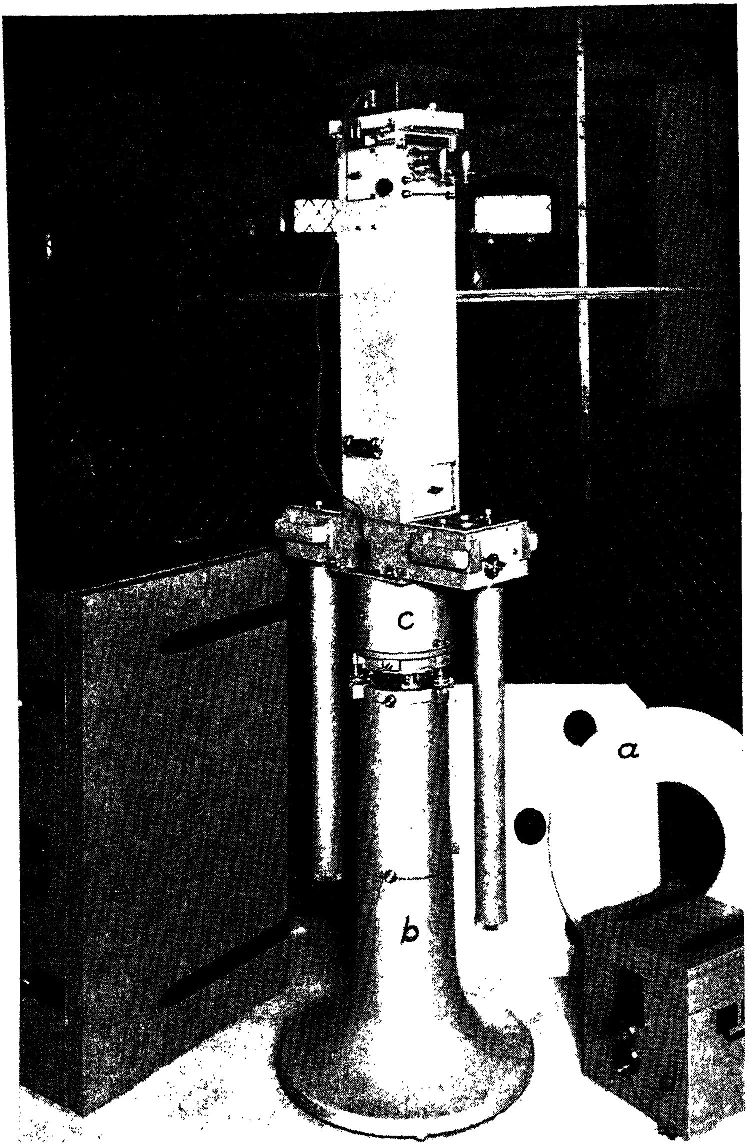


FIG. 107.—Exterior view of large torsion balance. (Courtesy of American Askania Corporation.)

a—Aluminum base plate
b—Pedestal
c—Turntable
d—Carrying case for turntable
e—Carrying case for balance

other below it, the center of gravity of the suspension system is about 2 cm. above the beam. The small equal weights have a mass of about 22 g. and the distance between them h is 45 cm.

The base and central section are identical with those of the large torsion balance, except that the base consists of one piece. The overall height of the Z-beam torsion balance, however, is only 120 cm. (47 in.) and its weight when set up is 104 pounds.

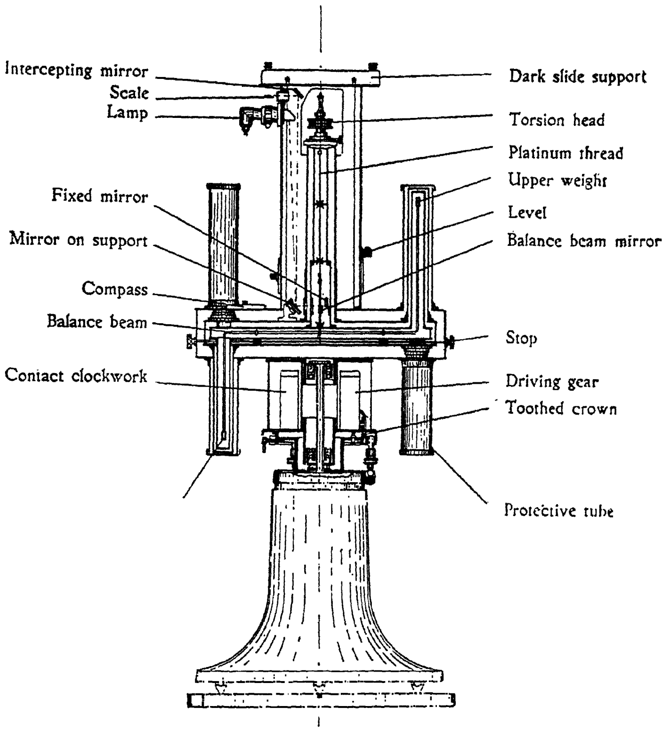


FIG. 108.—Small torsion balance (in section). (Courtesy of American Askania Corporation.)

The observation period or time for the balance beam to come to rest is 40 minutes. The sensitivity is about 0.6 Eötvös units per 0.1 of a scale division of the graduated plate. The angular sensitivity $\left(\frac{mhl}{\tau}\right)$ is 4.5×10^4 cm. for each c. g. s. unit of gradient, and the effective sensitivity $\left(\frac{\tau}{4fmhl}\right)$ * is 1.2×10^{-9} c. g. s. units of gradient for each 0.01 cm. deflection.

* 4f instead of 2f is employed in this formula because the balance beam mirror has been turned through 45° and another reflecting mirror inserted in the beam casing, thus doubling the deflection of the spots of light on the photographic plate.

The Z-beam torsion balance has been used for investigations of geologic structure and tectonic problems, and for locating salt domes, anticlines, and faults.

Torsion Balance with Inclined Beams

The central section and base of this instrument are similar to those of the large torsion balance and the Z-beam torsion balance except that the upper plate of the central section is designed to correspond with the base of the upper section. The center of gravity of the suspension system is 90 cm. above the base plate. The overall height of the inclined balance is 126 cm. (50 in.), and its weight when set up is 82 pounds.

An exterior view of the instrument is shown in Figure 109. The upper section of the inclined beam balance differs fundamentally from that of the large and Z-beam balances. It comprises a central casing for the optical system and two casings for the balance beams. Each of the balance casings can be removed independently of the other. The balance casings are double-walled as a protection against temperature changes, and each contains a torsion wire and torsion head in addition to the inclined balance beam. The torsion wire is 260 mm. long. The small masses weigh 40 g. each, the horizontal distance between them is 20 cm. and the vertical distance is 30 cm. The prism support is connected to the balance beam by a bifilar suspension which prevents the axial oscillations of the balance beam from being transmitted to the suspension prism. The balance beam can be locked by turning a knob.

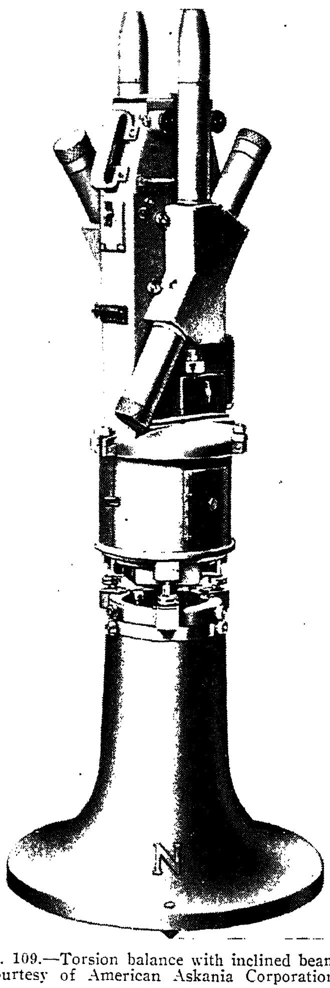


Fig. 109.—Torsion balance with inclined beams. (Courtesy of American Askania Corporation.)

The balance beam can be viewed through a glass window by removing the screw caps at the ends of the tube. The torsion head is provided with a horizontal slide and a fine vertical and azimuth adjustment. In addition, the height adjustment screw is hollow which permits a considerable increase in the length of the torsion wire without a corresponding increase in the overall height of the instrument.

To decrease the observation time, tungsten wires having a torsion coefficient of $\tau = 1.2$ c. g. s. units are used. The sensitivity is 0.1 Eötvös units per 0.1 scale division of the graduated plate. The sensitivity calculated from the constants is:

Angular sensitivity $= \dfrac{\int dm \cdot hl}{\tau} = 1.13 \cdot 10^{3}$ cm. for each c. g. s. unit of gradient and

Effective sensitivity $= \dfrac{\tau}{4f \int dm \cdot hl} = 2.9 \cdot 10^{-9}$ c. g. s. units of gradient for each 0.01 cm. deflection.

The sensitivity of the instrument can be doubled by replacing the tungsten torsion wires ($\tau = 1.2$ c.g.s.) by a wire with $\tau = 0.6$ and removing the damping plate. This however increases the observation period from 20 to 30 minutes.

The inclined beam balance, like the Z-beam balance, is used chiefly for investigations of geologic structure and tectonic problems, and for localizing salt domes, anticlines, and faults.

Reduction of Observations.—The gradient and differential curvature values given by the torsion balance are the vector sums of the effects of all the irregular mass distributions in the vicinity of the instrument which are sufficiently large to affect it.† These effects may be classified into three types: geologic, topographic and latitude. The geologic effects are those due to structural anomalies. The latitude effects are produced by the normal variation of gravity with latitude. The topographic effects, which are frequently of the same order of magnitude as the structural effects, are produced by irregularities in the distribution of mass due to hills and valleys and smaller mounds and depressions in the vicinity of the torsion balance.

Hence, before structural anomalies can be deduced from the torsion balance data, it is necessary to correct the values observed at each station for topographic and latitude effects.

Topographic Corrections

The magnitude of the topographic correction to be applied to the gravity gradient depends on several factors. For example, the correction for a small mass, say a boulder, would depend on: the horizontal distance and the vertical distance of the boulder from the torsion balance, the dimensions of the boulder, and its density relative to that of the surrounding media. The effect of the small mass on the gravity gradient is a maximum if the mass is situated within a space angle of 40° to 60° below or above the horizontal plane through the balance, and the effect is substantially zero when the small mass is either vertically below or level with the balance.

Except in very refined work, the effect on the gradient is negligible for irregularities of mass located within ±5° of the horizontal plane through the balance. Furthermore, the effect varies inversely as the cube of the distance of the mass from the balance.*

As a consequence of the topographic effects on the gradient outlined above, it is not practical to use the torsion balance in rugged areas for determinations of gravity gradients due to structure, unless the gradients due to the structures under investigation are very large.

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† D. C. Barton, "Eötvös Torsion Balance Method of Mapping Geologic Structure," *A.I.M.E. Geophysical Prospecting,* 1929, pp. 431-433.

* The practical significance of these facts in regard to field work may be stated as follows: Whenever feasible, the site chosen for the torsion balance station should be such that all appreciable mass irregularities are situated at moderate distances from the balance and within ±5° of the horizontal plane through the balance. Furthermore, if the balance were set up on an extensive hill sloping uniformly 1°, the effect of the slope on the gravity gradient would be approximately 14 Eötvös units, the precise value depending on the mean density of the topographic feature. Hence, because the gradients due to structure generally are of the order of 7 to 25 Eötvös units, it is preferable to choose station sites where the inclination of the ground is less than 1°.

The effect of a small mass on the differential curvature value approaches its maximum value within 10° of the horizontal plane through the center of gravity of the balance. Hence, it is difficult even in regions of moderate relief to choose station sites such that the effect of topography will not be excessively great. From a practical viewpoint, therefore, the differential curvature is of negligible value for predicting structure in areas where the topography is even mildly rugged.

It is common practice to classify topographic corrections as terrain and cartographic corrections respectively, and to apply somewhat different theoretical assumptions and field methods for the evaluation of the two types. The term terrain corrections, as used here, connotes corrections for effects due to irregularities in topography which are present over a circular area having the balance at its center and a radius equal to 100 meters. Cartographic corrections refer to corrections due to topographic irregularities located at distances greater than 100 meters from the balance. As applied to field methods for evaluating these effects, this classification is not rigid because identical field methods frequently may be applied for evaluating effects due to excesses and deficiencies of mass located at distances from the balance of 3 to 300 meters.*

Analytical treatments of topographic and cartographic corrections have been given by von Eötvös, Schweydar, Numerov, Nikiforov, Lancaster Jones, Heiland, Ansel and others. Graphical treatments have been given by Numerov, Jung, Haalck and others. Also, a detailed description of the type corrections utilized in the Imperial Geophysical Experimental Survey is given by Broughton Edge and Laby.

Corrections for mass irregularities located at distances from the balance greater than 100 meters are computed most conveniently by graphical methods. Corrections for topographic or terrain effects, i.e., excesses or deficiencies of mass situated at distances of less than a hundred meters, are computed conveniently in many cases by the Schweydar analytical method. The application of this method requires that the area surrounding the station site be surveyed in such manner that the levels are known accurately for eight points on each of several circles having points on the vertical line through the station site as centers and radii of 1.5, 3, 5, 10, 20, 30, 40, 50, 70 and 100 meters respectively, the eight points for each circle being located in eight directions and separated by an angular distance of 45°.**

In his first paper Schweydar † expresses the heights on the various circles having the station as center in terms of a Fourier series as follows:

$$z_n = a_n + b_n \sin \alpha + c_n \cos \alpha + d_n \sin 2\alpha + e_n \cos 2\alpha + \ldots \qquad (47)$$

* It is of some interest to note that many investigators classify the corrections due to topographic irregularities into three types: terrain (up to 100 feet from the site), topographic (from 100 to 1000 feet), and cartographic (over 1000 feet).

** In irregular country, the leveling is done in 16 azimuths instead of 8.

† W. Schweydar, *Zeit. für Geophysik*, Vol. 1, pp. 81-89 (1924).

where z_n is the difference in height between the station and a point on circle n, and a_n, b_n, c_n, etc. are parameters which do not involve z_n.

$$\pi \int_{-\pi} z_n \sin \alpha \, d\alpha$$

$$\cdot_n = \frac{1}{\pi} \int_{-\pi}^{\pi} z_n \cos \alpha \, d\alpha \tag{48}$$

$$d_n = \frac{1}{\pi} \quad z_n \sin 2\alpha$$

$$e_n = \frac{1}{\pi} \int_{-\pi}^{\pi} z_n \cos 2\alpha$$

Let*

(r, α) = the polar coordinates of an element of the terrain material referred to the station and the magnetic north.

σ = the density of the material.

h = the height of the center of gravity of the balance above the station site.

z = the height of the element of terrain above the station site.

The gradient in the x direction due to the element of terrain is

$$= 3G\sigma \frac{r^2 \cos \alpha \, (h - z) \, d\alpha \, dr \, dz}{} \tag{49}$$

where G is the gravitational constant. (G equals 6.68×10^{-8} in the centimeter-gram-second system of units.) The total gradient in the x direction due to the terrain is

$$\int_0^z \int_0^r \int_{-\pi}^{\pi} \frac{r^2 \cos \alpha \, (h - z)}{[r^2 + (h - z)^2]^{5/2}} \, d\alpha \, dr \, dz$$

Because $\dfrac{z^2 - 2hz}{r^2 + h^2}$ is small, only a small error is introduced by writing the expression for the gradient in the form

$$\int_0^r \int_{-\pi} \frac{r^2 \cos \alpha \, (hz_{nr} - \tfrac{1}{2} z_{nr}^2)}{} \, d\alpha \, dr$$

* See also Broughton Edge and Laby, *loc. cit.*, p. 309.

where z_{nr} is the value of z for a circle of radius r. Also, if the differences in height are small, the term $\frac{1}{2}z_{nr}^2$ is small relative to hz_{nr}, and the expression for the gradient $\frac{\partial^2 U}{\partial x \partial z}$ becomes

$$\frac{\partial^2 U}{} = 3G\sigma \int^{r} r^2 h \int_{-\pi}^{\pi} z_n \cos \alpha \, d\alpha \, dr$$

or

$$= 3\pi c_n G\sigma \qquad r^2 h \tag{50}$$

In similar manner, it can be shown that c_n is the only Fourier coefficient that must be known to determine

and that d_n and e_n are the only Fourier coefficients that must be known to determine the curvature quantity.

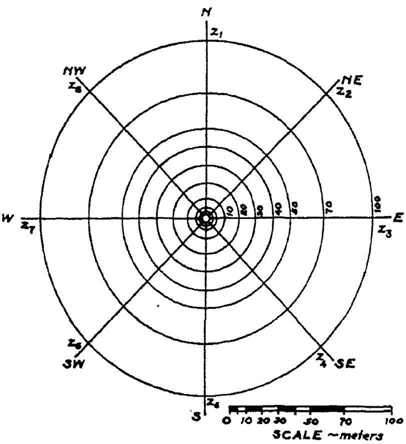

FIG. 110.—Layout for terrain leveling. (After Schweydar, *Zeitschrift für Geophysik.*)

In practical field work, it is impossible to specify z as a continuous function of α. Instead, Schweydar utilized 8 azimuths (0°, 45°, 90°, etc.). (For very rugged terrain, 16 or more azimuths may be used.) If the leveling is done in eight azimuths (0°, 45°, etc.) and if the differences in height on any circle are designated as z_1, z_2, z_3, etc. (Figure 110), substitution in the Fourier equation (47) yields:

$$\begin{aligned} 4b_n &= \frac{1}{\sqrt{2}}(z_2 \\ 4c_n &= \frac{1}{\sqrt{2}}(z_2 \\ 4d_n &= z_2 - z_4 \\ 4e_n &= z_1 - z_3 \end{aligned} \tag{51}$$

Schweydar now makes the assumption that the variations in height from one circle to the next in any given azimuth are proportional to the

TABLE 9

SCHWEYDAR TERRAIN CORRECTIONS

Distance	1.5 m	3 m	5 m	10 m	20 m	30 m	40 m	50 m	70 m	100 m
$\frac{1}{\sigma}\frac{\partial^2 U}{\partial x \partial z}$	2.36 E	0.643 E	0.239 E	0.082 E	0.0186 E	0.00467 E	0.00187 E	0.001204 E	0.00080 E	0.000428 E
$\frac{1}{\sigma}\frac{\partial^2 U}{\partial y \partial z}$	"	"	"	"	"	"	"	"	"	"
$\frac{1}{\sigma}\frac{\partial^2 U}{\partial x \partial y}$	3.302 E	1.962 E	1.343 E	0.844 E	0.357 E	0.147 E	0.0805 E	0.0686 E	0.0616 E	0.0472 E
$\frac{1}{\sigma}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)$	"	"	"	"	"	"	"	"	"	"

where σ is the average density of the ground around the station.

radial distances from the station. The values of the integral expressions for $\frac{\partial^2 U}{- - -}$, $\frac{\partial^2 U}{- - z}$, etc., obtained on this assumption are as follows:

$$\left.\begin{aligned} \frac{\partial^2 U}{\partial x \partial z} &= (K_1 c_1 + K_2 c_2 + \cdots) \\ \frac{\partial^2 U}{\partial y \partial z} &= (K_1 b_1 + K_2 b_2 + \cdots) \\ 2\frac{\partial^2 U}{\partial x \partial y} &= (L_1 d_1 + L_2 d_2 + \cdots) \\ \frac{U}{y^2} - \frac{\partial^2 U}{\partial x^2} &= -(L_1 e_1 + L_2 e_2 + \cdots) \end{aligned}\right\} \tag{52}$$

where c_1, b_1, d_1, e_1, refer to the first circle, c_2, b_2, . . . etc., refer to the second circle, and so on, and the K's and L's are constants which depend only on the values of h and the values of the successive radii.

Schweydar computed the terrain effects for h equal to 90 centimeters and obtained the results given in Table 9.

In his second paper Schweydar † gives a more rigorous treatment for the evaluation of terrain effects without neglecting the z^2 term.

For the evaluation of cartographic effects, contour lines based on reasonably detailed general topographic surveys are used chiefly. There are other methods, however, which are sometimes applied for the evaluation both of terrain and cartographic corrections.

Numerov,‡ for example, devised a convenient graphical method for the determination of gradients due to topographical features by assuming that the value of r is large (over 33 meters) and by computing the effect due to a prism of mean height z_1, the cross section of which is included between radii r_n and r_{n+1} which are in azimuths Q_m and Q_{m+1}. The principle of this method is to divide the area surrounding the station into zones such that the quantity $\left(\frac{1}{2r_n^2} - \frac{1}{2r_{n+1}^2}\right)$ $(\cos Q_{mh} - \cos Q_m)$ is constant. The application of the method is described by Broughton Edge and Laby. (*loc. cit.*)

Latitude or Planetary Corrections

The variation of *gravity* with latitude is given by Equation 3. That is

$$g_\phi = 978.039\,(1 + 0.0053 \sin^2 \phi - 0.000007 \sin^2 2\phi) \tag{3}$$

The variation of the *gravity gradient* with latitude can be obtained by differentiating Equation 3 with respect to x and y respectively.

† W. Schweydar, *Zeit für Geophysik*, Vol. 2, pp. 17-23 (1927).
‡ B. Numerov, *Zeit für Geophysik*, Vol. 1, pp. 367-371 (1925); Vol. 4, pp. 117-134 (1928).

Because the parallels of latitude are circles, gravity does not vary from east to west. That is, $\frac{\partial g}{\partial y} = U_{yz} = 0$. Also, $\frac{\partial g}{\partial x} = U_{xz}$ is approximately equal to $\frac{1}{R}\frac{\partial g}{\partial \phi}$ where ϕ is the latitude of the area under investigation and R is the radius of the earth at the equator. Hence,

$$\frac{\partial g}{\partial x} = U_{xz} = \frac{1}{R}\frac{\partial}{\partial \phi}\left[978.039\left(1 + 0.0053 \sin^2 \phi - 0.000007 \sin^2 2\phi\right)\right]$$

$$= \frac{1}{6.38 \cdot 10^8}\left[978.039\left(0.0053 \sin 2\phi\right)\right] \quad \text{(approximately)}$$

$$= 8.15 \sin 2\phi \cdot 10^{-9} \quad \text{(approximately)} \qquad (53)$$

The planetary correction for the curvature is given by the formula: *

$$U_\Delta = 5.15\, E\, (1 + \cos 2\phi) \cdot 10^{-9} \qquad (54)$$

Graphs showing the latitude effects are given in Figure 111.

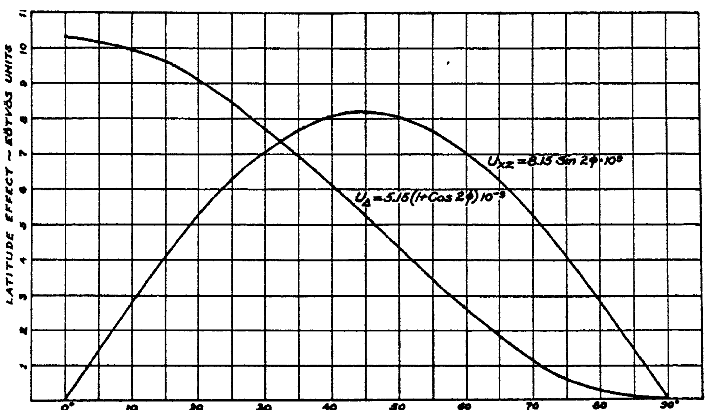

FIG. 111.—Effects of latitude on quantities measured by the torsion balance. (Courtesy of E. V. McCollum.)

Field Procedure.—The organization of the torsion balance field parties depends on many factors both technical and economical. In the early days of torsion balance prospecting in America, two-instrument parties having a crew of 6 to 8 men were common. At the present time,

* Compare also, H. Shaw and E. Lancaster Jones, "The Theory and Practical Employment of the Eötvös Torsion Balance," *Mining Magazine,* July, 1927.

a one-instrument party may comprise: 1 observer, 1 surveyor, and 2 helpers; a two-instrument party may comprise: 1 or 2 observers, 1 surveyor, and 3 helpers. A large, four-instrument party may include a total of 9 to 14 men.

Testing of Instrument

Prior to taking a torsion balance into the field, the instrument should be adjusted, checked and its constants determined. The work can best be done at the laboratory where adequate equipment and shop facilities are available.

Transportation of Instrument

The method of transportation to the area under investigation and from station to station depends on many factors: notably, climate, topography, the character of the roads and trails, surface conditions, and to some extent, the number of men and instruments in the party. On marshy ground, water boats and canoes may be used. On dry lands having relatively good roads and trails, the torsion balance is transported either in a trailer provided with a torsion balance shelter or in a passenger car equipped for this purpose.

FIG. 112.—Initial leveling of station site in immediate vicinity of the instrument. *a*, stadia rod equipped with level bubble for leveling; *b*, portions of collapsible house for instrument.

Choice of Site

The choice of the instrument site depends both on its preliminary location on the map to suit the general balance net to be used in the area under investigation and on its topographic details. The topographic details must be such that observations made with the instrument will be of value for predicting underground structure and ore bodies. The properties which the site should have in order to minimize the topographic effects on the gradient and curvature values were summarized on p. 217.

The area under and immediately adjacent the instrument is leveled to within one-half of a centimeter for a radius of at least one and one-half meters. (Figure 112.)

Surveying

The choice of method of surveying depends on the topography and on the accuracy of data desired. In areas of rugged topography, surveying may be carried along 16 azimuths and as far as 250 meters from the in-

strument. In many surveys, 8 azimuths and distances of from 70 to 100 meters are sufficient. (However, in areas of relatively mild topography, it is common practice not to carry the surveying for topographical details beyond 30 to 40 meters from the balance.)

The manner in which the stations are laid out depends on the type of subsurface structures expected in the area under investigation. For average reconnaissance work, the stations are placed from one-half to one-quarter of a mile apart. For detailed work, the station separation may be considerably less than one-quarter of a mile. The lines of stations are usually run in a manner to conform as far as possible to the roads and trails present in the area under investigation without altering to a large degree the station net agreed upon before starting the survey.

Instrument Set-Up

After arriving at the leveled station site, which is usually well marked by a surveying crew in advance, the observer drives three wood or steel stakes into the ground so that their tops are in the same horizontal plane. The height of the stakes, as projected from the ground, should always be of the same standard value. After the stakes have been leveled carefully with a hand level, the aluminum base plate is laid upon them with its north groove pointing north. The next step is to set up the hut, which is a portable shelter consisting usually of a light wood frame covered inside and outside with thin plywood. The space between the plywood layers and frame is filled with a heat insulating material. (Figure 113.) After the hut has been set up, the torsion balance is placed on the aluminum base plate. The turntable is then placed on the pedestal and clamped in position. Next, the upper part of the balance is placed on the turntable, and the turntable is leveled and oriented.

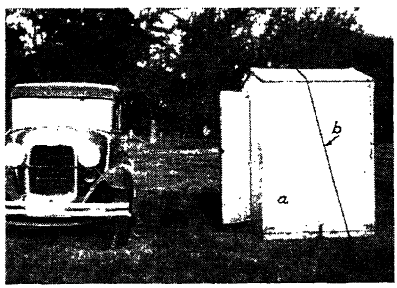

FIG. 113.—Final set-up of torsion-balance station. *a*, collapsible house; *b*, snub-lines to hold structure against wind.

After a new photographic plate has been inserted and the electric light connections and the performance of the electrical contact mechanism checked, the instrument is left on the station site to record automatically. (If a visual torsion balance is used, the observer reads the instrument and rotates it into the next position at certain fixed intervals until a satisfactory series of readings has been obtained.)

The instrument is usually kept at each station long enough to have a repeated observation in one azimuth at least.

Station No.:

Area:

Date:

TORSION BALANCE SURVEY

Terrain Corrections

Density = 2.2

Instrument No.:

Observer:

Surveyor:

	1	2	3	4	5	6	7	8	I	II	III	IV	V	VI	VII	VIII	c	b
Dist. meters	0°	45°	90°	135	180	225	270	315	2-6	8-4	I+II	I-II	.707 III	.707 IV	1-5	3-7	VII+V	VIII+VI
1.5	-0.5	-2.0	+0.5	+1.5	+1.5	+2.5	+2.0	+1.5	-4.5	0	-4.5	-4.5	-3.2	-3.2	-2.	-1.5	-5.2	-4.7
3.	0	+1	+0.5	+2	+3	+5	+4	+3	-4.0	+1.0	-3.0	-5.0	-2.1	-3.5	-3.0	-3.5	-5.1	-7.0
5.	0	0	-2	+4	+6	+8	+8	+7	-8	+3	-5	-11	-3.5	-7.7	-6	-10	-9.5	-18
10	+1	0	0	+2	+10	+15	+20	+12	-15	+10	-5	-25	-4.0	-18	-9	-20	-13	-38
20	0	-5	+1	+10	+20	+30	+35	+20	-35	+10	-25	-45	-18	-32	-20	-34	-38	-66
30	-3	-10	-1	+10	+30	+45	+55	+35	-55	+25	-30	-80	-21	-56	-33	-56	-54	-112
40	-10	-20	-3	+15	+35	+55	+80	+45	-75	+30	-45	-105	-32	-74	-45	-77	-77	-151
50	-25	-35	+10	+20	+40	+70	+90	+60	-105	+40	-65	-145	-46	-102	-65	-80	-111	-182
70	-60	-65	+15	+25	+50	+95	+105	+85	-160	+60	-100	-220	-70	-154	-110	-90	-180	-242
100																		

		c'	b'	I'	II'	III'	IV'	d	e		d'	e'
Dist. meters	Const.	c'	b'	2-4	6-8	1-3	5-7	I'+II'	III'+IV'	Constant	d'	e'
1.5	0.59	-3.1	-2.8	-3.5	+1.0	-1.0	-.5	-2.5	-1.5	0.8255	-2.0	-1.2
3.	0.161	-0.8	-1.1	-1.0	+2.0	-.5	-1.0	+1.0	-1.5	0.4405	+0.5	-0.7
5.	0.05975	-0.6	-1.1	-4.	+1.	+2.	-2.	-3.	0	0.3357	-1.0	0
10	0.0205	-0.3	-0.8	-2	+3	+1	-10	+1	-9	0.211	+0.2	-1.8
20	0.00465	-0.2	-0.3	-15	+10	-1	-15	-5	-16	0.0892	-0.4	-1.5
30	0.00116	-0.1	-0.1	-20	+10	+2	-25	-10	-23	0.0367	-0.4	-0.9
40	0.00461	0	-0.1	-35	+10	-13	-45	-25	-58	0.0201	-0.5	-1.2
50	0.0003	0	-0.1	-55	+10	-35	-50	-45	-85	0.01715	-0.7	-1.3
70	0.0002	0	0	-90	+10	-75	-55	-80	-130	0.0150	-1.2	-2.0
100												

↓Sum	↓
-5.5	-10.6

$\left(\text{Corr.}\ \frac{\partial^2 U}{\partial x \partial z}\right) \; -5.6 = 1.1 \qquad \overset{\text{Sum}}{-5.1 - 6.4} \quad 1.1 = -7.0 \; \left(\text{Corr.}\ \frac{\partial^2 U}{\partial z \partial y}\right)$

$\left(\text{Corr.}\ 2\frac{\partial^2 U}{\partial x \partial y}\right) -6.1 = 1.1 \qquad -5.5 - 10.6 - 1.1 = +11.7 \; \left(\text{Corr.}\ \frac{\partial^2 U}{\partial^2 y} - \frac{\partial^2 U}{\partial^2 x}\right)$

LEGEND FOR TERRAIN CORRECTION TABLE

Distance	
1, 2, 3, 4, 5, 6, 7	Differences in elevation between the instrument point and the points situated on the intersections of the circles and the azimuths drawn from the instrument point. The differences in elevation are expressed in centimeters.
I	Differences in elevation between two points of the same circle situated on azimuths 2 and 6
II	Differences in elevation between two points of the same circle situated on azimuths 8 and 4
III	Sum of I and II computed for the same circle
IV	Difference of I and II computed for the same circle
V	0.707 times III
VI	0.707 times IV
VII	Differences in elevation between two points of the same circle on azimuths 1 and 5
VIII	Differences in elevation between two points of the same circle on azimuths 3 and 7
c	Sum of the data recorded in the columns VII and V for the same circle
b	Sum of the data recorded in the columns VIII and VI for the same circle
c′	Values recorded in the column *c* times respective constants
b′	Values recorded in the column *b* times respective constants
I′	Differences between two points of the same circle situated on azimuths 2 and 4
II′	Differences between two points of the same circle situated on azimuths 6 and 8
III′	Differences between two points of the same circle situated on azimuths 1 and 3
IV′	Differences between two points of the same circle situated on azimuths 5 and 7
d	Sum of the data recorded in columns I′ and II′ for the same circle
e	Sum of the data recorded in columns III′ and IV′ for the same circle
d′	Values recorded in the column *d* times respective constants
e′	Values recorded in the column *e* times respective constants

Computation of Final Corrections *

Summation of all the values recorded in the column c' times $\frac{1}{2}\sigma$ = correction to be applied to

Summation of all the values recorded in the column b' times $\frac{1}{2}\sigma$ = correction to be applied to $\frac{\partial^2 U}{\partial y \partial z}$

Summation of all the values recorded in the column d' times $\frac{1}{2}\sigma$ = correction to be applied to $2\,\frac{\partial^2 U}{\partial x \partial}$

Summation of all the values recorded in the column e' times $-\frac{1}{2}\sigma$ = correction to be applied to $\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x}$

where σ is the density of material.

* In computing the final corrections σ has been set equal to 2.2.

Station No.: **TORSION BALANCE SURVEY** Instrument No.

Area: Gradients and Curvatures

Date: Observer:

	Azimuth	A Beam (left)				
		N	N_0	N_1	N_2	N_3
1	0°	73.8	(75.2)	−(1.4)		
2	120°	73.9	74.9		−1.0	
3	240°	77.0	74.6			+2.4
	1	72.9	74.3	−1.4		
	2	73.1	(74.0)		−0.9	
	3					
	1					
	2					
		Mean		−1.4	−1.0	+2.4
	A_l =	$N_2 - N_3 = -3.4$				
	B_l =	$N_2 + N_3 = +1.4$				

	Constant	
$A_l - A_r = -1.4$	+2.62	$+2.62(-1.4) = -3.7 = U_{xz}$
$A_l + A_r = -5.4$	−5.07	$-5.07(-5.4) = +27.4 = U_\Delta$
$B_l - B_r = -1.4$	+4.52	$+4.52(-1.4) = -6.3 = U_{yz}$
$B_l + B_r = +4.2$	−8.79	$-8.79(4.2) = -37.0 = 2U_{xy}$

	Beam Values	corrections negative: Terrain	Cartog	Latitude		Final Results	Remarks
U_{xz}	−3.7	−5.6	−	+8.1		−6.2	
U_{yz}	−6.3	−7.0	−			+0.7	
U_Δ	+27.4	+11.7	−	+6.1		+9.6	
$2U_{xy}$	−37.0	−6.1	−			−30.9	

	Azimuth	B Beam (right)				
		N	N_0	N_1	N_2	N_3
1	0°	31.2	34.0	−2.8		
2	120°	34.4	34.0		+0.4	
3	240°	36.4	34.0			+2.4
	1	31.2	34.0	−2.8		
	2	34.3	33.9		+0.4	
	3					
	1					
		mean:		−2.8	+0.4	+2.4
	A_r	$= N_2 - N_3 = -2.0$				
	B_r	$= N_2 + N_3 = +2.8$				

$2\lambda = 252°$

$\pi = 126°$

$R = 32.0$

$$\frac{\partial^2 U}{\partial x \partial z} = U_{xz}$$

$$\frac{\partial^2 U}{\partial y \partial z} = U_{yz}$$

$$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = U_\Delta \; ; \; \frac{\partial^2 U}{\partial x \partial y} = U_{xy}$$

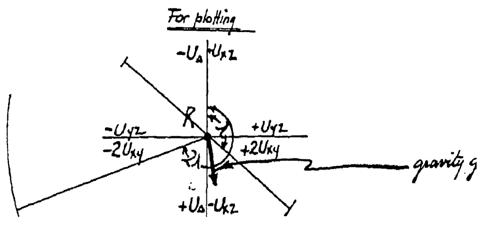

FIG. 114.

LEGEND FOR GRADIENT AND CURVATURE COMPUTATIONS

Column **_A Beam_**

N Values read off torsion balance plate and expressed in units of a transparent reading plate

N_0 Zero values arrived at by summing the recorded values in three azimuths and dividing the sum by three. For example, 74.9 is the average of 73.8, 73.9 and 77.0

N_1 Difference between the zero value in N_0 column and the recorded value in N column for the first azimuth only

N_2 Same as in N_1 column for the second azimuth only

N_3 Same as in N_1 column for the third azimuth only

B Beam

Identical procedure and computation.

To obtain the observed (uncorrected) gradients and curvatures the differences and sums $A_l - A_r$, $A_l + A_r$, $B_l - B_r$, $B_l + B_r$ are multiplied by their respective constants.

To obtain the final (corrected) gradient and curvature values the terrain and latitude corrections expressed in Eötvös units are subtracted from the observed values according to a small table included in the survey form.

The final values are plotted according to the sketch given at the bottom of the torsion balance form. An accepted convention is to plot R (curvature) at the balance point along a λ azimuth where λ is defined by the relation

$$\tan 2\lambda = \frac{-2U_{xy}}{\text{[illegible]}}$$

The R value is represented by a *line segment* having a length proportional to $\sqrt{U_\Delta^2 + (2U_{xy})^2}$. The $\frac{dg}{ds}$ or gradient value is represented by a vector having a length proportional to $\sqrt{(U_{xz})^2 + (U_{yz})^2}$.

Number of Stations Occupied Per Day

The number of stations occupied per day depends on several factors such as type of instrument used and its condition, topography, climate, precision desired in the observations, and the number of men in the crew. Under ordinary conditions, a four-instrument crew can occupy 12 to 14 stations a day. The daily average of a two-instrument party may be 6 or 7 stations. A one-instrument party consisting of two men generally will occupy 2 stations per day at most.

Field Computations and Graphs

It is common practice to compute the observations pertaining to the evaluation of the gradient and curvature values at the field headquarters, preferably on the same day in which the observations were obtained. The computation and plotting of the results are done under the supervision of the party chief. After the data obtained in a day's work have been computed, the graphs and computed data usually are sent to a home office immediately.

A survey form for terrain correction data is given on p. 226. The legend accompanying the form is self-explanatory.

Figure 114 shows a survey form which includes gradient and curvature values, corrections, and a scheme for plotting the corrected data. The figure is accompanied by a self-explanatory legend.

Interpretaton of Gravity Gradient and Curvature Data.—The technique of interpreting gravity gradient and curvature data is complex and can not be described in a standardized "step by step method," because each survey usually involves a large number of variable factors: such as, the type of underground structure, its dimensions and depth, general geology, surface topography, precision of the survey, etc.

Generally, the first step in the interpretation consists in plotting the gravity gradient and curvature values on a map or on two separate maps. In some cases, these maps can be used to interpret the data directly. In other cases, field gradient and curvature maps alone are inadequate and it is necessary to draw isanomalic contours and profiles and utilize the isanomalic curves in the interpretation.

The construction of a gradient map consists in plotting a series of points corresponding to station sites and drawing vectors of appropriate magnitude and direction through these points. Generally, a scale of one or two millimeters per one Eötvös unit is used.

The gravity gradients when plotted on the map will point toward a relatively dense subsurface anomaly and away from a relatively light subsurface anomaly. (Figure 115.)

Over the top of a symmetric anomaly, the gravity gradient due to the subsurface anomaly will be zero and the curvature due to the subsurface anomaly will be a maximum.

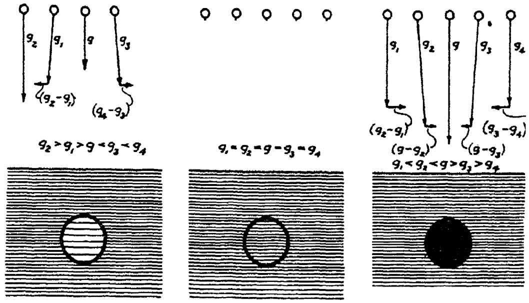

FIG. 115. — Diagrammatic sketches showing gravity vectors and gradients.
a. Subsurface body lighter than surrounding rock.
b. Homogeneous subsurface.
c. Subsurface body denser than surrounding rock.

Curvature values may be shown on a gradient map by line segments which pass through the extremities of the gradient vectors and have a magnitude proportional to the magnitude of the curvature at that point and a direction parallel to a λ azimuth defined by the relation: $\tan 2\lambda = \frac{-2U_{xy}}{U_\Delta}$

Curvature maps have a somewhat limited application in extended structural surveys for oil, because curvature values are influenced by topographic irregularities to a larger degree than the gravity gradients. In many reconnaissance surveys the curvature values are neither computed nor plotted.

Profiles and Contours

To obtain a gravity gradient profile along an arbitrary line passing through certain balance points, it is necessary only to project the gradient vectors on this arbitrary line and scale off the vector component of the gradients along this line.

The drawing of contours in reconnaissance surveys is based on the selection of a few stations through which a suitable closed traverse will be passed, and on the computation of differences in gravitational anomaly between the traverse stations and an arbitrarily selected base station. The procedure may be explained by referring to Figure 116 and its accompanying legend. A closed traverse is passed through stations 1, 2, 3, 4 and 5, station 1 being chosen as the base station. At each station, a vector is constructed which has a length proportional to the magnitude of the observed gradient at that station and a direction parallel to the direction of the observed gradient. Next, the gradient vector at each station is projected on the closed traverse and the magnitude of the component along the traverse is computed. (Component vectors having the same direction as that in which the traverse is traced are taken as positive, and those having the opposite direction are taken as negative.) The next step is

to determine average values of the components by adding components corresponding to two adjacent stations and dividing by two. For example, the average value of the components corresponding to stations 1 and 2 is $\frac{4+16}{2}$, 4 being the component of the gradient at station 1 along the traverse 1, 2 and 16 the component at station 2 along the same traverse. After the average value of the gradient component has been computed, the difference in the force of gravity between any two stations is computed by multiplying the average gradient component by the distance between the two stations. Thus, in Figure 116, it is assumed that the gravity anomaly at station 1 (base

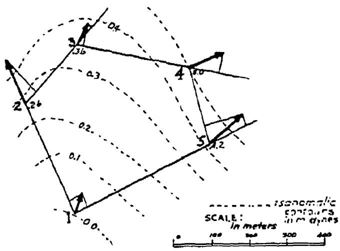

Fig. 116

APPROXIMATE METHOD FOR DRAWING GENERAL ISANOMALIC CONTOURS IN RECONNAISSANCE SURVEY

1	2	3	4	5	6	7
Gravity Gradients in E.U.	Station No.	Distances in cm.	Average Gravity Gradient	Difference in Gravity Eotvos Units	Corrected Difference Eotvos Units	Accumulated anomalies in m. gals.
7						0
		30000	$\frac{}{2} = +10$	$+300000$	$+260000$	
16						$+.26$
		20000	$\frac{5+7}{2} =$	$+120000$	$+100000$	
8						$+.36$
		30000	$\frac{2.5+10}{2} = +6$	$+180000$	$+140000$	
12						$+.50$
		20000	$= -3$	-60000	-80000	
14						$+.42$
		40000	$\frac{-5-13}{5-13} = -9$	-360000	-410000	
7						$+.01$
				$+180000$	$+10000$	

station) is zero. The anomaly at station 2 is equal to the distance between stations 1 and 2 times the average gradient component = 30,000 cm. x 10 E/cm. = 300,000 E.

It is now necessary to adjust the gravity differences for error in closure. The corrections applied in Figure 116 depend on the ratio of the distance between any two stations to the total length of the closed traverse. Thus, the correction for station 2 is $-\frac{30,000}{140,000} \times 180,000 = -40,000$ (approximately). The corrected difference, therefore, is $300,000 - 40,000 = 260,000\ E$. (The corrected *differences* are shown in column 6 of the legend and the corrected *anomalies* or accumulated anomalies relative to the base station are shown in column 7.)

After adjusting the magnitudes of the gravitational anomalies at the traverse stations, the remaining stations plotted on the map are tied into the traverse. Various methods are employed. An approximate method for tieing in a station not too distant from the closed traverse already drawn is to connect this station with a neighboring station on the closed traverse by a straight line and to compute an average gradient component as before.

Finally, the stations of equal gravitational anomaly are connected by continuous curves. (Dotted curves of Figure 116.) The isanomalic contour intervals are usually made from 0.1 to 0.5 of a milligal; however, smaller or larger intervals are not uncommon.

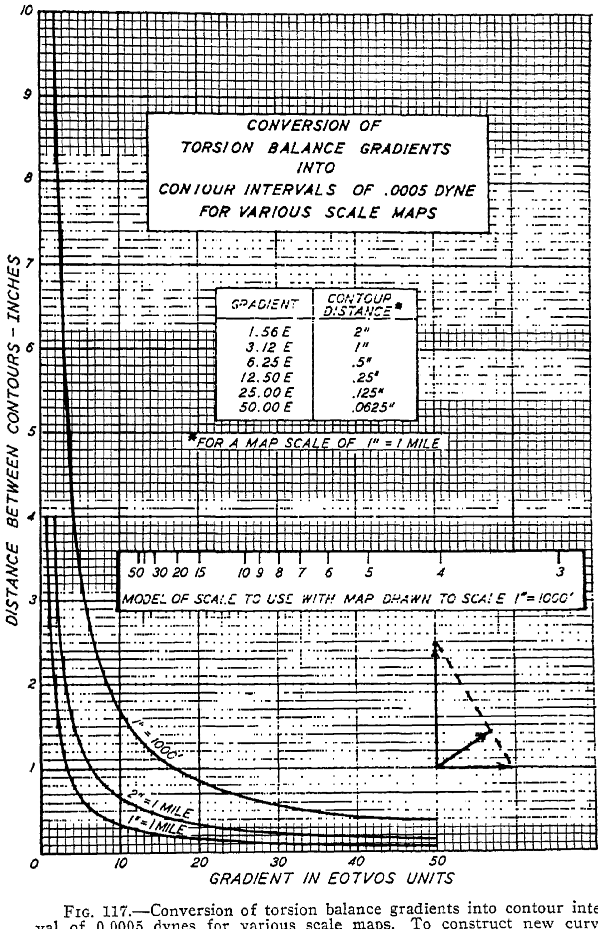

FIG. 117.—Conversion of torsion balance gradients into contour interval of 0.0005 dynes for various scale maps. To construct new curve, measure number of inches on particular map being used. Multiply this number by values in right hand side of above tabulated values, and plot against those on left hand side of table.

If so desired, a scale may be constructed from the curve which gives distance between contours but is graduated in terms of gradient units.

An alternative method for converting a gradient map into a contour map is summarized in Figure 117.

In detailed reconnaissance surveys, one of several elaborate schemes based on an application of the method of least squares to torsion balance data may be used. An illustration of the application of least squares to torsion balance data is given by Roman. †

Detailed Interpretation

Detailed surveys require a more precise evaluation of data than reconnaissance surveys. For example, detailed interpretation generally requires the compiling and drawing of additional profiles and maps and the comparison of the experimental data obtained in the area under investigation with theoretical results obtained by computing the effects produced by various simple geometrical forms and simplified geological structures. Moreover, the final interpretation requires the application of quantitative or graphical and short-cut methods, or a combination thereof, in order to combine the data obtained with the torsion balance and the geological possibilities as derived from a detailed study of regional and local geology.

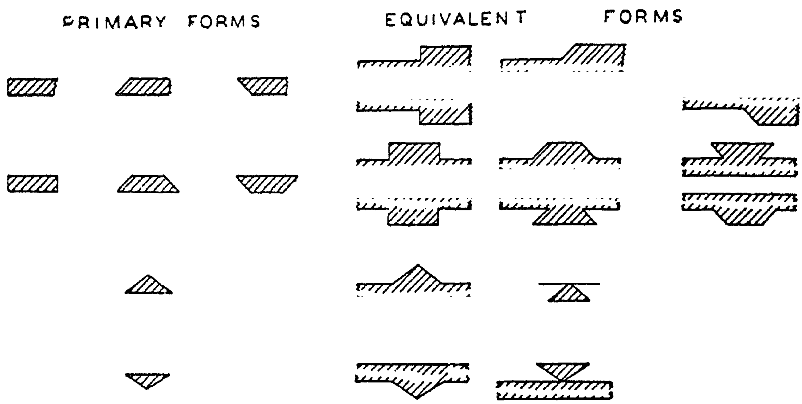

FIG. 118.—Simple geometric forms which correspond to many common types of geologic structure. (Barton, *The Science of Petroleum.*)

Effects Produced by Simple Geometrical Forms and Structures.—A great number of petroliferous structures resemble approximately some one of the simple bodies shown in Figure 118. A horizontal layer of infinite extent alters the absolute value of gravity, but produces no effect on the relative gravity, the gradient, or the differential curvature. Hence, the presence of such an infinite layer tangent to the top or bottom of one of the basic forms will not affect the observed anomalies; i.e., the "equiva-

† I. Roman, "Least Squares in Practical Geophysics," *A.I.M.E. Geophysical Prospecting,* 1932, p. 460.

lent forms" of Figure 118 produce anomalies which are identical with those of the corresponding basic or primary forms.

The gradient and differential curvature anomalies produced by certain simple structures are shown in Figure 119 where the curves are drawn for the case that the structures are denser than the surrounding media. (If the anomalous geologic structures are less dense, the anomalies will have the same form but will be inverted.)

Convenient rules of thumb for determining the approximate depth to the top of the geologic structure are given by the following two relations: (1) For fairly symmetrical anomalies, e.g., type A, the horizontal distance

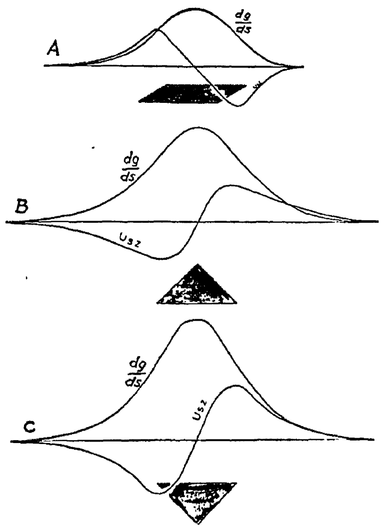

FIG. 119.—Anomalies produced by bodies of simple geometric form.
A. Relative gravity and gradient profiles for a plate-like prism.
B. Relative gravity and gradient profiles for a prism of triangular cross section, apex up.
C. Relative gravity and gradient profiles for prism of triangular cross section, apex down.
D. Differential curvature profile for horizontal plate-like prism.
E. Differential curvature profile for prism of triangular cross section, apex up.
F. Differential curvature profile for prism of triangular cross section, apex down.
(Barton, *The Science of Petroleum.*)

between two points at which the gradient is one-half the maximum gradient is equal to twice the depth to the top of the body; (2) for anomalies of the type B and C, the horizontal distance between the two points having the greatest absolute values of the gradient is equal to twice the depth to the top of the geologic body. For anomalies of the type B, rule (1) is used either with the right hand portion or the left hand portion of the anomaly.

Formulas for Computing Effects of Simple Forms.—The derivations of the analytical expression for anomalies associated with geologic bodies of simple geometric form utilize the same potential theory used in

the solution of magnetostatic and electrostatic problems. The characteristic feature of the potential theory as employed in gravitational exploration work is that three types of quantities are evaluated: (1) the first derivative of the potential, i.e., the component of gravity in a particular direction; (2) second derivatives of the form U_{xx}, i.e., the rate of change of the x component in the x direction; (3) second derivatives of the form U_{yx}, i.e., the rate of change of the x component in the y direction.

The total gravitational effect of any mass is the algebraic sum, or integral, of the individual effects of its constituent elements. The elements usually employed are particles of mass, i.e., point elements, or thin cylindrical masses, i.e., line elements.

The gravitational potential due to a particle of mass m (point element) at a point P located at a distance r from m follows directly from the definition of the potential. That is,

$$' = \int_r G \frac{m}{\ } dr = \frac{Gm}{\ } \qquad (55)$$

where G is the gravitational constant ($6.68 \cdot 10^{-8}$ c.g.s. units.) The effect of this potential is to produce a differential curvature and gravity gradient at the torsion balance which may be computed as follows:

and

$$\frac{x}{r^5} - G\ \frac{m}{\ }$$

Similarly

$$\frac{y^2}{r^5} - G\ \frac{m}{r^3}$$

and (56)

Also

and (57)

A line element is equivalent to a thin cylinder of uniform cross section and density (Figure 120). The cross section of the line element PN will be designated by δ and its density by σ. 0 is the center of the balance.

The equations for the Eötvös effects in this case are:†

$$\frac{\partial^2 U}{} \quad \partial^2 U - 3G\sigma\delta\,(b^2 - a^2)\int_0^c \frac{dc}{r^5} = G\sigma\delta\frac{b^2 - a^2}{(b^2 + a^2)^2}\left(\frac{3c}{r}\right.$$

$$\frac{\partial^2 U}{} - 3G\sigma\delta ab\int_0^c \frac{dc}{r^5} = G\sigma\delta\,\frac{ab}{(b^2 + a^2)^2}\left(\frac{3c}{r} - \frac{c^3}{r^3}\right)$$

$$3G\sigma\delta a\int_0^c \frac{cdc}{r^5} = G\sigma a\delta\left[\frac{1}{(b^2 + a^2)^{3/2}} - \frac{1}{r^3}\right]$$

$$3G\sigma\delta b\int_0^c \frac{cdc}{r^5} = G\sigma b\delta\left[\frac{1}{(b^2 + a^2)^{3/2}} - \frac{1}{r^3}\right]$$

Utilizing the above formulas for point and line elements, it is possible to derive the Eötvös gravity effects: (a) for structures bounded by plane surfaces, e.g., infinite layers of finite rectangular cross section, semi-infinite layer with sloping edge, etc., and (b) for various irregular structures which can be represented, approximately, as the sum of several regular bodies.

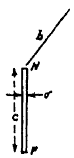

FIG. 120.—Coordinates of attracting line element NP referred to center of balance 0.

The more usable formulas for the gradient and the differential curvature anomalies produced by different types of bodies in different orientations are given below.* In these formulas, G is the gravitational constant and σ is the relative density of the block, i.e., $\sigma = \sigma_1 - \sigma_2$ where σ_1 is the density of the block and σ_2 that of the medium in which the block lies.

(a) Infinite horizontal slab (Figure 121) bounded by $x = x_1$ and $+\infty$, $y = \pm\infty$, $z = z_1$ and z_2 $(z_2 > z_1)$.

$$+ z_2^2 \tag{59}$$

$$\tan^{-1}\frac{z_2}{x_1} - \tan^{-1}\frac{z_1}{x_1} \tag{60}$$

$$= U_{xy} =$$

† E. Lancaster Jones, "Computation of Eötvös Gravity Effects," *A.I.M.E. Geophysical Prospecting*, 1929, pp. 506-509.

* The summary of formulas given here is taken from D. C. Barton, "Calculations in the Interpretation of Observations with the Eötvös Torsion Balance," *A.I.M.E. Geophysical Prospecting*, 1929, pp. 481-486. The derivations of the formulas are given by Lancaster Jones in the article cited above, pp. 517-529.

(b) Infinite horizontal slab bounded by an inclined face

$$U_{xz} = G\sigma \left[\sin^2 \phi \log_e \frac{x_2^2 + z_2^2}{x_1^2 + z_1^2} - (\theta_2 - \theta_1) \text{ si} \right. \tag{61}$$

(c) Infinite horizontal prism with vertical faces bounded by $x = x_1$ and x_2, $y = \pm \infty$, $z = z_1$ and z_2.

(63)

$- \tan^{-1}$

(d) Infinite horizontal prism with inclined faces

$$U_{xz} = G\sigma \left[\sin^2 \phi_1 \log_e \frac{x_2^2 + z_2^2}{x_1^2 + z_1^2} - \sin^2 \phi_2 \log_e \cdot \right.$$

$$- (\theta_2 - \theta_1) \sin 2\phi_1 + (\theta_4 - \theta_3) \sin 2\phi \tag{65}$$

$$U_\Delta = - G\sigma$$

$$\left. - \theta_1) \sin^2 \phi_1 - 2(\theta_4 - \theta_3) \sin^2 \phi_2 \right] \tag{66}$$

(e) Finite rectangular prism with vertical and horizontal faces parallel to the axes, bounded by $x = x_1$ and x_2, $y = y_1$ and $-y_1$ (y_2), $z = z_1$ and z_2.

$$= G\sigma \quad \frac{{}^2 + z_1^2}{{}_1^2 + y^2} \quad \frac{- y}{y^2 + z_2^2 + y}$$

$$y^2 +$$

(67)

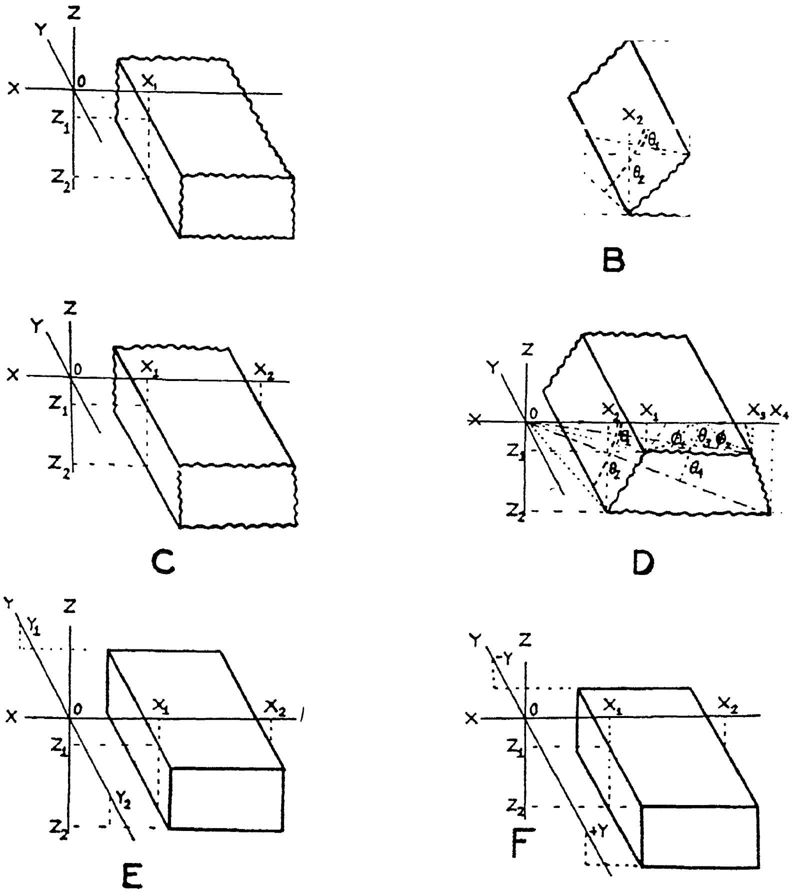

FIG. 121.—Relations of blocks and slabs to coordinate axes. Origin 0 coincides with the center of the torsion balance. (Barton, *A.I.M.E. Geophysical Prospecting*, 1929.)

$$
\begin{aligned}
U_\Delta = 2G\sigma \Bigg[& \tan^{-1} \frac{yz_2}{x_2\sqrt{x_2^2+y^2+z_2^2}} - \tan^{-1} \frac{x_2z_2}{y\sqrt{x_2^2+y^2+z_2^2}} \\
& - \tan^{-1} \frac{yz_1}{x_2\sqrt{x_2^2+y^2+z_1^2}} + \tan^{-1} \frac{x_2z_1}{y\sqrt{x_2^2+y^2+z_1^2}} \\
& + \tan^{-1} \frac{yz_1}{x_1\sqrt{x_1^2+y^2+z_1^2}} - \tan^{-1} \frac{x_1z_1}{y\sqrt{x_1^2+y^2+z_1^2}} \\
& - \tan^{-1} \frac{yz_2}{x_1\sqrt{x_1^2+y^2+z_2^2}} + \tan^{-1} \frac{x_1z_2}{y\sqrt{x_1^2+y^2+z_2^2}} \Bigg] \qquad (68)
\end{aligned}
$$

$$U_{xy} = U_{yz} = 0$$

(f) Same as (e) except $y = y_1$ and y_2 where $|y_2| \neq |y_1|$

$$U_{xz} = G\sigma \left[\log_e \frac{\sqrt{x_1^2 + y_1^2 + z_1^2} + y_1}{\sqrt{x_1^2 + y_2^2 + z_1^2} + y_2} \cdot \frac{\sqrt{x_1^2 + y_2^2 + z_2^2} + y_2}{\sqrt{x_1^2 + y_1^2 + z_2^2} + y_1} \right.$$

$$\left. + \log_e \frac{\sqrt{x_2^2 + y_2^2 + z_1^2} + y_2}{\sqrt{x_2^2 + y_1^2 + z_1^2} + y_1} \cdot \frac{\sqrt{x_2^2 + y_1^2 + z_2^2} + y_1}{\sqrt{x_2^2 + y_2^2 + z_2^2} + y_2} \right] \qquad (69)$$

U_{yz} obtained from U_{xz} by interchanging x and y

$$U_{xy} = G\sigma \left[\log_e \frac{\sqrt{x_1^2 + y_1^2 + z_1^2} + z_1}{\sqrt{x_1^2 + y_1^2 + z_2^2} + z_2} \cdot \frac{\sqrt{x_1^2 + y_2^2 + z_2^2} + z_2}{\sqrt{x_1^2 + y_2^2 + z_1^2} + z_1} \right.$$

$$\left. + \log_e \frac{\sqrt{x_2^2 + y_1^2 + z_2^2} + z_2}{\sqrt{x_2^2 + y_1^2 + z_1^2} + z_1} \cdot \frac{\sqrt{x_2^2 + y_2^2 + z_1^2} + z_1}{\sqrt{x_2^2 + y_2^2 + z_2^2} + z_2} \right. \qquad (70)$$

$$U_\Delta = -G\sigma \left[\tan^{-1} \frac{y_2 z_2}{x_2\sqrt{x_2^2 + y_2^2 + z_2^2}} - \tan^{-1} \frac{x_2 z_2}{y_2\sqrt{x_2^2 + y_2^2 + z_2^2}} \right.$$

$$+ \tan^{-1} \frac{y_1 z_1}{x_2\sqrt{x_2^2 + y_1^2 + z_1^2}} - \tan^{-1} \frac{x_2 z_1}{y_1\sqrt{x_2^2 + y_1^2 + z_1^2}}$$

$$+ \tan^{-1} \frac{y_2 z_1}{x_1\sqrt{x_1^2 + y_2^2 + z_1^2}} - \tan^{-1} \frac{x_1 z_1}{y_2\sqrt{x_1^2 + y_2^2 + z_1^2}}$$

$$+ \tan^{-1} \frac{y_1 z_2}{x_1\sqrt{x_1^2 + y_1^2 + z_2^2}} - \tan^{-1} \frac{x_1 z_2}{y_1\sqrt{x_1^2 + y_1^2 + z_2^2}}$$

$$- \tan^{-1} \frac{y_1 z_1}{x_1\sqrt{x_1^2 + y_1^2 + z_1^2}} + \tan^{-1} \frac{x_1 z_1}{y_1\sqrt{x_1^2 + y_1^2 + z_1^2}}$$

$$- \tan^{-1} \frac{y_2 z_2}{x_1\sqrt{x_1^2 + y_2^2 + z_2^2}} + \tan^{-1} \frac{x_1 z_2}{y_2\sqrt{x_1^2 + y_2^2 + z_2^2}}$$

$$- \tan^{-1} \frac{y_1 z_2}{x_2\sqrt{x_2^2 + y_1^2 + z_2^2}} + \tan^{-1} \frac{x_2 z_2}{y_1\sqrt{x_2^2 + y_1^2 + z_2^2}}$$

$$\left. - \tan^{-1} \frac{y_2 z_1}{x_2\sqrt{x_2^2 + y_2^2 + z_1^2}} + \tan^{-1} \frac{x_2 z_1}{y_2\sqrt{x_2^2 + y_2^2 + z_1^2}} \right] \qquad (71)$$

One of the simple formulas not included in the above list is that for a sloping plane: namely,

$$U_{xz} = 2G\pi\sigma m \text{ (approximately)} \qquad (72)$$

where m = slope.

This formula is accurate for gentle dips and is quite useful in computing terrain effects as well as subsurface effects.

Mathematical Treatment of Geologic Structures and Ore Bodies. —To calculate the gravitational effects of actual geologic structures and ore bodies, it usually is necessary to consider the structures as composed of a series of simple bodies for which formulas of the type 59 to 72 are available and not too complicated. The gravity effects are calculated for each of the constituent simple bodies and then summed up to get the effect

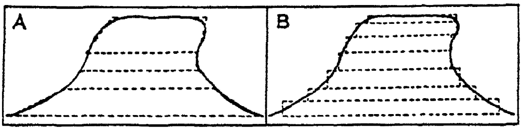

FIG. 122.—Schematic representation of a structural ridge. *A* shows approximation by four prisms, *B* by seven prisms. (Barton, *A.I.M.E. Geophysical Prospecting,* 1929.)

of the whole body. For example, in certain cases, an irregular ridge may be represented by four prisms (Figure 122A) or by seven rectangular prisms (Figure 122B), and the gravity effects of the ridge can be computed as the sum of the effects of the prisms.

The calculations are obviously quite lengthy.† For example, suppose the structure is split into several infinite horizontal prisms. Each evaluation of the appropriate formula would give the gradient for a single rectangular block at a single station. The minimum number of points to determine, approximately, a limited section of profile such as shown in Figure 123 would be five. Good accuracy would require at least nine points. For the differential curvature curve, seven points would be the minimum and thirteen, or more, preferable. To obtain the gradient profile corresponding to Figure 122A, Equation 65 would have to be calculated at least 20 and preferably 36 times; to obtain the differential curvature, Equation 66 would have to be calculated 28 and preferably 52 times.

FIG. 123.—General type of gradient and differential curvature profiles produced by structural ridges. (Barton, *A.I.M.E. Geophysical Prospecting,* 1929.)

Quantitative Methods: Trial and Error Calculations.—Actually, the geophysicist is interested in the inverse problem of that described above: namely, it is desired to infer from the observed data, the form, dimensions, and depth to the structure which gives the observed gravity effects. The quantitative, and very rarely used, procedure is as follows:† A tentative cross section as suggested by visual inspection of the results is sketched. This tentative structure is split into blocks, as shown in Figure 122, and the gravity effects are calculated block by block. The calculated anomalies are then compared with the observed anomalies. In order to make the observed and computed anomalies agree, it usually will

† Barton, *loc. cit.*

be necessary to add and subtract blocks from the cross section and repeat the calculations for each addition or subtraction. This procedure is extremely tedious; for example, 3 months of steady calculations of this sort were required to obtain a satisfactory structural picture of the Hoskins Mound Salt Dome, Brazoria County, Texas. The time element, therefore, precludes any appreciable practical use of these interpretative methods.

Short-Cut Methods.†—The most extensive development of these methods has been done by Karl Jung ‡ who has devised formulas and graphical methods for the recognition of certain simple bodies and the determination of their depths and dimensions. Jung utilizes the abscissas of the numerical maxima and minima of the gradient and differential curvature profiles. With the exception of the sphere, the geologic bodies covered by the Jung formulas and graphical methods are assumed: (a) to be infinite at right angles to the vertical plane of the cross section, (b) to have a cross section of simple geometric shape, (c) to be homogeneous in density, and (d) to be surrounded by a homogeneous medium. The methods, therefore, have two obvious limitations: (1) In general, the abscissa of the point of maximum gradient or differential curvature cannot be determined with great accuracy. (2) Most geologic structures and ore bodies cannot be treated as infinite at right angles to the plane of the section; also, their gravity effects do not correspond to that of bodies having a simple geometric cross section.

Graphical Methods.—These methods utilize sets of standard graphs, each graph representing a vertical section along a line of symmetry. In constructing these graphs, formulas of the type 59 to 72 are employed. For example, graphs are based on formulas 63 and 65 when infinite prisms are used as the building blocks and on formulas 67 and 68 when finite prisms are used. In one convenient standard set, the following relations are assumed. Depth to top : depth to bottom : length of each prism $= a:b:c$, where the ratio $a:b$ is retained constant and c is varied. † Graphs constructed according to this formula correspond to anticlines. (A complete discussion of the graphical method is given by Barton in the article cited.)

Torsion Balance Surveys.—A salt dome usually produces a large, clearly defined density anomaly. Salt domes in the Gulf Coast usually comprise a frustrum of a cone of salt capped by a cylinder or thimble-like mass of lime rock-anhydrite-gypsum, intruded into 20,000 to 30,000 feet of Ter-

† Barton, *loc. cit.*

‡ Karl Jung, "Die Bestimmung von Lage und Ausdehnung einfacher Massenformen unter Verwendung von Gradient und Krümmung's Grösse," *Zeit. für Geophysik*, 1927, Vol. 3, p. 257.

tiary and Cretaceous sands and clays.† The diameter of the top of the salt core is usually between 1 and 3 miles; the height of the salt core above its base is 3 to 6 miles; and the difference between the densities of the salt and the surrounding beds range from 0 to +0.2 grams per cubic centimeter at the surface to − 0.5 grams per cubic centimeter (estimated) at a depth of 20,000 feet. The vertical thickness of the cap is usually between 200 and 500 feet, but on a few domes it varies between 900 and 1000 feet. The difference between the density of the cap and the surrounding sediments is + 0.5 to + 0.7 grams per cubic centimeter.

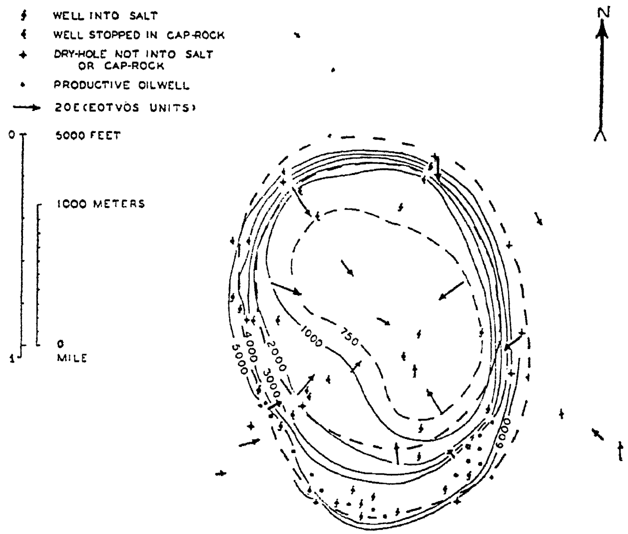

FIG. 124.—Gravity maximum over a shallow salt dome (Nash dome) located in the Gulf Coast, Brazoria and Fort Bend Counties, Texas. (Barton, *A.I.M.E. Geophysical Prospecting,* 1929.)

A characteristic shallow salt dome in the Gulf Coast produces a composite anomaly consisting of a small maximum within a large minimum. The cap, due to its relatively large positive density, produces a gravity maximum. The amplitude of the maximum, which is of the general order of 0.6 milligals, is large relative to the regional variations of gravity. Also, the maximum lies directly above the top of the dome and is only slightly wider than the cap.

The lower half of the salt core, due to its relatively negative density, produces a gravity minimum. The amplitude of the minimum depends on the diameter of the salt core and the downward flare of the flanks. In the Gulf Coast it usually varies between 2.5 and 3.5 milligals, but may reach 7.0 milligals. The values of the gradients of these salt dome minima are of the same general magnitude as those of regional features. Hence, the

† D. C. Barton, "Gravitational Methods of Prospecting," *Science of Petroleum,* Vol. I, pp. 374-375.

center of the minimum may be shifted considerably. Most shallow domes show both the maximum and the minimum; one or the other, however, may be too small to be detected.

A deep salt dome produces a gravity minimum similar to that produced by the lower half of a shallow salt dome. The gravity anomaly produced by the cap of a deep dome is not detectible.

Figures 124 and 125 show the gravity maximum of a shallow salt dome and the minima of a deep dome respectively. In Figure 124, the gradient arrows are superimposed on the structural contours on the top of the cap-salt. The Nash dome is of particular interest as the first Gulf Coast salt dome to be discovered by geophysical methods. It was predicted from the gradient arrows shown. The two heavy dashed lines are 500—900 feet and 4,000—5,000 feet contours on the cap-salt predicted prior to any drilling.

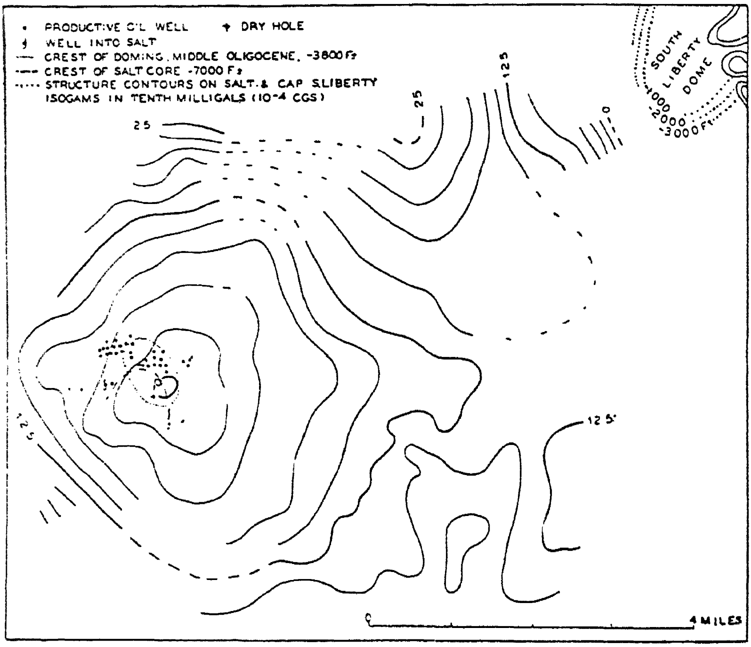

FIG. 125.—Isogams of Esperson salt dome minima. (Barton, *The Science of Petroleum.*)

The convergent gradient arrows well away from the dome are presumably due to a large minimum to the west, north, and east rather than to the Nash dome.

Figure 125 shows isogams due to (a) the minimum of a deep salt dome, Esperson, the top of whose salt core lies at a depth of 7,000 feet; (b) part of the minimum around a shallow dome, South Liberty, Texas, the top of whose cap is less than 320 feet from the surface; (c) the maximum ridge between the two minima.

The shift of the center of the minimum eastward from the crest of the dome is presumed to be due to asymmetry of the salt mass.

Anticlines and Faults

A map showing the gradient and curvature values over an anticline in northern Mexico is shown in Figure 126.

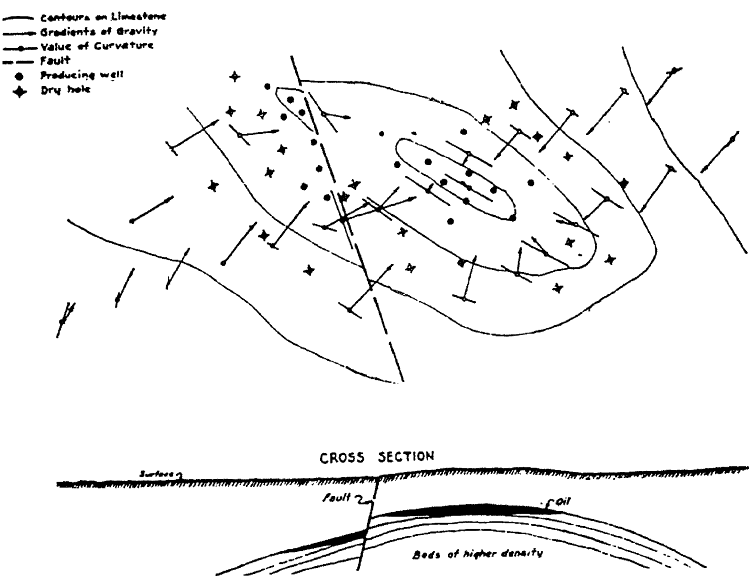

FIG. 126.—Gravity gradient and curvature values over an anticline in northern Mexico. (Courtesy of the American Askania Corporation.)

Mining

The torsion balance has not been used extensively in mining exploration due to the difficulty of interpretation caused by the usual rough topography and complex subsurface geology. Experimental investigations over zinc and lead deposits in areas of moderate topographic relief have given encouraging results.†

COMPARISON OF GRAVIMETER METHOD AND TORSION BALANCE METHOD

The modern gravimeter is superior to the torsion balance in regard to the rapidity with which a survey can be carried out. Under normal conditions, a crew using one gravimeter occupies 12 to 20 stations per day, whereas the usual two-instrument torsion balance crew occupies 6 or 7 stations per day. As a consequence of this factor, a gravimeter survey generally is much cheaper than a torsion balance survey.

The accuracy of gravity maps obtained with the torsion balance is influenced more by the character of the terrain than those obtained with gravimeters. In regions where

† P. W. George, "Experiments with Eötvös Torsion Balance in the Tri-State Zinc and Lead District," *A.I.M.E. Geophysical Prospecting*, 1929, p. 561.

there is considerable relief or irregular surface materials, the use of a gravimeter is preferable. Thus, a distant mass which produces comparable effects on the two instruments, say one Eötvös unit on the torsion balance and one-tenth millidyne on the meter, will produce a considerably greater effect on the balance than on the meter when close to the two instruments.† This is illustrated in Figure 127 which gives two

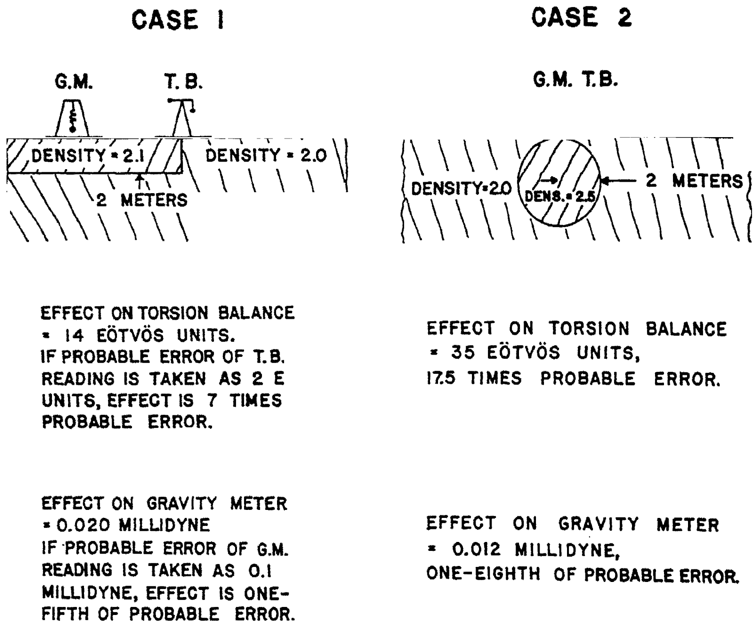

FIG. 127.—Comparison of the effect of disturbing masses on the gravimeter and on the torsion balance. (L. M. Mott-Smith, *Geophysics*.)

examples of the effect of local disturbances on the two instruments. Case 1 illustrates the effect of a thin layer which is located at or near the surface and has a greater density than the normal soil. This layer is assumed to begin as shown and extend forward, backward, and to the left for a considerable distance. Both the gravimeter (G.M.) and the torsion balance (T.B.) are placed at the least favorable position for each. The greatest effect on the meter occurs when it is directly over the bed and far removed from the edge; the greatest effect (maximum gradient) is experienced by the T.B. when it is set up over the edge as shown. The calculated effects, which are summarized in Fig. 127, show that the gradient effect is large, being about seven times the probable error of the T.B.; while the relative gravity effect is altogether negligible.

Case 2 deals with a boulder, which for simplicity has been taken as a sphere located just below the surface. The calculated effects corresponding to the least favorable positions of the two instruments are summarized in Fig. 127. They show again, that in terms of the probable errors of the instruments, the effect on the balance is considerably greater than on the meter. These calculations indicate that the gravimeter should be capable of obtaining usable results in regions where shallow irregularities of density would make torsion balance work practically worthless.

† L. M. Mott-Smith, "Gravitational Surveying with the Gravity Meter," *Geophysics*, Vol. II, No. 1, pp. 30-32.

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CHAPTER V

ELECTRICAL METHODS: *Potential and Resistivity Methods*

Electrical prospecting is the technique of measuring certain properties of electrical fields of force and then utilizing such data to determine the subsurface deposits or structures. Usually, the electrical methods depend for their operation upon the effects produced at the surface of the earth by the flow of an electric current through subsurface formations.

There are several methods by which the electric field may be created. Natural electrochemically-generated ground currents create a field which may be utilized, as in the so-called self-potential method. More commonly, artificial means are employed wherein an electric current is conductively or inductively caused to flow in the portion of the subsurface to be investigated. Electric field properties which may be measured include: potential distribution at the earth's surface, ratio of surface potential to energizing current, phase shift, electromagnetic field strength and direction, distortion of wave front and polarization effects.

Often, the electrical properties of different rocks will vary from other physical properties, and then the electrical methods may be of advantage under conditions where other methods cannot be employed successfully. Also, they may be utilized to obtain data which will supplement the data given by other methods. Conversely, electrical methods will not give data of the most efficient type when the conditions are not favorable to studies of the electric field.

ELECTRICAL PROPERTIES OF ROCKS

The magnitude and the distribution of current flow in the subsurface depend upon the effective *resistivity*, or its reciprocal, conductivity, of the subsurface materials. The resistivity of a material is defined as the resistance in ohms between opposite faces of a unit cube of the material. The units of resistivity commonly employed are the ohm-centimeter and the ohm-meter.† The corresponding conductivity units are the

† Compare S. F. Kelly, "A Uniform Expression for Resistivity," *A.I.M.E. Geophysical Prospecting*, 1932, pp. 141-143.

mho-centimeter and the mho-meter. Different rocks exhibit marked differences in resistivities, and in a great majority of rocks, a variation in electrical resistivity will be accompanied by a discernible variation in lithology.

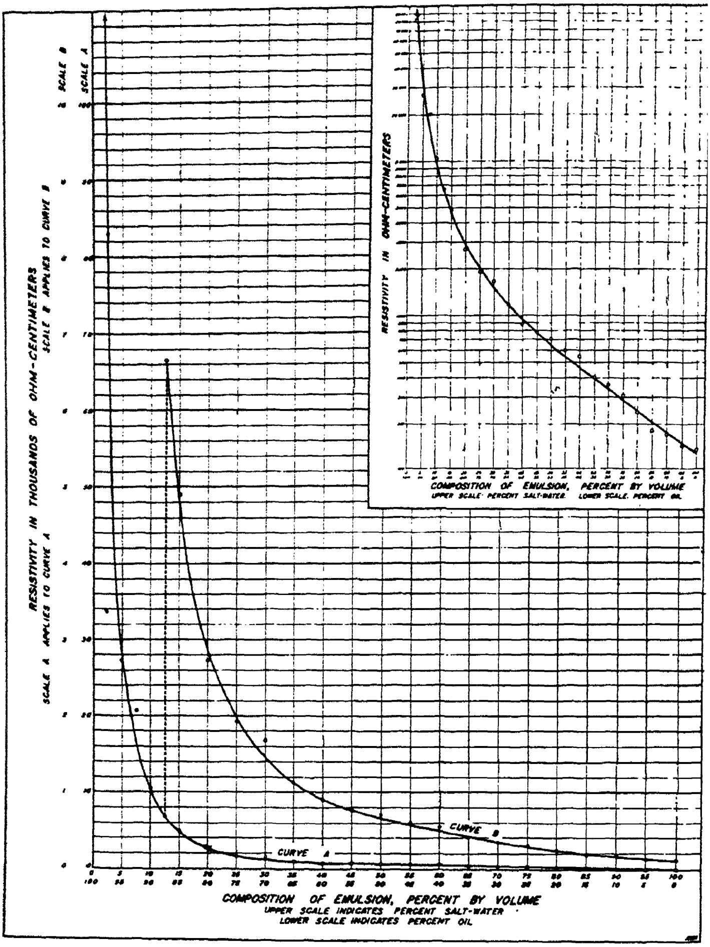


FIG. 128.—Resistivities plotted against oil-water compositions. Curve *B* is a more detailed plot of the lower part of curve *A*. The insert shows curve *A* drawn with a logarithmic ordinate scale. (Jakosky and Hopper, *Geophysics.*)

The flow of an electric current in non-metallic rocks is chiefly *electrolytic*. Practically all rocks are porous † and contain moisture, and it is

† A report on sand porosity studies is given by C. S. Slichter, "Theoretical Investigation of the Motion of Ground Waters," U. S. Geol. Survey, 19th Annual Report, Part 2, 1897-1898. See also L. C. Uren, *Petroleum Production Engineering* (Oil Field Exploitation) (McGraw-Hill, 1939) p. 5.

due to this moisture that rocks are fair conductors despite the fact that their constituent minerals (quartz, feldspar, etc.) are poor conductors, or even excellent insulators.

Laboratory studies of both petroliferous and non-petroliferous rocks have shown that for high values of moisture content, the conductivity approaches the conductivity of the absorbed electrolyte (moisture), while

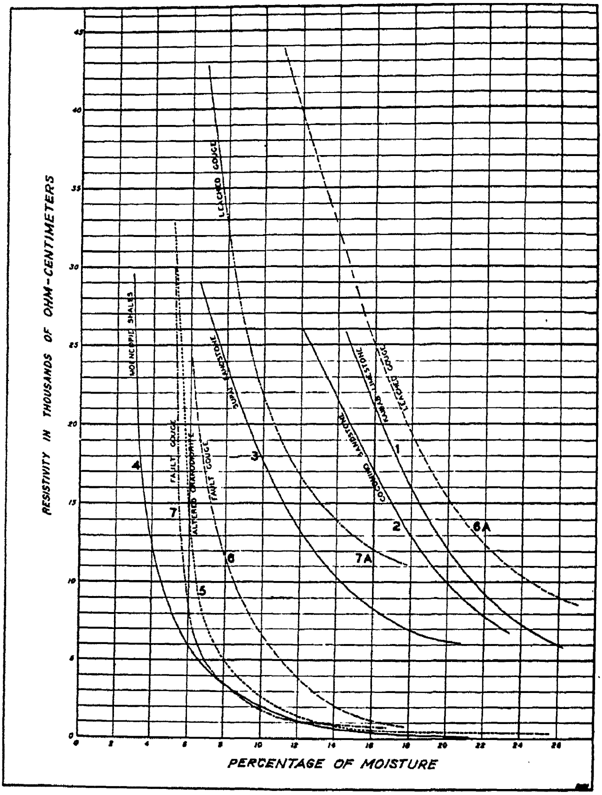


FIG. 129.—Resistivities plotted against moisture percentages for various rocks. (Jakosky and Hopper, *Geophysics.*)

for very low values of moisture content the resistivity values are high and are governed by the properties of the rock materials. †

The decrease in resistivity with increased moisture content is of special significance with regard to the possible use of electrical methods in the direct location of petroliferous rocks. Crude oil is an almost perfect insulator against electrical current; hence, it might be expected that a

† J. J. Jakosky and R. H. Hopper, "The Effect of Moisture on the Direct Current Resistivities of Oil Sands and Rocks," *Geophysics,* Vol. 2, No. 1, pp. 33-35, 1937.

porous rock containing oil would have a very high resistivity. However, practically all oil reservoir rocks contain both oil and water in their pores,† and the presence of dissolved salts in this water tends to lower the resistivity of the rock.

Representative graphs of experimental values of the resistivities of emulsions of oil and water as a function of the percentage of water content are shown in Figure 128. Similar graphs for rocks impregnated with various percentages of moisture are shown in Figure 129.

Magnitudes of the Resistivities.—The electrical constants of rocks may be obtained by *laboratory* measurements, using rock specimens, and by *field* measurements in which average resistivities of subsurface materials and outcrops are measured in place.

The electrical resistivities of earth materials vary within very wide limits. For example, the resistivity of certain metallic elements is about 10^{-6} ohm-centimeters and the resistivity of certain igneous and metamorphic rocks is greater than 10^7 ohm-centimeters.

The accompanying tables list some resistivity values obtained in field explorations in the United States and Canada and resistivity values for laboratory samples. Figure 130 is a graphical correlation of effective resistivities of earth materials of different geological periods.

CLASSIFICATION OF METHODS

The electrical methods are more diversified than any of the other geophysical methods, and a rigid classification is difficult. The following classification is one which corresponds best to field practice.

(1) ***Conductive Methods***

These include all methods in which both the energizing and the measuring electrodes make direct contact with the ground. The energizing current may be direct or alternating. Two types may be enumerated:

(a) *Methods Applicable Under Steady State Conditions.*

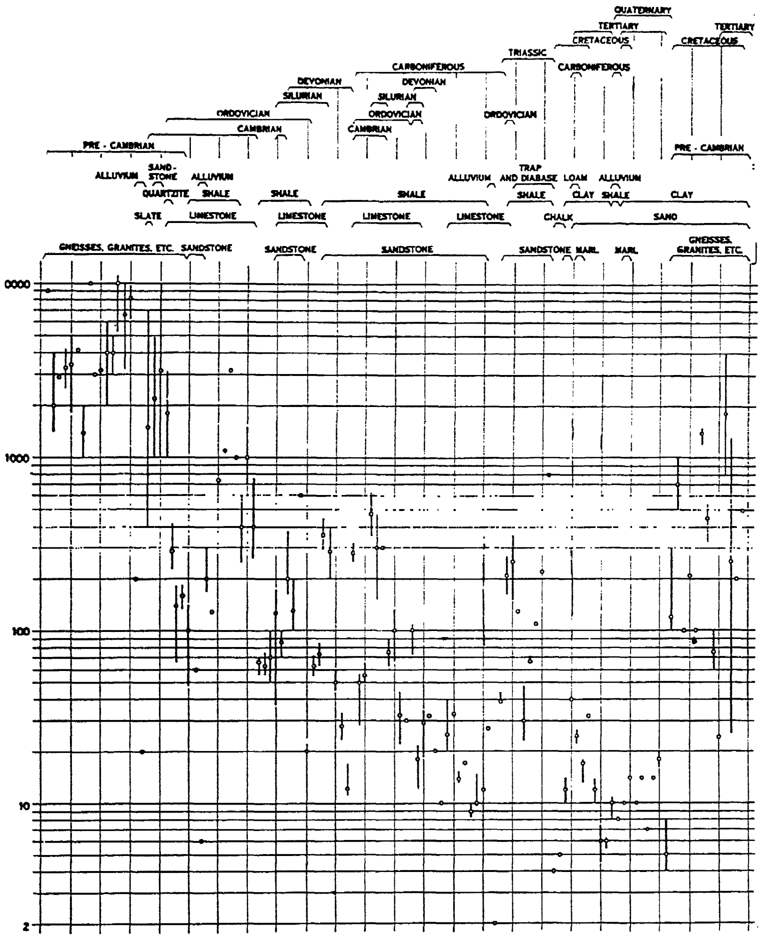
These include the self-potential method, the direct current and low frequency alternating current equipotential line methods, and the various modifications of the resistivity methods. The characteristic feature of these methods is that the diagnostic variable, resistivity, for example, is determined under conditions such that steady state relationships exist, and the typical alternating current phenomena (real and imaginary components, phase differences, etc.) need not be considered in the interpretation of the data.

(b) *Methods Applicable Under Moving Field Conditions.*

These include all methods which employ alternating or variable current of sufficiently high frequency to produce significant inductive phenomena. Analysis of the data obtained with these methods must take into account such variables as phase shift, polarization ellipse, impedance (instead of resistivity), redistribution of current caused by inductive effects, etc. All

† C. Pyle and T. H. Jones, "Quantitive Determination of the Connate-Water Content of Oil Sands," *Oil and Gas Journal*, Nov. 12, 1936. Paper delivered at 17th annual meeting, American Petroleum Institute, Los Angeles.

methods employing medium and high frequency alternating currents and transient currents fall within this classification. The typical medium frequency alternating current methods are those in which frequencies ranging from about 50 to 500 cycles per second are employed. The typical high frequency methods employ alternating currents having frequencies from a few thousand cycles per second to the radio frequencies.



The heavy lines indicate the range of effective resistivities for tests in which measurements were made in several subsections or exploring wires; the mean effective resistivities are indicated by circles. Isolated circles indicate effective resistivities where only one measurement was made

The sequence in which the various types of materials are listed is not necessarily the time sequence in which these materials occur in the geological structures, although the oldest materials are, in general, on the left

The overburden is not indicated except in a few cases where material thicknesses of alluvium are present

"Limestone" as used in connection with silurian and earlier formations includes, in many cases, dolomites as well

FIG. 130.—Correlation of effective resistivities with geological periods and materials. (R. H. Card, "Earth Resistivity and Geological Structure," *Electrical Engineering*, Nov. 1935, p. 1156.)

TABLE 10

RESISTIVITIES OF MATERIALS FOUND IN THE OUTER CRUST OF THE EARTH

Igneous and Metamorphic Rocks	*Resistivity ohm-cm*
Basalt	$2x10^6$
Crystalline rock of normal physical character (igneous, gneiss, schist)	$2x10^4$ - $2x10^6$
Diabase	$2x10^3$ - $2x10^6$
Diorite	$5x10^6$
Gabbro	$1x10^4$ - $1.5x10^6$
Gneiss	$2x10^4$ - $3.4x10^6$
Granite	$3x10^4$ - $>10^6$
Lava	$1.2x10^4$ - $5x10^6$
Marble	$1x10^4$ - $1x10^7$
Meteoric iron	$1x10^{-6}$ - $3x10^{-6}$
Porphyry	$6x10^3$ - $1.5x10^6$
Quartzite	$1x10^3$ - $2x10^7$
Schist	$5x10^2$ - $1x10^6$
Serpentine	$2x10^4$ - $3x10^5$
Syenite	$1x10^4$ - 10^7
Trachyte	$1x10^3$ - $1x10^7$
Trap Rock	$1.5x10^4$ - $3x10^5$

Sedimentary Rocks	*Resistivity ohm-cm*
Alluvium and Silt	$2.5x10^3$ - $1.5x10^5$
Clay-Shales	$4x10^{-2}$ - $9x10^4$
Clay	$5x10^2$ - $1.5x10^5$
Glacial sediments	$8x10^{-2}$ - $9.5x10^5$
Conglomerate	$2.5x10^3$ - $1.5x10^6$
Consolidated sedimentary rocks (slates, shales, sandstones, limestones, etc.)	$1x10^3$ - $5x10^4$
Graywacke	$2x10^5$ - 10^6
Limestone	$6x10^3$ - $5x10^7$
Loams	$1x10^3$ - $4.5x10^4$
Marls	$0.5x10^2$ - $7x10^3$
Sand	$9.5x10^1$ - $5x10^5$
Sandstone	$3x10^3$ - $1x10^7$
Shales	$8x10^2$ - $1x10^6$
Slate	$6x10^4$ - $8x10^4$
Soil	$2x10^2$ - $1x10^6$
Unconsolidated and recent formations (marls, clays, sands, alluvial deposits, etc.)	$5x10^1$ - $1x10^4$

RESISTIVITIES OF MATERIALS FOUND IN THE OUTER CRUST OF THE EARTH (Cont.)

Minerals and Ores	*Resistivity ohm-cm*
Anhydrite	10^5 - 10^7
Arsenopyrite	$2 x 10^1$
Bornite	0.5 - $5 x 10^1$
Calcite	$>10^7$
Chalcocite	0.1 - $6 x 10^1$
Chalcopyrite	$1.5 x 10^{-2}$ - $3.5 x 10^1$
Chalcopyrite-Hematite	5.5
Chalcopyrite-Sphalerite	1
Chalcopyrite-Pyrrhotite	<0.1
Chromite	$1 x 10^2$ - $2 x 10^6$
Coal (Bituminous)	$6 x 10^1$ - 10^7
Coal (Anthracite)	$1 x 10^2$ - $2 x 10^7$
Coal (Lignite)	$9 x 10^2$ - $2 x 10^4$
Cobalt Iron	$5 x 10^{-2}$
Copper	$1.5 x 10^{-6}$ - $1.5 x 10^{-1}$
Copper-Iron	0.7
Covellite	<0.1
Cuprite	$3 x 10^4$
Diamond	$1 x 10^{14}$
Galena	$3 x 10^{-3}$ - $2 x 10^1$
Galena-Sphalerite	6 - 10^4
Graphite	$8 x 10^{-4}$ - 6
Hematite	$5 x 10^4$ - 10^7
Hematite (specular)	0.4
Limonite	$1 x 10^5$ - $1 x 10^7$
Magnetite	0.6 - $5 x 10^3$
Marcasite	1 - $3.5 x 10^2$
Meteoric iron (oxidized)	$>10^3$
Mica	$9 x 10^4$ - $9 x 10^7$
Molybdenite	0.1 - $5 x 10^1$
Nickel	$1 x 10^{-5}$ - $1.5 x 10^{-1}$
Nickel-Cobalt	$5 x 10^{-2}$ - $6 x 10^{-1}$
Pyrite	$5 x 10^{-2}$ - $1 x 10^{-4}$
Pyrite-Chalcopyrite	<0.1
Pyrite-Pyrrhotite	<0.1
Pyrolusite-Psilomelane (mixed)	0.5
Pyrrhotite	$5 x 10^{-2}$ - 5.0
Quartz	$>10^7$
Rock Salt	$3 x 10^3$ - $>10^7$
Serpentine	$2 x 10^4$ - $3 x 10^5$
Siderite	$7 x 10^3$
Sphalerite	$1.5 x 10^2$ - $1.5 x 10^6$
Stibnite	$>10^5$
Sulphur	$>10^7$
Wulframite	$1 x 10^3$ - $1 x 10^7$

(2) ***Electromagnetic Methods***

This group includes those methods wherein the properties of the magnetic field associated with the flow of an electric current are utilized. Two groups are distinguished:

(a) *Methods Employing Conductive Energizing Means.*

In these methods, direct or alternating current is passed into the ground between two electrodes, and its subsurface distribution is studied by means of the magnetic field associated with the flow of current. If medium or high frequency alternating current is utilized, a direction-finding or search coil with appropriate amplifying and phase compensating apparatus may be employed. If direct current or low frequency alternating current is used for energizing the ground, various forms of variometers, magnetometers, etc., may be used for measuring the magnetic field associated with this flow of current.

(b) *Methods Employing Electromagnetic Energizing Means.*

The characteristic feature of this group is that an alternating current is induced in subsurface bodies by passing high or medium frequency alternating current through an energizing coil or loop mounted on the surface of the earth and oriented, usually, in a horizontal or vertical position. Studies of the subsurface distribution of current may be made by different means: such as, search coils and surface electrodes.

SPONTANEOUS POLARIZATION OR SELF-POTENTIAL METHOD

This method utilizes the natural flow of current in the earth. The general subsurface distribution of the natural earth current is determined from studies made of the lines of equipotential at the surface of the ground. From this information, and a knowledge of the geology of the district, predictions can be made regarding the presence of an oxidizing ore body at depth. This method is one of the simplest, and perhaps the oldest, geophysical process utilizing electrical phenomena. In recent years, the spontaneous polarization method has been used to locate corroding pipe lines and other extended metal structures in contact with the earth.

OPERATING PRINCIPLE

The method operates on the fundamental premise that an ore body undergoing oxidation is a source of electric current. Water seeping downward from the surface carries absorbed oxygen. Such water coming in contact with a sulfide ore body creates, as a result of the oxidizing process, a natural large-scale galvanic cell, with the top or upper portion of the ore as the positive pole and with the weak acid formed in the oxidation process as the electrolyte. The potential differences at the surface of the earth resulting from the chemical activity of the ore body vary with the electrolytic properties, the size and configuration of the ore body, and with the depth of the body below the surface. In some cases this potential difference may be as great as 500 to 1000 millivolts. The electric current flow is usually downward within the ore body, and then

outward and upward through the surrounding earth. The return currents spread outward for considerable distances, due to the relatively high resistance of the earth. At the surface of the earth, the current flow is toward a point, usually above the ore body, which is called the "negative center."

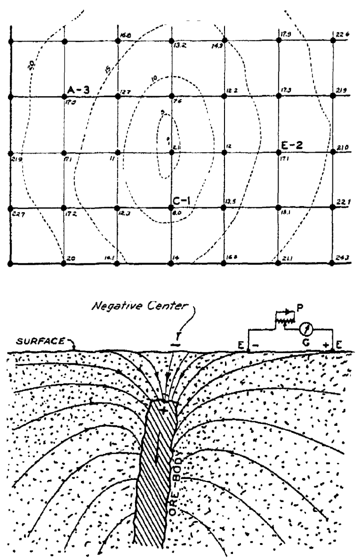


FIG. 131.—Equipotentials and negative center over a vertical ore body.

The negative center may be found by measurements over the surface of the ground: (1) by locating points at the same potential; (2) by measuring the earth potentials at regularly spaced intervals and drawing the equipotential contours; (3) by obtaining potential profiles in a direction across the ore body.

In the theoretically ideal case of a vertical cylindrical ore body surrounded by a homogeneous medium having a uniform distribution of moisture or subsurface water, the *equipotential curves* * measured on the surface of the earth would be a set of concentric circles and the center of the circles would be the negative center.

In practical cases, the negative center is directly over the ore body only if the topography of the region under investigation is relatively flat. (Compare Figures 131 and 135.)

FIELD EQUIPMENT AND PROCEDURE

Location of Points of Equal Surface Potential.—The apparatus for this work comprises: (a) indicating meter or galvanometer, (b) electrodes for making contact with the earth, and (c) two insulated cables, one about twenty feet and the other about two hundred fifty feet in length.

Galvanometer.—Various types of sensitive direct current galvanometers may be employed. The galvanometer should be of rugged construction and have a sensitivity of 0.25 to 2 microamperes per readable unit of scale. The resistance of the indicating meter should be 1000 ohms, or higher, in order to minimize the effects of the contact resistance between the probe electrodes and the earth.

A number of commercial galvanometers are available for this work. It is preferable to employ a rugged pivotless instrument of the pointer type

* An equipotential curve or an equipotential is a curve such that every point on it is at the same potential.

with the coil suspended between taut bronze ribbons. The suspension type instruments must be properly leveled while readings are made.

Electrodes.—The simplest ground contact would consist of a metal rod driven into the earth. This type of contact is unsuitable because of electrochemical effects. Due to the dissolved salts present in the ground, the moist earth acts as an electrolyte and a potential difference is created between the metal electrode and the earth. The magnitude of the electrochemical potentials depends on the metal constituting the electrodes, the concentration of the electrolyte, and the temperature of the electrodes. For instance, temperature differences between electrodes may create potentials of considerable magnitude when one electrode is in the hot sun and the other electrode in the shade.

Non-Polarizing Electrodes

One type of electrode whose potential is not appreciably affected by the chemical properties of the soil consists of a metallic electrode immersed in a supersaturated solution of one of its metallic salts, the solution being contained in a semi-permeable or porous cup that is in contact with the earth. This type of electrode is described in detail on p. 329.

Testing Electrodes in the Field.—During field work, the non-polarizing electrodes should be tested occasionally by immersing both of them in a single earthenware or glass container and observing the difference in potential with the aid of a galvanometer or a potentiometer. This difference in potential should be less than one millivolt if chemically pure materials have been used. While not in use, the electrodes should be kept immersed in a non-metallic jar (glass or glazed earthen ware) containing the same strength solution as in the porous cups. Distilled water should be used for mixing all solutions, if possible. In field work it may be difficult to obtain pure water, and should it be necessary to use water containing appreciable quantities of dissolved minerals, care must be taken to keep the solutions in both electrodes the same. This may be accomplished by emptying both cells into a large container and refilling with the mixed solution once or twice a day. Any difference in potential between the two cells must be compensated by adding or subtracting from the field reading. The compensation may be positive or negative depending upon the relative polarity of the electrodes and the field readings.

Field Procedure for Locating Equipotential Lines

In surveying an area, an arbitrary starting point, say station 1, is selected. The meter and tripod are set up near this station and connected by a short length of wire to the non-polarizing electrode buried at the station. The other electrode, which is connected to the meter by the long length of insulated wire, is contacted with the earth's surface at a point approximately 100 to 200 feet from the first electrode. The polarity and magnitude of the potential difference between the electrodes are noted, and

the electrode is then moved to another location. If the reading is lower, the movement has been in the right direction; the electrode is then moved to various locations until a point is found where no potential difference exists. This point is now marked with a suitable stake and constitutes the second station on the equipotential curve. The galvanometer and its electrode (which were at station 1) are now moved to the new location (station 2) and the process repeated to find another equipotential station. By continuing this procedure, the equipotential stations will eventually close in on the first arbitrarily selected point. This completes one group of equipotential stations and a line drawn through these points is an equipotential curve. Another arbitrary starting point is selected about 100 feet inside the curve and the steps outlined above are repeated, thereby locating another group of equipotential stations. On continuing this procedure, the

FIG. 132.—Photographic view of equipment for measuring earth potentials. *a*, non-polarizing electrode; *b*, potentiometer; *c*, tripod; *d*, anchor stake; *e*, reel.

enclosures will become smaller in size; the smallest enclosure, theoretically a point, is the negative center. The various stations are now located on a suitable map, prepared from data obtained by an alidade and plane table survey, and are connected by closed lines so as to form equipotential curves. The various curves will be concentric with the negative center as their origin.

Field work of this type is relatively slow, both as regards tracing out the points of equipotential, and the time required for surveying-in the equipotential stations with their irregular locations. However, if the field

work is carefully done, the method often is more accurate than that described in the following section.

Measurements of Earth Potentials at Regularly Spaced Intervals.—The equipment required for the direct measurement of potentials is somewhat more involved than that described for locating points of equipotential. It comprises a potentiometer, two non-polarizing electrodes, and a long insulated flexible connecting wire which is wound on a reel to facilitate handling. A photographic view of the complete equipment and operators is shown in Figure 132.

The direct current potentiometer should have a range of 0 to 1000 millivolts. A double-pole double-throw switch is provided for reversing the input. The entire apparatus should be mounted in a hardwood or metal waterproofed box equipped with suitable carrying straps. A short, sturdy tripod for the potentiometer and a camp stool for the operator facilitates the field work.

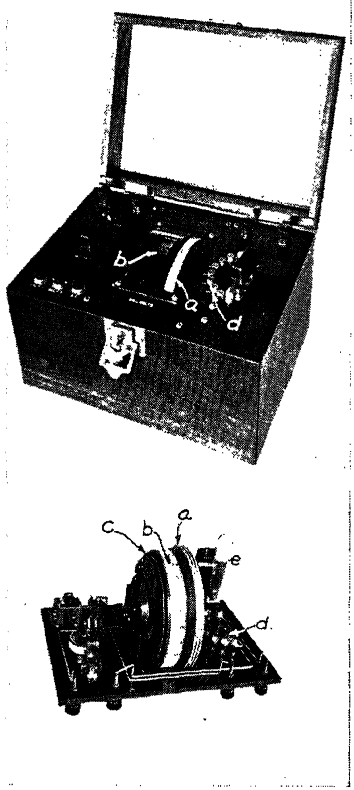


FIG. 133.—Interior and exterior views of potentiometer showing: *a*, control dial; *b*, calibrated scale; *c*, variable and *d*, fixed resistors; and *e*, standard cell.

Direct Current Potentiometer.—The potentiometer is a basic instrument used in many types of quantitative direct current measurements. The theory of the potentiometer is given in any standard text on general physics or electricity. Constant potentials of any value within the range of the instrument may be obtained by proper combination of a dial or drum setting, which controls a variable resistor and a tap switch. Photographic views of the exterior and interior of a potentiometer are shown in Figure 133.

Field Procedure for Potential Measurements

The measurements are conducted to determine the relative potential and polarity between certain pre-selected stations in the area. Usually, the first step consists in surveying two base lines at right angles to each other (lines OF and 04 in Figure 131) and then surveying two series of regularly spaced traverse lines, each series being parallel to one of the base lines. Their intersections form a grid-work of stations. A satisfactory

field procedure consists in setting up the potentiometer at a station on one of the base lines and measuring the difference in potential between that reference point and other points along a traverse line which passes through the station and is parallel to the other base line. (To connect the potentiometer with the distant base line, it is convenient to use a reel containing about 1000 feet of insulated wire.)

Upon completion of the traverse line the wire is disconnected from the instrument and anchor stake and is wound up by the reelman as he returns to the instrument. The instrument is then carried to the next station, set up, and the potential measured between the new base station and the previous station. The reel is now moved along the new traverse and the procedure repeated. Should the work extend over a distance greater than 1000 feet, the potentials between the various base stations should be checked once or twice daily, so that corrections can be made for marked potential fluctuations.

An average lineman, who handles the reel and digs holes (to a depth of 3″ to 6″) for the non-polarizing electrode, usually can contact from 10 to 15 stations, at one hundred foot intervals, per hour. Slightly greater speed is obtained by having an assistant dig the electrode holes.

Sources of Error

Considerable potential variations will oftentimes occur in an area due to natural earth currents, rains, changes in temperature (especially in regions where the nights are cool and the mid-days hot), and freezing and thawing weather. Additional potential variations caused by industrial or mining operations will be found in many localities, and these are usually the most common source of error in obtaining accurate field data.

Earth Currents.—The phenomenon designated as "earth currents" is due to a great variety of causes. The potential difference between two grounded electrodes consists of several components; some change with time relatively slowly, and others fluctuate rapidly and irregularly. The earth potential components of preponderantly direct-current character include: the electrode potentials already described; * potentials due to oxidizing ore bodies; a regional gradient in the area which amounts, according to place and time of year, to some 10 to 100 millivolts per kilometer.

In addition to these more or less steady components, various fluctuating potentials occur. Earth currents have strong diurnal variations, magnetic storm fluctuations, and other variations with a period longer than one day. † Figure 134 shows a comparison of the variations in earth currents, magnetic activity, and sunspot numbers over a period of several years. ‡ The curves are necessarily "smoothed out"; that is, monthly, daily, and hourly

* The electrode potential variations are, of course, minimized when non-polarizing electrodes are used.

† W. J. Rooney, "Earth Current Variations with Periods Longer than One Day," *Terr. Mag.* 42, No. 2, p. 166, June, 1937.

‡ W. J. Rooney, "Earth Currents," p. 291 *Terrestrial Magnetism and Electricity* (Edited by J. A. Fleming) (McGraw-Hill, 1939.)

variations that would give the curves a saw-toothed character are not shown. Hourly earth-current records are shown by Gish. †

The errors due to varying earth currents are minimized by employing relatively short separations between the measuring electrodes. In average areas, the effects of varying earth currents are of minor importance at separations less than 500 feet.

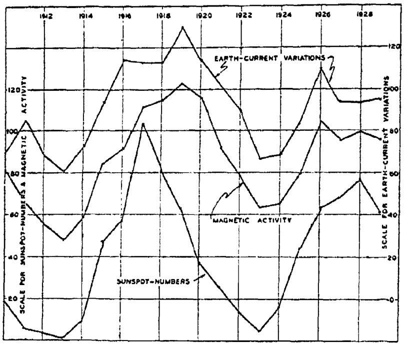


FIG. 134.—Comparison of variations in earth currents, magnetic activity, and sunspot numbers. (Rooney, "Earth Currents," *Terrestrial Magnetism and Electricity*.)

Traverses Across Vein Conductors.—The self-potential method can often be employed to advantage in studying veins and parallel vein systems undergoing oxidation by running traverses across the area at right angles to the vein system. Work of this type is much more rapid than the equipotential studies previously outlined. For this work, a potentiometer, reel, and non-polarizing electrode system are employed. Readings are made at suitable intervals along the traverse line. Plots may be made of the differences in potential between the stations as ordinates and the traverse distances as abscissas. Due to the change in the direction of current flow above the oxidizing ore body, the potential curve has a minimum value above the negative center. Non-symmetry of the curve about the negative center may be caused by a dipping ore body, differences in the electrical conductivity of the rock on each side of the vein system, variations in ground water elevation and distribution, or the regional gradient. These variables must be evaluated by proper geologic control and measurement technique.

INTERPRETATION OF SPONTANEOUS POLARIZATION STUDIES

The field work involved in spontaneous polarization measurements is relatively simple, yet its interpretation is oftentimes complicated due to several factors. In some cases, it is of interest to compare observed potential

† O. H. Gish, "Electrical Messages from the Earth, their Reception and Interpretation," *Journal of the Washington Academy of Sciences*, Vol. 26, No. 7, July 15, 1936.

data with theoretical potential profiles. † Final interpretation should give careful consideration to: (1) effects of topography, (2) geological and structural conditions, (3) spurious earth currents, (4) ore occurrence in the district, and (5) regional gradient.

In areas where rugged topography prevails, interpretation is usually complicated by an irregular distribution of surface potentials. In many cases, the negative center will be shifted, with a resultant shift in the predicted location of the oxidizing ore zone. This effect is apparent when the topographic contour map of the area is studied in conjunction with the potential map. As a general rule, the negative center in fault studies will be shifted toward the hanging-wall side of the ore body.

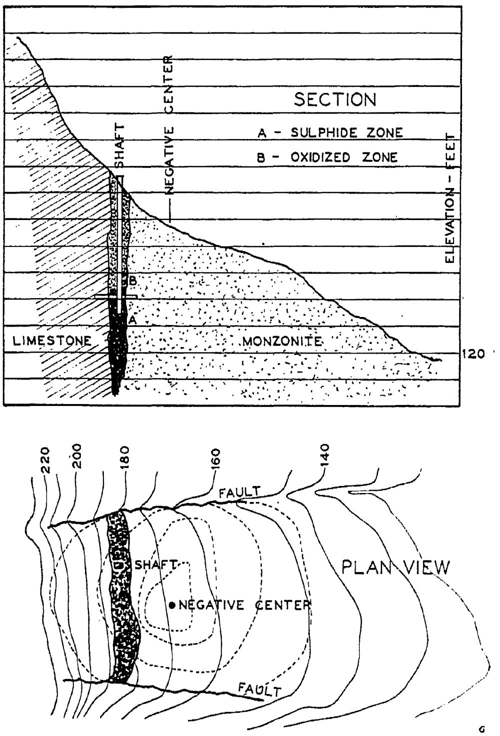


FIG. 135.—Sketch illustrating the displacement of the negative center with respect to the top of the ore body. Equipotential lines, dotted; topographic contours, solid.

Interpretation should never be attempted without proper geological control. Areal maps should be drawn to the same scales as the geophysical potential contours. Contacts of different materials containing ground waters

† Theoretical potential profiles produced by (1) a polarized rod and (2) a polarized sheet are discussed by Broughton Edge and Laby, *Geophysical Prospecting* (Cambr. Univ. Press, 1931), pp. 243-246.

of different chemical properties often give rise to earth potentials which may not be related to ore occurrence. In addition, different geological formations usually possess different electrical conductivities; hence, they cause a distortion of the normal regional ground currents with a resultant redistribution of the surface potentials. Fault zones filled with wet clay gouges, or other conducting materials, cause severe distortion of the surface potentials.

The effects of topography and different materials may best be seen by reference to Figure 135 which shows the surface potentials existing over a pyrite vein formation in Arizona. The sulfides exist at a depth of approximately 45 feet. A general cross section of the vein is shown in the upper portion of the figure. The mineralization occurs at the contact of the limestone and a monzonite intrusion. The limestone has an electrical resistivity of 85,000 ohm-cm. while the monzonite has a value of 30,000 to 50,000 ohm-cm. As shown on the plan view, the mineralization is localized between two cross faults. The faults are filled with a clay gouge material which when wet has a very low resistivity, approximately 8,000 ohm-cm. Laboratory measurements of the pyrite vein material gave values of 2,000 to 6,000 ohm-cm. The equipotential lines are shown dotted in the plan view in the lower portion of the figure. It will be noted that the current flow was concentrated chiefly between the faults in the lower resistance monzonite. The negative center was found to be approximately 40 feet to the east of the actual vein itself.

The self-potential method is capable of furnishing important data, provided the interpretation is made with due regard to ore occurrence in that particular district. Final utilization of the survey results is accomplished by subsequent drilling or other direct exploration to ascertain the commercial value of the ore. There is no known physical relationship between galvanic currents or surface potentials and the economic value of the mineralization.

In regions where the water table is considerably below the surface and where the upper near-surface material is highly resistant (as is the case in many desert areas), the magnitude of the galvanic currents which penetrate the highly resistant overlying materials may be too small to create a measurable potential difference at the surface. Under such conditions, the self-potential method will not give interpretable results. Also, application of the self-potential method in certain localities will quickly show that many persistent earth potentials are present which are not related to mineralized areas or ore occurrence.

Artificial conductors such as car rails, pipe lines (especially those extending from the surface to underground workings), etc., are all possible, and usually probable sources of earth currents. In areas where mining operations are in progress and direct current is used for power, difficulty will usually be met due to leakage and "ground-return" power circuits.

Oftentimes, these leakage currents are encountered at distances of a mile or more from the mine and cause erratic self-potential data. In one case, failure to correct for leakage currents led to an erroneous conclusion. (Fortunately, the error was disclosed on checking the area by resistivity measurements prior to development of the "indication.")

FIELD RESULTS

Surveys of Sulphide Ore Bodies.—A well known self-potential survey is that made by Schlumberger in 1913 on the Sain-Bel ore body. †

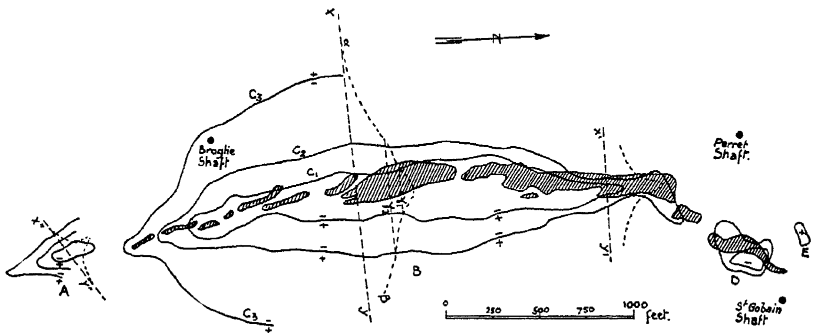


FIG. 136.—Self potential survey of Sain-Bel ore body. (After Schlumberger, *Etude de la Prospection Electrique du Sous-Sol.*)

(Figure 136.) The potential difference between the negative center and a distant point of the surrounding terrain is of the order of 220 millivolts. The hatched area represents a cross section of the ore body at a depth of about 300 feet. It will be noted that the equipotential curves outline the area of mineralization quite accurately. The dotted profiles show the variation of the potential in the transverse direction across the ore body. At *A* there was located a negative center which was later proved to lie over a lens of fine pyrite.

Self-potential surveys of sulphide ore bodies have also been described by Kelly, ‡ Mason § and others. ††

Surveys Over Anthracite Coal Deposits.—Anthracite coal quite frequently shows the phenomenon of spontaneous polarization. It is found that a *positive* instead of a negative center occurs over the upper part of the anthracite beds. The potentials encountered are usually much smaller than those found over sulphide ores.

† C. Schlumberger, *Etude de la Prospection Electrique du Sous-Sol.* (Gauthier-Villars, Paris 1920).
‡ S. F. Kelly, *Engineering and Mining Journal,* Vol. 114, Oct. 7 and Oct. 14, 1922.
§ M. Mason, "Geophysical Exploration for Ores," *A.I.M.E. Geophysical Prospecting,* 1929, p. 27.
†† Broughton Edge and Laby, *Geophysical Prospecting,* pp. 81-84, 98-100.

Corrosion Surveys.—The oxidation of pipe lines is an important commercial problem which has been the object of numerous investigations.[†] The types of corrosion that may affect a buried metallic conductor may be enumerated as follows:[‡]

1. Soil corrosiveness. The conductor is attacked by the surrounding soil.

2. Autogalvanic corrosion. If a metallic conductor connects two regions of the ground in which the electrolytes have a different composition, an electric current will be generated, and certain zones of the conductor will be oxidized.

3. Electrolytic corrosion. Stray currents, due to power lines for example, may enter the conductor in certain sections and leave it in others, thus causing oxidation in certain zones.

The effects produced at the surface due to autogalvanic and electrolytic corrosion are entirely analogous to the effects produced by oxidizing ore bodies. Hence, spontaneous polarization methods may be employed to locate the corrosive regions.

EQUIPOTENTIAL POINT AND LINE METHODS

The equipotential point and line methods both utilize artificially created potential fields. In favorable cases, the use of these methods permits mapping subsurface deposits of anomalous conductivity, such as a highly conductive ore body in a less conductive formation.

OPERATING PRINCIPLE

When an electric potential is applied between two points, or between two parallel line conductors, on the surface of the ground, an electric current will flow. The potential distribution produced by flow of electric current in a homogeneous medium can be readily calculated. (See Figure 148.) Where the ground is not homogeneous, the potential distribution will not follow the pattern calculated for the homogeneous medium. Hence, it is always possible, at least theoretically, to detect the presence of an inhomogeneity by comparing the measured potential distribution with the theoretical distribution for a homogeneous medium.

In practice the usual procedure is to note any deviation of the potential distribution from a regular pattern and to attribute this to subsurface inhomogeneities, without recourse to comparison of theoretical and observed distributions.

The most practical type of energizing current is direct current or low frequency alternating current. From a theoretical viewpoint, the concept of equipotential lines in the case where alternating fields are used has sig-

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[†] F. N. Speller, *Corrosion Causes and Prevention* (McGraw-Hill, 1935).
O. P. Watts, "Electrochemical Theory of Corrosion," *Trans. Electrochem. Soc.*, Vol. 64, pp. 125-153, 1933.
T. G. Elliott, R. J. Sarjant, and W. Cullen, "Special Alloy Steels As Applied to Chemical Engineering," *J. Soc. Chem. Ind.*, Vol. 51, pp. 502-531, 1932.
Scott Ewing, "Electrical Methods for Estimating the Corrosiveness of Soils," *Amer. Gas Assoc. Monthly*, Vol. 14, pp. 356-360, 1932.
W. G. Heltzel, "The Maintenance of Oil Pipe Lines," *Proc. Amer. Petroleum Inst.*, Vol. 14, pp. 168-191, 1933.
G. N. Scott, "Report of A.P.I. Research Associate to the Committee on Corrosion of Pipe Lines," *Proc. Amer. Petroleum Inst.*, Vol. 14, pp. 204-220, 1933.
J. M. Pearson, "The Value of Soil-Survey Methods," *Oil and Gas Journal*, Nov. 18, 1938, p. 100.

[‡] C. and M. Schlumberger and E. G. Leonardon, "Location and Study of Pipe Line Corrosion by Surface Electrical Measurements," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 476 (1932).

nificance only with reference to mean effective values of the potential. However, for low frequencies the field distribution is represented with sufficient accuracy by the D.C. field distribution.

When the frequency is not sufficiently low, appreciable phase shifts will occur. The phase shifts are produced by the inductive effects of the alternating field in the conductive materials, e.g., the ground, mineral bodies, *et al,* lying within the area investigated. When alternating current potential profiles are to be determined in regions of high conductivity where appreciable phase shifts occur, use is made of some form of phase compensator, as described later.

D.C. galvanometers and potentiometers or A.C. galvanometers and vacuum tube voltmeters may be used with direct current or low frequency alternating current, respectively, for determining the lines of equipotential. If audio-detecting means (usually headphones with vacuum tube amplification) are used, the frequency should only be high enough to give a clear cut signal, because the effective depth of penetration decreases as the frequency increases. Frequencies from 100 to 500 cycles are often employed for work of this type.

The potential methods are chiefly useful for general reconnaissance purposes. In simple cases, such as sulphide lodes extending to within 50 feet or so of the surface, these methods will locate the conducting bodies with sufficient accuracy for practical purposes. † For deeper deposits, electrical methods of better resolving power are necessary.

FIELD PROCEDURE AND APPARATUS

Direct Current Method.—The energizing electrodes are positioned to include the area under investigation. Sufficient current is caused to flow between the electrodes to create a measurable potential field. After this field has been created, the field investigations are quite similar to those made in the spontaneous potential method. In the previously described self-potential method, only one datum is necessary at each point of measurement: namely, the value of the natural earth field. In the D.C. equipotential line method, two data should be obtained: namely, (a) the undisturbed natural earth field and (b) the resultant field due to the natural earth current and the artificial current. The readings of (a) are employed for correcting the values of (b).

Power may be supplied to the energizing electrodes by a gasoline-driven, direct current generator. The generator should have an output of 1500 to 2000 watts and a voltage up to 220 volts for extensive surveys. The contact resistance in the energizing circuit should be minimized by use of large extended electrodes.* Because of the losses by polarization and electrolysis or contact potentials adjacent the electrodes, a larger power supply is necessary when employing direct current than when employing alternating current.

† Compare, also, Broughton Edge and Laby, *Geophysical Prospecting,* p. 48.
* The electrodes are described in detail on p. 331, *et seq.*

Alternating Current Audio Method.—The methods employing A.C. are more common in field practice than those employing D.C. for several reasons:

(a) increased portability of the movable circuit, consisting of metallic electrodes, headphones, and an amplifier;
(b) absence of non-polarizing electrodes which are difficult to handle;
(c) decreased power of generator necessary when vacuum tube amplification is used.

The equipment used in the A.C. audio-frequency potential methods comprises essentially: two power electrodes; insulated connecting wire on reels; power source for energizing the ground; two search electrodes or "probes"; amplifier; headphones, or other balance indicating instruments; and flexible, rubber-insulated connecting cables.

Electrodes.—The multipoint electrodes commonly employed in the energizing circuit are described on page 331. The probe electrodes are preferably made of duralumin and have a diameter of ½ inch and a length of 3 feet. They are pointed at the ground end and provided with an insulated handle for the operator.

Power Sources.—Portable gasoline-engine driven alternators are commonly used as a power source. The alternator should develop from 250 to 1000 watts output at a frequency well up in the audio range, preferably from 100 to 500 cycles per second. The load voltage output of the alternator should have a value of 100 to 200, or a suitable output transformer should be employed to allow proper impedance matching between the alternator and the load.

Amplifiers. — A two- or three-stage transformer-coupled, audio-frequency amplifier may be used. It should be well shielded electromagnetically and electrostatically. The amplifier should be rugged, portable and non-microphonic. Plate supply may be small "B" batteries, and "A" batteries may be used to heat the filaments of dry-cell tubes. The design of the amplifier follows that of conventional audio-frequency equipment.

Headphones.—The headphones should be a light-weight, rugged type and should be provided with rubber cushions to minimize extraneous interfering noise, such as that due to the wind.

A.C. Methods Based on Comparison of In-Phase Current Components. — Equipotentials are obtained in a manner analogous to that used in the D.C. method. The points of equipotential are traced out on the surface of the ground by moving one probing electrode until a minimum of sound is detected in the headphones.*

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* If the frequency of the alternating current is not sufficiently low, the investigation must take into account the phase relation between the energizing current and the potential field at the surface of the ground. Zuschlag † has proposed a method wherein an alternating electric or electromagnetic ground field is created, and measurements are then made to determine the ratio between the electric potentials or the electromagnetic fields at desired points in the area under investigation.

The A.C. Potential Ratio Method.—The principle of the A.C. potential ratio method developed by Broughton Edge † may be illustrated by reference to Figure 137. The method consists essentially in employing a bridge circuit which makes contact with the ground at three points A, B, C, the point B being chosen such that AB equals BC. Each of the two ratio arms AO and CO contain a condenser and resistance connected in series. An audio frequency alternating current (about 500 cycles per second) is passed through the ground between distant earthed conductors.

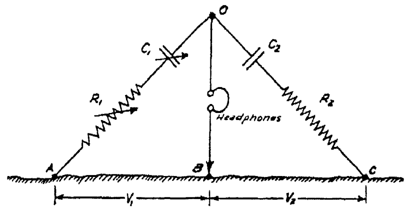

FIG. 137.—Alternating current bridge method, with ratio bridge.

This arrangement is used to compare the potential drops V_1 and V_2 between the pairs of contacts AB and BC and also to determine the difference in phase angle between them.

When in complete balance, i.e., no sound in headphones,

$$V_2 \qquad \frac{R_2 \sin \tan^{-1} \frac{R_1}{X_1}}{\sin \tan^{-} \frac{R_2}{X_2}}$$

where R_1 and R_2 are the resistances and X_1 and X_2 are the capacitative reactances of the two arms of the bridge; also,

$$\theta_2 - \theta_1 = \tan^{-1} \frac{R_2}{X_2} - \tan^{-1} \frac{R_1}{X_1}$$

where $\theta_2 - \theta_1$ is the difference in phase angle between V_2 and V_1. This difference is positive when V_2 leads V_1 and vice versa.

In applying this method, it is essential that the earth at contact B be intermediate in potential with respect to contacts A and C. Observations are usually made along straight line traverses. From data obtained with this bridge both potential and phase variations can be plotted.

A.C. Method Utilizing Three or More Current Electrodes.—In this method, several current electrodes are used and the strengths of the currents in each circuit are made to vary in a predetermined manner. ‡ When three power electrodes are used, the total current I_1, which enters the earth through a common power electrode, will be distributed in the subsurface so that currents of magnitude I_2 and I_3, respectively, reach the other two power electrodes. The ratio of the currents I_2 and I_3 is varied by varying the effective impedance of each branch circuit.

The characteristic and important advantage claimed for the method is that the depth of maximum current density in the ground depends on the ratio $\frac{I_2}{I_3}$. Hence, because the potential difference is a function of the ratio $\frac{I_2}{I_3}$ and because the ratio $\frac{I_2}{I_3}$ can be varied so as to penetrate to any preassigned depth, the measured potential difference theoretically can be made to correspond to a current penetration to any desired depth.

† Broughton Edge and Laby, *loc. cit.*, p. 50.

‡ H. M. Evjen, "Electrical Method of Geophysical Exploration," U. S. Patent 2,169,685; issued Aug. 15, 1939. U. S. Patent 2,172,557; issued Sept. 12, 1939.

Method Employing Commutated Current.—Serious limitations of the D.C. methods outlined above are the errors introduced by undesired earth potentials and the necessity of employing non-polarizing electrodes which are difficult to handle, except under favorable terrain conditions. Marked disadvantages of the audio-frequency A.C. methods are their poor penetrating power and the indefiniteness of the minima detected with headphones when working in conducting regions where marked out-of-phase conditions are encountered. These limitations are minimized by using the very low frequencies developed by commutated direct current.

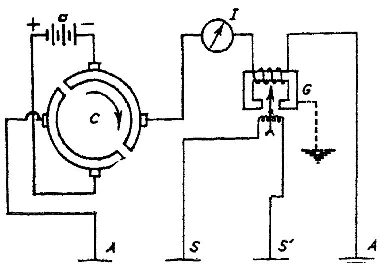

FIG. 138.—Circuit employing commutated direct current for mapping potential profiles.

A schematic diagram of a circuit employing commutated currents is shown in Figure 138. The commutator C is connected to a direct current generator or to a bank of heavy duty batteries B. One lead from the commutator is connected to one of the power electrodes A; the other commutator lead is series-connected to the second power electrode A' through a current meter I and through the field coil of a galvanometer G. (The galvanometer should have a fairly long period, 2 to 5 seconds, to give smooth operation.) The two search electrodes are connected to the movable coil of the galvanometer. The current is reversed periodically by the commutator, which is connected by suitable gears or belt drive to the generator.

The chief disadvantage of this method is the necessity of connecting the galvanometer field coil with the power supply, because this requires handling extra wires during the field operations. From the viewpoint of apparatus design, another undesirable feature is introduced: namely, the necessity of insulating the moving or potential coil of the galvanometer from the effects of the field coil. This is a marked disadvantage, because it is difficult to prevent electrostatic and electromagnetic coupling between the two circuits.

Usually frequencies of from 5 to 10 cycles/sec. are employed. These frequencies are sufficiently high that earth current variations, which ordinarily have a frequency of less than 1 cycle/sec. will not affect the readings. Also, the frequencies are sufficiently low to preclude appreciable phase shift phenomena. A direct current, direct-coupled amplifier may be employed for greater sensitivity; however, this is seldom necessary. Ordinary iron electrodes are used for all contacts.

Low Frequency Voltmeter Method.—For shallow investigations the most satisfactory form of apparatus for alternating current work com-

prises: (1) low frequency (25 cycle) gasoline-driven alternator, (2) ordinary iron electrodes for both the energizing and the measuring circuits, and (3) one or two stages of audio-frequency amplification feeding a vacuum tube voltmeter.

This type of apparatus, while more complicated than the simple D.C. galvanometer method, eliminates the errors and difficulties due to earth currents and electrode-contact phenomena. For shallow investigations, the frequency is low enough to avoid bothersome inductive effects and phase-shift phenomena. In addition, no electrical connection need be employed between the energizing circuit or power supply and the detecting or measuring circuit.

The complete apparatus is illustrated in Figure 139. The alternator for supplying power may be of conventional design, with an output of about 1000 watts at 110 volts. The output is controlled by the rheostat in the excitor circuit. The gasoline engine should preferably be of the standard 2-cycle type which is made of cast iron and has water-bath cooling. The light - weight, high - speed, air-cooled engines, frequently advocated because of their portability, usually are not satisfactory for continuous, heavy duty work. An output transformer must be employed for proper matching of the load impedance, which varies over very wide limits, due to variations in stake resistance.

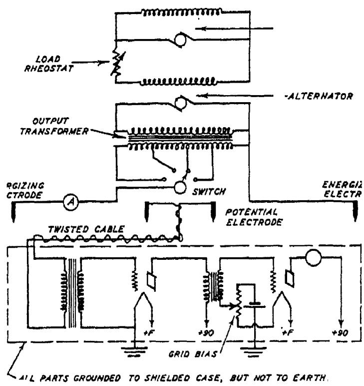

FIG. 139.—Low frequency voltmeter method.

The vacuum tube voltmeter consists of one stage of 25-cycle amplification and a rectifying tube or detector which functions as a vacuum tube voltmeter. A grid-bias control is provided for proper initial adjustment of the meter. Details of vacuum tube voltmeter design and theory are given in many texts. †

INTERPRETATION OF EQUIPOTENTIAL DATA

Simple calculations suffice for converting the field readings into usable data. Usually, the only corrections necessary are those for natural earth potentials (when using the D.C. methods) and instrumental or meter characteristics. The stations occupied during the survey are plotted accurately on a map, preferably with alidade and plane table while the survey is in progress. The equipotential lines are plotted later by drawing in the best-fitting contours.

† J. H. Morecroft, *Principles of Radio Communication* (John Wiley & Sons, 1927).
A. Hund, *Phenomena in High-Frequency Systems* (McGraw-Hill, 1936).
F. E. Terman, *Measurements in Radio Engineering* (McGraw-Hill, 1935).

A map which shows surface topography and areal geology is necessary for final interpretation of the electrical data. This map should be drawn to the same scale as the equipotential contour map.*

The interpretation is largely empirical in character and must be based on previous experience with the method and knowledge of the local geology. Usually, it is relatively simple to make a qualitative interpretation of the results by outlining the indicated conductive zones and determining by inspection which of them are best defined and are in best accord with the geologic possibilities. In general, potential data alone will not permit determinations of the depth of the conductive body, its strike, length, or width. Usually, shallow deposits give narrow and pronounced potential peaks, while the deeper lying bodies produce broader and less pronounced peaks. The steepness and the width of the peak are thus oftentimes an index as to the depth of the body.

Various modifications of the general field technique may be employed to suit special conditions. If an ore body is partly accessible, one of the energizing electrodes may be connected to the ore body, thereby making the potential of the surface of its conductive portion the same as that of the energizing electrode. The shape of the ore body may then be inferred from the equipotentials observed at the surface of the earth. Another modification takes advantage of the anistropy of sedimentary rocks. The dip and strike of formations may be indicated from the increased conductivity in the direction of bedding planes. †

Model Experiments.—The interpretative techniques of the equipotential point and line methods may be facilitated by experiments with small scale models or test tanks. The tanks usually are filled with layers of different conductive materials, such as moist sand, clay, etc.; or they are filled with a weak electrolytic solution (such as soluble salt added to fresh water) in which are immersed model ore bodies made of a conductive material. The model may conveniently be made by forming a block of wood into the desired shape and covering it with a layer of 20 to 24 gauge sheet copper, with joints soldered.

Laboratory model experiments are of value because they show the type of anomalies which would be obtained under the simple conditions

* Generally the potential irregularities produced by topographic features are evaluated empirically, although in certain cases these anomalies can be computed by making a sufficient number of simplifying assumptions.

† E. G. Leonardon and S. F. Kelly, "Some Applications of Potential Methods to Structural Studies," *A.I.M.E. Geophysical Prospecting*, 1929, pp. 180 to 186.

C. and M. Schlumberger and E. G. Leonardon, "Electrical Measurements in Anisotropic Media," *A.I.M.E. Geophysical Prospecting*, 1934, pp. 159-181.

selected for the experimental work. However, because actual geological conditions are usually quite complex, the application of model results to the solution of field problems is valid only to the extent that the simple conditions pertaining to the laboratory work exist in the field.

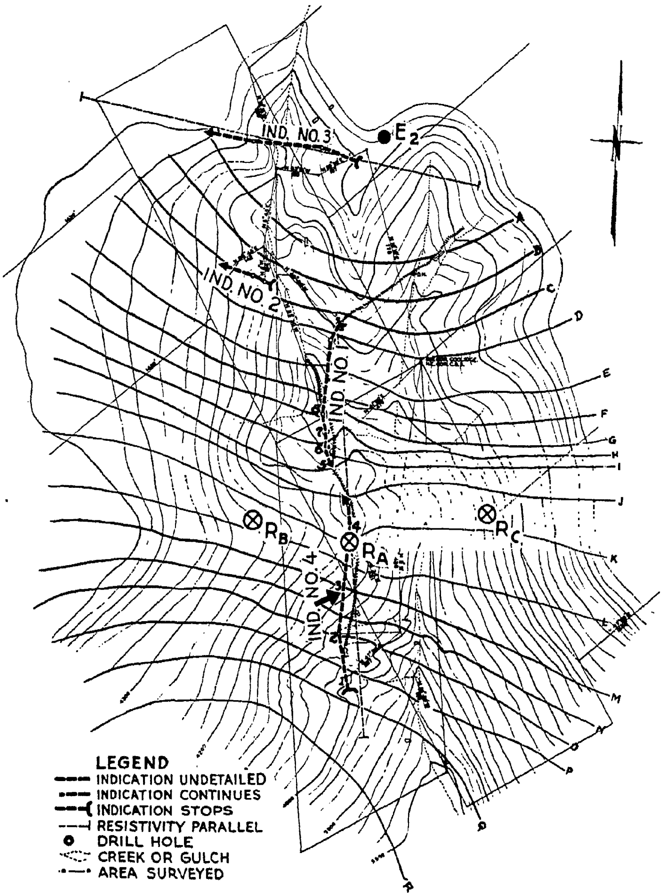

FIG. 140.—Results obtained in an alternating current equipotential line survey in Pinal County, Arizona.

FIELD RESULTS

A number of equipotential surveys for the location of ore have been described by Lundberg † and by Broughton Edge and Laby. ‡ The sur-

† Hans Lundberg, "Recent Results in Electrical Prospecting for Ore," *A.I.M.E. Geophysical Prospecting*, 1929, p. 105, p. 110, pp. 120-122.

‡ A. Broughton Edge and Laby, *loc. cit.*, pp. 74, 83, 86, 92-94, 111, 116, 122.

veys of the latter investigators (with one exception) were made in the eastern part of Australia (Queensland, New South Wales, Victoria and Tasmania).

Figure 140 shows the results obtained in an alternating-current equipotential survey in Pinal County, Arizona. The ground was energized with a 25-cycle gasoline-driven alternator. The iron power electrodes were positioned at locations E_1 and E_2. The lines of equipotential were traced with two probing electrodes connected to the voltmeter type of equipment illustrated in Figure 139.

The topography in this area is moderately rough. The formations are well exposed and the surface contacts can be traced easily. Within the area covered by the survey are rocks of diabase, hornblende porphyry, and quartzite schist. The area is cut by numerous faults, and the ore usually occurs in the north-south fault or fracture system. The ore is associated with quartz, which fills the fractures. The ore is galena accompanied by sphalerite, tetrahedrite, chalcopyrite, and bornite. The chalcopyrite and bornite are usually found in the diabase close to the fracture zones. The galena ore is an excellent electrical conductor.

The location of the fractured mineralized zones was known from geological and exploratory work. The chief purpose of the equipotential studies was to ascertain the zones of high electrical conductivity, because it was predicted that the best commercial values would be found in such zones.

Three types of electrical studies were conducted: (1) equipotential, (2) high frequency inductive, and (3) resistivity. The high frequency studies were conducted first, and the "indications" followed the main fracture system. The strongest effects were obtained between stations 1 to 8. The electrodes E_1 and E_2 for energizing the ground for the equipotential studies were located so as to allow the current path to include the main fracture system. The lines of equipotential A to R showed their maximum deviation from the normal along contours G to L, in the vicinity of stations 4 to 8. The anomalies in the contours may be seen best by holding the book so the plane of Fig. 140 is level with the eye and sighting along the contours.

Because both the high frequency and the equipotential methods are "indicative" only, it was decided to obtain more detailed information regarding the probable depth of the mineralized area by use of resistivity-depth studies. The centers of three resistivity-depth stations were located at R_B, R_A and R_C, with the lines oriented parallel to the fracture zone (approximately in a north-south direction). Subsequent drilling established the existence of a good grade commercial ore zone at a depth of approximately 400 feet.

RESISTIVITY METHODS

The resistivity methods allow quantitative electrical data to be obtained from the field measurements. Calculations may be made of the average resistivity of the portion of the subsurface included in the measurements. The methods have a greater resolving power than the regular potential methods,* because the field work can be conducted so as to introduce a depth variable which relates the electrical data obtained at the surface with the variation of effective resistivity with depth.

OPERATING PRINCIPLES

The field procedure employed in resistivity determinations consists in passing a measured current through a selected portion of the earth and measuring the potential drop, or some other electrical quantity associated with this flow of current. This is usually accomplished by passing the current between energizing electrodes placed at two selected points and measuring the potential difference between two or more auxiliary electrodes placed at other points in the area under investigation. For convenience in the interpretative analysis, all the electrodes are usually placed in a straight line. From the observed values of the current and potential, the apparent resistivity of the material included within the zone of measurement can be calculated for any given electrode configuration.

The effective depth of measurement is governed, to a limited extent, by the spacings of the various electrodes involved in the measurements and by the relative resistivities of the various geologic strata included in the measurement. In a particular area it is necessary to evaluate the effective depth of measurement empirically, using whatever is known of the local geology as a control.

Generally, the subsurface is heterogeneous and the potential distribution at the surface of the ground is affected by the size, shape, composition, and relative positions of the subsurface rock masses. Because these are the elements of geologic study, the geoelectrical problem is to infer the subsurface geology from the surface measurements of electrical quantities. Interfering factors are always present. These include natural ground currents, polarization phenomena, uneven topography, and non-structural near-surface resistivity variations. The effect of such factors must be evaluated or minimized by proper technique in the field work or in the interpretation.

Resistivity investigations as usually conducted may be divided into two general classifications: (1) studies wherein the specific subsurface conditions are to be deduced; (2) studies wherein relative subsurface conditions at one locality (station) are to be compared with the conditions existing at another locality.

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* Potential surface methods include all summation potential methods, such as the magnetic, gravitational, and the self-potential and equipotential line methods.

In (1), the field work and the interpretative technique must be so conducted that deductions may be made regarding the configuration, mass, distribution, and depths of the materials which constitute the subsurface. Analysis of the data is based on relationships derived for certain relatively simple conditions. The interpretative technique includes the following steps: (a) deriving theoretical curves for various assumed subsurface conditions; (b) comparing the theoretical curves with the actual field curves; (c) inferring from an evaluation of all available data the combination of assumed subsurface conditions which would produce electrical anomalies that approximate the observed anomalies. The field procedure and interpretative technique of these methods are applicable for relatively simple subsurface conditions only.

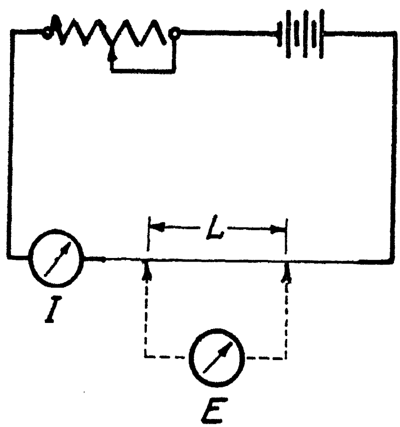

FIG. 141.—Circuit illustrating how Ohm's law may be used to determine the resistance by measuring the voltage drop across any known length L of a conductor of uniform area.

In (2), a quite different type of field procedure and interpretative technique is employed. The field work is conducted primarily to obtain general resistivity-depth relationships from which a characteristic or type curve may be obtained for some sequence of subsurface conditions, whatever they may be. If approximately similar subsurface conditions exist at two localities in a given area, the type curves will generally exhibit recognizable similarities; if the geological conditions are different, the type curves will generally show variations which may be correlated with the geological variations. This type of analysis may be applied best in areas where lateral variations in geology are restricted largely to changes in the depth to certain characteristic beds or "markers." Due to the complexity of beds forming a sedimentary series, deep structural investigations may be handled more successfully by this method than by method (1).

DERIVATION OF FUNDAMENTAL FORMULAS

Ohm's Law.—The elementary and familiar form for Ohm's law is

$$I = \frac{E}{R} \quad \text{or} \quad R = \frac{E}{I}$$

where I = current in amperes
E = electromotive force in volts
R = resistance in ohms

For a conductor (such as a wire)

$$R = \rho \frac{L}{A}$$

where A = cross-sectional area
L = length
ρ = resistivity or the resistance of a cube of unit length.

By combining these two equations, it is evident that

$$E = \rho \frac{LI}{A} \quad \text{or} \quad I = \frac{EA}{\rho L}$$

The application of Ohm's law to the measurement of the electrical resistance of a wire is illustrated in Figure 141, wherein the current is measured by the ammeter I and the voltage by the voltmeter E.

The last equation is the starting point for determining the laws governing the current distribution in an infinite or semi-infinite conductor. Let E represent the potential difference between two points at which the potentials are V_1 and V_2; then

$$E = V_2 - V_1$$

Also, the elementary Ohm's law becomes

$$I = \frac{(V_2 - V_1)\,A}{\rho L} \tag{1}$$

This law gives the magnitude of the current in a conductor of cross section A sq. cm., length L cm., and resistivity ρ ohm-cm. when a potential difference of $(V_2 - V_1)$ volts exist between the ends of the conductor.

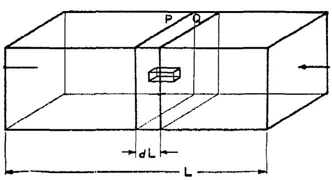

FIG. 142.—Homogeneous parallelopiped. Normal cross sections P and Q are equipotential surfaces.

Differential Form of Ohm's Law

Ohm's law can be used to give the value of current which flows between two normal cross sections such as Q and P at an infinitesimal distance dL apart. (Figure 142.) It is assumed that the value of the potential is the same at any point on a cross section normal to the axis of the parallelopiped; i.e., the normal cross

sections are assumed to be equipotential surfaces. If a difference of potential dV exists between the equipotentials Q and P, Ohm's law states that

$$I = - \frac{}{\rho} \quad dL$$

This equation gives the current which flows through the whole cross section, i.e., through A sq. cm. The current through one square centimeter is

$$\overline{dL}$$

If the distance dL is not measured in the direction of current flow, Equation 3 must be modified. In this case,

$$(2)$$

where i is the current per square centimeter of area normal to L and $\frac{\partial V}{\partial L}$ is the rate of change of potential in the direction of L. In particular,

$$i_y = - \frac{-}{\rho} \cdot$$

where i_x, i_y and i_z are the components of current density in the directions x, y, and z respectively, and $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, and $\frac{\partial V}{\partial z}$ are the partial derivatives of V with respect to and z respectively.

Flow of Current in a Continuous Medium.—When a steady current flows through a conductor, it behaves very much like an incompressible fluid in that the total current which flows into any closed surface within the conductor is equal to the total current which flows out of that surface. In order to express this fact mathematically, it will be convenient to consider a small cube within the conductor. (Figure 143.)

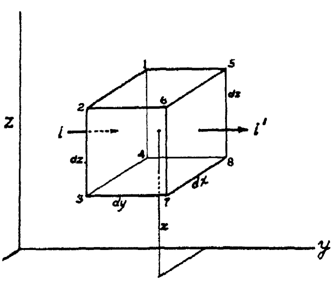

FIG. 143.—Steady state currents through a small cube.

x, y, and z are the coordinates of the center of the cube and dx, dy, dz are the edges of the cube.

Let $i_y dx dz$ be the total current entering the cube at the face $\overline{1234}$ and $i_y' dx dz$ be the total current leaving at the face $\overline{5678}$. Then the excess of current entering at the face $\overline{1234}$ over that leaving the face $\overline{5678}$ is

$$\Delta i_y = i_y\, dxdz - i_y'\, dxdz = (i_y - i_y')\, dxdz = \left(\frac{1}{\rho}\frac{\partial V}{\partial y} - \frac{1}{\rho}\frac{\partial V'}{\partial y}\right) dxdz$$

or

$$\Delta i_y = \frac{1}{\rho}\left(\frac{\partial V}{\partial y} - \frac{\partial V'}{\partial y}\right) dxdz$$

Also,

$$\Delta i_z = \frac{1}{\rho}\left(\frac{\partial V}{\partial z} - \frac{\partial V'}{\partial z}\right) dxdy$$

and

$$\Delta i_x = \frac{1}{\rho}\left(\frac{\partial V}{\partial x} - \frac{\partial V'}{\partial x}\right) dydz$$

If there are no sources or sinks within the cube, the total current entering the cube must equal that leaving it; hence, the sum of the currents Δi_y, Δi_z and Δi_x must equal zero. That is,

$$\frac{1}{\rho}\left(\frac{\partial V}{\partial x} - \frac{\partial V'}{\partial x}\right) dydz + \frac{1}{\rho}\left(\frac{\partial V}{\partial y} - \frac{\partial V'}{\partial y}\right) dxdz +$$
$$\frac{1}{\rho}\left(\frac{\partial V}{\partial z} - \frac{\partial V'}{\partial z}\right) dxdy = 0$$

Division of this equation by $\left(\frac{dxdydz}{\rho}\right)$ yields

$$\left(\frac{\partial V}{\partial x} - \frac{\partial V'}{\partial x}\right)\frac{1}{dx} + \left(\frac{\partial V}{\partial y} - \frac{\partial V'}{\partial y}\right)\frac{1}{dy} + \left(\frac{\partial V}{\partial z} - \frac{\partial V'}{\partial z}\right)\frac{1}{dz} = 0$$

The quantity $\left(\frac{\partial V}{\partial x} - \frac{\partial V'}{\partial x}\right)\frac{1}{dx}$ represents the average rate of change of $\frac{\partial V}{\partial x}$ in the direction of x. When dx is made to approach zero, this quantity approaches the second partial derivative of V with respect to x. That is,

$$\lim_{dx \to 0}\left(\frac{\partial V}{\partial x} - \frac{\partial V'}{\partial x}\right)\frac{1}{dx} = \frac{\partial^2 V}{\partial x^2}$$

Also

$$\lim_{dy \to 0}\left(\frac{\partial V}{\partial y} - \frac{\partial V'}{\partial y}\right)\frac{1}{dy} = \frac{\partial^2 V}{\partial y^2}$$

and

$$\lim_{dz \to 0}\left(\frac{\partial V}{\partial z} - \frac{\partial V'}{\partial z}\right)\frac{1}{dz} = \frac{\partial^2 V}{\partial z^2}$$

Hence, in the limiting case

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad (3)$$

This equation is known as Laplace's equation. It is frequently written in the form

$$:0 \qquad (4)$$

where the operator ∇^2 is defined by the relation

$$\nabla^2 \equiv \frac{\partial^2}{} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{}$$

It is clear from the derivation of this equation that whatever the shape of the conductor through which a steady current flows or whatever the particular conditions of the problem, the potential V must satisfy Equation 4. This is true everywhere except at points where there is either a source or a sink of current.* Equation 4 alone does not determine the solution of any particular problem because it expresses only one of the conditions which V must satisfy.

Furthermore, Laplace's equation (4) holds only for steady currents in isotropic, homogeneous media.

In any particular problem there are "boundary conditions" which must be satisfied. The boundary conditions require: (1) at any boundary separating two media of different resistivities, $V_1 = V_2$ where V_1 and V_2 are the potentials on opposite sides of the boundary, and (2) the normal component of the current entering the boundary through one side is equal to the normal component of the current leaving through the opposite side. That is,

$$\rho_1 \quad \partial n \qquad \rho_2 \quad \partial n$$

Any solution of Laplace's equation which also satisfies the boundary conditions constitutes the unique solution of the given problem.**

Solutions of Laplace's Equation

Solutions of Equation 4 for several simple cases of current flow in media of uniform resistivity ρ will be discussed in the following paragraphs. For exploration purposes, certain portions of the earth's crust are approximately homogeneous; other portions approximate the mathematical ideal of two or three homogeneous layers of uniform resistivities; still other portions comprise, approximately, two homogeneous, semi-infinite media separated by a bounding plane (fault), etc. The flow of current through such structures may be described by a solution of Laplace's equation which satisfies the boundary conditions at all the boundaries.

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* Points where current flows out of a conducting medium or into a conducting medium from an external source.

** For a proof that V is determined uniquely by Laplace's equation and the boundary conditions see J. H. Jeans, *The Mathematical Theory of Electricity and Magnetism* (Cambridge University Press).

Case I.—Consider a small source of current surrounded by an infinite isotropic homogeneous conductor of resistivity ρ. From considerations of symmetry, it is clear that the potential V will be a function only of the distance r from the current source.

If the origin of rectangular coordinates is chosen at the source,

$$r^2 = x^2 + y^2 + z^2$$

Also,

$$2r\frac{\partial r}{\partial x} = 2x \quad \text{or} \quad \frac{\partial r}{\partial x} = \frac{x}{r}$$

and

$$\frac{\partial^2 r}{\partial x^2} = \frac{r - x\left(\dfrac{x}{r}\right)}{r^2} = \frac{1}{r} - \frac{x^2}{r^3}$$

Furthermore,

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial r} \cdot \frac{\partial r}{\partial x}$$

and

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial r^2}\left(\frac{\partial r}{\partial x}\right)^2 + \frac{\partial V}{\partial r}\left(\frac{\partial^2 r}{\partial x^2}\right)$$

Hence,

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial r^2}\frac{x^2}{r^2} + \frac{\partial V}{\partial r}\left(\frac{1}{r} - \frac{x^2}{r^3}\right)$$

Similarly,

$$\frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2}\frac{y^2}{r^2} + \frac{\partial V}{\partial r}\left(\frac{1}{r} - \frac{y^2}{r^3}\right)$$

and

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 V}{\partial r^2}\frac{z^2}{r^2} + \frac{\partial V}{\partial r}\left(\frac{1}{r} - \frac{z^2}{r^3}\right)$$

Substitution of these values of $\dfrac{\partial^2 V}{\partial x^2}$, $\dfrac{\partial^2 V}{\partial y^2}$, and $\dfrac{\partial^2 V}{\partial z^2}$ into Equation 4 yields

$$\frac{\partial^2 V}{\partial r^2}\left[\frac{x^2 + y^2 + z^2}{r^2}\right] + \frac{\partial V}{\partial r}\left[\frac{3}{r} - \frac{x^2 + y^2 + z^2}{r^3}\right] = 0$$

or

$$\frac{\partial^2 V}{\partial r^2} + \frac{\partial V}{\partial r} \cdot \frac{2}{r} = 0$$

Since r is the only independent variable in the problem, the partial derivatives may be replaced by total derivatives. That is, the last equation may be written in the form

$$\frac{\quad}{dr^2} + \frac{\quad}{r}\frac{\quad}{dr} = 0$$

This equation can be readily integrated by multiplying by r^2, thereby forming an exact differential. That is,

$$\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0 = r^2\frac{d^2V}{dr^2} + 2r\frac{dV}{dr}$$

and

$$\int\left(r^2\frac{d^2V}{dr^2} + 2r\frac{dV}{dr}\right)dr = \int d\left(r^2\frac{dV}{dr}\right) = r^2\frac{dV}{dr} = \text{const.} = S$$

Hence,

$$\frac{dV}{dr} = \frac{S}{r^2}$$

and

$$V = -\frac{S}{r} + C$$

The values of S and C must be determined from boundary conditions of the problem. If it is assumed that the value of V at infinity is zero, C vanishes. Also, the constant S may be expressed in terms of the total current I which flows out of the source. Consider a small sphere surrounding the source. The current which flows through one sq. cm. of the surface of the sphere in an *outward* direction is

$$-\frac{1}{\rho}\frac{dV}{dr} = -\frac{1}{\rho}\frac{S}{r^2}$$

The total current is

$$I = -4\pi r^2\left(\frac{1}{\rho}\frac{S}{r^2}\right) = -\frac{4\pi S}{\rho}$$

and

$$S = -\frac{I\rho}{4\pi}$$

Hence

$$V = \frac{I\rho}{4\pi r} \tag{5}$$

Equation 5 expresses the value of the potential at any point in an infinite isotropic homogeneous medium due to a small source of current.

It will be noticed that the solution is of the form $V = S/r$. (The particular value given for the constant: viz., $S = I\rho/4\pi$, is due to the geometric form of the electrode.) The general conclusion which can be drawn from the above analysis, therefore, is that the potential due to a small current source or sink is S/r, where S is chosen to satisfy the boundary conditions.

Practically, this condition is approximately realized in that type of electrical bore-hole exploration in which one current electrode is lowered

a considerable distance into the earth while the other current electrode is fixed at a great distance from the first.

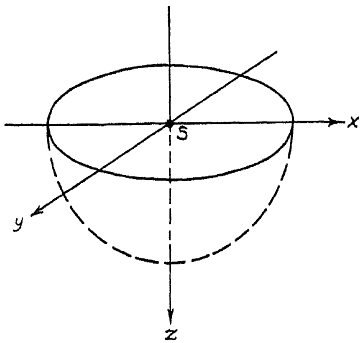

FIG. 144.—Current source S in a plane xy bounding a semi-infinite conductor.

Case II.—Consider next the case of a current source located in a plane P which bounds a semi-infinite, isotropic, homogeneous conductor of resistivity ρ. (Figure 144.) The boundary conditions in this case are $V=0$ at $r=\infty$ and $-\dfrac{1}{\rho}\dfrac{\partial V}{\partial z}=0$ at $z=0$. The solution of this problem is

$$V=\frac{S}{r}$$

for this solution satisfies Laplace's equation. Also, $V=0$ when $r=\infty$ and

$$-\frac{1}{\rho}\frac{\partial V}{\partial z}=-\frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{S}{r}\right)=\frac{1}{\rho}\frac{Sz}{r^3}=0 \text{ when } z=0$$

The value of S in this case is not the same as that in Case I. Since the total current I flows out of a hemisphere,

$$I=-2\pi r^2\frac{1}{\rho}\frac{\partial V}{\partial r}=\frac{2\pi S}{\rho}$$

and

$$S=\frac{I\rho}{2\pi}$$

Hence,

$$V=\frac{I\rho}{2\pi r} \tag{6}$$

By using Equation 6, it is possible to calculate the resistivity of an isotropic, homogeneous medium from the values of the potential observed at the boundary surface and the total current I flowing from the source.

Suppose V is measured at a distance a and again at a distance $2a$ from the current source. Let V_1 and V_2 be the values obtained at these two points. From Equation 6

$$V_1 = \frac{I\rho}{2\pi}\frac{1}{a} \text{ and } V_2 = \frac{I\rho}{2\pi}\frac{1}{2a}$$

In particular, if the source and the two points (a and $2a$) lie in a straight line,

$$V_1 - V_2 = \frac{I\rho}{2\pi}\left[\frac{1}{a} - \frac{1}{2a}\right] = \frac{I\rho}{4\pi a} \tag{7}$$

or

$$\rho = 4\pi a \frac{V_1 - V_2}{I}$$

In practice the conditions of this problem are approximately satisfied by placing the current or energizing electrodes at two widely spaced points and measuring the potential difference $V_1 - V_2$ between two points which are close to one of the energizing electrodes. This method is known as the uni-electrode method for measuring resistivity. †

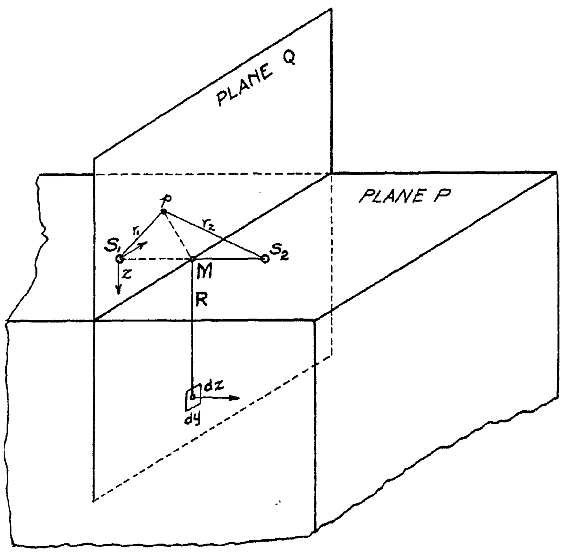

Fig. 145.—Source and sink in a delimiting plane.

Case III.—The conditions of Case II cannot be exactly satisfied in practice, because it is impractical to separate the two energizing electrodes by a sufficiently large distance. Practical measurements of resistivity, there-

† F. W. Lee, J. W. Joyce and P. Boyer, "Some Earth Resistivity Measurements," *U. S. Bur. of Mines Inform. Circ.* 6171 (1929).
L. Gilchrist, "Measurements of Resistivity by the Central Electrode Method at the Abana Mine, Northwestern Quebec, Canada," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 386 (1931).

fore, are based on the solution of a mathematical problem in which a source and a sink of current are assumed to exist at a finite distance from each other. Consider the following problem. A source S_1 and a sink S_2 are located in a plane P delimiting a semi-infinite, isotropic, homogeneous conductor of resistivity ρ. (See Figure 145.)

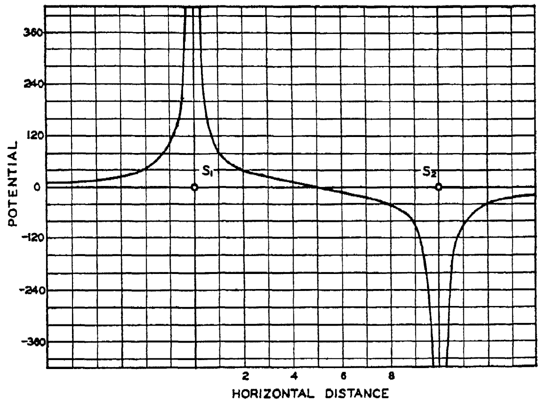

FIG. 146.—Variation of potential along line through current electrodes S_1 and S_2. (It should be noted that the potential curve is not correctly described by Equation 8 in the immediate vicinity of S_1 and S_2. It is evident from physical considerations that the potential must be finite at these points.)

The potential at any point p located at distances r_1 and r_2 from S_1 and S_2 is obtained by adding the potentials due to S_1 and S_2. The potential due to source S_1 is $V_1 = \frac{S_1}{r_1}$, and the potential due to sink S_2 is $V_2 = \frac{S_2}{r_2}$. Hence, the total potential V at p is

$$V = V_1 + V_2 = \frac{S_1}{r_1} + \frac{S_2}{r_2}$$

Since both V_1 and V_2 satisfy Equation 4, their sum satisfies Equation 4 at all points except the points where the source S_1 and the sink S_2 are located.

Also,
$$\frac{1}{\rho}\frac{\partial V}{\partial z} = 0 \text{ at } z = 0$$

If it is assumed that the total current I entering at the source S_1 is equal to that leaving at the sink S_2, evaluation of the constants yields $S_1 = -S_2$ and $S_1 = \frac{I\rho}{2\pi}$ as in case II. Hence, the potential may be expressed by the equation

$$V = \frac{S_1}{r_1} + \frac{S_2}{r_2} = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{8}$$

Equation 8 may be used to describe the distribution of potential and current in some detail. The variation of potential along the line through the electrodes S_1 and S_2 is plotted in Figure 146. Two features of this curve should be noted: (1) the large drop of potential that occurs near each electrode due to the high current densities in the immediate vicinity of the electrodes and (2) the fairly flat portion of the curve midway between the electrodes.*

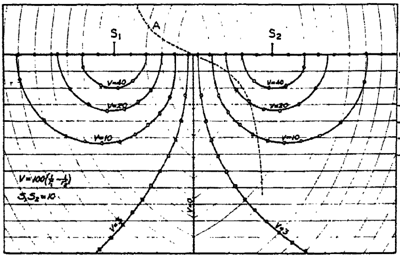

FIG. 147.—Vertical section of equipotential surfaces surrounding current electrodes S_1 and S_2. (Curve A shows the variation of potential along the surface of the ground.)

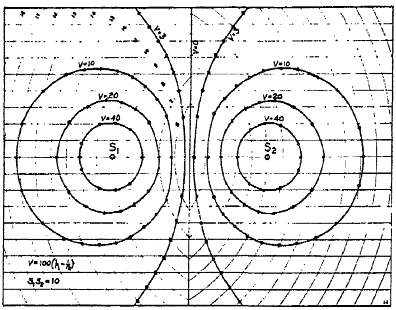

FIG. 148.—Distribution of potential in a plane P which contains a source S_1 and a sink S_2.

The vertical section of the equipotential surfaces is shown in Figure 147 and the distribution of potential in the boundary plane containing the electrodes is shown in Figure 148.

Figure 149 shows the current per sq. cm. flowing through a vertical plane Q which intersects the line joining the two energizing electrodes at its midpoint M as a function of the vertical distance from M. Figure 150 shows the fraction of the total current which penetrates below a plane C located at a depth of d units as a function of the ratio of the depth d to the electrode separation.†

The equations needed for plotting Figures 149 and 150 may be derived as follows:

Let r_1 and r_2 be the distances of a point on a vertical plane Q from S_1 and S_2 respectively. Since the vertical plane Q is located midway between S_1 and S_2, the distance r_2 is equal in magnitude to the distance r_1. The current at the point (r_1, r_2) is

$$-\frac{1}{\rho}\frac{\partial V}{\partial x} = -\frac{S}{\rho}\frac{\partial}{\partial x}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{S}{\rho}\left(\frac{x}{r_1^3} - \frac{x-L}{r_2^3}\right)$$

or

$$-\frac{1}{\rho}\frac{\partial V}{\partial x} = \frac{SL}{r^3}$$

where L = distance between S_1 and S_2.

* Field work is usually planned so that the observational points (points at which the probe or potential electrodes are inserted into the ground) lie in the flat portion of the curve.

† See also W. Weaver, "Certain Applications of the Surface Potential Method," *A.I.M.E. Geophysical Prospecting*, 1929, p. 70.

From this equation the graph given in Figure 149 is readily obtained:

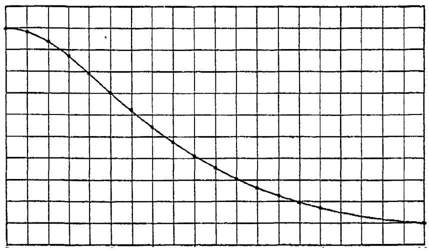

FIG. 149.—Current density versus depth in a plane Q which is normal to and midway between the energizing electrodes.

In order to calculate the fraction of the total current flowing below plane C, consider an element $dy\,dz$ of plane Q. The current flowing through this element is

$$-\,dy\,dz \cdot \frac{1}{\rho}\,\frac{\partial V}{\partial x}$$

Hence, the total current flowing below plane C is

$$-\int_d^{\infty}\int_{-\infty}^{+\infty}\frac{1}{\rho}\,\frac{\partial V}{\partial x}\,dy\,dz \quad \frac{SL}{\rho}\int_d^{\infty}\int_{-\infty}^{+\infty}\frac{dy\,dz}{r_1{}^3}$$

where

$$r_1{}^3 = \left[\left(\frac{L}{2}\right)^2 + y^2 + z^2\right]^{3/2}$$

To carry out the integration, it is convenient to make the following substitutions. Set

$$a^2 = \left(\frac{L}{2}\right)^2 + z^2$$

and set

$$y = a\tan\theta;\; dy = a\sec^2\theta\,d\theta$$

Then

$$I_d = \frac{SL}{\rho}\int_d^{\infty}\left[\int_{-\pi/2}^{\pi/2}\frac{a\sec^2\theta\,d\theta}{a^3\sec^3\theta}\right]dz = \frac{SL}{\rho}\int_d^{\infty}\frac{2}{a^2}\,dz$$

$$= \frac{2SL}{\rho}\int_d^{\infty}\frac{dz}{(L/2)^2 + z^2} = \frac{4S}{\rho}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2d}{L}\right)\right]$$

Or on substituting the value for S and dividing through by I

$$\frac{I_d}{I} = \frac{2}{\pi}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2d}{L}\right)\right]$$

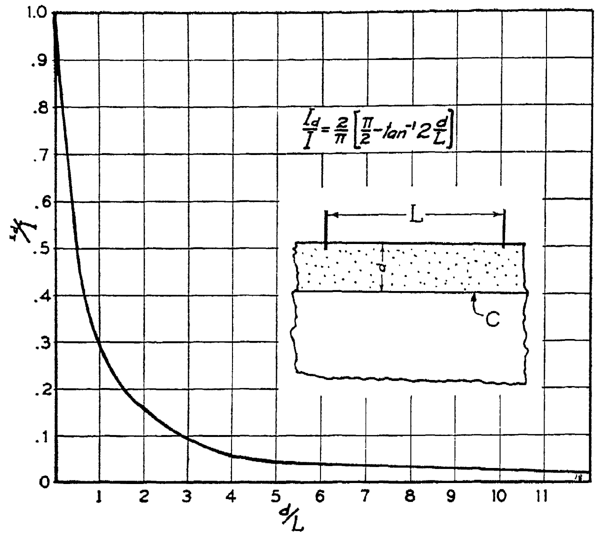

FIG. 150.—Plot of the fraction of the total current penetrating below a plane C located at a depth of d units versus the ratio of the depth d to the electrode separation L. Ratio of conductivity of upper to lower layer materials is 1:1.

Case IV: Layers of Different Materials.—The discussions of cases I, II, and III yielded expressions for the effective resistivity ρ of a homogeneous medium as a function of potential differences observed at the surface, distances measured along the surface (e.g., electrode separations), and the total current I passing between the energizing electrodes. In the present case (IV), a new concept appears: namely, variation of effective resistivity with depth.

Suppose, for example, that for a given area explored by one of the methods described under Case III, it is found that the calculated values of ρ are the same for all electrode separations. Hence, if a plot of the values of the resistivity as ordinate against the values of the electrode separation as abscissa is a horizontal, straight line, one may infer that the subsurface structure is uniform. Generally, however, a resistivity graph of field data yields a complicated curve indicating that the average resistivity varies with depth below the surface. In this case, the description or deduction of subsurface conditions from the geoelectrical data may be quite difficult.

Physical considerations alone may yield useful information. Consider, for example, a semi-infinite medium of resistivity ρ_2 covered by a layer of

resistivity ρ_1 and of thickness A. The source and the sink are placed at the boundary of the upper layer as shown in Figure 151. The following simple considerations indicate the type of solution to be expected: Suppose that the distance between S_1 and S'_2 is very small compared with A; only a small amount of current will flow into medium ρ_2 and the measurement of the resistivity by one of the methods described in Case III will yield a value of ρ nearly equal to ρ_1.

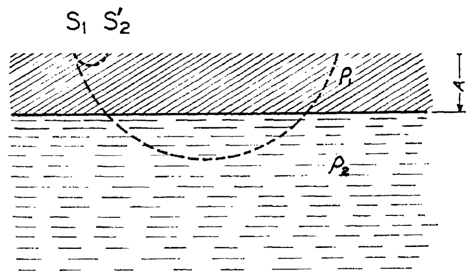

FIG. 151.—Current lines between two electrodes in a layered medium.

If, on the other hand, the electrode spacing is very large compared with the thickness A, the effect of the now relatively thin upper layer will generally be small, and the value of ρ obtained by one of the methods described in Case III will be nearly equal to ρ_2. The curve obtained on plotting ρ against $\overline{S_1S_2}$ resembles that shown in Figure 152. The region of transition AB occurs at values of $\overline{S_1S_2}$ comparable in magnitude with A.

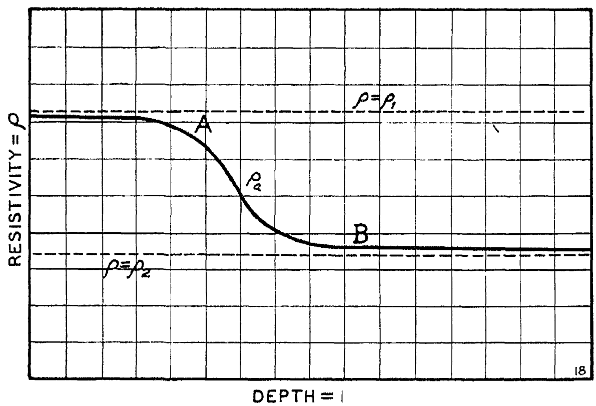

FIG. 152.—Schematic resistivity curve for two-layer structure.

Thus, in addition to enabling one to recognize a simple resistivity distribution, such considerations indicate the order of magnitude of the depth of the buried medium. It would be very difficult, however, to determine the exact depth from such physical considerations. It is much safer to refer to the exact mathematical solution of the two-layer problem.

Before considering the two-layer case, it will be of some interest to illustrate the use of the exact method by solving two simple problems:

(1) A source of current S is placed at a distance A below a plane xy which delimits a semi-infinite medium of resistivity ρ. (Figure 153.) To obtain the solution of Laplace's equation, it will be convenient to introduce an image source S' (equal to S) at a distance A on the other side of the xy plane. It will also be convenient to assume, for the time being, that the medium of resistivity ρ extends beyond the xy plane. If S' is set equal to S, the value of the potential due to the two sources may be written

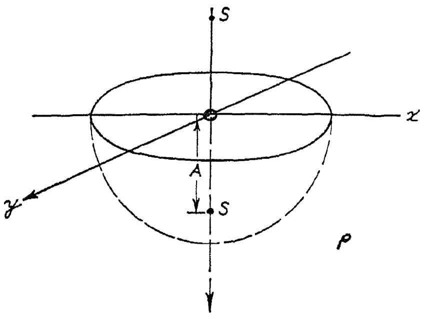

FIG. 153.—Current source S located in a semi-infinite medium of resistivity ρ at a distance A below the xy plane.

$$V = S \qquad \frac{1}{\quad + (z - A)^2]^{1/2}} \qquad \frac{1}{\quad} \Big\} \tag{9}$$

where $r^2 = x^2 + y^2$.

This value of the potential satisfies the boundary condition that the current crossing the xy plane is zero, for

$$\frac{(z - A)}{\quad} \perp \frac{(z + A)}{\quad}$$

$$\rho \quad \partial z \,/\, z=0 \qquad \rho$$

Also, Laplace's equation $\nabla^2 V = 0$ is satisfied throughout the region below the plane xy except at S. The value of V given by the right hand side of Equation 9 does not hold for the region above the plane xy, because it makes V infinite at S'. Therefore, for the region investigated, i.e., for the region below the plane xy, Equation 9 is the required solution.

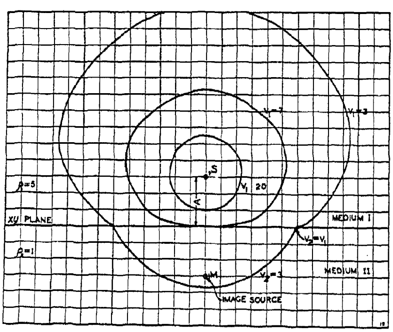

FIG. 154.—Small source of current ${}_1S$ placed at a distance A from the xy plane.

It is of interest to point out that this same procedure may be used to deduce the solution of the problem of two electrodes, a source and a sink, both buried at a distance A below a horizontal plane and thus arrive at a generalized solution for the problem treated under Case III.

(2) A plane xy separates two media having resistivities ρ_1 and ρ_2. (Figure 154.)

$$(\rho = \rho_1 \text{ for } z < 0, \quad \rho = \rho_2 \text{ for } z > 0)$$

A small source of current ${}_1S$ is placed at a distance A from the xy plane

in the negative direction. To arrive at the potential distribution, assume an image source $_1M$ on the other side of the plane.* The potential V_1 for the region $z < 0$ may be written

$$V_1 = \frac{_1S}{[r^2 + (z + A)^2]^{1/2}} + \frac{_1M}{[r^2 + (z - A)^2]^{1/2}}$$

where $r^2 = x^2 + y^2$

The potential V_2 for the region $z > 0$ is a function of the distance from the source only; that is,

$$V_2 = \frac{_2S}{[r^2 + (z + A)^2]^{1/2}} \tag{10a}$$

where $_2S$ is a constant to be determined from the boundary conditions.

The boundary conditions are

$$V_1 = V_2 \text{ for } z = 0$$

$$\rho_1 \quad \partial z \qquad \rho_2 \quad \partial z$$

If the boundary conditions are combined with Equations 10 and 10a, the following relations are obtained:

$$_1S + {_1M} = {_2S} \tag{11}$$

and

$$\frac{1}{\rho_1}\left\{-\frac{_1S\ (z + A)}{[r^2 + (z - A)^2]^{1/2}} - \frac{_1M\ (z - A)}{[r^2 + (z + A)^2]}\right.$$

or

$$\frac{_1SA}{\rho_1} - \frac{_1MA}{\rho_1} = \frac{_2SA}{\rho_2} \text{ and } \frac{_1S}{\rho_1} - \frac{_1M}{\rho_1} = \frac{_2S}{\rho_2}$$

For Equations 11 and 12 to be true simultaneously, it is necessary that

$$\rho_1 + \rho_2) \quad S \text{ and } {_1M} = \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) {_1S}$$

Hence, the potential may be expressed by the two equations

$$V = \quad {_1S} \quad \left(\rho_2 - \rho_1\right) \quad {_1S} \qquad z < 0$$

$$\rho_2$$

The vertical cross sections of the equipotential surfaces close to $_1S$ are plotted in Figure 154 and their behavior is typical of what happens at the boundary of two conductors of different resistivities. In plotting these curves, values of $\rho_1 = 5$, $\rho_2 = 1$ and $_1S = 60$ were assumed.

* Note that in this case it is not assumed that $_1M$ and $_1S$ are equal in magnitude.

Mathematical Solution of the Two-Layer Problem

Consider a uniform layer (I) of thickness d and resistivity ρ_1 overlying a semi-infinite homogeneous medium (II) of resistivity ρ_2. This geological two-layer structure corresponds to an electrical three-layer problem, the third layer (0) being air. The "layer" of air lies above medium I and has a resistivity of $\rho_0 = \infty$. Assume that a source S_0 and a sink $-S_0$ are located at the boundary of media 0 and I. (Figure 155.)

As in case III, the solution is facilitated by considering the effects of the source and sink independently and then combining the effects. Further-

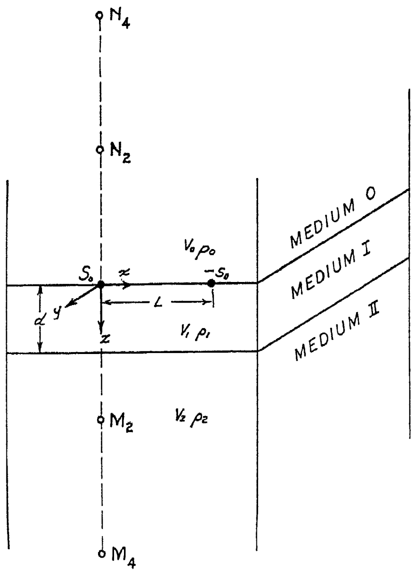

FIG. 155.—The case of three layers.

more the solution for the potential due to the sink $-S_0$ at any point on the line passing through the source and sink can be obtained from the solution for the potential due to the source S_0 by replacing x (the distance of the point from S_0) by $L - x$ where L is the distance between the source and sink. Thus the problem reduces to calculating the potential due to the source S_0. This is most readily accomplished by assuming that in addition to S_0 there exists an infinite number of images of S_0. It will be evident from the preceding problem that the relative magnitudes of the current source and its electrical images will depend on the values of the resistivities of the media in which the images are assumed to exist.

Thus the solution of the problem consists, essentially, in determining the relative magnitudes of these images.

Designate the potential values in the media 0, I, and II by V_0, V_1, and V_2 respectively. Let M_2 be the electrical image of S_0 in medium II, N_2 the electrical image of M_2 in medium 0, M_4 the electrical image of N_2 in medium II and so on *ad infinitum*, all the images in medium II being denoted by M, those in medium 0 by N and the subscript denoting the distance of the image from the plane bounding media 0 and I in terms of the thickness d of the layer I as unity.

V_0 may be expressed as the sum of potentials due to S_0 and the images M in medium II; i.e.,

$$V_0 = \frac{S_0}{[r^2+z^2]^{1/2}} + \frac{M_2}{[r^2+(2d-z)^2]^{1/2}} + \frac{M_4}{[r^2+(4d-z)^2]^{1/2}} + \dots$$

where $r^2 = x^2 + y^2$ and S_0, M_2, M_4, etc., are constants to be evaluated later.

Similarly, V_2 is the sum of the potentials due to S_0 and the images N in medium 0. V_1, however, is due to the source S_0 and to the images M and N in medium II and medium 0. Thus, the potentials in the three media are given by the equations:

$$V_0 = \frac{{}_0S_0}{[r^2+z^2]^{1/2}} + \frac{{}_0M_2}{[r^2+(2d-z)^2]^{1/2}} + \frac{{}_0M_4}{[r^2+(4d-z)^2]^{1/2}}$$

$$+ \dots = \frac{{}_0S_0}{[r^2+z^2]^{1/2}} + \sum_{k=1}^{\infty} [r^2+(2kd-z)^2]^{1/2}$$

$$\overline{]^{1/2}} + \frac{{}_2N_2}{[r^2+(2d+z)^2]^{1/2}} +$$

$$\dots \quad [r^2+z^2]^{1/2} + \sum_{k=1} [r^2+(2kd+z)^2]^{1/2}$$

$$V_1 = \frac{{}_1S_0}{\phantom{[r^2+z^2]^{1/2}}} + \frac{{}_1M_2}{\phantom{[r^2+(2d-z)^2]^{1/2}}} + \frac{{}_1M_4}{\phantom{[r^2+(4d-z)^2]^{1/2}}} + \dots$$

$$[r^2 + \qquad\qquad + \frac{{}_1N_4}{[r^2+(4d+z)^2]^{1/2}} + \dots$$

$$ {}_1S_0 \quad \overline{]^{1/2}} + \sum_{k=1}^{\infty} \frac{{}_1M_{2k}}{[r^2+(2kd-z)^2]^{1/2}} + \sum_{k=1}^{\infty}$$

All of these potentials satisfy Laplace's equation (4). If it can be proved that they also satisfy the boundary conditions, they will constitute the exact and unique solution of the problem. The boundary conditions to be satisfied at the boundary of medium 0 and medium I are:

$$V_0 = V_1$$

and

$$\frac{1}{\rho_0}\frac{\partial V_0}{\partial z} = \frac{1}{\rho_1}\frac{\partial V_1}{\partial z} \text{ at } z = 0,\ r \text{ arbitrary}$$

Likewise at the boundary of medium I and medium II,

$$V_1 = V_2$$

and

$$\frac{1}{\rho_1}\frac{\partial V_1}{\partial z} = \frac{1}{\rho_2}\frac{\partial V_2}{\partial z} \text{ at } z = d,\ r \text{ arbitrary}$$

Each of these four conditions results in a number of equations involving the unknown parameters S_0, M_k, and N_k. The first condition at $z=0$, namely $V_0 = V_1$, requires that

$$\frac{{}_0S_0}{r} + \sum_{k=1}^{\infty} \frac{{}_0M_{2k}}{[r^2+(2kd)^2]^{1/2}} = \frac{{}_1S_0}{r} + \sum_{k=1}^{\infty} \frac{{}_1M_{2k}}{[r^2+(2kd)^2]^{1/2}} + \sum_{k=1}^{\infty} \frac{{}_1N_{2k}}{[r^2+(2kd)^2]^{1/2}}$$

For this equality to hold for all values of r the corresponding terms of the summation, i.e., those having the same denominators, must be equal. This gives

$${}_0S_0 = {}_1S_0$$

$${}_0M_{2k} = {}_1M_{2k} + {}_1N_{2k} \text{ for } k = 1, 2, 3 \cdots$$

The second condition, $\dfrac{1}{\rho_0}\dfrac{\partial V_0}{\partial z} = \dfrac{1}{\rho_1}\dfrac{\partial V_1}{\partial z}$, requires that

$$\frac{1}{\rho_0}\left\{-\frac{{}_0S_0\, z}{[r^2+z^2]^{3/2}} + \sum_{k=1}^{\infty} \frac{{}_0M_{2k}\,(2kd-z)}{[r^2+(2kd-z)^2]^{3/2}}\right\}_{z=0} = \frac{1}{\rho_1}\left\{-\frac{{}_1S_0\, z}{[r^2+z^2]^{3/2}}\right.$$

$$\left. + \sum_{k=1}^{\infty} \frac{{}_1M_{2k}\,(2kd-z)}{[r^2+(2kd-z)^2]^{3/2}} - \sum_{k=1}^{\infty} \frac{{}_1N_{2k}\,(2kd+z)}{[r^2+(2kd+z)^2]^{3/2}}\right\}_{z=0}$$

or

$$\frac{1}{\rho_0}\sum_{k=1}^{\infty} \frac{{}_0M_{2k}\,(2kd)}{[r^2+(2kd)^2]^{3/2}} = \frac{1}{\rho_1}\left\{\sum_{k=1}^{\infty} \frac{{}_1M_{2k}\,(2kd)}{[r^2+(2kd)^2]^{3/2}} - \sum_{k=1}^{\infty} \frac{{}_1N_{2k}\,(2kd)}{[r^2+(2kd)^2]^{1/2}}\right\}$$

The corresponding terms in both members of the equation have the same denominators; hence

$$\frac{1}{\rho_0}\,{}_0M_{2k} = \frac{1}{\rho_1}\,{}_1M_{2k} - \frac{1}{\rho_1}\,{}_1N_{2k}$$

The boundary conditions at $z = d$ yield another set of equations between the constants. The condition $V_1 = V_2$ at $z = d$ requires that

$$\frac{{}_1S_0}{[r^2+d^2]^{1/2}} + \sum_{k=1}^{\infty} \frac{{}_1M_{2k}}{[r^2+(2k-1)^2 d^2]^{1/2}} + \sum_{k=1}^{\infty} \frac{{}_1N_{2k}}{[r^2+(2k+1)^2 d^2]^{1/2}}$$

$$= \frac{{}_2S_0}{[r^2+d^2]^{1/2}} + \sum_{k=1}^{\infty} \frac{{}_2N_{2k}}{[r^2+(2k+1)^2 d^2]^{1/2}}$$

Equating corresponding terms of the summation yields

$${}_1S_0 + {}_1M_2 = {}_2S_0$$

and

$${}_1M_{2k} + {}_1N_{2(k-1)} = {}_2N_{2(k-1)} \text{ for } k = 2, 3, 4 \cdots .$$

The condition $\dfrac{1}{\rho_1}\dfrac{\partial V_1}{\partial z} = \dfrac{1}{\rho_2}\dfrac{\partial V_2}{\partial z}$ at $z = d$ requires that

$$\frac{1}{\rho_1}\left\{-\frac{{}_1S_0\, z}{[r^2+z^2]^{3/2}} + \sum_{k=1}^{\infty} \frac{{}_1M_{2k}\,(2kd-z)}{[r^2+(2kd-z)^2]^{3/2}} - \sum_{k=1}^{\infty} \frac{{}_1N_{2k}\,(2kd+z)}{[r^2+(2kd+z)^2]^{3/2}}\right\}_{z=d}$$

$$= \frac{1}{\rho_2}\left\{-\frac{{}_2S_0\, z}{[r^2+z^2]^{3/2}} - \sum_{k=1}^{\infty} \frac{{}_2N_{2k}\,(2kd+z)}{[r^2+(2kd+z)^2]^{3/2}}\right\}_{z=d}$$

or

$$\frac{1}{\ }\int \frac{{}_1S_0\, d}{\ } - \sum^{\infty} \frac{{}_1M_{2k}\,(2k-1)\, d}{\ } \quad \sum_{k=1}^{\infty}$$

Equating corresponding terms of the summation yields

$$\frac{{}_1S_0}{\rho_1} - \frac{{}_1M_2}{\rho_1} = \frac{{}_2S_0}{\rho_2}$$

and

$$\frac{1}{\rho_1}\{-(2k- \qquad (2k- \qquad \frac{1}{\rho_2}\,(2k-1)\, d\, {}_2N_{2(k-1}$$

$$\rho_1 \qquad \rho_1 \qquad \rho_2$$

Thus, the boundary conditions have yielded the following relations between the constants S_0, M_k and N_k:

(a) ${}_0S_0 = {}_1S_0$

(b) ${}_0M_{2k} = {}_1M_{2k} + {}_1N_{2k}$ for $k = 1, 2, 3 \cdots$

(c) $\dfrac{{}_0M_{2k}}{\rho_0} = \dfrac{{}_1M_{2k}}{\rho_1} - \dfrac{{}_1N_{2k}}{\rho_1}$ for $k = 1, 2, 3 \cdots$

(d) ${}_1S_0 + {}_1M_2 = {}_2S_0$

(e) ${}_1M_{2k} + {}_1N_{2(k-1)} = {}_2N_{2(k-1)}$ for $k = 2, 3, 4 \cdots$

(f) $\dfrac{{}_1S_0}{\rho_1} - \dfrac{{}_1M_2}{\rho_1} = \dfrac{{}_2S_0}{\rho_2}$

(g) $-\dfrac{{}_1M_{2k}}{\rho_1} + \dfrac{{}_1N_{2(k-1)}}{\rho_1} = \dfrac{{}_2N_{2(k-1)}}{\rho_2}$ for $k = 2, 3, 4 \cdots$

On combining Equations (b) and (c), we obtain

$${}_0M_{2k} = \left(\frac{2\rho_0}{\rho_0 + \rho_1}\right) {}_1M_{2k} \quad \text{and} \quad {}_1N_{2k} = \left(\frac{\rho_0 - \rho_1}{\rho_0 + \rho_1}\right) {}_1M_{2k}$$

From Equations (e) and (g), we obtain

$${}_2N_{2(k-1)} = \left(\frac{2\rho_2}{\rho_1 + \rho_2}\right) {}_1N_{2(k-1)} \quad \text{and} \quad {}_1M_{2k} = \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) {}_1N$$

From Equations (d) and (f), we obtain

$${}_2S_0 = \left(\frac{2\rho_2}{\rho_2 + \rho_1}\right) {}_1S_0 \quad \text{and} \quad {}_1M_2 = \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) {}_1S_0$$

Let

$$Q_1 = \frac{\rho_0 - \rho_1}{\rho_0 + \rho_1} \quad \text{and} \quad Q_2 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

Then

$$\frac{2\rho_0}{\rho_0 + \rho_1} = 1 + Q_1$$

and

$${}_0M_{2k} = (1 + Q_1)\, {}_1M_{2k} \text{ for } k = 1, 2, 3 \cdots$$
$${}_1N_{2k} = Q_1\, {}_1M_{2k} \text{ for } k = 1, 2, 3 \cdots$$
$${}_2N_{2(k-1)} = (1 + Q_2)\, {}_1N_{2(k-1)} \text{ for } k = 2, 3, 4$$
$${}_1M_{2k} = Q_2\, {}_1N_{2(k-1)} \text{ for } k = 2, 3, 4 \cdots$$
$${}_1M_2 = Q_2\, {}_1S_0$$
$${}_2S_0 = (1 + Q_2)\, {}_1S_0$$

Also,

$${}_1S_0 = {}_0S_0$$

These equations may be expressed in terms of one parameter S_0 as follows:

$${}_1N_{2k} = Q_1\, {}_1M_{2k} = Q_1 Q_2\, {}_1N_{2(k-1)} = Q_1^2 Q_2\, {}_1M_{2(k-1)} = (Q_1 Q_2)^2\, {}_1N_{2(k-2)}$$

or

In particular, for $k-n=1$ or $n=k-1$

$$_1N_{2k} = (Q_1 Q_2)^{k-1}\,{}_1N_2 = Q_1 (Q_1 Q_2)^{k-1}\,{}_1M_2 = (Q_1 Q_2)^k\, S_0 = (Q_1 Q_2)^k$$

Also,

$$\frac{_1N_{2k}}{Q_1} = \frac{(Q_1 Q_2}{Q_1}$$

$$_0M_{2k} = (1+Q_1)\,{}_1M_{2k} = (1+Q_1)\,(Q_1^{k-1} Q_2^{k})\,{}_0S_0$$

It remains now to find ${}_0S_0$ itself. Consider a very small sphere surrounding the real source of current S_0 or ${}_0S_0$. The total current is the sum of two currents: namely, I_0 which flows into medium 0 through a hemisphere and I_1 which flows into medium I also through a hemisphere. If the radius r_1 of the sphere is negligible relative to d, all terms except the first may be neglected in the expressions for V_0 and V_1. That is,

$${}_0S_0 \qquad {}_0S_0$$

and

$$V_1 = \frac{_1S_0}{[r^2+z^2]^{1/2}} = \frac{_0S_0}{r_1}$$

where r_1 = radius of sphere = $(r^2+z^2)^{1/2}$

The current flowing into medium 0 is

$$I_0 = -\frac{1}{\rho_0}\frac{\partial V_0}{\partial r_1}\cdot 2\pi r_1^2 =$$

while the current flowing into medium I is

$$I_1 = -\frac{1}{\rho_1}\frac{\partial V_1}{\partial r_1}\cdot 2\pi r_1^2 = 2\pi\frac{_0S_0}{\rho_1}$$

The total current is thus

$$\rho_1$$

so that

$${}_0S_0 = \frac{I}{2}$$

or

All the potentials can now be written in terms of I, Q_1, and Q_2. We shall only make use of the expression for V_0. Evidently,

$$V_0 = \frac{_0S_0}{[r^2+z^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{_0M_{2k}}{[r^2+(2kd-z)^2]^{1/2}}$$

$$= \frac{_0S_0}{[r^2+z^2]^{1/2}} + \sum^{\infty}\frac{(1+Q_1)\,Q_1^{k-1}Q_2^{k}\,{}_0S_0}{[r^2+(2kd-z)^2]^{1/2}}$$

In particular, at the boundary between layers 0 and I ($z=0$)

$$\sum_{k=1}^{\infty}\frac{(1+Q_1)\,Q_1^{k-1}Q_2^{k}}{[r^2+(2kd)^2]^{1/2}}\Bigg] \qquad (13)$$

Equation 13 is the general solution for the potential produced by the current source S_0 at the boundary separating layers 0 and I.

When medium 0 is air, $\rho_0 = \infty$. Also,

$$\frac{1-\frac{\rho_1}{\rho_0}}{1+\frac{\rho_1}{\rho_0}} = \frac{1-\frac{\rho_1}{\infty}}{1+\frac{\rho_1}{\infty}} = 1$$

Hence, V_0 can be written as follows:

$$_0 = \frac{I}{2} \rho_1 \Big| \quad \frac{1}{\cdot} \; +2$$

or

$$\cdot = \frac{I}{-} \frac{\rho_1}{-} \quad 1+2 \tag{14}$$

Equation 14 expresses the potential at any point on the surface of the ground due to a current source S_0.*

If a sink of strength $-S_0$ is now added at a distance $x = L$ where x is measured along the line passing through the source and sink (Figure 155), the potential V_0' produced by the sink at any point on the x axis is:

$$2\pi \qquad 1 \qquad +2 \sum_{=1}^{\infty} \tag{15}$$

The total potential produced by the source and sink is obtained by adding V_0 and V_0'.

* For calculations involving this formula it is convenient to use the tables given by Irwin Roman in an article entitled "How to Compute Tables for Determining Electrical Resistivity of Underlying Beds and Their Application to Geophysical Problems." (Technical Paper 502 U. S. Department of Commerce.)

In the Roman tables, values of $W = \qquad \sum_{k=1}$ are given for the arguments Q and a.

The use of these tables is facilitated by noting that

$$\sum Q^k \left\{ \frac{1}{k} - \frac{1}{\sqrt{k^2+a^2}} \right\}$$

$$\sum \frac{Q^k}{a} \left\{ - \frac{}{\sqrt{1+\left(\frac{k}{a}\right)^2}} \right. \qquad \cdot - W = -\log(1-Q^k) - W$$

Whence

$$\sum_{k=1}^{\infty} \frac{Q^k}{\sqrt{1+\frac{k^2}{a^2}}} = -a\log(1-Q^k) - a\,|$$

The left-hand member of this equation is identical with the expression

$$\infty \qquad \frac{Q_2^k}{}$$

appearing in Equation 14 provided one sets $a = \frac{r}{2d}$

For most calculations only the difference in potential between two points is used and the logarithmic term will enter the equations as an additive constant independent of r. Hence, the logarithmic term can always be dropped, because the potential is not unique to an additive constant.

The potential V at any point on the x axis is

$$\frac{1}{x} \quad 1 \quad +2\sum_{k=1}^{\infty}\frac{Q_2^k}{[x^2+4k^2d^2]^{1/2}} - 2 \quad {}_{k=1}$$

where I = current leaving the source at S_0 (Figure 155).
ρ_1 = resistivity of overburden.
ρ_2 = resistivity of subsurface layer.

ρ
x = distance between S_0 and the point on the surface at which the potential is V.
L = separation of energizing electrodes (S_0 and $-S_0$).
d = thickness of overburden.

Application to Wenner Electrode Configuration.—The Wenner configuration consists of four electrodes mounted in a straight line along the surface of the ground at distances 0, a, $2a$, and $3a$ respectively. If the power is applied between the outer two electrodes, the expression for the potential produced at either of the two inner electrodes may be obtained from Equation 16 by replacing L by $3a$. That is,

$$2\pi \Big(x \quad \frac{1}{3a-x} \quad \sum_{k=1} [x^2+4k^2d^2]^{1/2} \quad \sum_{k=1} [(3a-x)$$

The potential difference between the two inner potential electrodes is found by substituting $x=a$ and then $x=2a$ into the above formula and forming the difference of the two resulting equations. That is,

$$= V_a - V_{2a} = \frac{I\rho_1}{2\pi}\Big\{\frac{1}{a}-\frac{1}{2a} \quad \sum_{k=1} \quad -2\sum_{k=1}\frac{Q_2^k}{[4a^2+4k^2d^2]^{1/2}}$$

$$-\frac{1}{2a}+\frac{1}{a}-2\sum_{k=1}^{\infty}\frac{Q_2^k}{[4a^2+4k^2d^2]^{1/2}}+2\sum_{k=1}^{\infty}\frac{Q_2^k}{[a^2+4k^2d^2]^{1/2}}\Big\}$$

or

$$=\frac{I\rho_1}{2\pi}\Big\{\frac{1}{a}+4\sum^{\infty}\frac{Q_2^k}{[a^2+4k^2d^2]^{1/2}}-2\sum^{\infty}\frac{Q_2^k}{[a^2+k^2d^2]^{1/2}}\Big\} \qquad (17)$$

Equation 17 is the generalized formula which applies when the geological structure consists of a semi-infinite medium of resistivity ρ_2 overlain by a homogeneous overburden of resistivity ρ_1.

In particular, if $\rho_1=\rho_2=\rho_a$, $Q_2=0$, and

$$E=$$

or

$$\rho_a = 2\pi \quad E \qquad (18)$$

Equation 18 is known as Wenner's formula.* Evidently this formula gives an average resistivity ρ_a for the area under investigation in terms of measurable quantities (E/I and a). The average or apparent resistivity ρ_a is a mean or weighted value of the resistivity which is equal to the actual resistivity only if the ground is isotropic and homogeneous.

In interpreting experimental data obtained with a Wenner configuration of electrodes, the conventional procedure is to plot the average resistivity calculated from Equation 18 as a function of the electrode separation.** For an ideal two-layer case, the curve has the equation

$$\rho_a = 2\pi a \frac{E}{I} = a\rho_1 \left\{ \frac{1}{a} + 4 \sum^{\infty} \frac{Q_2^k}{[a^2 + 4k^2d^2]^{1/2}} - 2 \sum \frac{}{[a^{\cdot} \quad \cdot k^2d^2]} \right.$$

or

$$\frac{\rho_a}{\rho_1} = \left\{ 1 + 4 \sum \right.$$

The quantities ρ_a and ρ_1 can be obtained from measurements taken at the surface, since ρ_a is equal to ρ_1 for $a << d$ (electrode separation small relative to the thickness of the overburden).

Tagg's Graphical Method.—Figure 156 represents a two-layer structure comprising a bed of resistivity ρ_1 and thickness d overlying a medium of resistivity ρ_2 and infinite extent.† A Wenner electrode configuration for measuring the apparent resistivity is shown schematically in the upper portion of the figure.

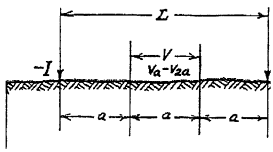

FIG. 156.—Wenner configuration applied to a two-layer structure.

The depth d to the lower formation is obtained by making use of Equation 19 and the relation $Q_2 = \frac{}{\rho_2 + \rho_1} - \frac{1 -}{1 + \rho_1/}$. It is clear that the two "unknowns" in Equation 19 are Q_2 and d/a. Also, it follows from the relation $Q_2 = \frac{1 - \rho_1/\rho_2}{1 + \rho_1/\rho_2}$ that Q_2 depends only on the ratio ρ_1/ρ_2. Also, Q_2 can only take on values between $+1$ and -1.

* Equation 18 may also be obtained by computing the potential difference between the two potential electrodes from Equation 8.

** In investigations not employing the Wenner configuration, Equation 18 does not obtain. However, it is always possible to plot the average resistivity as a function of the electrode separation by deriving an equation which expresses the apparent resistivity ρ_a as a function of measurable quantities (E/I and electrode separations). (Compare p. 326.)

† G. F. Tagg, "Interpretation of Resistivity Measurements," *A.I.M.E. Geophysical Prospecting*, 1934, pp. 135-145.

The computed values of ρ_1/ρ_2 corresponding to various assumed values of are given in Table 11.

TABLE 11.

COMPUTED VALUES OF ρ_1/ρ_2 CORRESPONDING TO VARIOUS ASSUMED VALUES OF Q_2.†

Q_2	ρ_1/ρ_2	Q_2	ρ_1/ρ_2
1.0	1/∞	− 1.0	∞
0.9	1/19	− 0.9	19
0.8	1/9	− 0.8	9
0.7	1/5.67	− 0.7	5.67
0.6	1/4	− 0.6	4
0.5	1/3	− 0.5	3
0.4	1/2.33	− 0.4	2.33
0.3	1/1.85	− 0.3	1.85
0.2	1/1.5	− 0.2	1.5
0.1	1/1.33	− 0.1	1.33
0.0	1/1	0.0	1

It is possible to calculate the ratio of the average resistivity to the resistivity in the upper stratum, ρ_a/ρ_1, by *assuming* various values for Q_2 and the ratio d/a. (Compare Equation 19.) This is done most conveniently by a graphical method. When Q_2 is positive, i.e., when ρ_2 is greater than ρ_1, it is convenient to use the ratio of the conductivities σ_a/σ_1, instead of ρ_a/ρ_1. (σ_a is the apparent conductivity and is equal to $1/\rho_a$, while σ_1 is the conductivity of the upper stratum and is equal to $1/\rho_1$.)

Families of curves showing the relationship between σ_a/σ_1 and d/a when Q_2 is positive and the relationship between ρ_a/ρ_1 and d/a when Q_2 is negative, for a given set of conditions, are shown in Figure 157.

The use of these curves for determining the depth to a horizontal stratum will be illustrated for data taken from an experimental survey conducted by Tagg ‡ in Gloucestershire, England. The surface material is limestone overlain by about six inches of loam; the depth of the limestone varies from 50 to 266 feet, and underneath the limestone is either sand or clay. The sites chosen were practically level. The resistivity ρ_1 of the surface limestone was obtained from measurements at electrode intervals to 70 feet. The averaged value was 6703 ohm-inches. The apparent resistivity ρ_a is shown plotted as a function of the electrode separation in Figure 158.

† Tagg, *A.I.M.E. Geophysical Prospecting*, 1934.
‡ *Loc. cit.*

A series of values of the apparent resistivity corresponding to several values of the electrode separation is read off the experimental curve. The apparent resistivities are then divided by the surface resistivity ρ_1 to determine the ratios ρ_a/ρ_1. Because the values of the ratio ρ_a/ρ_1 are greater than unity, the reciprocal ratios σ_a/σ_1 are also formed. (Table 12.)

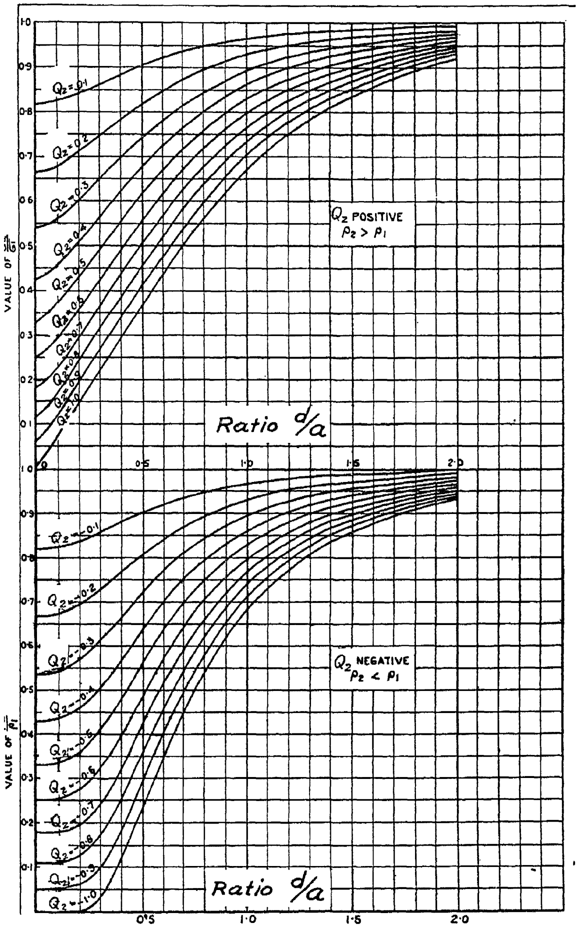

FIG. 157.—Computed curves showing the relationship between σ_a/σ_1, ρ_a/ρ_1, and d/a for various values of Q_2. (Tagg, *A.I.M.E. Geophysical Prospecting*, 1934.)

The values of d/a and Q_2 corresponding to each of the values of the ratio σ_a/σ_1 given in Table 12 are now read off Figure 157. The values thus obtained are shown in Table 13.

The next step in determining the depth consists in plotting a series of curves of Q_2 as a function of d, one curve for each value of σ_a/σ_1, —i.e.,

one curve for each electrode interval. These are shown in Figure 159. Th six curves intersect within the dotted circle, and the center has the coordi

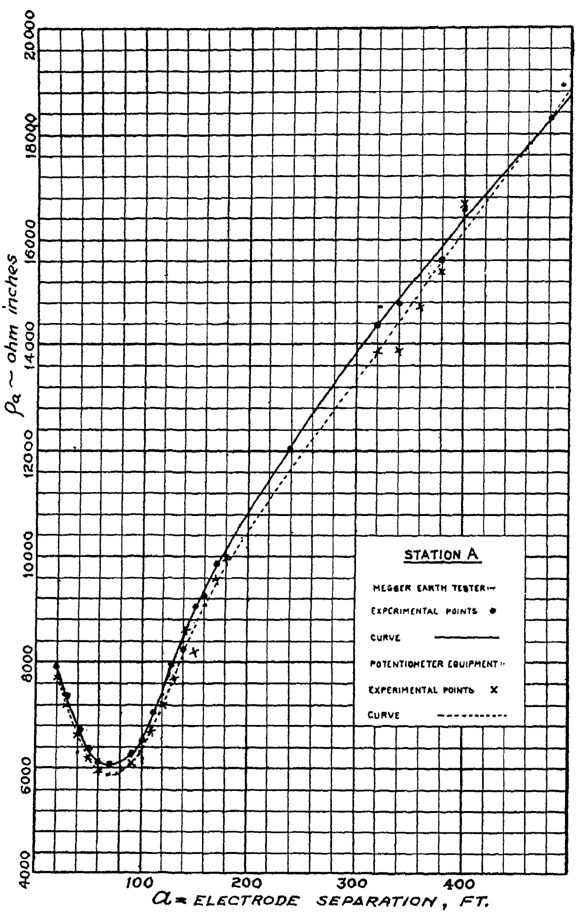

FIG. 158.—Experimental values of the apparent resistivity as a function of electrode separation. Solid curve is experimental curve obtained with Megger Earth Tester and broken curve is experimental curve obtained with potentiometer equipment. (Tagg, *A.I.M.E. Geophysical Prospecting*, 1934.)

nates $d = 142$ and $Q_2 = 0.0702$; that is, the intersection of the curves gives 142 feet as the value of the depth and 0.0702 as the value of the reflection factor Q_2.

TABLE 12

RATIO OF THE RESISTIVITIES AT VARIOUS ELECTRODE SEPARATIONS†

Electrode Separation, Feet	Apparent Resistivity, Ohm-inches	Ratio, ρ_a/ρ_1	Ratio, σ_a/σ_1
150	8,960	1.338	0.748
200	10,740	1.601	0.625
250	12,320	1.840	0.544
300	13,860	2.068	0.483
350	15,220	2.270	0.441
400	16,480	2.460	0.407

† Tagg, *loc. cit.*

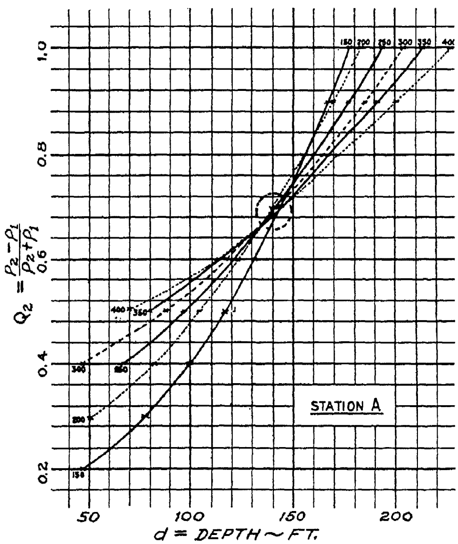

FIG. 159.—Curves showing the reflection factor Q_2 vs. the depth d for various values of σ_a/σ_1. (Tagg, *A.I.M.E. Geophysical Prospecting*, 1934.)

TABLE 13

VALUES OF Q_2 AT VARIOUS ELECTRODE SEPARATIONS†

Value of Q_2	150 Ft.		200 Ft.		250 Ft.		300 Ft.		350 Ft.		400 Ft.	
	$\sigma_a/\sigma_1 = 0.748$		$\sigma_a/\sigma_1 = 0.625$		$\sigma_a/\sigma_1 = 0.544$		$\sigma_a/\sigma_1 = 0.483$		$\sigma_a/\sigma_1 = 0.441$		$\sigma_a/\sigma_1 = 0.407$	
	d/a	d°	d/a	d	d/a	d	d/a	d	d/a	d	d/a	d
1.0	1.19	179	0.915	183	0.770	193	0.675	202	0.61	214	0.560	224
0.9	1.12	168	0.850	170	0.705	176	0.610	183	0.545	191	0.500	200
0.8	1.045	157	0.775	155	0.640	160	0.545	163	0.485	170	0.435	174
0.7	0.96	144	0.700	140	0.565	141	0.478	143	0.41	144	0.36	144
0.6	0.87	130.5	0.620	124	0.485	121	0.390	117	0.325	114	0.28	112
0.5	0.785	118	0.525	105	0.39	97.5	0.295	74	0.226	78	0.17	68
0.4	0.66	99	0.42	84	0.27	67.5	0.16	40	0.06	21		
0.3	0.525	79	0.26	52	0.03	7.5						
0.2	0.315	47										
0.1												

° d is obtained by multiplying d/a by a.

† Tagg, *A.I.M.E. Geophysical Prospecting*, 1934.

Also, it is of interest to point out that when Q_2 and ρ_1 are known, the resistivity ρ_2 of the lower formation can be determined from the relation

or

$$\rho_2 = \quad 1 - Q_2$$

Substitution of the values given above for ρ_1 and Q_2 into the last relation yields

$$\rho_2 = \frac{6703\ (1.0702)}{0.298} = 38300 \text{ ohm-inches.}$$

Resume.—The preceding sections have been concerned chiefly with the mathematical relationships governing the current flow in the subsurface. Generally, however, a rigid mathematical treatment of resistivity data obtained from measurements made at the surface of the earth is not possible because many variable factors affect the apparent resistivity.

The measured values are influenced chiefly by two mutually dependent phenomena: first, the actual path of the current flow, and second, the surface potentials at the points where the potential electrodes make contact with the surface of the earth. The current path is dependent chiefly upon the distance between the current electrodes and the relative conductivities of the strata constituting the subsurface. Although small portions of the current spread out to great distances in all directions in the earth, a considerable portion is confined to the subsurface volume included between the two current electrodes. Furthermore, only a relatively small amount of the current flows along the uppermost surface of the ground, and the potentials measured in the resistivity methods are only those created by this current flow at the surface.

The relative magnitudes of the effects produced by these factors are discussed below.

MEASUREMENTS OF NEAR-SURFACE INHOMOGENEITIES

In any geophysical method it is necessary that the anomalies in the measured physical property be of sufficient magnitude to manifest themselves in an unequivocal manner, despite any irrelevant variables which may be included in the measurements. In measurements of potentials, the effect of a disturbing factor increases rapidly with the depth of measurement. Very small lateral variations in the conductivity of the ground near the surface will produce effects comparable to those produced by structural variations of large magnitude at some depth below the surface. The lateral

variations may be demonstrated by maintaining the current electrodes in a fixed position and measuring the E/I ratios as the potential electrodes are moved to different locations. Frequently the individual readings deviate fifty per cent or more from the computed theoretical values. Structural changes, however, seldom cause a variation in the computed resistivity value of more than a few per cent, except in unusual cases. It is evident, therefore, that the near-surface variations tend to mask the absolute resistivity variations that are associated with changes in the path of current flow caused by subsurface structural changes.

The greatest lateral variations occur in the aerated layer and are associated primarily with ground water movements. Local zones of higher resistivity are usually associated with descending fresh water movement. The more highly conductive areas are usually found in regions where ascending solutions and concentration of mineralization occur—due to surface evaporation—and in regions where local moisture content is increased by surface or near-surface impoundment of rain or drainage water. These conditions often manifest themselves visibly by alkali beds and changes in vegetation. Frequently, however, visual inspection fails to account for certain shallow anomalies, and therefore it is necessary to conduct the field work in a manner which will permit differentiating between the near-surface and the "deep" variations.

Lateral Investigations.—Lateral investigations are usually conducted to determine lateral variations associated with fault zones, contacts of different formations, variations in depth to "marker" beds, etc. Lateral resistivity measurements generally are carried out by moving the electrode configuration as a unit along a traverse line that crosses the area. The variations in resistivity are plotted against the traverse distance.*

The manner in which near-surface inhomogeneities affect the apparent resistivity values as the electrodes move over the area may be illustrated by referring to Figure 160. Positions of the electrodes are shown with reference to a zone of relatively high conductivity. The energizing or power electrodes are designated by 1 and 4. The electrode configuration shown is the conventional Wenner arrangement, wherein the separation between adjacent electrodes is equal. The lines of current (not shown in the sketches) are concentrated in the zone of high conductivity and the equipotentials are distorted so as to be displaced away from this zone.

Referring to diagram *A*, as the electrode configuration approaches the zone of high conductivity, the distortion of the equipotentials away from the conductor decreases the potential difference between 2 and 3. Hence, because the apparent resistivity is proportional to $(V_2 - V_3)/I$, the

* Measurements of this type are often termed "constant depth" traverses.

apparent resistivity is diminished with respect to the value it would have in the absence of the conductor. (Compare the resistivity-distance curve in the lower portion of the figure.) For the electrode positions shown in sketch *B*, the potential drop between 1 and 2 is very small due to the short circuiting effect of the conductor. Hence, the potential between 2

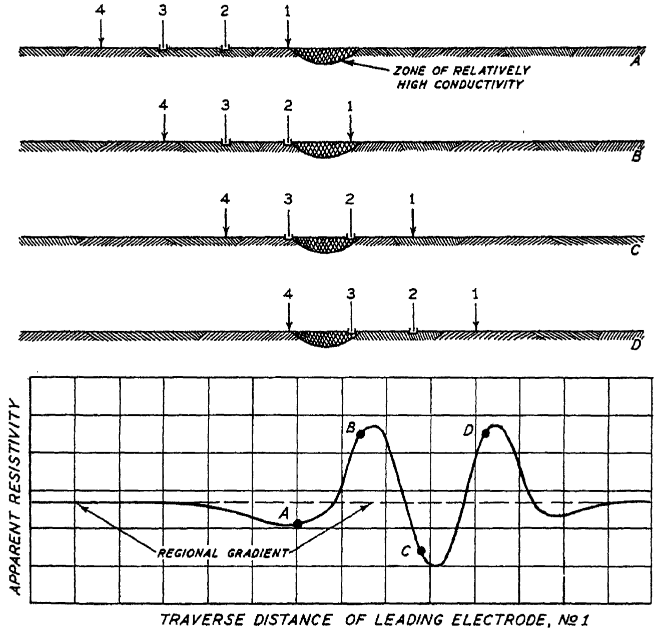

FIG. 160.—Lateral investigations and near-surface effects.

and 3 is large and the apparent resistivity has a maximum value. At the electrode positions shown in *C*, the potential difference is measured across the conductor and hence is very small. Thus for this position of the potential electrodes, the apparent resistivity is a minimum. In diagram *D*, the relative positions of the potential electrodes are the same as in *B*. Hence, the apparent resistivity again reaches a maximum. As the configuration continues its movement, the curve at first decreases slightly and then coincides with the normal regional resistivity profile.* This type of anomaly is typical for the movement of such an electrode configuration across a zone of relatively high conductivity. It is the usual anomaly encountered in conjunction with fault zones and other conductive zones of relatively nar-

* The *normal regional resistivity* profile or the *regional resistivity* profile is a smoothed-out plot of resistivity versus distance for a given region. The best fitting line through the regional resistivity profile is sometimes called the *regional gradient.*

row width. By like reasoning it can be seen that a zone of lower conductivity would give an inverted type of curve having the same general characteristics.

An unsymmetrical electrode configuration which is suitable for lateral investigations is illustrated in Figure 161. In this configuration, the potential electrodes 2 and 3 are closer to power electrode 1 than to power electrode 4. The distance between electrodes 3 and 1 is about one-third that between 3 and 4. This configuration has the advantage that the surface effect at one of the energizing electrodes is minimized. Thus, the predominant resistivity variation occurs adjacent the energizing electrode which is near the potential electrodes.

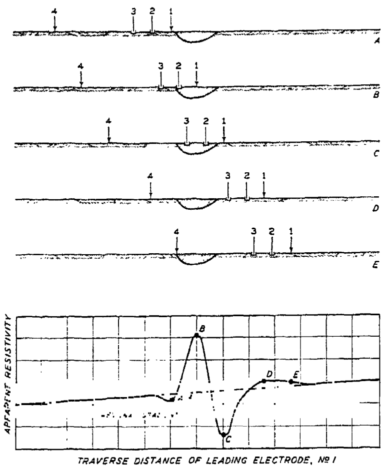

FIG. 161. — Unsymmetrical electrode configuration for lateral exploration.

Movement of this configuration across the zone of better conductivity will give the type of anomaly indicated in the lower part of the figure. The greatest deflection occurs when the electrode 1 crosses the zone of better conductivity. A much smaller anomaly is produced when electrode 4 crosses the zone.

It will be shown later that if electrodes 1, 2, and 3 are maintained stationary and if the depth of measurement is varied by moving only electrode 4, a vertical depth exploration may be made with a minimum surface effect. Hence, this type of unsymmetrical electrode configuration furnishes the basis for a surface correction method useful in vertical exploration.

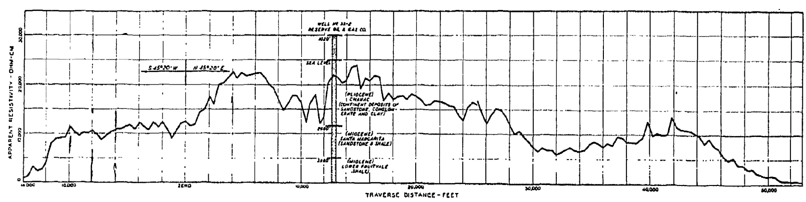

FIG. 162.—Resistivity profile in Tejon Ranch Area, San Joaquin Valley, California.

Figure 162 shows a regional resistivity profile plotted from data obtained with the unsymmetrical electrode configuration shown in Figure 161. The separation of the energizing electrodes was 2400 feet and the

entire configuration was moved as a unit, with readings made each 400 feet along the traverse.

Vertical Investigations.—Vertical depth investigations are oftentimes conducted by moving the electrodes symmetrically outward from a central point along a straight line. Figure 163 illustrates the anomaly usually obtained when the zone of better conductivity is located asymmetrically with reference to such electrode movement. The center of the electrode configuration, usually designated as the station, is located as illustrated in the figure, and electrodes 1 and 2 move outwardly in one direction from this central point of reference while electrodes 3 and 4 move outwardly in equal increments of distance in the opposite direction. If the subsurface is homogeneous, with the exception of the zone of high conductivity, movement of electrodes 3 and 4 outwardly will not produce any appreciable change in the apparent resistivity. Therefore, the present analysis may be confined to the movement of electrodes 1 and 2. As these electrodes move outward, the apparent resistivity at first decreases a small amount, then rises sharply. (Compare the resistivity-distance curve in the lower portion of the figure.) After reaching a maximum value, the resistivity gradually decreases and finally takes on the normal value it would have in the absence of the conductor.

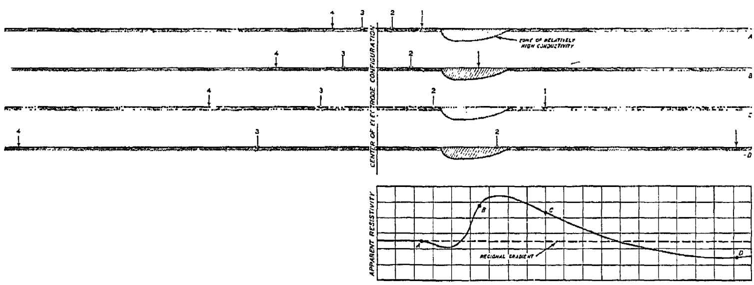

Fig. 163.—Vertical investigations and near-surface effects.

It is clear that near-surface inhomogeneities existing along the traverse to the left of the center of the configuration would yield a similar resistivity-distance curve due to the movement of electrodes 3 and 4. Evidently, it is difficult to determine the near-surface correction from data obtained with this type of electrode movement, because the near-surface disturbances may occur on either or both sides of the center point.

Investigations of Asymmetrical Subsurface Inhomogeneities.—The problem of ascertaining the subsurface structure when the subsurface inhomogeneities are asymmetrical has been attacked by numerous methods.

One of the first methods for detecting anomalies produced by asymmetrical inhomogeneities is due to F. H. Brown.† Brown's apparatus consisted essentially of a differentially wound receiver and three electrodes. The electrodes were mounted in a straight line with the "central" electrode midway between the two exploring electrodes. One winding of the receiver was connected to the central electrode and one of the exploring electrodes, and the other winding was connected to the central electrode and the other exploring electrode. This method operated on the principle of causing current to traverse different depths of the earth forming one circuit, and at the same time causing current to flow through the other earth circuit. The relative strengths of the two currents could be compared audibly by means of the differential coil in the receiver.

Schlumberger ‡ utilized equipotential studies around the power electrode to determine inhomogeneities. A non-symmetrical distribution of subsurface current flow was indicated by a distortion of the equipotential lines.

Lee § has proposed a method wherein the ground is "partitioned," preferably into symmetrical parts, with the station center at the midpoint as in the Brown arrangement. Measurements are made on each side of the midpoint for the purpose of comparing one side with the other. The comparison may be made of the computed resistivity values or any function of these values, such as measured potentials, currents, or resistances. This arrangement measures the resultant effect produced by local zones of anomalous conductivity and dip of structure.

The potential ratio methods generally utilize a central electrode as a reference point. Zuschlag†† illustrates and describes one method for comparing electrical potentials about the midpoint of the electrode configuration. Methods of this type indicate a non-symmetry of position, but they do not isolate the near-surface effect or locate the region of its occurrence along the traverse line of measurement.

Experimental results show that the potential electrodes are subject to a far greater disturbance as a result of near-surface effects than the power, or energizing, electrodes. Hence, the near-surface effects can be minimized by employing an electrode system wherein the two potential electrodes remain in a fixed position, while the two power electrodes move outward. This condition is illustrated in Figure 176 f.

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† F. H. Brown, "Electrical Apparatus for Determining the Location of Metallic Ores," U. S. Patent 817,749, issued April 17, 1906.

‡ C. Schlumberger, "Process for Determining the Nature of the Subsoil by the Aid of Electricity," U. S. Patent 1,163,468, issued Dec. 7, 1915.

§ F. W. Lee, "Method of Conducting Geological Survey," U. S. Patent 1,951,760, issued Mar. 20, 1934.

†† T. Zuschlag, "Electrical Prospecting," U. S. Patent 1,951,387, issued Mar. 20, 1934.

An accentuation of near-surface effects occurs when the electrode movement is such that the power electrodes remain stationary and the potential electrodes move. It is this accentuation of near-surface effects which so greatly handicaps the potential ratio methods and other methods wherein only the potential electrodes are moved.

Double - Depth Investigations. — The differentiation of the near-surface effects from the deeper structural effects can oftentimes be accomplished by comparing measurements at two different depths of penetration.

Double-depth investigations may be divided into two general groups: (a) lateral exploration and (b) vertical exploration. Illustrations of both types will be given to indicate the general principles involved in the field technique. Various other modifications will occur to the reader.

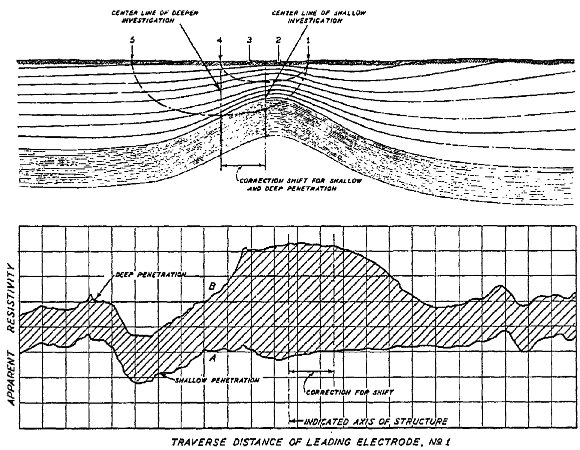

Fig. 164.—Illustration of surface corrections in the lateral exploration method.

Lateral Exploration

The operating principle of the lateral exploration method will be evident from a consideration of Figure 164. An electrode system comprising electrodes 1, 2, 3, and 4 is moved as a unit along a traverse line. The separation between power electrodes 1 and 4 is only sufficient to give an effective penetration which includes the upper near-surface beds. A third energizing electrode 5 trails the electrode system (consisting of 1, 2, 3, and 4) at a constant distance which depends on the desired depth of effective penetration into the deeper lying beds. Two sets of resistivity measurements are made for each position of the electrodes: In one, the

energizing current flows between electrodes 1 and 4; in the other, the energizing current flows between electrodes 1 and 5.

The apparent resistivity data are plotted with the resistivity values as ordinates and the traverse positions of the leading power electrode as abscissas. (Lower portion of Figure 164.) Curve *A* corresponds to the case that the energizing current flows between electrodes 1 and 4, and curve *B* to the case that 1 and 5 are the energizing electrodes. In this particular area, the curves show that the shallow material affecting the measurements at the small separations has a lower resistivity than the deeper lying material which affects the measurements at the large separations.

The curves also exhibit a general similarity, because the potential electrodes occupy the same surface positions in both the shallow and the deep measurements. If the current penetrates the earth uniformly, the effective zone of measurement for the symmetrical shallow electrode configuration will lie approximately midway between electrodes 1 and 4. For the deep investigations, the effective zone of penetration lies approximately midway between electrodes 1 and 5, but due to the unsymmetrical electrode arrangement the zone of effective measurement is shifted toward the potential electrodes. Hence, in comparing the shallow and deep investigations it is necessary to make a shift correction. The amount of this shift will depend on: (a) electrode configuration, (b) the relative conductivities of the deep and shallow zones, and (c) the changes in dip or thickness of section between the two zones.

The data utilized in the interpretation are obtained by making the shift correction and subtracting corresponding ordinates of the two resistivity curves.

Vertical Exploration

The operating principle of the vertical exploration method will be evident from Figure 165. The electrodes are placed along the traverse line with the potential electrodes 2 and 3 positioned at certain fixed distances, usually one and four units, from the stationary energizing electrode. The moving energizing electrode † is started a distance of about four and one-half units from the stationary energizing electrode. The unit length depends upon the desired depth of measurement and the relative conductivities of the strata in the area. The zone of investigation increases in depth as the energizing electrode 4 moves out. (Figure 165A.) If direct current or very low frequency alternating current is employed, the data recorded during this movement of the electrode are the values of the potential, current, and electrode separation.

† J. J. Jakosky, "Method and Apparatus for Electrical Exploration of the Subsurface," U. S. Patent 2,192,404, issued March 5, 1940.

Upon completion of the series of measurements, the entire electrode configuration is moved forward along the traverse line about 1000 feet. Recordings are then made of the potential, current, and electrode spacing as the moving electrode proceeds inwardly. (Figure 165B.) Upon completion of this series of measurements, the entire configuration is again moved another 1000 feet forward along the traverse line, and the entire procedure repeated. The technique of making two series of measurements (outward and inward) at one location constitutes the occupation of one station. As explained later, the stations are spaced to allow an overlap of about three stations in order to facilitate proper evaluation of near-surface effects.†

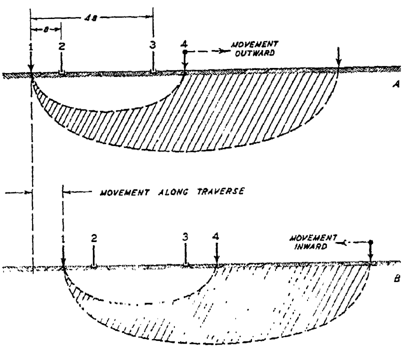

FIG. 165.—Electrode arrangement for vertical exploration.

The potential-current ratios for each station are now reduced by an appropriate formula * and are plotted with resistivity as ordinate and electrode separation as abscissa. The near-surface resistivity anomalies

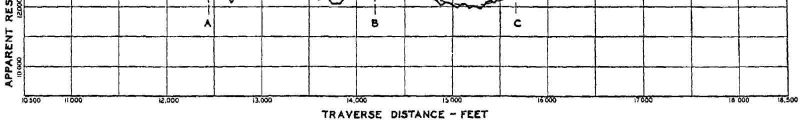

FIG. 166.—Correlation of four overlapping stations on a traverse line.

appear when the moving energizing electrode traverses the inhomogeneities. Hence, the anomalies are present at the same traverse point on all of the overlapping curves,** and the initial step in the interpretation process is to "match" or "correlate" each of the apparent resistivity curves at its respective traverse location.

† J. J. Jakosky, "Method and Apparatus for the Electrical Exploration of the Subsurface." U. S. Patent 2,211,125, Aug. 13, 1940.

* See p. 326.

** The *shapes* of corresponding near-surface anomalies which occur in curves of overlapping stations are generally not identical.

The correlation of four overlapping stations located on a short section of traverse line in the Permian basin near Lovington, New Mexico, is shown in Figure 166.* The irregularities produced in the resistivity profile by the near-surface inhomogeneities are quite pronounced. Prominent anomalies which occur on all of the curves at the same traverse location are marked at *A, B,* and *C.* Many minor anomalies will be seen by more detailed study of the curves.

Generally the shallow near-surface effects are very apparent due to their abruptness and short lateral extent. Oftentimes, however, near-surface effects exist over a considerable distance and are not easily differentiated from the deeper effects.

The dotted curves of Figure 166 are the diagnostic curves utilized in the interpretation. These curves are obtained by "smoothing-out" the observed data so as to minimize the near-surface variations. (Experience is required in drawing the best-fitting "smoothed-out" curve.)

Alternative Configuration.—A useful modification of the electrode system described above is obtained by placing the stationary current electrode midway between the stationary potential electrodes. With this arrangement, the electrical fields produced at the potential electrodes by the fixed current electrode are the same, provided the earth between the two power electrodes is homogeneous. Hence, variations in the measured potential difference depend only on the relative position of the moving power electrode. This arrangement also has a greater sensitivity than the arrangement previously described wherein both power electrodes are located outside the potential electrodes.

Dip Determination at a Single Station.—Determination of the dip at single stations may often be accomplished by comparing the curves obtained from measurements made in three or more directions from the station hub.† The measurements are usually made by a tri-directional system, as illustrated in Figure 167. The three lines of measurement are laid out at 120° angles, with the corresponding points of measurement on each line being equidistant from the station hub. Current is passed into

* In practice the individual station curves are not replotted as was done for this illustration, but are matched by using a light box and superposing the curves at their proper traverse separations.

† J. J. Jakosky, "Methods of Determining Underground Structure," U. S. Patent No. 2,138,818, Dec. 6, 1938.

J. J. Jakosky and C. H. Wilson, "Electrical Mapping of Oil Structures," *Mining and Metallurgy,* May 1936, pp. 231-237.

J. J. Jakosky and C. H. Wilson, "Prospecting for Oil Structure by Electrical Methods," *The Petroleum Engineer,* Feb. 1937, pp. 143-149.

the ground between electrodes I_A and I_B, then between I_A and I_C, and and then between I_B and I_C, the points of measurement moving progressively outward as the depth of penetration is increased. It will be recognized that this procedure is a combination of lateral and vertical investigation.

The subsurface distribution of current can be predicted by one of two types of measurement: (a) potential measurements at a given spacing from the energizing electrodes or (b) electromagnetometer measurements wherein the strength of the magnetic field associated with the subsurface current flow is measured at the hub.

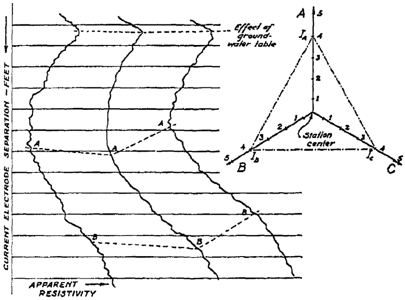

FIG. 167.—Tri-directional system of measurement for determining dip at a single station.

Analysis of the multi-directional measurements is usually done by some form of curve correlation procedure. The curves for each line of measurement are plotted as shown in the figure and are correlated by the method outlined on p. 317.

ANALYSIS OF RESISTIVITY DATA

Various interpretative procedures have been developed for inferring the subsurface structure from resistivity data. The *mathematical methods* have been discussed by Tagg, Roman, † Hummel, ‡ Watson § and others. †† The method of Tagg has already been considered. Apart from the mathematical methods, the interpretative procedures commonly employed may be classified into three groups: (1) rule of thumb or empirical theorems; (2) curve correlation methods which often can be applied successfully in

† Irwin Roman, "How to Compute Tables for Determining Electrical Resistivity of Underlying Beds and their Application to Geophysical Problems," U. S. Bur. of Mines, Tech. Pub. 502; "Some Interpretations of Earth Resistivity Data," *A.I.M.E. Geophysical Prospecting,* 1934, pp. 183-201.

‡ J. N. Hummel, "Theoretical Study of Apparent Resistivity," *A.I.M.E. Geophysical Prospecting,* 1937, Tech. Pub. 496; "A Theoretical Study of Apparent Resistivity in Surface Potential Methods," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 418, 1931.

§ R. J. Watson, "A Contribution to the Theory of the Interpretation of Resistivity Measurements Obtained from Surface Potential Observations," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 518, 1934.

†† D. O. Ehrenburg and R. J. Watson, "Mathematical Theory of Electrical Flow in the Stratified Media," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 400, 1931.

S. Stefanesto, in collaboration with C. and M. Schlumberger, "Sur la Distribution Electrique Potentielle Autour D'Une Prise de Terre Ponctuelle Dans un Terrain A Couches Horizontales, Homogenes et Isotropes," *Le Journal de Physique et le Radium,* Vol. 1, 1930, p. 132.

many cases of complex structure where the more basic mathematical analysis would be either impractical or impossible; (3) small scale experiments.

Rule of Thumb Theorems for Determining Effective Depth of Measurement.—In an attempt to simplify the calculations necessary to interpret field data, many empirical rules have been devised. The literature of electrical prospecting contains numerous illustrations of the application of such rules to actual problems. Many of these rule theorems are valid for the particular problems for which they were derived. However, the blind application of such rules to general exploration problems usually results in errors because the conditions under which the empirical rules were developed may not obtain in the particular area under investigation. Two of the many "short cuts" in interpretation will be discussed here.

Many literature references contain discussions of the "potential bowl" theory.† The apparent validity of this method of interpretation derives from the following considerations. In an isotropic, homogeneous medium, all points equidistant from the current electrode lie on an equipotential surface. Hence, the potential at a point P, at any fixed distance along the surface of the ground from the current source S_1, will define the surface trace of the particular equipotential hemisphere having a radius r_s equal to the distance PS_1. (Figure 168.) It is assumed that any subsurface condition which may alter the potential value at any point a distance r_s from the point S_1 will cause a like change in the surface potential measured between P and S_1.

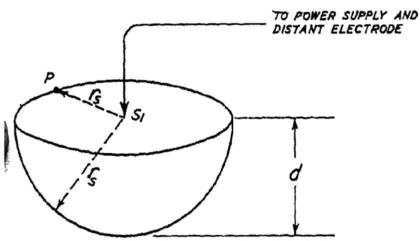

FIG. 168.—Assumed equipotential bowl surrounding electrode S_1.

In other words, the equipotential bowl is assumed to be a rigid, non-distorted surface; and if this were true, the depth of investigation d would be equal to the distance r_s.

Obviously, this equipotential bowl theory must be modified to correct for the distortion of the equipotential surface caused by the effect of the distant power electrode, and also for the distortion of the equipotential surface caused by the subsurface inhomogeneities, including the effect of the distortion due to the layer whose depth is to be determined. In extreme cases (where an excellent conductor exists at depth) the depth of investi-

† See, for example, A. S. Eve and D. A. Keys, *Applied Geophysics* (Cambr. Univ. Press), 1938, pp. 95-97.

gation may be greater than the surface radius of the equipotential bowl, while in other cases it may even be less than $\frac{1}{5}$ of the surface radius. With these limitations it is obvious that the theory must be applied empirically.

Empirical formulas relating the electrode spacing at the surface of the ground with the depth to an underlying stratum have also been derived for the Wenner electrode arrangement. (Figure 169.) Again it is assumed that the equipotential surfaces about the two power electrodes are undistorted hemispheres and that the depth of measurement is equal to the distance *a*; i.e., the effective depth of measurement is assumed to be one-third the separation of the energizing electrodes.

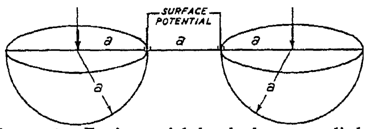

FIG. 169.—Equipotential bowl theory applied to the Wenner configuration.

Generally, the empirical "short cuts" just discussed are in error, because they neglect the ever present distortion of the potential bowls surrounding the source and sink of the current. For instance, in areas where the resistivity increases with depth, the effective path of current flow is always materially less than the $\frac{1}{3}$ value cited for the Wenner configuration.† Usually the depth of measurement varies from about $\frac{1}{4}$ to $\frac{1}{9}$ the distance between the power electrodes. Moreover, the depth of measurement usually is not a constant fraction of the distance between the power electrodes; instead, it depends on such factors as the relative conductivities of the component layers constituting the subsurface and the lateral variations in conductivity. In general, therefore, the depth of penetration is dependent upon many factors, only one of which is the separation and configuration of the electrodes.

Interpretation by Curve Correlation.—This method of interpretation is based upon the fact that in an area of extended flat structures, any given multi-layer structure composed of layers of different conductivity usually produces a characteristic type of resistivity curve.‡ The general shape of this curve is dependent upon the relative thickness, conductivity, and sequence of the layers or components included in the measurements. It has been found that each area usually has its own characteristic type of curve. This curve will differ from that of another area if the subsurface structure differs so as to produce electrical variations when the depth of current penetration changes.

Extensive field work has shown that in favorable cases a characteristic pattern in one portion of the curve may often be followed through a series

† H. M. Evjen, "Depth Factors and Resolving Power of Electrical Measurements," *Geophysics*, 1938, pp. 78-95.

‡ J. J. Jakosky, C. H. Wilson and J. W. Daly, "Geophysical Examination of Meteor Crater, Arizona," *A.I.M.E. Geophysical Prospecting*, 1933, pp. 63-97.

J. J. Jakosky, "Continuous Electrical Profiling," *Geophysics*, Vol. 3, No. 2, Mar. 1938.

of stations even though variations in thickness occur in portions of the geologic section. In one application of the method for subsurface structural mapping, a certain group of markers within a given depth interval is correlated.*

The measurements are made to include a given depth interval by starting with the power electrodes at some fixed distance apart and increasing their separation until the desired depth interval has been measured. The similarity of the pattern at various stations in an area will depend on the lateral uniformity of the subsurface layers. In areas where rapid lateral changes are predominant, such as in the lenticular and overlapped structures which prevail in the San Joaquin valley of

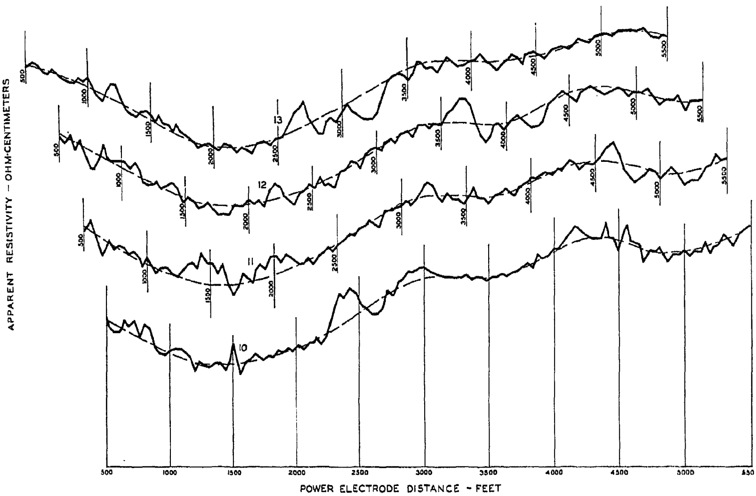

FIG. 170.—Correlation of resistivity curves from four adjacent stations along a traverse line.

California, the curves will vary rapidly in character; under such conditions, it will be found that curve to curve correlation will be difficult even when the stations are placed close together. In areas where sufficient resistivity depth variations occur and where the lithology is fairly uniform, with resultant small variations in the characteristics of the curves, reliable correlations often may be made between stations which are many thousands of feet apart. The latter condition prevails in many parts of the Mid-Continent area and in the Permian basin area of New Mexico and Texas.

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* Correlations may be carried out even when the upper sedimentary beds overlying the group vary in thickness, because the variation usually is not sufficient to mask the characteristics of the curve due to the lower part of the section.

To correlate curves, the field data for each station are plotted with the resistivity values as abscissas and the separations of the current electrodes as ordinates on a separate sheet of transparent cross-section paper. Corrections for surface effects are made as illustrated in Figure 166. The corrected curves corresponding to the various stations are then matched over a light box, i.e., correlated as a unit. The interpreter first correlates the curves for the first two stations at one end of the traverse and then proceeds to successive stations. The correlation of four curves is shown in Figure 170. Before evaluating the subsurface interval between stations, it is necessary (a) to correct for variations in surface topography and elevation, and (b) to determine the effective depth of measurement.

The topography along the traverse is taken by transit and stadia rod, preferably during the initial laying-out of the traverse line, and the elevations of the electrode locations are reduced to a mean datum plane.

The effective depth of measurement varies with the area and electrode configuration used and can seldom be predicted or calculated from known theoretical relationships. Usually it is determined by control data taken from geologic logs of wells in the area. When work is done in areas where proper well control is not available, a penetration factor is assumed, the magnitude of which is based on experience under similar geological and electrical conditions. In these cases, the results should be considered as qualitative until proper control is available.

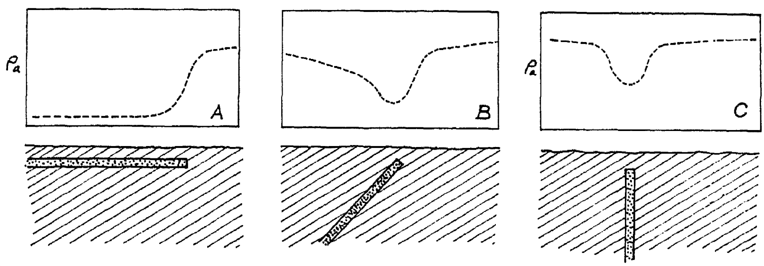

FIG. 171.—Type anomalies produced by model formations of various dips. Resistivity value obtained by moving a fixed Wenner configuration of electrodes normal to the strike.

Small Scale Experiments.—In small scale experiments designed to determine the effects of ore bodies, conductors (model conductors) of various shapes are immersed in tanks partially filled with water of appropriate conductivity. The model ore bodies may be made by covering any convenient material with copper foil. The tank should be relatively large and should have dimensions about ten times the maximum dimension of the model object contained in the tank.

Extensive experimental work has been done with small scale models. † A few general types of anomalies characteristic of model formations are shown in Figures 171 and 172, taken from tests by Clyde H. Wilson.

Representative results of tank tests to determine the zone of influence in a Wenner electrode configuration are shown in Figure 172. In these tests a glass partition, oriented at right angles to the line of electrodes, was moved in a direction along the line of electrodes.* Curve No. 1 was

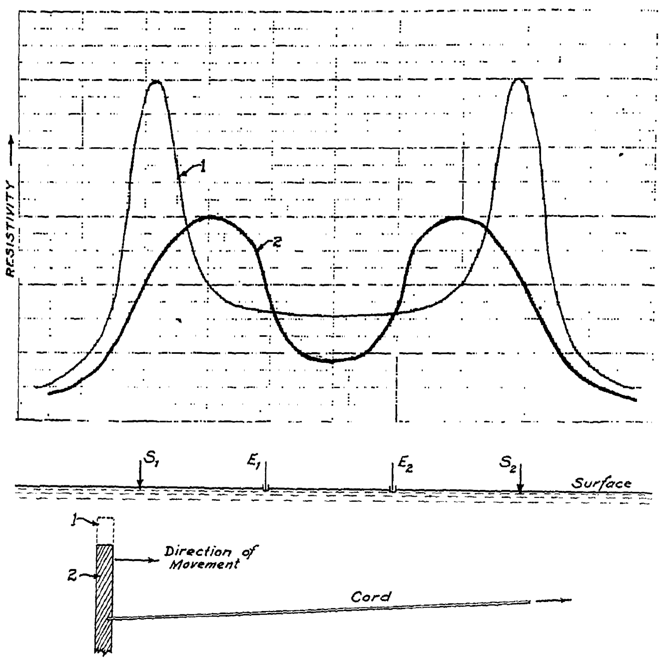

FIG. 172.—Resistivity variations over high resistance partition, using Wenner configuration.

obtained for the higher partition, and curve No. 2 for the lower partition. The results show that the zone of influence is associated with the distorted potential bowl surrounding each of the energizing electrodes. The peaks of the curves shift farther away from the power electrodes at the greater depth. The results indicate that a distorted potential bowl

† M. King Hubbert, "Theory of Scale Models as Applied to the Study of Geologic Structures," *Bull. Geological Society of America*, 1937, Vol. 48, pp. 1459-1520.
M. King Hubbert, *A.I.M.E. Geophysical Prospecting*, 1934.
C. A. Heiland, *Colorado School of Mines Bulletin*, 1929-1930.
J. H. Swartz, "Resistivity Measurements upon Artificial Beds," *U.S. Bureau of Mines, Information Circular* 6445, Feb. 1931.

* It will be seen of course that movement of the partition will produce the same result as would be obtained if the partition were to remain stationary and the electrodes moved in a fixed configuration. For experimental tests, movement of the partition is more convenient and far more rapid.

exists around each source and sink (the power electrodes) and the actual potential as measured is the resultant of the effects in the vicinity of each.

This phenomenon explains the limited resolving power of the Wenner configuration because the results are dependent upon the sum of the effects in the vicinity of the two energizing electrodes. If, for example, a deep investigation were attempted and the energizing electrodes spaced 20,000 feet apart, the two effective zones of measurement probably would be from 10,000 to 15,000 feet apart. At these great distances, many changes could occur, especially in areas of steeply dipping structures.

The instruments employed for small scale investigations may be those used for the large field studies provided they have a sufficiently large scale range to enable the observer to read the very low current (usually 1 to 10 milliamperes) and the relatively high potentials (usually over 100 millivolts) encountered in the small scale investigations.

Small scale, non-polarizing electrodes may be constructed by using small diameter glass tubing. The bottom of the tube is plugged with a short saturated roll of chamois skin. The tube is filled with copper sulphate solution in which a bare copper wire is partially immersed. Ordinary electrodes may be made from small diameter graphite (lead-pencil carbons) or carbon rods similar to those employed for brushes in small motors. Chemically active electrodes, such as iron or copper, should be avoided. Finally, the electrodes should not extend into the conducting material of the tank to a distance greater than one per cent of the minimum electrode separation employed in the investigations.

Tank or small scale experiments may supply considerable information which will be indicative of the results to be expected in field work. However, the tank experiments usually do not yield the same curve characteristics obtained in field work. This may be due to the absence of polarization and related phenomena at the interface of strata in the small scale tests. It is evident, therefore, that considerable care must be taken in using tank results as an aid in interpretation.

FACTORS TO BE CONSIDERED IN INTERPRETATION

1. Depth of Measurement and "Detectability."—In all geophysical investigations in which it is desired to map a certain formation or structural marker with reference to a horizontal datum plane, the most accurate results will be obtained when the variation of the diagnostic quantity measured at the surface is proportional to the depth of the marker. In the electrical methods, however, the magnitude of the diagnostic variable does not depend solely on the vertical depth of the marker. This will be evident from a consideration of Figure 173. Part *A* of the figure illustrates a small, relatively steep, dipping structure. To produce a detectable variation in the resistivity value, it is necessary that the structure comprise a certain portion of the subsurface included in the measure-

ments. The effect of the structure is a weighted or mass effect, depending primarily upon the effective volume included in the zone of measurement rather than upon depth. Part *B* shows a flat lying structure, the weighted or mass effect of which is equivalent to that of part *A*. Thus, both types of structure give comparable electrical anomalies, despite the fact that their reliefs are different. Analysis of resistivity data, therefore, must proceed from the viewpoint of a weighted average, rather than from the viewpoint of a direct measurement.*

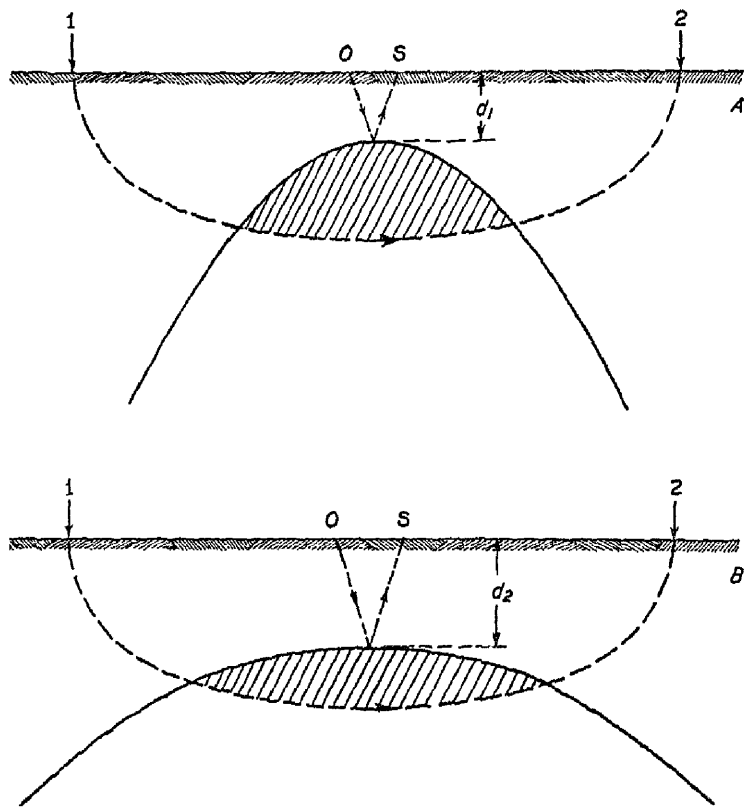

FIG. 173.—Sketch illustrating concepts of depth of penetration and detectability.

2. Variations in Effective Penetration.—The effective depth of current penetration is dependent to a large extent on the relative conductivities of the strata which constitute the volume included in the path of current flow. The ratio of the effective depth of penetration to the distance between the two power electrodes is termed the "penetration factor." This factor usually is not constant in any one area or even for any one station but varies with changes in the ratio of the conductivities of the near surface and the deeper strata.

The reason for the changes in the penetration factor will be evident from the simplified diagram given in Figure 174. The diagram shows a layer of high resistivity underlain by an excellent conductor. † At electrode separations less than *d*, the effects of the good conducting layer is

* The other potential methods, gravitational and magnetic, also depend on mass or weighted effects.

† See also, Warren Weaver, "Certain Applications of the Surface Potential Method," *A.I.M.E. Physical Prospecting*. 1929, p. 68.

small and the penetration factor is relatively small. The highest penetration factor will be obtained when the distance L is slightly more than $2d$, due to the bending down of the current lines toward the good conductor. As the electrode separation is made greater than the distance $2d$, the current lines tend to be confined within the highly conducting layer and the penetration factor decreases again.

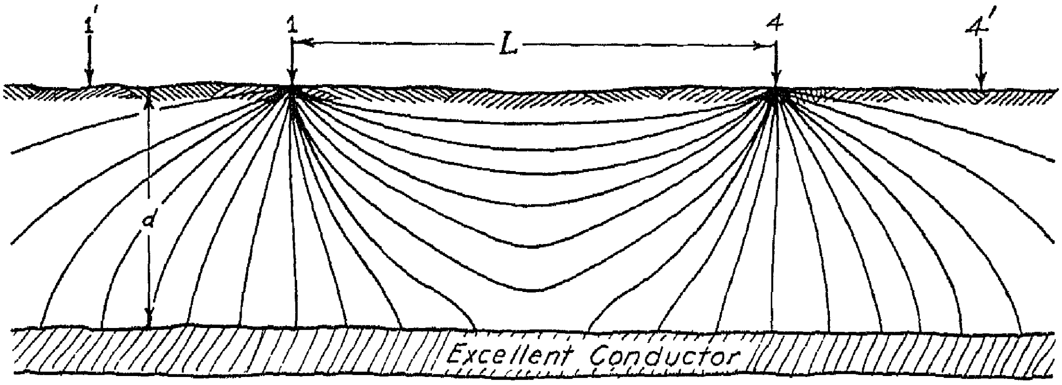

FIG. 174.—Sketch illustrating current paths for a structure comprising a layer of high resistivity underlain by a good conductor.

3. **Anisotropic Media.**—In many cases, the formations investigated by the resistivity methods possess a marked electrical anisotropy which influences the data obtained in surface measurements. Theoretical investigations on the propagation of electric current in anisotropic media have been reported by Schlumberger and Leonardon† and by Maillet and Doll.‡ It is well known that in stratified rocks electric current is propagated more easily along the strike than in a direction perpendicular to the strike. This phenomenon may be treated quantitatively by introducing a coefficient of anisotropy λ defined by the relation

$$\lambda = \sqrt{\frac{r_t}{r_l}}$$

where r_t denotes the resistivity along the strike, "transverse resistivity," and r_l the resistivity perpendicular to the direction of the beds, "longitudinal resistivity."

Figure 175 shows a cross section of an equipotential surface by a vertical plane that is perpendicular to the bedding plane and passes through a point source of current 0 located in a homogeneous medium. If the medium is isotropic as well as homogeneous, the equipotential surfaces surrounding the current source 0 are spheres. (Compare p. 316.) If the medium is anisotropic and homogeneous, the equipotential

† C. and M. Schlumberger and E. G. Leonardon, "Some Observations Concerning Electrical Measurements in Anisotropic Media, and their Interpretation," *A.I.M.E. Geophysical Prospecting*, 1934, pp. 159-181.

‡ R. Maillet and H. G. Doll, "Sur un théorème relativ aux milieux électriquement anisotropes, et ses applications à la prospection électrique en courant continu," *Ergänzungshefte für angewandte Geophysik*, Vol. 3, No. 1, 1932.

surfaces surrounding 0 are ellipsoids of revolution around an axis through 0 perpendicular to the strike. It may be shown that the ratio of the semi-axes of the ellipse ($0A/0B$) is equal to the coefficient of anisotropy λ.

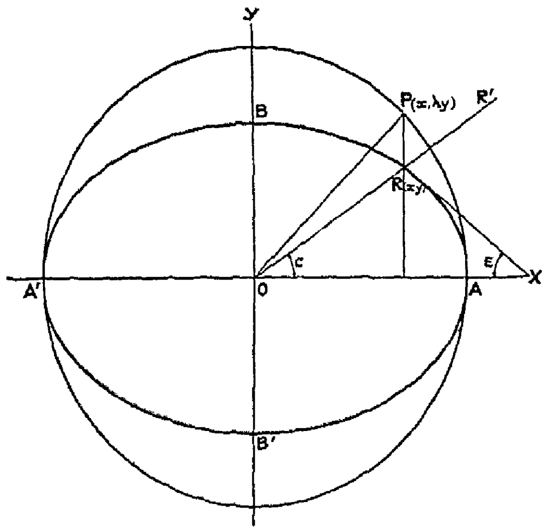

FIG. 175. — Cross section of equipotential surface by a vertical plane passing through a point source of current 0 located in a homogeneous, horizontal stratum. (After Schlumberger and Leonardon, *A.I.M.E. Geophysical Prospecting*, 1934.)

The ellipticity of the equipotential surfaces in anisotropic media has important consequences as regards the interpretation of resistivity data. One of these consequences, the so-called paradox of anisotropy, is that in the case of a stratified formation tilted vertically, the observed, *apparent* transverse resistivity is smaller than the observed, *apparent* longitudinal resistivity, while the inverse proposition holds for the true resistivities. Another consequence is that formulas derived for a stratified medium comprising several parallel, homogeneous, isotropic layers hold for a medium comprising anisotropic layers provided the resistivity of each layer is set equal to $\sqrt{r_l r_t}$, where r_l and r_t denote the longitudinal and transverse resistivities of the layer respectively, and the thickness of each layer is set equal to λd where d denotes the actual thickness of the layer and λ its coefficient of anisotropy. Evidently from a physical viewpoint these consequences affect the interpretation of resistivity data in that the values of the apparent resistivity must be modified to take into account the anisotropy of the formations being investigated.

FIELD PROCEDURE AND EQUIPMENT

Electrode Configurations for Measuring Resistivities.—Various electrode spacings and configurations may be employed in resistivity measurements. As a general rule, the configurations employed are chosen because of their symmetrical arrangement which allows simplification of formulas. The literature, with special reference to many professional-type papers, is replete with various configurations which reputedly are superior to all other configurations. There is no theoretical basis for judging one configuration superior to another. In practice, however, it has been found that certain configurations, used in conjunction with the proper field technique, may permit better evaluations of certain factors that introduce errors. Briefly stated, these undesirable factors are introduced chiefly by: (1) near-surface effects, (2) variations in depth of current penetration, (3) lateral changes in geology.

The electrode configuration employed should preferably be such that the maximum flow of current will take place in the depth range desired in the measurement. Also, the potential electrodes should be so positioned with respect to the current electrodes that they receive the maximum influence from the subsurface flow of current.

Obviously, an electrode arrangement wherein the current electrodes are very close together will be very ineffectual for deep subsurface investigations, because the percentage of current penetrating to the desired depth range will be small and its variations will not be measurable at the surface. In practice, the separation between the current electrodes usually should be from three to six times the desired depth of measurement. The optimum value for any given electrode configuration must be determined experimentally for any given area.

The location of the potential electrodes is important, because if the potential electrodes are positioned too near the current electrodes, they will be predominately influenced by shallow effects. As the distance between the potential and the current electrodes is increased, the deeper zones have a greater effect on the measurement, but at the same time the observed potentials become smaller and the effects of near-surface variations become proportionally greater. Usually, the best "detectability" is obtained when the separation between the potential and the current electrodes is from one-third to one-half the desired depth of investigation. In areas where the lateral variations in resistivity are relatively small, it is usually advantageous to employ the larger separations. In all cases, however, the particular characteristics of the area under investigation must be taken into account.

One of the chief reasons for the indifferent results obtained in many electrical resistivity investigations has been the blind application of a fixed configuration to areas in which the relative conductivities of the upper and deeper materials make that configuration unsuitable.

The more widely employed electrode arrangements are illustrated in the following sketches. † Whatever configuration is employed, the relation between the resistivity and the observed quantities (E/I and electrode separation) may be obtained by using Equation 8 twice.

The technique of applying Equation 8 will be illustrated for the Wenner electrode configuration. The formulas for the other electrode arrangements shown in Figure 176 may be derived in a similar manner.

Wenner Configuration ‡ *(Figure 176a)*

The four electrodes are arranged in a straight line, the current electrodes being separated by a fixed distance, $3a$, while the potential

† See also J. N. Hummel, "A Theoretical Study of Apparent Resistivity in Surface Potential Methods, *A.I.M.E. Geophysical Prospecting*, 1932, p. 392.

‡ Frank Wenner, "A Method of Measuring Earth Resistivity," *Bull. U. S. Bureau of Standards*, Vol. 12, 1916.

H. Gish and W. J. Rooney, "Measurements of the Resistivity of Large Volumes of Undisturbed Earth," *Terr. Mag.* 30, 1925, pp. 161-188.

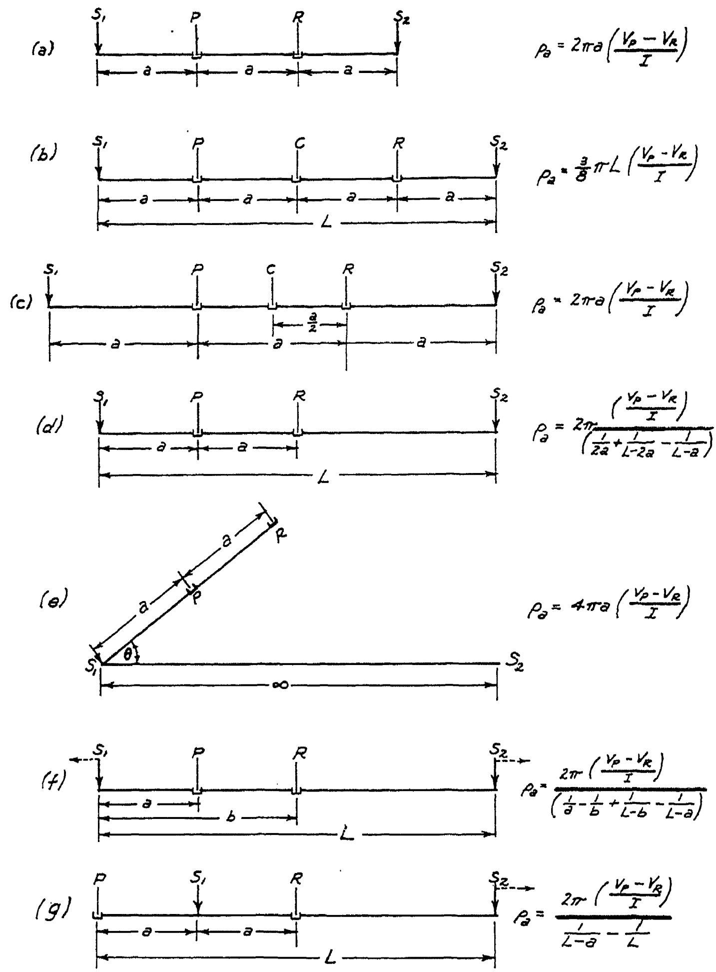

FIG. 176.

(*a*) Wenner configuration
(*b*) Five-electrode method using four equal spacings
(*c*) Lee partitioning method
(*d*) Unsymmetrical method: energizing electrodes separated by a finite distance
(*e*) Unsymmetrical method: one energizing electrode at infinity
(*f*) Method employing moving current electrodes
(*g*) Method employing one moving current electrode and one fixed current electrode positioned midway between the potential electrodes.

electrodes are situated at points distant a and $2a$ from the source respectively. The potential at P due to the source and sink is obtained by substituting a for r_1 and $2a$ for r_2 in Equation 8. That is,

$$V_P = \frac{I\rho_a}{2\pi}\left(\frac{1}{a} - \frac{1}{2a}\right)$$

The potential at R due to the source and sink is obtained by substituting $2a$ for r_1 and a for r_2. That is,

$$V_R = \frac{I\rho_a}{2\pi}\left(\frac{1}{2a} - \frac{1}{a}\right)$$

Hence, the potential difference between the two points is

$$V_P - V_R = \frac{I\rho_a}{2\pi}\left[\left(\frac{1}{a} - \frac{1}{2a}\right) - \left(\frac{1}{2a} - \frac{1}{a}\right)\right] = \frac{I\rho_a}{2\pi a}$$

and

$$\rho_a = 2\pi a\left(\frac{V_P - V_R}{I}\right)$$

It is of interest to point out that theoretically Wenner's formula holds irrespective of whether the potential electrodes are inside or outside of the current electrodes. † Practically, however, there is a decided decrease in detectability when the current electrodes are placed inside of the potential electrodes.

Five-Electrode Method Using Four Equal Spacings (Figure 176b)

$$\begin{aligned}
\rho_a &= \frac{3}{8}\pi L\left(\frac{V_P - V_R}{I}\right) = \frac{3}{2}\pi a\left(\frac{V_P - V_R}{I}\right) \\
&= \frac{3}{4}\pi L\left(\frac{V_P - V_C}{I}\right) = -\frac{3}{4}\pi L\left(\frac{V_C - V_R}{I}\right) \\
&= 3\pi a\left(\frac{V_P - V_C}{I}\right) = -3\pi a\left(\frac{V_C - V_R}{I}\right)
\end{aligned}$$

Lee Partitioning Method (Figure 176c)

$$\begin{aligned}
\rho_a &= 4\pi a\left(\frac{V_P - V_C}{I}\right) = 4\pi a\left(\frac{V_C - V_R}{I}\right) \\
&= 2\pi a\left(\frac{V_P - V_R}{I}\right)
\end{aligned}$$

† R. J. Watson, "Interpretation of Resistivity Measurements," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 518, pp. 6-7.

Unsymmetrical Method: Energizing Electrodes Separated by a Finite Distance (Figure 176d)

$$V_P - V_R = \frac{I\rho_a}{2\pi}\left(\frac{1}{2a} - \frac{1}{L-a}\right) - \frac{I\rho_a}{2\pi}\left(\frac{1}{2a} - \frac{1}{L-2a}\right)$$

or

$$\rho_a = \frac{2\pi\left(\dfrac{V_P - V_R}{I}\right)}{\dfrac{1}{2a} + \dfrac{1}{L-2a} - \dfrac{1}{L-a}}$$

Unsymmetrical Method: One Energizing Electrode at an Infinite Distance (Figure 176e)

The power electrode S_2 is assumed to be at infinity. (In practice, this assumption is justified when the separation S_1S_2 is 5 to 10 times that of the farthest potential electrode from S_1.)

$$V_P - V_R = \frac{I\rho_a}{2\pi} \cdot \frac{1}{a} - \frac{I\rho_a}{2\pi} \cdot \frac{1}{2a}$$

or

$$\rho_a = \frac{V_P - V_R}{}$$

At sufficiently great spacings of the power electrodes, S_1 and S_2, the angle θ may take on any value and the formula will be correct. If the distance S_1S_2 is not much greater than S_1R, however, the effect of S_2 may not be neglected, and the formula must be modified.

Method Employing Moving Current Electrodes (Figure 176f)

$$\rho_a = \frac{2\pi\left(\dfrac{V_P - V_R}{I}\right)}{\dfrac{1}{a} - \dfrac{1}{b} + \dfrac{1}{L-b} - \dfrac{1}{L-a}}$$

If S_1 is fixed and S_2 moves, L, $L-a$, and $L-b$ are variable quantities, and a and b are constants. If S_2 is fixed and S_1 moves, L, a, and b are variable quantities, and $L-a$ and $L-b$ are constants.

Method Employing One Moving Current Electrode and One Fixed Current Electrode Positioned Midway Between the Potential Electrodes (Figure 176g)

$L - a \quad L$

Electrodes.—In conductive methods of electrical prospecting, contact with the earth is made by electrodes imbedded or driven into the ground. The investigation of lateral or depth variations in a given area is accomplished by means of a series of measurements made at different positions or spacings of the electrodes. Usually the electrodes are moved along a traverse line. The movement may be in uniform increments of distance (intermittent readings) or at a continuous rate of progression (continuous recording). The type and design of the electrodes depend on the electrical method employed and on field conditions.

Electrodes may be divided into two general classes: (a) non-polarizing electrodes and (b) polarizing electrodes constructed of base metals.*

Non-Polarizing Electrodes

As far as practical application to geophysical work is concerned, the only type of electrode which does not change its properties appreciably with the passing of small quantities of current consists of a porous cup which contains a rod or sheet of metal and a saturated solution of a salt of the metal. In this type of electrode the metal will go into solution or will be precipitated, depending upon the direction of the current. The electrochemical process is reversible, and the chemical relationships do not change with the passage of a moderate amount of current. Electrodes of this type do not completely eliminate electrolytic or contact potential phenomena, because there must be a contact between the electrolyte seeping through the porous cup and the earth. However, contact potential

* Attempts have been made to employ simple electrodes constructed of chemically inactive materials, in order to avoid the inconveniences attached to employing the usual type of non-polarizing electrode. For example, investigations have been made of carbon and graphite electrodes, as well as electrodes constructed of base metals plated with platinum, iridium, gold, silver, etc. Due to their relatively small electrochemical activity, these materials produce smaller electrolytic potentials at the electrode. They do not, however, have any advantages over the base metals, with regard to polarization phenomena associated with the flow of current.

effects between the two electrolytes, the electrolyte or natural moisture in the earth and the electrolyte of the electrode, is very much smaller than the contact potentials between a metal and the earth electrolyte.*

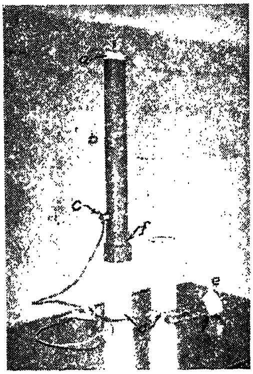

FIG. 177.—Simple non-polarizing electrode.
a—Expansion stopper and cap
b—Handle and reservoir
c—Terminal
d—Porous cup
e—Expanding stopper and guard chain
f—Rubber sleeve

A non-polarizing electrode of simple design is shown in Figure 177. The hollow bakelite tube functions as a convenient handle and also acts as a reservoir for the saturated copper sulphate ($CuSO_4$) solution. The solution is added by removing the expanding rubber stopper at the top of the hollow bakelite tube handle. Contact with the solution is made by means of a pure copper bar, 1/8″ x 3/4″ in cross section, with the lower end doubled back to give a larger surface area in the solution and hence a lower contact resistance.

Electrodes of the non-polarizing type have two chief disadvantages in field operation: (a) Precautions must be taken to insure that the materials used are chemically pure and that the materials and electrolyte of one electrode do not differ from those of the other electrode. (b) The electrodes are fragile and cannot be subjected to rough treatment.

In using non-polarizing electrodes, it is necessary to make a small hole in the ground, usually by means of a small hand or garden trowel. The porous cup portion is placed in the hole. A small quantity of water or dilute salt solution should be poured around the electrode and the earth lightly tamped to give better electrical contact.**

These operating details make the use of non-polarizing electrodes in field work rather tedious. This is especially marked in those methods wherein the potential electrodes must be moved rapidly from one set-up to another. Attempts have been made, therefore, to eliminate the use of non-polarizing electrodes by appropriate field technique and design of equipment. Various types of commutating arrangements have been devised to minimize the effects of spurious potentials at the potential electrodes. These will be described later in conjunction with the commutator methods.

Metal Stake Electrodes

Stake electrodes may be made from any type of structural or bar metal available. Angle-, H- or T-section iron bars, pipes, round and square rods, etc., have all been used by various workers. For practical use,

* The electrolytic or contact potential between two electrolytes is usually negligible in commercial work.

** The procedure described in this sentence is frequently termed "wetting down."

however, it has been found that square steel rod is most satisfactory. Steel, although costing slightly more, will outlast ordinary cold rolled iron many times, and its longer field life more than offsets its initially higher cost. The initial electrode length is preferably 36 to 40 inches. The electrodes are made preferably from ¾″ square rod, which is sharpened at one end.

The electrodes are driven into the ground to a depth sufficient to contact the moist layer immediately underneath the surface. This depth will vary with local conditions but usually will be from two to twelve inches below the surface. In areas where moist earth cannot be contacted within that depth of the surface, the electrode resistance will be relatively high, and more than one electrode may be necessary. The number of electrodes is governed by the voltage drop which can be tolerated in the energizing circuit. Obviously, an increased contact resistance in the energizing circuit necessitates high voltages to create a sufficient flow of current.

As a general rule, from 85% to 95% of the total potential drop in the circuit takes place in the immediate vicinity of the grounded electrode contacts. In dry earth it is often necessary to reduce the contact resistance by "wetting down" or by employing more than one electrode. Contact with the ground is a typical point contact between the irregular earth particles and the smooth electrode surface. Point contacts of this type develop notoriously high resistances. A lower effective resistance between the electrode and the ground may be obtained by using (a) a larger extended contact area or (b) an electrolyte which fills the interstices between the electrode surface and the earth's particles and thereby increases the contact area. If an electrolyte is used, "wetting down" solutions may be carried in ordinary canvas water bags, such as are employed extensively for desert travel. About one-half pint of saturated salt solution per electrode is usually sufficient.

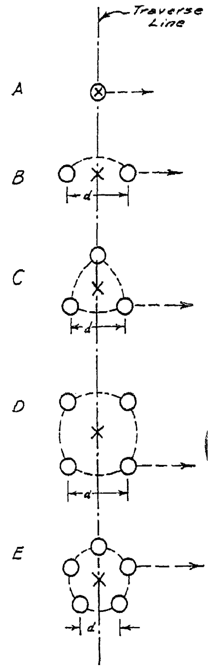

FIG. 178.—Recommended arrangements for stake electrodes. (d is depth the electrodes are imbedded in the ground.)

A—Single electrode placed at point of measurement, marked x.

B—Two electrodes placed on each side of point of measurement.

C—Three electrodes arranged in equilateral triangle with point of measurement at center.

D—Four electrodes arranged in square with point of measurement at center.

E—Five electrodes, or more, arranged on circle, with point of measurement at center.

Location of Multi-Point Electrodes.—A single stake electrode should be driven into the earth at the desired point of measurement as shown at *A* in Figure 178. When two electrodes are employed, it is advisable to place them as shown at *B*. The individual electrodes should be separated a distance

approximately equal to the depth to which they are driven into the ground; as previously mentioned, this depth will vary with the local conditions. If three electrodes are necessary to give the desired current flow, the measuring point should be approximately at the center of an equilateral triangle formed by the electrodes, as shown at *C*. Other electrode arrangements are illustrated at *D* and *E*. In each case an attempt is made to have the electrodes approximate a large extended circular electrode, because a circular arrangement of electrodes produces a minimum distortion of the current lines for a given voltage drop.

Multi-point electrodes are connected electrically by means of "jumpers" made from short pieces of the same wire used for field lines. It is convenient to use jumpers which are made of lengths of wire approximately three feet long and provided with clips at each end and at the center. A jumper of this type will connect three electrodes. The contact clips are the conventional heavy-duty battery clips, which are employed for storage battery charging. A clip which has strong spring-actuated jaws is preferable.

The electrodes are driven into the ground with heavy hammers and are removed from the ground upon completion of a given set of measurements by a special type of stake-puller, or by hammering the side of the electrode to loosen it.

The field work is greatly speeded up if two full sets of stakes are employed for each moving electrode. Two electrode men are stationed along the line of movement of the moving electrode. While one set of electrodes is being utilized in the measurement, one electrode man will move and drive in his set of electrodes at the station to be read next. With an arrangement of this type, the speed of operation in the field depends upon the spacing between electrodes and the energy of the instrument man. Even in areas of fairly rough terrain, electrode moves may be made in time intervals of two to five minutes.

Mobile Stake Electrodes

Various procedures have been tried for obtaining readings at closer electrode spacings without increasing unduly the time required for a series of measurements. In areas of fairly even topography and moderately hard surface materials, success has been obtained with an intermittent type of electrode mounted upon a light truck. By means of a gear drive connected to a power take-off on the truck motor, a steel rod 1½ inches in diameter is forced into the ground to a depth of penetration which depends on contact conditions and varies from one foot to three feet. Upon completion of the measurement, the power is reversed and the rod is withdrawn from the earth. The truck is then moved forward to the next point of measurement and the procedure repeated. The reel which contains the field wire is mounted upon the truck, and the wire is auto-

matically played-out as the truck moves forward. Contact is maintained by means of a suitable slip ring on the reel. The first measurement is made at some predetermined electrode separation, and subsequent measurements are made at uniform increments of distance. This type of arrangement has proved to be somewhat faster than the stake method described above.

The points where contact is to be made are usually measured and marked on the ground by means of stakes (pieces of lath 12 inches long) on which are written the traverse distance. (Because the resistivity formulas for calculating resistivity values include "distance," it is necessary to measure the electrode separations with considerable accuracy.)

Continuous Contact Electrodes

To minimize the time and labor necessary for closely spaced intermittent readings, a mobile continuous contact electrode has been developed.† The mobile electrode obviates the necessity of driving electrodes into the

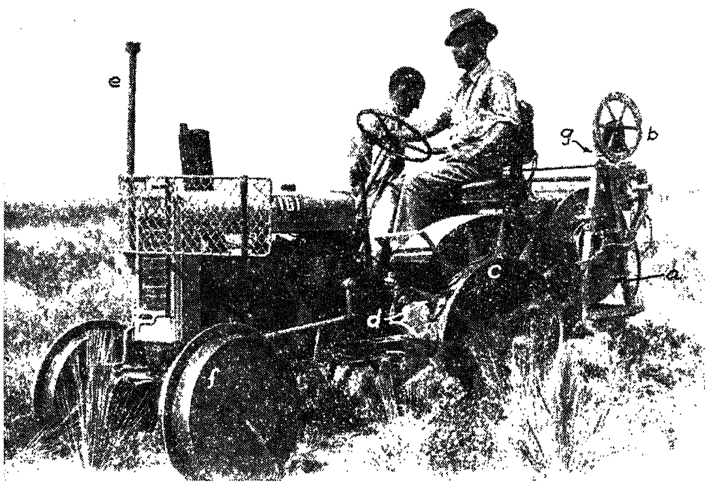

FIG. 179.—Continuous mobile electrode. *a*, reel of flexible insulated wire; *b*, measuring sheave wheel; *c*, steel tractor wheel; *d*, steel contact stakes; *e*, guide for reeling-in cable; *f*, weighted flanged wheels; *g*, contact ring which actuates measuring signal system. (Courtesy of International Geophysics, Inc.)

ground at each predetermined electrode position and of surveying-in the various stations. Also, because the electrode is always in contact with the ground, oscillographs may be employed for continuous recording of the potential and current.

† J. J. Jakosky, "Method and Apparatus for Electrical Exploration of the Subsurface," U. S. Patent 2,105,247, issued Jan. 11, 1938; Canadian Patent 374,475, issued June 14, 1938; other U. S. and foreign patents pending.

The electrode is a small tractor which moves forward at a constant speed which is determined by a governor on the engine. (Figure 179.) The uniform movement of the tractor allows accurate control of the time of current flow. The rear wheels of the tractor have a steel rim to which short steel stakes are bolted. The stakes are approximately ten inches long in order to make good contact with the subsoil. Usually four stakes on each wheel are in continuous contact with the ground. The tractor is equipped with special low-gear ratios. A large reel holding about 20,000 feet of heavy insulated wire is mounted on the tractor as shown in Figure 179. A carriage, mounted on top of the reel, supports a measuring sheave wheel and wire spooling device. The measuring sheave is connected to a relay signalling system mounted on the tractor. The relay system automatically transmits to the recording truck the exact footage traversed by this mobile electrode, and this footage, at intervals of ten feet, is recorded on the chart simultaneously with the potential and current values.

A carrier current telephone system is provided to allow communication between the operator of the tractor and the operator of the recording truck. With a potential difference between the electrodes of approximately 2000 volts, an electric current of 1.5 to 5 amperes usually can be maintained. Similar continuous type measurements may be made in submerged areas by using small power-driven cruisers or boats as the moving electrode.

The mobile electrode has greatly increased the effectiveness of electrical methods of prospecting. Usually, when the intermittent type hand-driven stake electrodes are used, three to five hours are required for a series of readings at a station, which consists of measurements made at intervals of every fifty feet along a 5000 foot traverse movement. With the mobile type of electrode these same readings may be made in approximately fifteen minutes. The mobility of the moving electrode has been found to be very good, even in areas of extremely "tough going." The mobile electrode usually requires two operators, one for driving the tractor and maintaining telephonic communication and the other for handling the reel and measuring equipment.

The continuous contact electrode is employed most conveniently in areas where the surface conductivity is fairly high and the surface is not too rocky. In areas where the surface is rocky, the contact will be irregular.

Semi-Continuous Electrode System

A semi-continuous electrode system is illustrated in Figure 180. This system has operating characteristics intermediate between the intermittent stake electrode method and the continuous contact mobile electrode method.

The reel contains approximately 10,000 feet of wire and is mounted on a shaft supported by two large steel wheels. These wheels have a diameter of approximately 30 inches and are similar to those used on concrete "buggies." A small hand crank, connected by means of a sprocket chain drive, is employed for reeling-in the wire at the completion of a run. As will be seen from the diagram of connections, the stationary end of the reel is connected to a commutator. The commutator has a single brush contact connected to a transformer. The other terminal of the transformer is connected to the two insulated handles for contacting the electrodes. The secondary of the transformer is connected to a double-pole, double-throw, key switch. In one position, the switch closes a

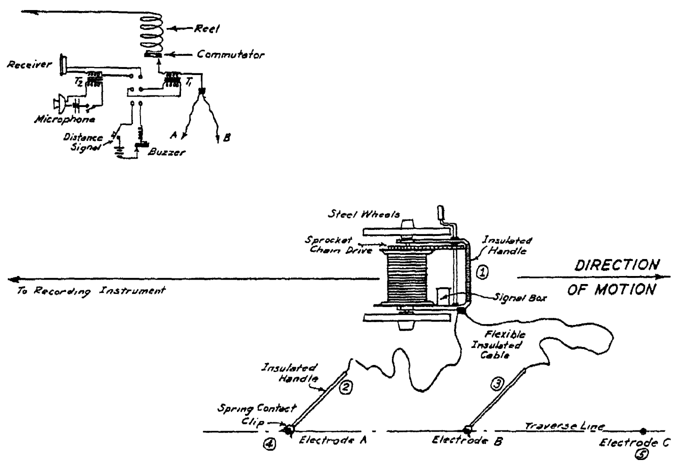

FIG. 180.—Electrical connections for semi-continuous electrode system. (Courtesy of International Geophysics, Inc.)

buzzer circuit which is employed for signalling the instrument truck. In the reverse position, the switch closes a telephone circuit which includes a hand-set for voice communication.

Three electrodes, or sets of electrodes, A, B, and C, are placed along the traverse line at the desired distance apart. By means of insulated handles, provided with spring contact clips, electrode men stationed at 2 and 3 connect the electrodes A and B, respectively, with the lead wires. The flexible lead wires connecting the reel terminal box with the electrode handles are of equal length, and this length is approximately the distance which the electrodes are to be placed along the traverse line. The operator stationed at 1 slowly moves the reel forward; after a certain distance has been traversed, the flexible wire connecting the rear electrode A becomes taut and continuous movement forward of the reel pulls the clip from the electrode. At this time the operator stationed at 2 walks past the operator

at 3 and makes contact with the next electrode C. The operator stationed at 4 removes electrode A from the ground and drives it into the ground on the traverse line beyond electrode C, at the desired spacing. As the reel continues to move forward, electrode B will automatically become disconnected, and the same procedure is repeated. By this procedure, one ground electrode, then two, then one again, and so on, are always connected to the recording equipment.

This system of semi-continuous electrode movement necessitates continuous recording equipment similar to that employed for the continuous contact mobile electrode. The position of the reel is signalled to the recording truck by the operator stationed at 1 who closes a key switch each time the reel comes abreast of an electrode. These signals are recorded at the instrument truck in a manner similar to that employed for the continuous moving electrode. The method requires at least five operators for satisfactory operation. In areas of rough terrain, an additional operator may be required for handling the reel. With sufficient personnel, the use of this electrode is almost as rapid as that of the mobile electrode.

Insulated Wire for Field Use.—A stranded conductor covered with bare rubber insulation is preferred. Cloth braid and other protective coatings for the rubber have not proved satisfactory, principally because the cloth abrades very fast when the wire is drawn across rocks and through brush. In addition, when cloth braid is used, it is difficult to detect breaks in the conductor.

A single conductor of No. 16 American wire gauge is satisfactory for short lines. One wire of this size on the market consists of 13 strands of hard drawn copper, each of which has a diameter of 0.0142 inches and is covered with a thirty per cent rubber insulation of approximately 3/64 inch thickness. For the longer high voltage energizing circuits in which the voltage may rise to 2000 volts or more, it is recommended that a single conductor No. 14 American wire gauge, consisting of 19 strands of 0.0142 inch hard drawn copper with 5/64 inch thirty per cent rubber insulation, be employed. This wire has an estimated tensile strength of approximately 190 pounds. The estimated break-down voltage for the 5/64 rubber insulation is 24,000 volts, while for the 3/64 inch rubber insulation it is approximately 15,000 volts.

During field work in which the conductor is subjected to considerable strain, a single conductor having twelve strands of 0.0126 inch tinned steel and 14 strands of 0.0126 inch tinned copper, bunch stranded with 2-inch pitch, has proved highly satisfactory. This conductor is covered with a 5/64 inch thick, thirty per cent rubber insulation and has a tensile strength of 350 to 400 pounds.

In dry areas where potentials of less than 100 volts are used, standard flexible fixture wire may be employed for the potential and energizing circuit leads. Fixture wire is usually No. 18 or No. 16 gauge stranded copper, covered with 1/64 inch rubber composition insulation and a cotton weatherproof, pitch-impregnated braid. This wire will stand a tensile pull of approximately thirty to fifty pounds.

Wire Splices

Several precautions should be observed when making splices for repairing breaks in the line wire. A layer of self-vulcanizing rubber tape, covered with a layer of friction tape, should be employed. The

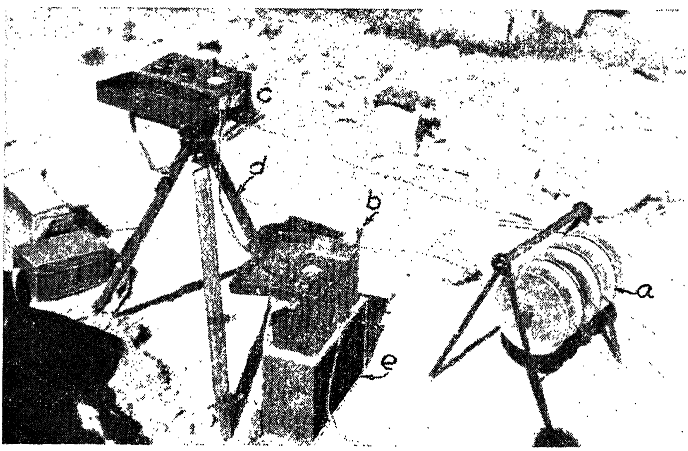

FIG. 181.—Equipment for shallow resistivity measurements. *a*, potential and energizing reels, mounted on common shaft; *b*, hand cranked commutator system and milliammeter; *c*, potentiometer; *d*, tripod; *e*, battery box.

usual width of friction tape is approximately three-quarters of an inch. It is advisable to split the tape to a width of approximately one-fourth to three-eighths of an inch. Prior to making measurements, all splices and field wire should be tested for line leakage. This can usually be accomplished in the field by disconnecting the ends of the wire from their respective electrodes and applying an overload voltage to the lines.

Reels

Many different types of reels are employed for handling the field wires. For shallow investigations, the length of lines usually does not exceed about five hundred feet, and it is often convenient to place all of the reels on a common stationary shaft and locate the shaft near the instrument. (Figure 181.) The potential and energizing lines are then carried out from this central reel system. This method, however, suffers

a disadvantage where long lines are necessary, due primarily to two factors: (a) high inductive coupling between the potential and power circuits and (b) undue strain on the lines as they are dragged over the surface of the ground.

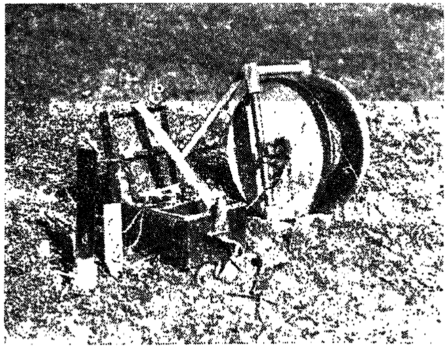

FIG. 182.—Equipment set-up at end of potential line. *a*, reel; *b*, portable field telephone and buzzer signal; *c*, stake marking traverse distance; *d*, non-polarizing electrode. (Courtesy of International Geophysics, Inc.)

For deeper investigations, it is advisable to employ separate reels, preferably of the hand type illustrated in Figure 182. These hand reels hold from 1500 to 2000 feet of 5/32-inch to 3/16-inch diameter wire and weigh approximately 55 pounds when loaded. The flanges of the reel have a diameter of 18 inches and are spun from No. 8 gauge sheet aluminum or No. 12 gauge sheet steel. The flanges are separated by a core and are mounted 5 inches apart on a steel axle, one end of which is extended and bent to form a crank. An "A" frame built of 1-inch

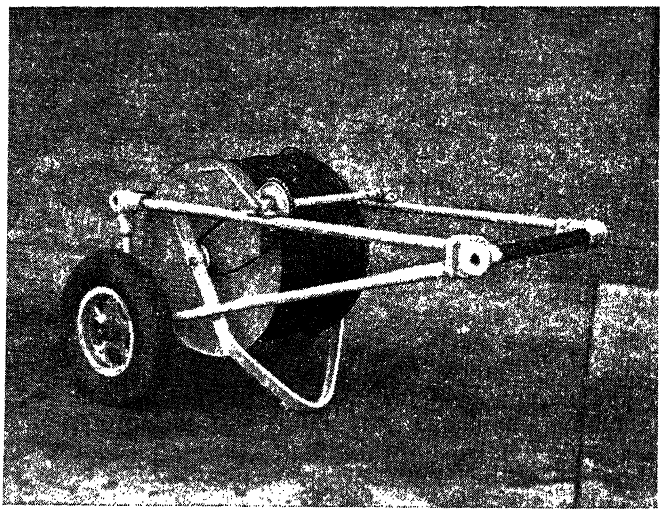

FIG. 183.—Mobile reel for handling field lines. (Courtesy of International Geophysics, Inc.)

seamless, welded, steel tubing is used for supporting the reel. The upper portion of the frame serves as a handle for carrying the reel. The core

consists of a 5-inch piece of 3-inch diameter Shelby steel tubing. The inside end of the wire passes through an insulating bushing in the flange and connects to a well-insulated terminal block. This terminal is connected to the electrode by means of a short flexible lead provided with a spring contact.

For deep subsurface investigations requiring long lines, the reel illustrated in Figure 183 is very satisfactory. This reel has flanges 20 inches in diameter which are separated by a piece of 6-inch diameter tubing 20 inches long. The pneumatic tires are the conventional type employed for concrete wheelbarrows and other industrial uses: namely, 4-ply, 4 inch x 8 inch I. D. These reels hold from 5500 to 6000 feet of the 3/16-inch diameter wire. The frames are made of 1¼-inch welded steel tubing. Roller bearings are employed for mounting the reel and the wheels. These reels weigh approximately 350 pounds when loaded, but can be handled by one man in areas of fairly even terrain.

FIG. 184.—Mobile reel in operation. *a*, small portable telephone; *b*, steel electrodes; *c*, flexible connectors; *d*, previously used set of electrodes. (Courtesy of International Geophysics, Inc.)

Figure 184 shows the method of moving the reel between readings. In this view, the stakes for the next reading position have been driven and are ready for the reelman. An assistant is removing the previously used set of stakes and will have them in position at the next set-up.

Handling Field Lines

The lines are laid out by connecting the end of the wires to the proper terminals of the measuring instruments. The wire should be anchored on a convenient wood stake to prevent disturbing the instruments when

the wire is pulled. Usually one potential wire and one energizing wire will extend outward in each direction from the instruments. The potential and power lines should be separated 15 feet or more, in order to minimize mutual inductive effects and leakage between circuits.

After testing for line leakage and continuity of circuits, the initial reading may be made. The reels must then be moved consecutively to the various points of measurement. On reaching each of these points, the reelman drives in the required electrodes, carefully connecting each electrode with the reel, and then signals the instrument operator. Upon completion of the reading, the instrument operator signals the reelman and telephones the number, usually the distance in feet, of the next station. The reelman then removes the electrodes from the ground and proceeds to the next station, and the process is repeated.

Proper precautions must be taken to prevent the reelman from setting up at the wrong point. It is usually a good plan for the instrument operator to give the next point of reading before the reelman leaves, and for the reelman to give his location to the operator before a reading is made at the new point. Abnormal readings or trends in the recorded values should be noted and checked by additional field measurements, if necessary. Careful field work is a prerequisite to proper interpretation. The instrument operator, or preferably a computer, should calculate and plot the values obtained for each point as the field work progresses. In this manner abnormal readings can be detected and checked without undue delay. Although the various steps appear somewhat involved, the operations may be conducted in a rapid manner.

Instruments and Methods.—In the majority of the conductive electrical methods, the quantity measured is the potential created at the surface of the ground for some particular electrode arrangement. Obviously, a great many different types of measurement may be made; hence, a large ramification of methods, apparatuses, and field procedures exist. Limited space prevents a full treatment of the possible field procedures.

The methods discussed below are described because of their commercial use and because they are illustrative of certain general principles of operation. The descriptions are not intended to cover all possible modifications of any of the methods. Instead, an attempt has been made to incorporate in the description of the different methods, various ramifications of instrumental technique. Thus, in the simple volt-ammeter method which is described first, a potentiometer employing a null-point type of measurement is described. It is obvious that other accurate methods of measuring potentials could be employed.

Simple Volt-Ammeter Method

A schematic diagram of the electrical connections is shown in Figure 185. Power is usually furnished by a bank of 22½ volt, heavy duty "B" batteries, such as used for radio plate supply. For average areas the

potential need seldom be greater than 180 volts, while in dry desert areas potentials of 300 to 400 volts are often necessary. The batteries should be placed near the operator, who obtains the desired energizing current flow by connecting an insulated clip to the proper battery terminal. The energizing current is measured by a D.C. milliammeter having a range

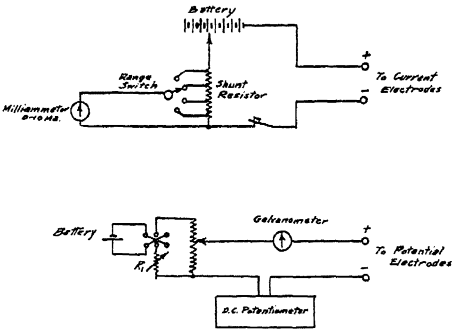

FIG. 185.—Apparatus for simple volt-ammeter resistivity measurements.

of 0-10 milliamperes and a shunt switch with multiplying ratios of 5, 10, and 100. A foot-controlled, sturdy contactor switch should be provided for opening and closing the circuit. The current electrodes for energizing the ground may be of the iron stake type.

The potentials are measured by means of a D.C. potentiometer having a range of 0-1000 millivolts. The potentiometer should be provided with a double-acting, closed circuit switch or push-button control. The galvanometer should be of the high resistance type, preferably 1000 ohms resistance, and should have a current sensitivity of at least 0.005 milliamperes per scale division. A convenient zero adjustment to bring the galvanometer needle to mechanical zero balance facilitates readings. The potential difference existing between the two potential electrodes due to natural earth potentials is neutralized or reduced to zero before each reading by means of a neutralizer or auxiliary potentiometer. This potentiometer is energized by a 1½-volt dry cell. A reversing switch allows the polarity of the impressed potential to be changed so as to oppose the polarity of the natural ground potentials. A resistor R_1 is employed to reduce the potential impressed across the neutralizer potentiometer to about 500 millivolts. Higher potentials than this make adjustment of the neutralizer unduly critical. The potential electrodes are of the non-polarizing type.

In using this method considerable care is necessary to avoid errors introduced by: (1) the erratic and unpredictable variations in ground potentials in the potential measuring circuit, (2) the rapid decline of the energizing current due to polarization and electrolysis phenomena

adjacent the current electrodes, and (3) failure to read the exact value of energizing current simultaneously with the adjustment of the potentiometer for a zero balance on the galvanometer. The effects of the ground current variations can usually be minimized by neutralizing the natural ground potentials and then immediately reading the current values for some setting of the potentiometer. As soon as the current value has been noted by the operator, he releases the foot switch, thereby breaking the current circuit, and then disconnects the potentiometer. If the galvanometer remains at balance it indicates that the natural ground potential has not varied, and the current and potential values read are the values desired. If an appreciable change in natural ground potentials has occurred, the ground potentials should again be neutralized, and the procedure repeated until check ratios of E/I are obtained.

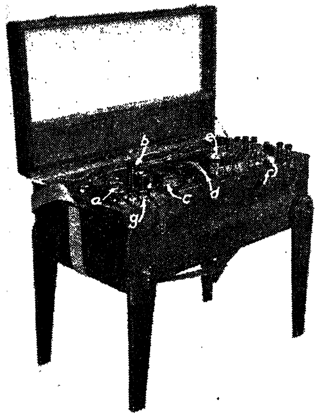

FIG. 186.—Simple millivolt-milliampere apparatus for resistivity measurements. *a*, potentiometer battery adjustment; *b*, high resistance galvanometer; *c*, ground current neutralizer; *d*, potentiometer dial; *e*, 0-10 milliampere meter; *f*, range switch for milliammeter giving values to 2000 milliamperes; *g*, galvanometer shunt and potentiometer switch keys.

Instruments using this principle may readily be made portable and convenient for field use. Figure 186 illustrates a successful form of apparatus. The folding legs bring the panel to a convenient height when the operator sits on a folding camp chair.

Controlled Potential Method

At depths of more than a few hundred feet, it is necessary that the electrical measurements have an accuracy greater than can be obtained with the simple volt-ammeter procedure described above. More accurate results can be obtained by bringing the potential (or current) up to some fixed value and measuring the corresponding current (or potential). Since a known potential is applied, the only reading that need be made is the current value.

In practice, it is customary to use a source of constant potential and vary the energizing current in order to obtain the set potential value.*

* Maintaining a constant value for the potential is preferable to keeping the current constant or bringing it to some predetermined value because, in the latter procedure, the current must first be adjusted to its desired value and then the potentiometer employed for measuring the potential must be adjusted. During the manipulation of the potentiometer the current ordinarily will have changed, partly because of polarization of the batteries supplying the current and partly because of changes in resistance in the earth in the immediate vicinity of the electrodes due to polarization.

If, during a series of readings, it becomes advisable to change from one fixed value of the potential to another, the effect of the change as a disturbing influence may be evaluated by taking double readings for both potentials at one or more positions of the electrodes. The ratio of E/I is obtained by dividing the potential by the instantaneous value of the current required to balance this impressed potential.

The apparatus shown diagrammatically in Figure 187 has a general similarity to the apparatus for the simple volt-ampere method, and corresponding parts will not be described again. The desired potential is regulated by means of a multi-point switch S_1 connected to a resistor, across which there is a potential drop of 100 m.v. This potential drop is maintained by adjusting a resistor R_1 to give a predetermined reading on the voltmeter E_1. A closed circuit push-button key K_2 controls the sensitivity

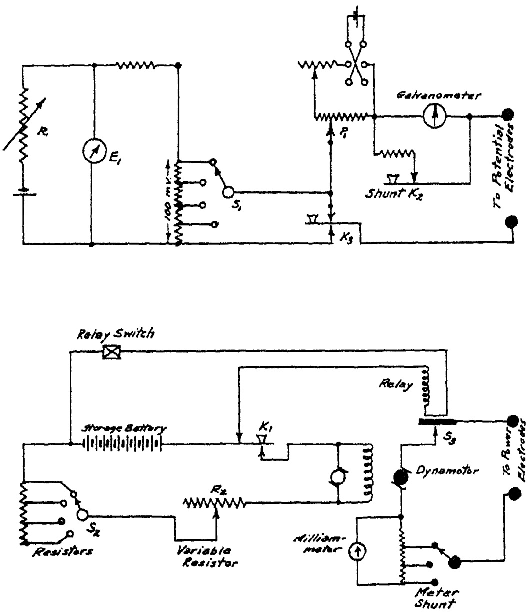

FIG. 187.—Resistivity apparatus for constant potential method.

of the galvanometer. A closed-circuit, double-acting key K_3 normally allows only the neutralizer potentiometer P_1 to be connected in the circuit, thus providing the necessary means for neutralizing the natural earth potentials.

When K_3 is depressed, a certain fixed potential, depending upon the setting of S_2, will be impressed on the circuit causing a deflection of the galvanometer. Key K_1, controlling the energizing current, is depressed simultaneously with K_3, and the energizing current is now adjusted by

rotating the dial R_2 to bring the galvanometer back to balance. (The energizing current is adjusted to the proper value to create a potential equal to and opposite in polarity to the applied fixed potential.)

The power is supplied by a direct current dynamotor which is connected to a storage battery through a heavy duty switch K_1. In series with the battery is a variable resistor R_2 and a bank of fixed resistors which may be tapped at various fixed values by switch S_2. The output of the dynamotor passes through a milliammeter having a range of 0-10 m.a. and a shunt having multiplying ratios 5, 10, and 100. A relay-controlled closed-circuit switch S_3 automatically breaks the circuit when the power switch K_1 is released. S_3 is made automatic in order to prevent the continued flow of current which normally would take place due to the inertia of the generator as it loses speed. The dynamotors are usually wound for 12 to 32 volts D.C. input and 500 to 1000 volts output. Full load output current should be 500 to 1000 milliamperes, intermittent service rating. By means of a tapped resistor and S_2, and the variable resistor R_2, the input current (and hence the output) may be controlled.

This constant potential method is more rapid and accurate than the volt-ammeter method. However, its use requires care on the part of the operator in reading the current at the time of galvanometer balance. In addition, the usual precautions must be taken to minimize errors due to natural ground potential variations. The rapidity with which the measurements are made, usually two to four seconds, prevents appreciable error due to ground currents except on occasional days when rapid magnetic and electrical changes occur.

Apparatus for Neutralizing Natural Ground Potentials

The very large potential spacings required when working to depths in excess of 1000 feet cause considerable difficulty due to the natural ground currents.† These currents produce irregular and varying uni-directional potentials which cannot be separated from the potentials created by the flow of the D.C. energizing current. Various methods have been proposed for eliminating or minimizing the effects of these ground potentials. Perhaps the best known of these methods is the one developed by Gish and Rooney.

The Gish-Rooney ‡ apparatus (Figure 188) employs a double commutating system so designed that the potential and current systems are reversed in synchronism. The power, which is supplied by suitable "B" batteries, passes through a direct-current milliammeter and into the commutating system. The polarity of the current is reversed periodically by the commutator at a frequency depending upon the speed of rotation—usually about 20-30 reversals per second. A second commutator mounted

H. Gish, "The Natural Electric Currents in the Earth," *Scientific Monthly*, July 1936, pp. [illegible]7.

O. H. Gish, "Electrical Messages from the Earth, their Reception and Interpretation," *[Jo]urnal of the Washington Academy of Sciences*, Vol. 26, No. 7, July 15, 1936.

‡ W. J. Rooney and O. H. Gish, "Results of Earth-Resistivity Surveys near Watheroo, Western [A]ustralia and at Ebro, Spain," *Terr. Mag.* 32, pp. 49-63 (1927).

on the same rotating shaft as the power commutator is used for reversing the potential periodically. This commutator changes the alternating potential created by the commutated energizing current to a pulsating uni-directional potential which can be measured by a standard direct-current potentiometer.

The double commutator system eliminates the variable ground currents from the measurements, because any difference in a uni-directional potential existing between the two inner electrodes is automatically nullified due to the rapid reversals.

The commutator may be either hand-cranked or motor-driven. A motor-driven commutator maintains constant speed which permits more accurate work and also eliminates the services of an assistant. Between the two commutators is a guard or slip-ring which is grounded as shown in the wiring diagram in order to prevent leakage from the current commutator to the potential commutator.*

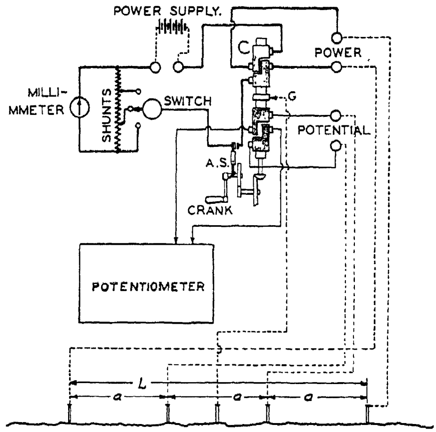

FIG. 188.—Diagram of Gish-Rooney double commutator method.

Precautions must be taken to prevent stray or leakage currents from influencing the potential readings. In addition, the lines and reels connected to the instrument should be laid in a manner which will minimize induction effects. Although direct current is secured from the power batteries, the "make" and "break" of this direct current, while being changed into a periodically reversed current, creates bad transient effects. These are usually minimized by having the power circuit "make" a short time interval before, and "break" a short time interval after the potential circuit. When this procedure is used, steady-state conditions exist during the time interval in which measurements are made.

The previously described constant potential system may be incorporated in the commutator method, with a resultant increase in general overall accuracy and speed of reading due to the fact that variations in ground potentials are removed by the commutator.

Photographic Method for Minimizing Effects of Ground Currents. —The current can be passed into the ground at predetermined time intervals, and the reoccurring pattern may be viewed with any of the conventional type oscillographic equipment. For field use, the cathode ray

* Constructional details for this equipment are available at the Carnegie Institution of Terrestrial Magnetism, Washington, D. C.

oscilloscope has been found particularly suitable. For this use an oscilloscope, with suitable amplification, is incorporated into the resistivity measuring instrument. The ground potentials are fed into an amplifier and then impressed across one pair of deflection plates. The energizing current is passed through deflection coils that have their axes at right angles to the potential deflection plates. The slope of the path of the resultant pattern depends upon the relative magnitudes of potential and current, and the instrumental constants.

Due to the persistence of vision, accurate measurements of slope may be made with pulses arriving as slow as one per second.

By use of suitable panchromatic film and a wide aperature (fast) lens, a photographic record of the pattern for each electrode position may be obtained. The exposure should be such as to give only a faint trace for each sweep of the beam. By recording a series of sweeps, the true slope is accurately shown by the record. Varying natural ground potentials merely cause a shift in the starting point of the trace, and do not affect the slope of the recorded patterns.

Alternating Current Methods

Various alternating current methods are employed for earth resistivity measurements. As indicated earlier, the characteristic feature of these methods is the use of an alternating current power supply instead of a direct or commutated current supply.

Medium Frequency Alternating Current Method.—Power for energizing the ground for this work is obtained by alternators driven by a small gasoline engine or a D.C. converter which operates on storage battery power. Frequencies in excess of 50 cycles per second are undesirable due to errors introduced by phase shift, etc.

The output terminals of the alternator are connected to the primary of an input transformer. The secondary of this transformer is tapped in order that the secondary impedance may be matched to the electrode-circuit impedance. By means of a multi-point switch and the field rheostat of the alternator, the energizing current may be controlled as desired. The alternating current potentials are measured by means of a vacuum tube voltmeter. The required sensitivity is determined by the electrode configuration used and the power available for the current circuit.

Vacuum Tube Voltmeter

The vacuum tube voltmeter shown in Figures 189 and 190, consists of a detector tube T_1 used to rectify the A.C. signal and a stage of direct coupled amplification which amplifies the D.C. output from the first stage. Calibration is dependent upon maintaining the proper grid voltages on the tubes. In the first stage, the grid voltage

is maintained by adjustment of R_1 to give a predetermined voltage, read on V_1. The variable resistor R_2 is used to adjust the total plate current of T_2 (read on milliammeter A_2) to its proper value with no A.C. signal input. The rheostat R_3 is then adjusted to bring the milliammeter A_1 to a zero position so that this meter can be used to read the changes in plate current of the tube T_2, which are caused by the A.C. signal input. By proper calibration the meter A_1 can be made to read the volts (or millivolts) input to the voltmeter directly.

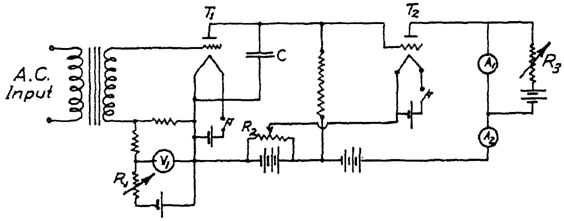

FIG. 189.—Circuit diagram of vacuum tube voltmeter for measuring alternating current potentials.

Low Frequency Alternating Current Methods.—The use of very low frequency alternating current (1 to 10 cycles per second) greatly minimizes the mutual inductance between the current and potential circuits and the phase shift effects which occur when the higher frequencies are used. The ordinary low frequency alternator is not practical for field use because of its large size.

Ambronn † utilizes harmonic current variations from a rotating resistor to produce the desired low frequency alternating current. The frequency depends entirely upon the rotational speed of the resistor and may have any desired value.

FIG. 190.—Vacuum tube A.C. voltmeter.

A low frequency harmonic power method is illustrated in Figure 191. The power supply, which may be any suitable D.C. source, such as storage batteries or an engine-driven generator, is connected across a commutator. The commutator comprises 100 or more segments which are connected to a closed ring of resistors. A rotating arm carries a contact brush at each end and connects the input winding of transformer T_1 to different segments of the commutator. (The commutator arm may be rotated by a gear train connected to the D.C. generator.) When the contact arms are at positions a-a', no potential difference is impressed across the transformer; at positions b-b', the full load voltage is impressed. Hence, the potential impressed across the

† Richard Ambronn. "Improvements in Process of and Devices for Electrically Exploring the Ground by Means of Alternating Currents of Very Low Frequency," British Patent 308,256, issued June 11, 1930.

transformer varies from zero to a maximum and passes through a complete A.C. cycle for each complete revolution of the rotating arm.

The voltage between the potential electrodes may be measured by the previously described vacuum tube voltmeter or by a sensitive alternating current galvanometer or millivoltmeter. The power supplied to the energizing circuit is measured by the milliammeter I_2 which is equipped with

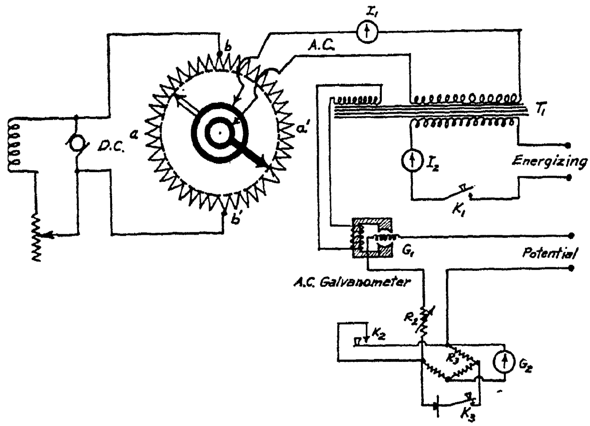

FIG. 191.—Electrical circuit for low frequency harmonic power method.

appropriate shunts. If an alternating current galvanometer or millivoltmeter is employed, the potential measuring circuit is current-operated (the current being measured in terms of fractions of a microampere), and a correction must be made for variations in the resistance of the circuit. This can be accomplished by two methods: (a) measuring the actual resistance of the circuit for each set-up of the potential electrodes or (b) selecting some arbitrary circuit resistance, e.g., 1000 ohms, and thereafter always adjusting the potential circuit to this value before each reading. The latter procedure simplifies the field operations and the computations.

The circuit shown in Figure 191 illustrates a conventional bridge which may be utilized for the type (b) method of operation. Adjustment of the potential circuit is made by depressing the closed-circuit key K_2, closing K_3, and then adjusting R_2 until the galvanometer G_2 indicates a balanced circuit. Flexibility of the equipment for use in different areas is obtained by making the resistor R_3 adjustable, with values of 500, 1000, and 2000 ohms. The lowest value is used in damp or wet areas; the mean value is used in areas where average surface conditions obtain; and the highest value is used in dry desert areas. Best sensitivity will be obtained by always operating at the lowest value possible in any given area. Should it be deemed advisable to change values during the progress of a survey, the

data may all be converted to a common basis by correcting for the circuit resistance changes.

Methods for the Direct Measurement of the Ratio E/I. — In the methods described thus far, separate measurements are made of the voltage and current simultaneously. However, only the ratio E/I is needed for calculating the apparent resistivity, and it is advantageous to use instruments which measure this ratio directly because only one reading need be taken, with a resultant increase in speed and accuracy.

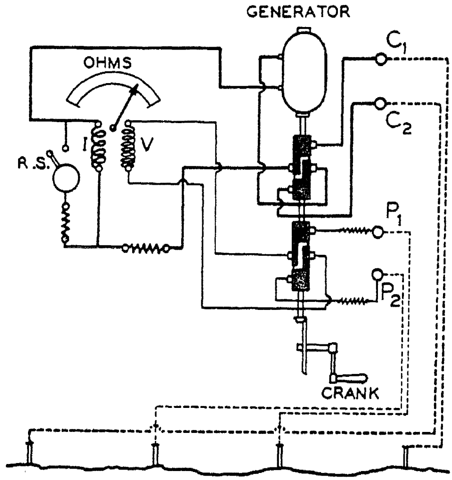

FIG. 192.—Diagram of connections for Megger circuit.

Many types of direct ratio reading instruments have been proposed. Of these, the Megger is the best known at present, although its use in geophysical measurements is limited, due to its inherently poor sensitivity.

The Megger

The Megger is a trade name applied to a type of instrument ordinarily employed for testing electrical insulation resistance and ground resistance. A modification of this instrument, the "Megger Ground Tester," may be used for shallow subsurface investigations.

The Megger Ground Tester † is illustrated diagrammatically in Figure 192. Turning the crank rotates a small direct-current generator. The output from this generator passes first through the current coil or ammeter element of an ohmmeter; the current then goes to a commutator mounted on the same shaft as the generator and is changed into commutated current of about 50 cycles per second. The current binding posts C_1 and C_2 are connected to the two energizing stakes. The two potential stakes are connected to the potential binding posts P_1 and P_2. The potential drop across these two stakes is measured by passing the current picked up by them through a second commutator, run synchronously with the first, which converts the current back into a uni-directional flow. The current then goes to the potential coil, or voltmeter element, of the ohmmeter.

The current coil and the potential coil of the ohmmeter are mounted on a common spindle (not shown in the sketch). The mounting is such that the torques exerted by the current and potential coil on the moving element of the ohmmeter oppose one another. Hence, the deflection of the ohmmeter element is proportional to the quotient of the potential divided by the current, and the scale of the Megger may be calibrated to read volts divided by amperes, i.e., ohms, directly. The instrument therefore indicates the effective *resistance* between terminals P_1 and P_2.

† B. Low, S. F. Kelly, W. B. Creagmile, "Applying the Megger Ground Tester in Electrical Exploration," *A.I.M.E. Geophysical Prospecting*, 1932, pp. 114-125.

Before calculating apparent resistivity values, various corrections must be applied to the E/I quotient. A small but appreciable current flows in the so-called potential circuit of the Megger, and this flow of current causes a potential drop due to electrode and circuit resistances. Corrections for these resistances are made by calibration. For one instrument, the following relationships were obtained:

TABLE 14

CALIBRATION DATA FOR MEGGER INSTRUMENT

Range in Ohms.	N Normal Resistance of Potential Circuit.	C Calibration Resistance.	Permissible Variation in Resistance for		
			1%	2%	3% error
0-3,000	150,000	2,000	500-3500	0-5000	0-6500
0- 300	50,000	1,000	500-1500	0-2000	0-2500
0- 30	20,000	400	200- 600	0- 800	0-1000
0- 3	10,000	300	200- 400	0- 500	0- 600

Normal resistance of potential circuit includes calibration resistance. Calibration resistance is the resistance assumed for the electrode stakes P_1 and P_2 in calibrating the instrument. Permissible variation means that the total resistance of the two electrodes must lie within the figures given for the error in the reading not to exceed the percentage indicated.

The procedure for correcting the electrode resistance is as follows:

P_1 and P_2 = actual resistances at potential stakes.

Actual resistance in potential circuit $= N + P_1 + P_2 - C$.

If $P_1 + P_2 = C$, the reading is correct.

If $P_1 + P_2$ is less than C, the reading is too high.

If $P_1 + P_2$ is greater than C, the reading is too low.

The correction factor is $\dfrac{N + P_1 + P_2 - C}{N} = F$

True resistance = Observed reading x F

$$\text{error} = F - 1 \times 100$$

Finally the corrected values of the *resistance* E/I may be changed to apparent resistivity values by the various formulas previously derived for the different electrode configurations wherein two current stakes and two potential electrodes are used.

RATIO INSTRUMENT

A recently developed vacuum tube instrument† for determining the ratio E/I possesses excellent flexibility, sensitivity, and ease of operation. Use is made of the linear relationship between the applied plate potential and the amplitude of the current in an oscillating vacuum tube circuit. In the usual geophysical application of the instrument, variations in the energizing current produce variations in the potential applied to the plate of an oscillating triode tube. These variations in plate potential cause corresponding variations in the oscillating current. The oscillating current is rectified and flows through a calibrated potentiometer. The potential drop across the potentiometer varies with the oscillatory current which, in turn, varies with the energizing current.

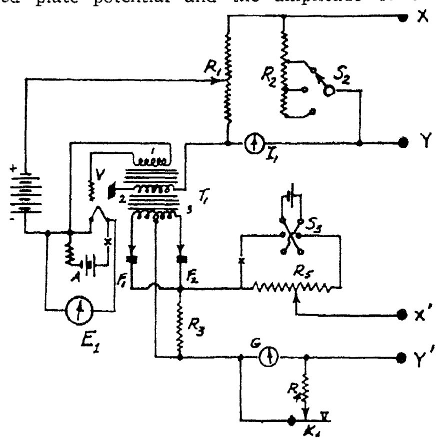

FIG. 193.—Circuit diagram for vacuum tube ratio instrument.

A schematic wiring diagram of the instrument is shown in Figure 193. The energizing electrodes and the power supply are connected to terminals X and Y which are connected to a heavy wire potentiometer R_1. A multipoint switch S_2 and resistor R_2 provide shunts for higher current ratings. The current through R_1 is indicated by the milliammeter I_1. The vacuum triode V is connected as an oscillator through transformer T_1. The potential applied to the plate of this oscillator is that of the battery B plus the potential drop across a desired part of the potentiometer R_1. Transformer T_1 must be of the shielded type, with special precautions taken to prevent leakage between the energizing current circuit which is connected to windings 1 and 2 and the potential circuit which is connected to winding 3.

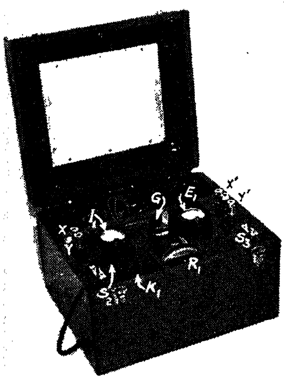

FIG. 194.—Vacuum tube ratio instrument. X and Y, current terminals; X' and Y', potential terminals; S_3, neutralizer reversing switch; E_1, filament and plate voltmeter; I_1, current meter; G, galvanometer; S_2, range switch for I_1.

Winding 3 of the transformer is connected to a full wave rectifier (of the copper-oxide type), and the rectified output passes through R_3. The drop across R_3 furnishes the potential for balancing against the E.M.F. existing between the two potential electrodes which are connected to the terminals X' and Y' of the diagram. The potentiometer R_5, with its small battery and reversing switch S_3, is used: (1) to neutralize the natural ground potentials and the potential drop across R_3 due to minimum plate current flow of the oscillator and (2) to compensate for the drift (usually negligible) of the oscillator output. The sensitivity of the galvanometer G is increased by depressing the closed circuit switch K_1 which removes the shunt R_4 from the galvanometer circuit.

† J. J. Jakosky, "Method and Apparatus for Electrical Exploration of the Subsurface," U. S. Patents 2,162,086; 2,162,087, issued June 13, 1939; "Method and Apparatus for Electrical Exploration of the Subsurface," Canadian Patent 374,475, issued June 14, 1938.

A photographic view of the instrument is shown in Figure 194. The panel controls are marked with the same designations used in the wiring diagram. Operation of the instrument is simple and rapid: (a) The desired current range is selected by means of the shunt switch S_2;* (b) the potentiometer R_5 is adjusted to neutralize the potentials existing in the potential electrode circuit; (c) the energizing circuit is closed, and the potentiometer R_1 is adjusted to give a null reading or balance on the galvanometer G. The reading of the calibrated potentiometer R_1 for null reading of G is the desired E/I ratio.

The instrument may be used with direct, commutated, or alternating current. In the latter two cases, an alternating current galvanometer, preferably of the rectifier type, is employed.

Potential Gradient Methods.—The theory of these methods is analogous to that of the gradient methods in gravimetric and magnetic explorations. Successful utilization of the gradient of the electrical potential would allow the field data to be presented in such a manner that those conductors which are better than the regional average could be plotted as positive gradients while conductors which are lower than the regional average could be plotted as negative gradients. (The contact between formations of different electrical conductivities would produce an inflection or zero gradient in the gradient profiles.) Furthermore, if it could be assumed that there is no distortion of the equipotential bowls surrounding an energizing electrode, the depth to a formation boundary would be indicated as the distance of the point of zero gradient from the energizing electrode. In practice it is found that this distance is a fraction of the potential bowl radius. This fraction is unfortunately not a constant but varies with different resistivities of the two layers.

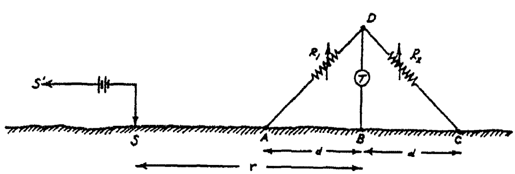

FIG. 195.—Potential drop ratio compensator (After Lundberg and Zuschlag, *A.I.M.E. Geophysical Prospecting*, 1932.)

In actual field application, these methods are subject to the same limitations as the resistivity methods, and the simple interpretation of the field data is applicable only in very limited cases.

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* The current range used depends on the resistivity and the contact conditions in the area where the measurements are being conducted.

Ratiometer Methods

In the ratiometer methods, potential drop ratio measurements are made with the aid of a modified bridge circuit.† A potential drop ratio compensator ‡ developed by Th. Zuschlag and called a "Racom" is illustrated schematically in Figure 195.

A source of power is connected to the current electrodes S and S', the electrode S' being located at such a distance that only the potential distribution due to electrode S need be considered. The bridge circuit makes contact with the ground at three points A, B, and C. The two ratio arms AD and DC contain variable known resistances R_1 and R_2 and the fixed contact resistances R_A and R_C of the electrodes A and C. When the bridge is balanced, i.e., zero deflection of the indicating instrument T, the following relation holds:

$$\frac{E_{AB}}{E_{BC}} = \frac{R_A + R_1}{R_C + R_2}$$

In this equation, the resistances R_A and R_C are unknown. Hence, a *compensation process* is introduced in order to obtain an expression for the potential drop ratio in terms of measurable quantities.

Suppose R_1 is arbitrarily changed to R_1', thus destroying the bridge balance. The bridge may be brought back to balance by adjusting R_2 to R_2'. The condition for balance becomes

$$\frac{E_{AB}}{E_{BC}} = \frac{R_A + R_1'}{R_C + R_2'}$$

On combining this equation with the last equation, we obtain

$$\frac{E_{AB}}{E_{BC}} = \frac{R_1' - R_1}{R_2' - R_2}$$

Although the contact resistances are eliminated from the final expression for the potential drop ratio, reliable results will be obtained only when contact resistances are negligible. This is true because the contact resistance is not a constant but is dependent on current flow and other factors which may change during a measurement.

The apparent resistivity at any station is proportioned to the product of the observed potential drop ratio and the resistivity of the surface layer.

The instrument described above is not suitable for use with alternating fields. A more recent form of the compensator is said to permit large phase adjustments and cover a wide range of contact resistances.

Low Frequency Potential Gradient Instruments

Many other potential gradient instruments have been developed § which utilize low frequency alternating current for energizing the ground. When low frequency energizing is used, the relatively detailed phase determinations that are required in the 500 and 250 cycle Ratiometers or Racoms are unnecessary, and depth penetration is increased. The circuit employed in a majority of these instruments corresponds to that of a simple alternating current bridge. Usually the zero instrument for indicating balance is a sensitive oscillograph galvanometer used with an amplifier.

As with other potential gradient instruments, depths to formation boundaries are indicated by zero gradients only if the layers are horizontal.

† C. Schlumberger, U. S. Patent 1,163,468.

H. Lundberg, "Potential Method for Elektrisk Malmletning," *Jernkontorets Ann.* 1919; see also *A.I.M.E. Geophysical Prospecting*, p. 50, 1932.

J. G. Königsberger, *Zeit für Geophysik*, 1930, Vol. 6.

A. Broughton Edge, British Patent Application 19120/30; see also Broughton Edge and Laby, *Geophysical Prospecting*, pp. 50-56 (Cambr. Univ. Press, 1931).

‡ H. Lundberg and Th. Zuschlag, "A New Development in Electrical Prospecting," *A.I.M.E. Geophysical Prospecting*, 1932, pp. 47-62.

§ For example, see C. A. Heiland, "Improvements in Geophysical Equipment for Foundation and Ground Water Research," presented at the A.I.M.E. meeting, New York City, February, 1937; "Method and Apparatus for Electrical Prospecting," U. S. Patent 2,189,377, issued Feb. 6, 1940.

Ground Current Compensation by Auxiliary Circuit.—As a general rule, the ground currents flow over large areas and show an approximately regional distribution. Advantage often may be taken of this fact to minimize ground potential variations in the measuring circuit.† A typical arrangement is illustrated schematically in Figure 196. Two potentiometers P_1 and P_2 which are mounted on the same control shaft C are connected to the three potential electrodes 3, 4, and 5. Electrodes 3 and 4 are so placed, with reference to the energizing electrode 1, that they are on the same equipotential line when energizing current flows in the power circuit. Electrodes 3 and 4 are spaced relatively close to 1 (usually less than ⅕ the total separation of the energizing electrodes) so that no significant shift of the equipotential line will take place when the energizing electrode 2 is moved. The spacing between 4 and 5 is some known

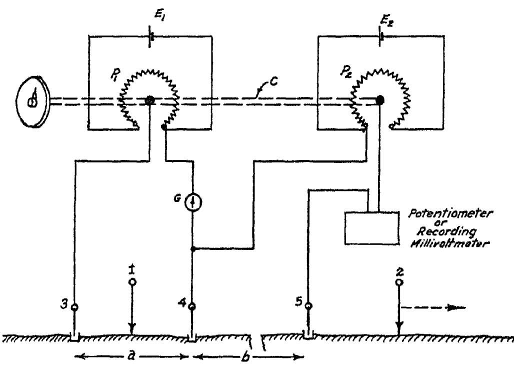


FIG. 196.—Auxiliary circuit for neutralizing variations in ground currents.

multiple of that between 3 and 4. The potentials E_2 and E_1 are in the same ratio as the electrode separations between 3 and 4 and between 4 and 5. Thus, if distance b is three times a, $E_2 = \frac{1}{3} E_1$. The distance b must be greater than a in order to minimize the effect produced by the movement of electrode 2 on the potential difference between 3 and 4.

Continuous adjustment of P_1, to keep the galvanometer G at balance, will automatically introduce a proportional neutralizing potential into the circuit containing P_2. Because the energizing current predominately affects the potential difference between 4 and 5, the readings in that circuit are employed for making the resistivity calculations. If desired, automatic compensation may be employed by means of a vacuum tube control wherein the potential difference between 3 and 4 is applied to the grid of the control tube, and the correspondingly varying output across a resistor in the plate circuit is connected to electrodes 4 and 5.

† J. J. Jakosky, "Method and Apparatus for Electrical Exploration of the Subsurface," U. S. Patent 2,162,087, issued June 13, 1939.

Continuous Recording Apparatus. — When continuous recording apparatus is used, the values of potential, current, and electrode movement are recorded photographically on a moving film. Upon completion of the run, the record is developed and the values of potential and current are scaled from the record and substituted into appropriate resistivity formulas for the particular electrode spacing employed. Graphs are then made of the apparent resistivity values at various electrode separations, ordinarily at regular intervals of about 50 feet, versus the traverse distance.

A portion of a typical field record which shows instantaneous values of potential E and corresponding values of the current I is given in Figure 197. The vertical lines mark each ten-foot movement of the mobile electrode.

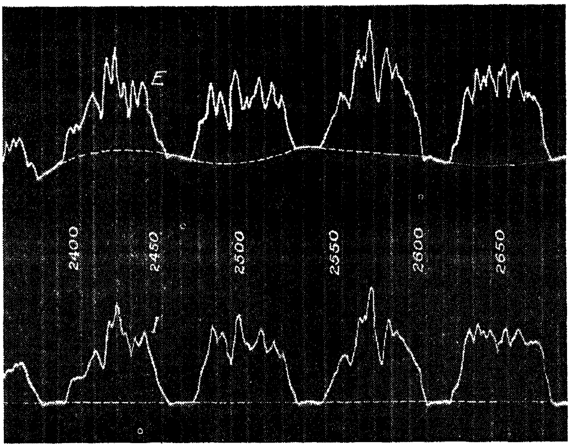


FIG. 197.—Typical record showing instantaneous values of potential E and corresponding values of the current I.

The current for energizing the ground is a uni-directional pulse having a controlled time interval † to govern polarization. During the intervals of time when the energizing current is off, the potential recorded is that created by the natural ground currents. Variations in natural ground potentials are shown by a curve which connects points obtained when the energizing circuit is open.

A view of the complete equipment for continuous profiling is shown in Figure 198. ‡ Cabinet 1 contains controls for the recording camera, the potential and current galvanometers, and a carrier-current relay-

† J. J. Jakosky, "Method for Determining Underground Structure," U. S. Patent 2,015,401, ıed Sept. 24, 1935.
‡ J. J. Jakosky, "Continuous Electrical Profiling," *Geophysics,* March 1938, pp. 130-153.

actuated measuring light; cabinet 2 contains the power control panel, the field rheostat for the generator, ratio instrument, etc.; cabinet 3 contains the direct current amplifier for the ground potentials; cabinet 4 contains telephone and signal apparatus, for communication between mobile and stationary electrode operators. The generators have a 2000 volt 2 ampere rating and may be connected in series or parallel. The control 6 governs the speed of the truck motor; the latter drives the generators through a conventional power take-off.

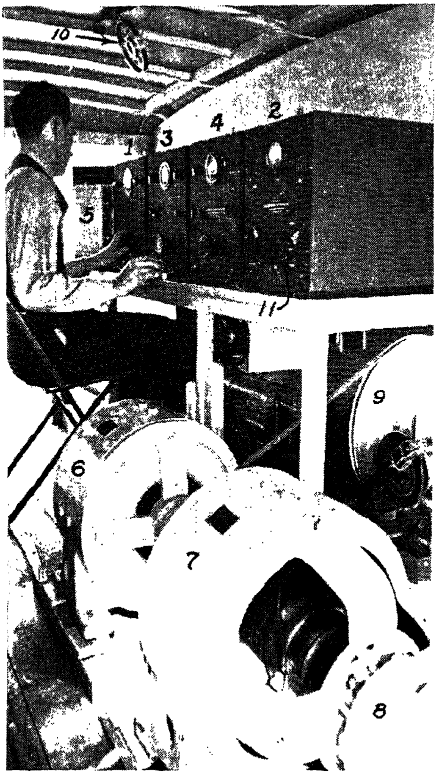


FIG. 198.—Complete equipment for continuous profiling: (1) control cabinet for recording equipment, (2) power control apparatus, (3) direct current amplifier for earth potentials, (4) communication, (5) recording camera, (6) and (7) high voltage direct current generators, (8) excitor, (9) commutator for pulse frequency, (10) microphone for communication with mobile electrode, (11) throttle control for truck engine driving the generators. (Courtesy International Geophysics, Inc.)

The recording camera, which is similar in design to the camera described below for seismic work, employs two recording traces and the distance measuring lines, as shown in Figure 197.

Transient Method.—When a *varying* current is caused to flow in the subsurface between grounded current electrodes, magnetic and electric fields varying with time are produced at the surface. These fields induce voltages in the potential or measuring circuit, which are added to the earth-conductivity voltages.

Consider, for example, that the electrode configuration comprises four electrodes with the two potential electrodes located on the *extension* of an imaginary line through the current electrodes.† Let $A(t)$ denote the transient voltage measured at the potential electrodes due to one ampere of direct current suddenly applied between the current electrodes and let $A'(t)$ denote the derivative of $A(t)$ with respect to time. ($A(t)$ and $A'(t)$ are simply the voltage oscillograms that are obtained during

† Gifford White, "Application of Rapid Current Surges to Electric Transient Prospecting," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 1216, 1940.

the measurements in surveys by the transient method.) It may be shown that if either of these functions is known the voltage due to any other driving current, however arbitrary, may be computed. That is, a single measurement of $A(t)$ or $A'(t)$ is sufficient to compute the response that would be observed for any other applied current surge, provided only that the surge is sufficiently short.*

In an early application of this method † it was desired to record the transient characteristics obtained when a direct current was established through a path, a portion of which included the area under investigation. For a single layer, a direct current transient will have a time constant ‡

$$T = \frac{Ah}{R}$$

where A is a constant, h the depth of the layer, and R the resistance of the layer.

Blau postulated that if more than one layer were present the build-up time for the first layer would be relatively unaffected, and each subsequent layer would contribute a transient whose duration would be proportional to its thickness. The various transient effects would therefore be superposed on one another and would produce a composite transient composed of "ripples." The various ripples would thus be indicative of particular subterranean strata. It was futher postulated that a thin layer would create a short ripple on the transient curve while a thick layer would create a long ripple.** Lateral changes in the thickness of any stratum would be indicated by corresponding changes in the time interval of the ripples observed at different locations.

Because the transient methods utilize a surge of current into the ground, studies may be made of both the time constant and the ratio of E/I. A paper by Karcher and McDermott § gave the results of experimental work in determining time constants and resistivities using a fixed current electrode spread and outlined the general limitations of the work.

* It may be shown also that the ordinary direct-current resistivity is equal to the area of the $A(t)$ oscillogram, measured by a planimeter or other means, divided by the total electric charge Q of the surge. This is true irrespective of the slowness or rapidity of the surge. Hence, by employing a surge generator to supply power and a ballistic type of meter to integrate the amplified voltage transient, readings can be taken rapidly and accurately without using calibrated potentiometers, non-polarizing electrodes, or photographic recordings.

† L. W. Blau, "Method and Apparatus for Geophysical Exploration," U. S. Patent 1,911,137, issued May 23, 1933.

L. W. Blau and L. Statham, "Apparatus for Recording Earth Current Transients," U. S. Patent 2,079,103, issued May 4, 1937.

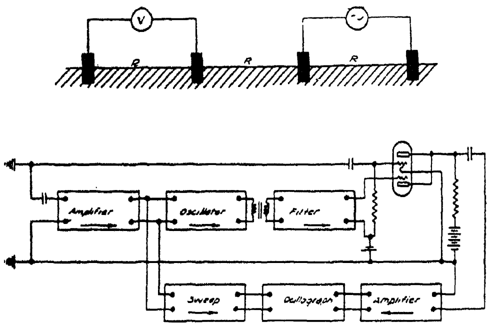
‡ Franz Ollendorff, "Elektromagnetische Ausgleichsvorgänge in Geschichtetem Erdreich," *Archiv Für Elektro-Technik*, Vol. 23, No. 3, pp. 261-278, 1930.

** The data would be plotted with the time duration of the ripple as ordinate and the distance on the surface from the point of reference as the abscissa, just as in seismic work where the times of arrival are plotted against distance from the point of reference.

§ J. C. Karcher and E. McDermott, "Deep Electrical Prospecting," *A.A.P.G. Bull.*, vol. 19, Jan. 1935, pp. 65-77.

A subsequent paper by Statham † described a modified configuration utilizing two current circuits so positioned that their effects nullified each other.

Investigations using alternating current of rectangular or other non-sinusoidal wave form were described by West. ‡ In this work, the two detecting electrodes were placed in line with, but outside of, the current electrodes. The changes in wave form caused by anomalous subsurface structures were determined by a null method in which the surface potential transient was balanced against an electromotive force produced by passing the output of an oscillator through an adjustable network. The electrical apparatus used by West is shown schematically in Figure 199.



B

FIG. 199.—Electrode and apparatus for transient prospecting studies. *A* shows the electrode configuration with the detecting electrodes at the left and the current electrodes at the right. *R* is usually 1000 feet. *B* shows the detecting and analyzing circuit, which is represented in diagram *A* as a simple voltmeter. Arrows indicate direction of control or current propagation. (West, *Geophysics.*)

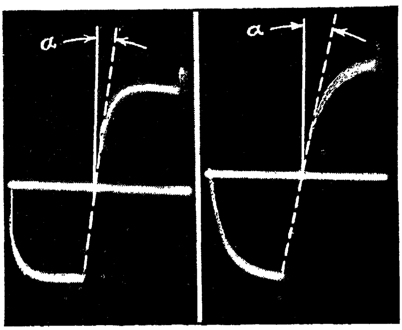
Studies were conducted by Hawley § to determine the characteristics of the transients associated with the flow of current when a source of constant voltage is suddenly applied between two electrodes imbedded in the ground. In particular, detailed studies were made of: (a) the transient in the current circuit and (b) the potential transient in the circuit which includes the potential electrodes. The chief purpose of the investigations was to determine whether the transients contained ripples which were associated with anomalous subsurface structural conditions.

By using a camera of the high speed revolving drum type to photograph the spot of light produced by a cathode-ray oscillograph tube, Hawley showed that the current build-up time in the energizing

† L. Statham, "Electric Earth Transients in Geophysical Prospecting," *Geophysics*, vol. 1, No. eb. 1936, pp. 271-277.
‡ S. S. West, "Electrical Prospecting with Non-Sinusoidal Alternating Currents," *Geophysics*, 3, No. 4, October 1938, pp. 306-314.
§ Paul F. Hawley, "Transients in Electrical Prospecting," *Geophysics*, vol. 3, No. 3, July

circuit varied from 30 microseconds for a 12,000 foot spread to 4.5 microseconds for a 3,000 foot spread. The voltage transient surge showed an almost instantaneous rise to its maximum value, followed by an exponential decrease. The voltage and current transients are typical of inductive, highly damped circuits. The work did not show the presence of any ripples on any of the curves, even with recording speeds sufficiently great to reveal ripples having a duration of only 10 microseconds.

In the present utilization of the transient methods, the original concept of the presence of ripples and their diagnostic value apparently has been abandoned, and interpretation is based on fundamental resistivity relationships. The spread of current through a bed of high resistivity is much faster than that through a bed of low resistivity. As a result, the current transient in a medium of high resistivity is steeper (less time interval) than that in a medium of low resistivity. The time constant and the shape of the wave therefore furnish a means for determining the relative resistivities of the materials included within the effective path of current flow. Two typical transient curves are shown in Figure 200.† The electrode and apparatus arrangement are essentially the same as that illustrated in Figure 199.



(a) (b)

FIG. 200.—Typical transient curves for (a) material of high resistivity and (b) material of low resistivity. (After Steinmann, *The Oil and Gas Journal.*)

As previously mentioned, the photographic or oscillographic method for measuring the wave form of reoccuring phenomena has an advantage in eliminating the effects of extraneous disturbances. For this work Klipsch ‡ stops down the recording camera lens until each single sweep of the oscillograph ray gives a faint image. The wave is repeated a sufficient number of times to build up a well exposed negative. If no extraneous potentials are present, the reoccurring wave pattern will produce a sharp, dense wave image, while if undesirable potentials are present (caused by earth currents, stray or line leakage, induction, etc.), the picture of the wave form will be a less distinct pattern, with the greatest exposure along the path of the reoccurring wave form. Analysis is based upon the more dense wave pattern thereby minimizing the effects of extraneous non-cyclic potentials.

† K. W. Steinmann, "Use of the Transient and Soil Analysis Methods in the Search for Oil," *The Oil and Gas Journal,* July 27, 1939, pp. 85-87.

‡ P. W. Klipsch, "Recent Developments in Eltran Prospecting," *Geophysics,* Oct. 1939, Vol. IV, No. 4, pp. 283-291.

A manual method of measuring the characteristics of the transient has been developed wherein the distortion of the wave form produced by the shallow surface layers is determined by employing an auxiliary circuit to transform the incoming potential to an easily recognized form (straight line) and measuring the additional distortion required. †

Studies have been made by various investigators to determine the relative merits of the transient and the direct-current steady-state resistivity measurements. The steady-state measurements usually give more reproduceable results, due to the absence of inductive and current redistribution effects necessarily present with the use of transients. Figure 201 shows

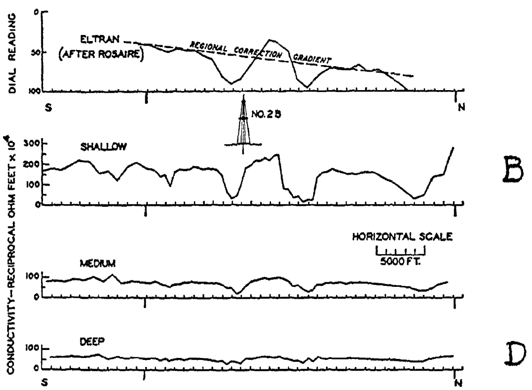


FIG. 201.—Comparison of a transient and a direct-current resistivity survey in Louisiana. *A*, transient data; *B*, *C*, *D*, direct current resistivity data for shallow, medium, and deep penetrations, respectively. (Blondeau, *Geophysics*.)

the comparison obtained by Blondeau ‡ between a transient and a direct-current resistivity survey in Louisiana. Curve *A* shows the transient results, while curves *B*, *C*, and *D* show the direct-current resistivity results at increasingly greater depths of penetration. The electrode configuration for curve *B* gave a current penetration of approximately 200 feet, which compares favorably with the results obtained by the transient method in curve *A*. The electrode spacing employed for the Eltran work would have given a much greater current penetration than any employed for the steady-state work if the penetration of the transient current were comparable to the steady-state current. Blondeau concludes that "the large observed variations in near surface resistivity can hardly be attributed to mineralization emanating upward over structure; and that the number of anomalies found is so great that their direct association with deep structure is highly improbable."

† R. Saibara, S. Bilinsky and W. G. McLarry, "Exploration by Incremented Wave Distortion," U. S. Patent 2,177,346, issued Oct. 24, 1939.

‡ E. E. Blondeau, "Shallow Resistivity Survey at South Elton, Louisiana," *Geophysics*, Oct. 1939. Vol. IV. No. 4. pp. 271-278.

APPLICATIONS OF GEOELECTRIC METHODS

The fields of applications of geoelectrical methods may be classified as follows: (1) prospecting for highly conductive materials; (2) structural investigations, which include prospecting for placer deposits, water supply problems, civil engineering construction problems, and oil structural mapping.

Prospecting for Highly Conductive Materials.—This application includes prospecting for certain veins, lodes, and dikes. Results obtained in vein deposit investigations will be discussed here.*

Vein Deposits

Geophysical methods are of special significance in the initial evaluation of a prospect, because they yield important data regarding subsurface conditions before purchasing or opening the property.

Prospecting for ore bodies should be done with the viewpoint that a great majority of the properties examined will be abandoned. This viewpoint implies that the risk in examining geophysically and geologically a large number of prospects is less than that in attempting to find an ore body by extensive development work on one property. The soundness of this concept may be illustrated by a practical case of a prospect in Sonora, Mexico. †

In a region characterized by extremely steep topography which is broken in many places by vertical cliffs, a 25-foot, copper-bearing ledge is exposed at the surface for a distance of 500 feet. (See upper part of Figure 202.) To the south, it is cut off by a fault having considerable displacement. To the north, it was traced to station E-1, where it is covered by talus and detritus.

The surrounding country rock is composed chiefly of an old series of sediments, portions of which had been highly silicified. These rocks in all probability are of pre-Cambrian age.

Prior to the geophysical investigations, a tunnel was driven 770 feet parallel to, and west of the ledge outcrop in an effort to cut the ledge at the 2250 foot elevation. Because ore was not encountered, the mine operators believed that the vein might dip farther to the west. A cross cut was driven westward for 310 feet, and failing to encounter the vein, an eastward cross cut was driven for 300 feet. At this stage of the development program, the geophysical work was employed.

The cross section in the lower portion of the figure is drawn from the results of the geophysical survey and shows the depth to which the ledge

* A general discussion of the applicability of geoelectrical methods in this field is given in the chapter entitled Geologic and Economic Background of Exploration Geophysics.

† J. J. Jakosky, "Geophysical Examination of Prospects," *Canad. Mining Journal*, Jan. 1934.

extends and its relation to the surrounding rock and structure. The limited amount of sulphide ore disclosed by the geophysical survey made the property of doubtful value due to high shipping and operating costs.

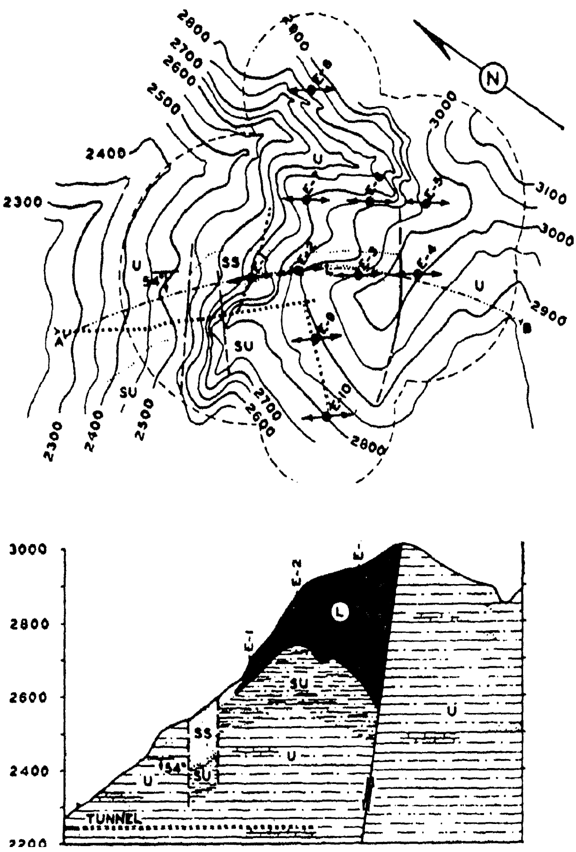


FIG. 202.—Cross section inferred from geophysical survey.
L—ledge
U—undifferentiated sediments
SU—silicified undifferentiated sediments
SS—sandstone
E—electrical stations
Electrical stations and topographic contours shown in upper portion of figure. (*Canad. Mining Journal.*)

Lateral Extension and Depth of Ore Body.—The application of geophysical work over known mineralized areas, where some development has already been done, allows important information to be obtained relative to the lateral extension of known ore bodies. Should the measurements indicate the possible extension of such bodies, it is economical to use a

few well-placed drill holes to check values. In many instances it has been found that where the geoelectrical work shows fairly constant magnitudes, the commercial values found in the drill holes may, for purposes of preliminary estimates, be used for estimating the deposit. Occasionally, the geophysical work is of value in indicating the nature of the deposit and may throw light on whether the deposit is of primary or secondary origin.

The most familiar geoelectrical result is that obtained over a highly conductive ore body of the sulphide type. When an electric current is passed through an area containing a conductive ore body, a decided change in values will be noted when in the vicinity of the ore body. A typical example is illustrated in Figure 203, which shows the general

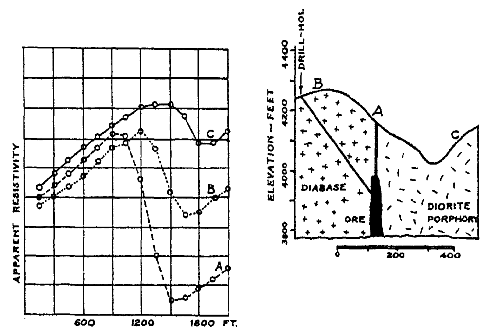


Fig. 203.—Variations of apparent resistivity on a traverse across a highly conductive ore body. (*Canad. Min. Journal.*)

surface conditions and the locations of the resistivity stations. The oxidized portion of a mineralized fracture zone between the diabase and the diorite porphyry outcropped at *A*. This vein showed good values in copper but was too narrow for economical mining, and no assurance could be had that commercial quantities of sulphide ores would be found at depth. A geoelectrical study of the area gave the results shown by the curves on the left of the figure. Because of the lower resistivity of the diabase, curve *B* starts with a lower initial value than either *A* or *C*. Because of the relatively low conductivity of the diorite porphyry, curve *C* has a higher initial apparent resistivity value than curves *A* or *B*, and maintains this higher value over the entire range of measurement. Under the geologic conditions existing in this district, it appeared most logical to assume that the drops in the resistivity profiles were due to the presence of a sulphide zone at a depth of about 300 feet.* Hence, diamond drilling was recommended and the hole shown in the figure penetrated a body of sulphides with high copper and silver values.

* Based on a depth of penetration of approximately 0.3 the current electrode separation.

Prospecting for Placer Deposits. †—This work may be illustrated by a description of a survey conducted in Trinity County, California. The property consisted of 10,000 acres of gravels which, on the surface and in occasional test pits, panned uniformly in gold. The problem was first to select, by a geological study and by preliminary geophysical work,

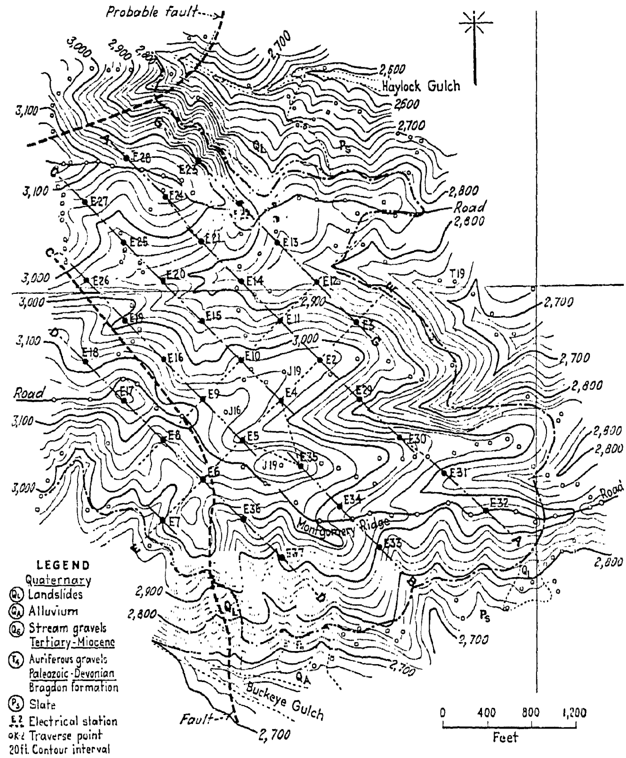


Fig. 204.—Map of placer area in Trinity County, California, showing surface geology and topography. (*Engineering and Mining Journal,* Feb. 1934.)

what part of the total acreage should first be studied; then, by detailed geological and geophysical work on that restricted acreage, to outline the gravels, give their depth, the contour of bedrock and attitude of bedrock surface, and if possible, locate the likely areas for gold concentration.* As a result of the survey, recommendations for further development and exploration were to be made.

† J. J. Jakosky and C. H. Wilson, "Examining a Placer by Geophysical Methods," *Engineering and Mining Journal,* February, 1934.

J. J. Jakosky and C. H. Wilson, "Geophysical Studies in Placer and Water Supply Problems," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 515, 1933.

* The likely areas for gold concentration would be indicated by "black sand" concentrations.

After careful geological reconnaissance of the whole property, between 500 and 600 acres were selected for detailed work. The lateral extent of the gravels, the areal geology, and the topography of this area were first determined by alidade and plane table and were then plotted in detail, as shown in Figure 204.

Magnetic measurements of the vertical intensity were made at 242 stations over the area, at intervals from 300 to 400 feet. Magnetic equipment for this work was the conventional Schmidt-type field balance. A study of the geological and magnetic maps did not allow a unique interpretation either as regards major concentration zones of the "black sands" or of bedrock conditions. Hence, the magnetic and geological work was supplemented with electrical studies.

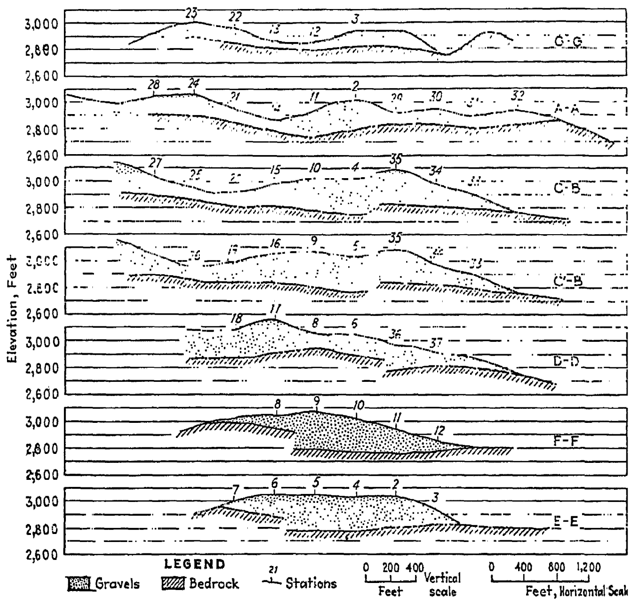


FIG. 205.—Cross sections of placer deposit along traverse lines. (*Engineering and Mining Journal*, Feb. 1934.)

Electrical resistivity measurements were made at 37 stations distributed at regular intervals. Proper spacing of stations is a matter of experience and is governed by the local geologic and topographic conditions. In this instance, most of the stations were located at 600-foot intervals along traverses having a northeast-southwest direction. In the southwestern portion, however, the stations were placed at 800-foot intervals, because location of the gravel contact was known on three sides from the surface evidence, and closer spacing was unnecessary. Resistivity measurements

were made with the constant potential apparatus and the five-electrode configuration previously described. Three curves were plotted for each station. All electrical curves showed the variation in apparent resistivity from the surface to an indicated depth of 600 feet. The first or main curve showed the variation in apparent resistivity along the traverse line. The two auxiliary resistivity curves showed the variations in two directions from the station and were used for evaluating topographic and near-surface effects.

Because of the difference in resistivity between the gravels and the more compact formations, such as the bedrock, the boundary or contact between the two materials at depth is indicated on the curves by a change in trend. The depth to bedrock at each station was determined by Tagg's method (p. 299) and correlation of curves (p. 317) using known geological conditions encountered in exploratory openings and projection of outcrops as control. By correlating the computed depths with the surface trace of the gravel-bedrock contact and with the surface elevation shown by the topographic survey, it was possible to plot subsurface bedrock contours.* The maximum thickness of gravels on this property was found to be about 300 feet.

The results of the electrical survey are illustrated in Figure 205 which shows cross sections of the deposit along the traverse lines. From these sections, the approximate total yardage of gravels was calculated. A striking feature of these sections is the evidence of faulting in the bedrock. In particular, sections *D-D*, *F-F*, and *E-E* indicate an escarpment in the bedrock of more than 100 feet average vertical displacement. Prior to the geophysical work, this faulting had not been suspected, because the trace of the fault was covered by alluvium over practically the whole property. After discovery of the fault by geophysical methods, its trace was found in a gulch at one point where erosion had slightly exposed it.

The knowledge of this escarpment materially affected plans for hydraulic mining. Before the geophysical survey was made, the mining company had perfected plans for beginning hydraulic operations in the northeast part of the area, because surface topography indicated that the bedrock most likely sloped toward this gulch. When the results of the geophysical work were known, the operations at that point were abandoned, and plans were made for work in the southern part of the area.

The complete bedrock contours predicted on the basis of the data obtained in the geophysical investigations are shown in Figure 206.

Water Supply Problems. †—As regards the application of geophysical work to water supply, the geological considerations are somewhat

* The depth to bedrock given by electrical work is an average or effective depth over an area having dimensions comparable to the depth of penetration.

† J. J. Jakosky and C. H. Wilson, "Geophysical Studies in Placer and Water Supply Problems," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 515, February, 1934.

similar to those involved in placer investigations. In general, the problems may be divided into two classes: (1) the determination of the existence of a permanent subsurface water table either in fill material (gravels,

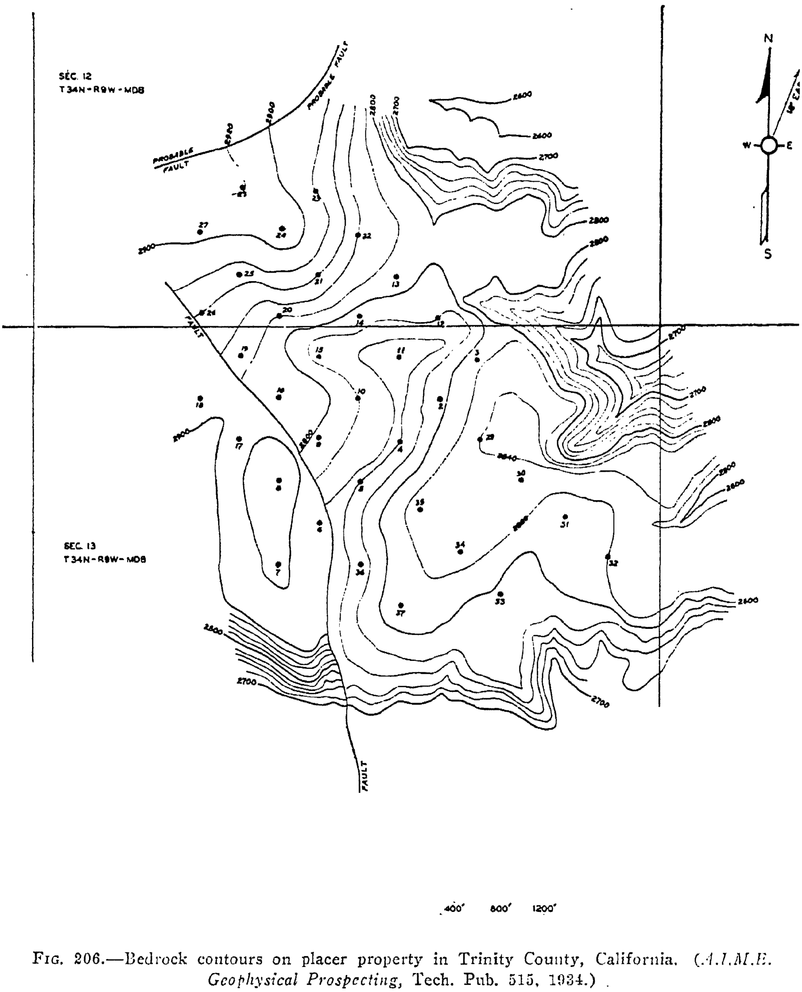


FIG. 206.—Bedrock contours on placer property in Trinity County, California. (*A.I.M.E. Geophysical Prospecting*, Tech. Pub. 515, 1934.)

sands, etc.) or more consolidated rock (2) the determination of subsurface structure that would affect ground water accumulations. In water supply investigations, as in other problems, the geophysical work is facilitated by direct correlation of the geophysical data with well logs.

Geophysical investigations for several different types of water supply problems are illustrated by descriptions of the following actual surveys.

In connection with some experiments * at the University of Arizona, water supply investigations were carried out on the Santa Rita Range Reserve at the northern end of the Santa Rita Mountains, southeast of Tucson, Arizona. The investigations were conducted in adjacent canyons or washes; namely, Saw Mill canyon, in which a small supply of water was already developed through a well to the bedrock, and in Florida canyon. The two areas have somewhat similar surface geologic and topographic features. The object of the geophysical work in the Florida canyon area was a determination of the depth and contour of the bedrock across the canyon and the electrical characteristics of the overlying fill material in order to establish the presence, or absence, of a water supply comparable to the known supply in Saw Mill canyon.

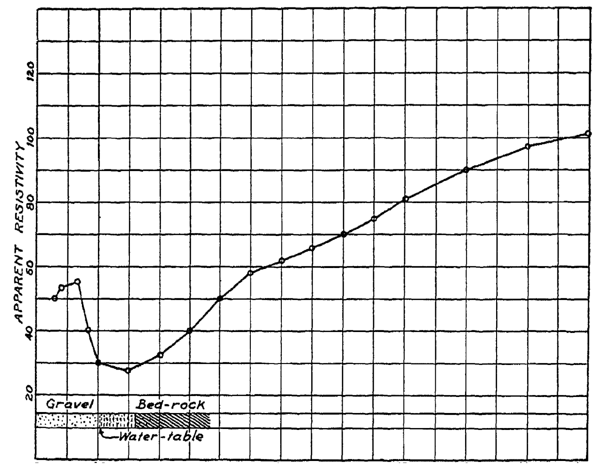


FIG. 207.—Curve showing effects of water-level and bedrock on resistivity. (*A.I.M.E. Geophysical Prospecting,* Tech. Pub. 515, 1934.)

Resistivity measurements in both areas produced results of the type shown in Figure 207. The curves showed inflections corresponding to (1) change from dry fill and stream wash to wet fill and top of water horizon and (2) change from fill material to the underlying bedrock. At this station the geophysical work placed the depth to the water level at 20 feet and the depth to bedrock at 29 feet. Direct measurement in an adjacent drill hole gave the water level as 20 feet; the depth to bedrock, 26 feet.

A combination of structural work and direct location of water table is illustrated by a survey conducted near Yuma, Arizona, for a mining company. The presence of a permanent water table in this area was known. The problem was to locate the nearest point to the mine at which

* Arranged by Dr. G. M. Butler, Dean of the College of Mines and Engineering, University of Arizona, Tucson.

a well would intersect the permanent water table in porous alluvium above the impervious bedrock which sloped downward from the foothills. Figure 208 summarizes the general results secured by the electrical survey. A well drilled near station 10 encountered the water table as predicted by curve correlation. According to the mine management, the well is capable of producing in excess of 500 gallons per minute.

In 1931 a municipality in Southern California was threatened with litigation by a group of land owners near the mouth of a river on which upstream development by the city was removing considerable water from the surface and subsurface stream flow.

It was the contention of the land owners that removal of the water from the subsurface channel would allow infiltration of sea water from

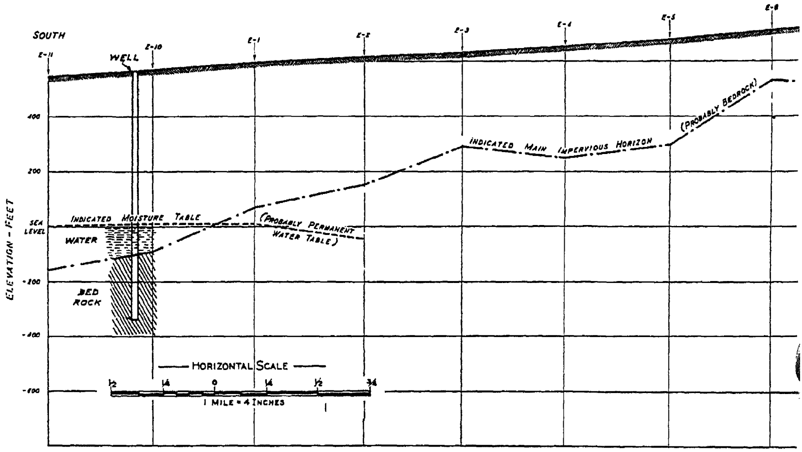


FIG. 208.—Location of water table and development of water supply, Yuma, Arizona.

the ocean which would be detrimental to their crops. The city claimed that a natural subsurface rock barrier existed across the narrow portion of the river valley and that this would effectively halt any damaging influx of sea water.

It was necessary to obtain data on the depth to bedrock and character of the subsurface contour of an old stream channel. The most obvious method was to secure data allowing the plotting of a subsurface contour map. These data could be obtained by a drilling program, or by employment of a geophysical survey over the area. The latter was chosen, and at 50 selected stations covering the area, geoelectrical data were obtained showing depth to bedrock, and depth to the boundary of coarse gravel, and finer silt and clay. These "electrical logs" were obtained at a small fraction of the cost that would have been required for ordinary drilling. In addition, magnetic work over the area as a part of the survey disclosed evidence on the character of the probable subsurface conditions underlying the stream valley.

This survey also illustrates the use of electrical methods in determining distribution of pervious and impervious materials. Figure 209 shows a typical profile of the subsurface conditions as disclosed by the geophysical work. The upper soil and clays are relatively impervious, while the lower gravels are very pervious and allow flow of a subterranean stream during the dryer portion of the year.

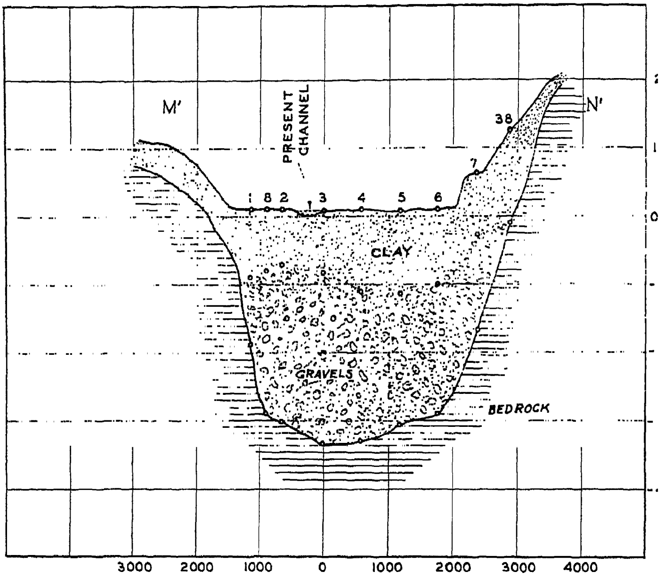


Fig. 209.—Bedrock contours and typical subsurface profile, Santa Ynez River, California. (*The Mining Journal*, Jan. 15, 1931.)

Civil Engineering Construction Problems.—Geophysical investigations are of especial value in the following engineering construction problems: (1) examination and location of dam and reservoir sites; (2) location of water wells; (3) examination and location of aqueduct and pipe line routes; (4) location of rock and gravel quarries; and (5) tunnel construction.

The application of geoelectrical methods will be illustrated by descriptions of two representative surveys: (1) a survey conducted at the Fort Peck Dam and Reservoir, Montana, and (2) a survey conducted in the vicinity of the Gillespie Dam on the Gila River near Phoenix, Arizona.

Fort Peck Dam

The primary purposes of the Fort Peck geophysical investigations were to determine: (1) the major structural features of the sedimentary rocks underlying the dam and reservoir sites, with special reference to the location of fault zones or other structural conditions that would influence

the location or construction of the proposed dam; (2) the thickness of overburden and fill materials overlying the shale bedrock at the dam site and the thickness of fill materials in the topographic depressions along the reservoir rims; (3) depth to and subsurface contour of the top of the Bearpaw shale (bedrock) at the dam site and along the reservoir rims; and (4) depths to and elevations of ground water levels in the proposed dam abutments and in the reservoir rims.

A generalized stratigraphic section of the area is shown in the right-hand portion of Figure 210. The surface is usually covered with clay which is oftentimes sandy. Beneath this clay layer is found glacial drift containing small or large boulders in a matrix of varying proportions of sand, clay, and small gravel. Underneath the glacial drift is a layer of clay, also often of a sandy nature. Beneath this clay layer there

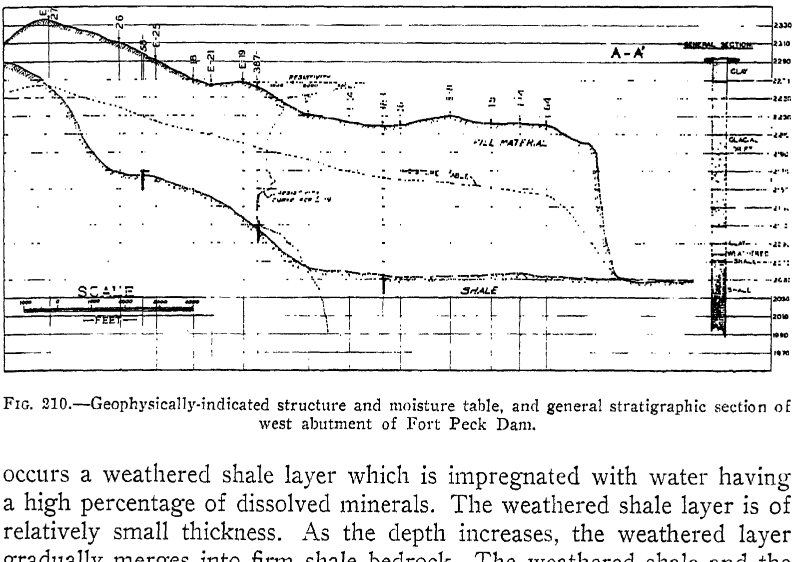
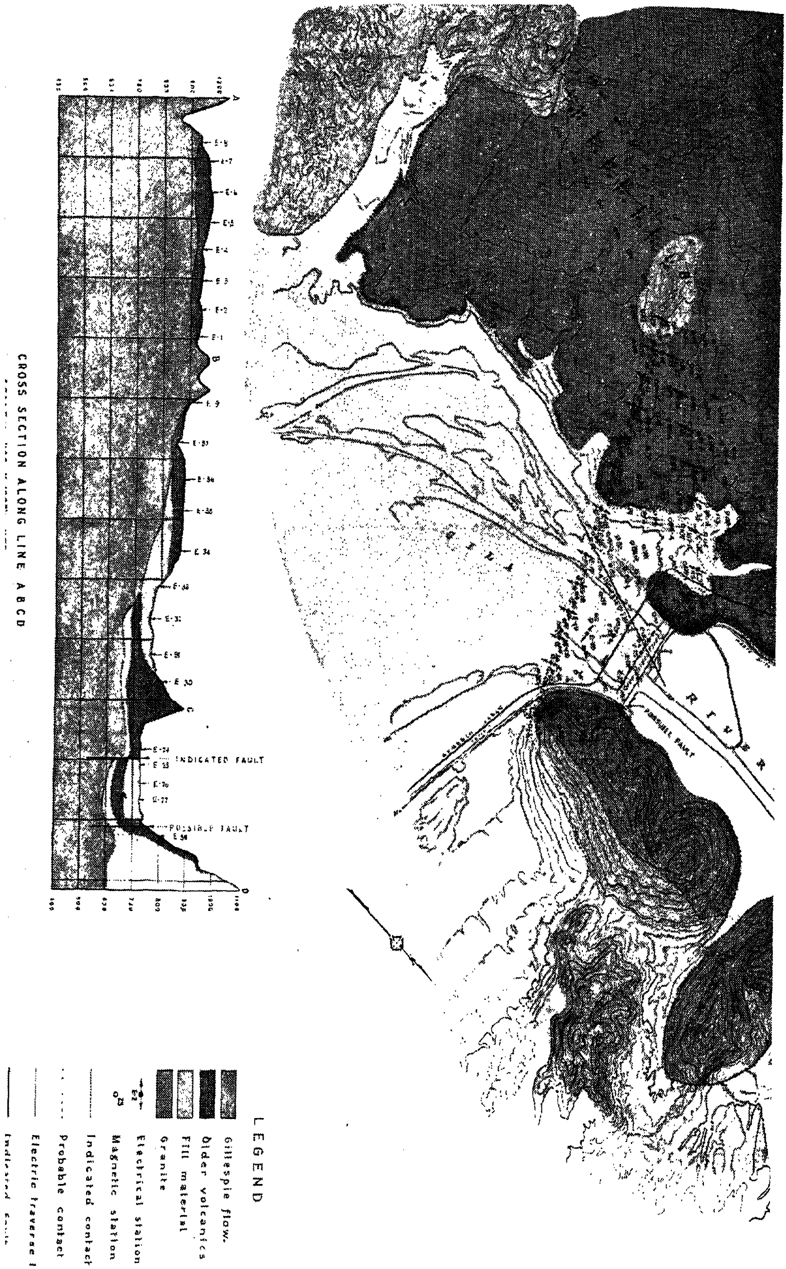


FIG. 210.—Geophysically-indicated structure and moisture table, and general stratigraphic section of west abutment of Fort Peck Dam.

occurs a weathered shale layer which is impregnated with water having a high percentage of dissolved minerals. The weathered shale layer is of relatively small thickness. As the depth increases, the weathered layer gradually merges into firm shale bedrock. The weathered shale and the firm shale layers constitute the upper portion of the Bearpaw shale. For purposes of dam site construction, the geophysical interpretation was directed toward determining the bottom of the weathered shale layer, rather than the theoretical "top" of the Bearpaw formation.

The typical stratigraphic section just described varies to a certain extent throughout the area, and in some cases there is a fairly distinct contact between the glacial drift and the shale formations, with no intervening clay, sand, or appreciable weathered shale formation.

Typical results of the geophysical studies conducted on the west abutment are shown graphically in Figure 210 and the survey may be summarized as follows:


CROSS SECTION ALONG LINE A B C D
A
B
C
D
INDICATED FAULT
POSSIBLE FAULT
RIVER
LEGEND
Gillespie flow
Older volcanics
Fill material
Granite
Electrical station
Magnetic station
Indicated contact
Probable contact
Electric traverse line

Structure

At the dam site, the correlation of isolated outcrops with the geophysical studies at the various geophysical stations indicates that the Bearpaw shale is quite uniform and that it dips slightly to the east, in conformity with the recognized regional dip. The geophysical work disclosed no evidence of abnormal structural conditions, such as major folding or faulting on the west abutment. Localized faulting is evidenced by small displacements of seams in the shale (six inches or less). However, as most of the exposures are on steep slopes, many of these minor displacements are probably due to hill-side "creep."

Bedrock Contours and Profiles

The bedrock contour map plotted from the geophysical data indicates that the overburden-shale contact is comparatively flat over a large central area, but dips steeply to the east in the western part of the area. There is some evidence that this flat central area may represent a bedrock valley or bench underlying the ridge. This indicated subsurface depression might possibly have existed as a pre-glacial drainage channel. The uniform attitude of the contact underneath this portion of the west abutment ridge is probably a unique and local condition. Geophysical results on other parts of the area indicate that the shale surface is quite irregular.

Ground-Water Table

The geophysically-indicated moisture table in the west abutment area is shown in the profile. This moisture table marks the approximate location of the transition zone between the relatively dry near-surface material and the moisture-impregnated deeper material.

Comparison of Geophysical Results with Logs of Core Holes

To provide a basis for correlation of the electrical bedrock determinations, preliminary studies were conducted at existing core hole locations for which logs were available. As a consequence of these preliminary studies, direct correlation could be made between the electrically indicated depth to firm shale and the depth as indicated by drillers' logs. The electrical studies were then utilized to extend the depth determinations into adjacent areas.

The close check between the electrical determinations and drill hole results is shown on Profile *A-A'* on which subsequently determined bedrock depths are plotted for drill holes 58, 387, and 383.

Gillespie Dam

The survey* conducted on the Gila River is an example of the use of the electrical method to investigate complex local geology at the site of the

* For permission to publish this material, the author is indebted to Raymond A. Hill, Supervising Engineer of Quinton, Code and Hill; Leeds and Barnard, Engineers, Consolidated, Los Angeles; Salt River Valley Water Users Association, Phoenix, Arizona.

Gillespie Dam. Figure 211 shows a portion of the area covered and a typical cross section obtained during the survey.

The Gila River flows between high lava abutments situated in Gillespie Gap, a natural pass formed by the Gila Mountains to the west and the Buckeye Hills to the east. The present course of the river is controlled by the high granitic peaks of the Buckeye Hills on the east and a flow of late Tertiary lava, known as the "Gillespie Flow" on the west. The Gila River channel, both ancient and present, is filled with a series of volcanic flows and various phases of flood plain detritus deposited by the river.

The granitic rocks and cropping on the east and west of the dam abutments and underlying the area of the "gap" form the boundaries which limit the flow of subsurface water. Above the granite, in order from oldest to youngest, lies a series of relatively more porous formations: (1) older volcanic rocks forming the dam abutments, (2) older flood-plain material, (3) younger volcanic rocks, (4) younger flood-plain material, and (5) recent alluvium.

The general purpose of the investigation was to determine the distribution and structural relationship of the above-named formations with particular attention to the effect they might have on subsurface flow of water. Specifically, it was desired to determine the location and cause of leakage that was taking place around the dam.

A study of the areal geology indicated that possible subsurface water flow was limited to two gaps bounded by the outcropping granite rocks: (1) that between the points where later were placed the geophysical stations *E*-1 and *E*-8, and (2) that extending from station *E*-9 eastward across the present channel of the Gila River to the granitic outcrop on the east abutment. The geophysical study was confined, therefore, to a study of subsurface conditions of these two locations.

The results of the geophysical studies are shown in Figure 211 by the cross section.

Cross section ABCD: From stations *E*-1 to *E*-8 the geophysical work indicated that the lava capping is relatively thin and that no appreciable amount of porous material such as gravel or alluvium lies between the lava and the underlying granite. From stations *E*-9 to *E*-23 a much more complicated subsurface condition was indicated. The most important structural features are the presence of the fault paralleling the stream channel and the presence of porous materials below the lava which forms the dam abutments.

The examination comprised geological, geolectrical, and geomagnetic studies. As a result of the work it appears that the leakage occurred through the subsurface underneath the dam. The leakage probably occurred through a layer of porous material which is indicated to underlie the lava flow upon which the dam is built.

Structural Investigations.

Structural Survey in East Vacuum Area, New Mexico

This survey was conducted during 1938 and is of particular interest because drilling subsequent to the survey has provided a good opportunity to evaluate the work.* Although complete structural information has not yet become avaliable, the present information shows a remarkably close agreement between the structure predicted from the geoelectrical survey and the structure as indicated by well log correlation. The accuracy of the geoelectrical work is evidenced by: (1) close agreement between

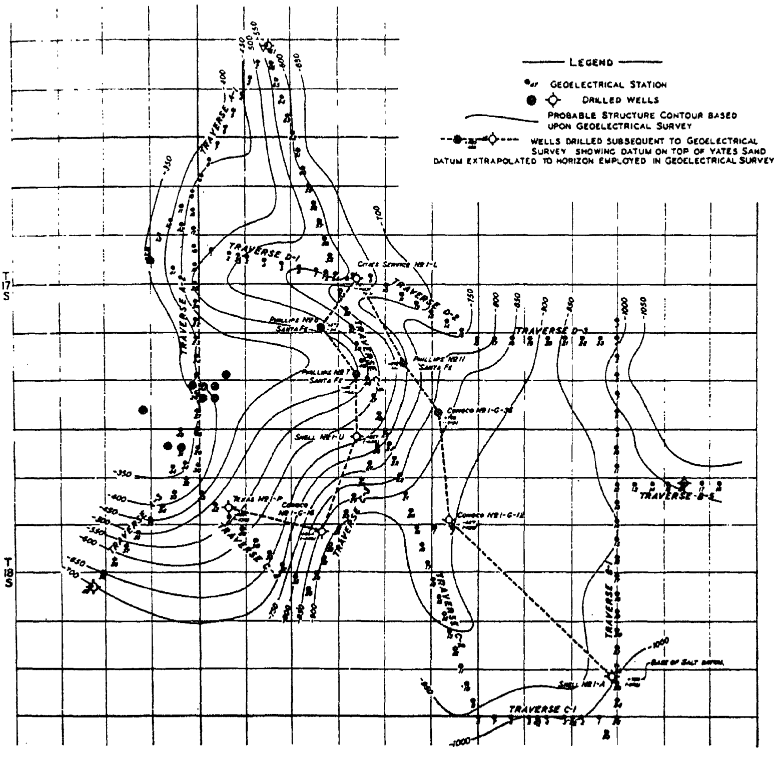


FIG. 212.—Structure contour map based on geophysical studies. (East Vacuum Area, Lea County, New Mexico.)

the proved boundaries of the oil pool and the probable boundary as predicted by structure contours plotted from the geoelectrical data, and (2) close agreement between the geoelectrical and the well log subsurface datums at the ten well locations for which information is available.

* The interpretation of the geophysical data utilized the method of curve correlation described on p. 317.

The accompanying figures illustrate (a) the results of the geoelectrical survey and (b) a comparison of these results with available data obtained from subsequent drilling. Figure 212 is a portion of the geoelectrical structure contour map. The group of ten representative wells for which well log information was available are connected by a discontinuous line. At each well location, the well log elevation of the top of the Yates sand is shown opposite the well symbol. The bracketed figure represents an extrapolation of the Yates sand datum to the deeper horizon employed in the geoelectrical correlations. This bracketed figure is directly comparable, therefore, to the subsurface values that may be interpolated from the geoelectrical structure contour map.

This comparison is better illustrated by Figure 213 which shows the subsurface values plotted on a discontinuous profile between the wells shown in Figure 212.

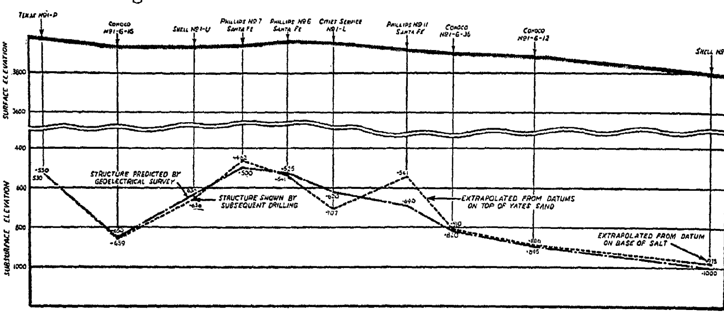


FIG. 213.—Structure profiles showing comparison of geoelectrical survey results with results of subsequent drilling. (East Vacuum Area, Lea County, New Mexico.)

The survey covered a total area of approximately 200 square miles and required approximately six months for completion.* Subsurface geoelectrical determinations were made over an approximate depth range of 3000 to 5000 feet, at 350 separate stations.

The geoelectrical survey results were submitted in terms of structure contours and profiles showing the average structural trend in the section included between the top of the Yates sand and the top of the White lime. Because of the generally greater reliability of the deeper geological marks, greater weight was attached to the geoelectrical correlation of the lower part of the measured section.

The available well log data are for the top of the Yates sand which occurs at a somewhat shallower horizon than that employed in the geoelectrical work. To facilitate the comparisons illustrated in the accompanying figures, it was necessary therefore to extrapolate from the Yates sand values, and also to interpolate the geoelectrical values from the nearest geoelectrical station to the location of the check well.

* A portion of the results only are shown in Figure 215.

Of especial interest in connection with this work are: (1) the substantiation of the indicated structurally low ground to the northeast of the Phillips No. 6-Santa Fe Location and (2) substantiation of the low and relatively flat structure between the Conoco No. 1-G-12 and the Shell No. 1-A wells in the southeastern part of the area.

The indication of low structure in the latter case is particularly significant inasmuch as it disproved the theory of a possible connection between the Monument Structure to the southeast and the East Vacuum pool. The Shell No. 1-A well is reputed to have been drilled as a result of recommendations based upon a seismic survey in this vicinity. This area is particularly difficult for seismic operations due to two factors: (a) irregular velocities in the near-surface beds, and (b) large variations in thickness of the anhydrite zone overlying the limestone.

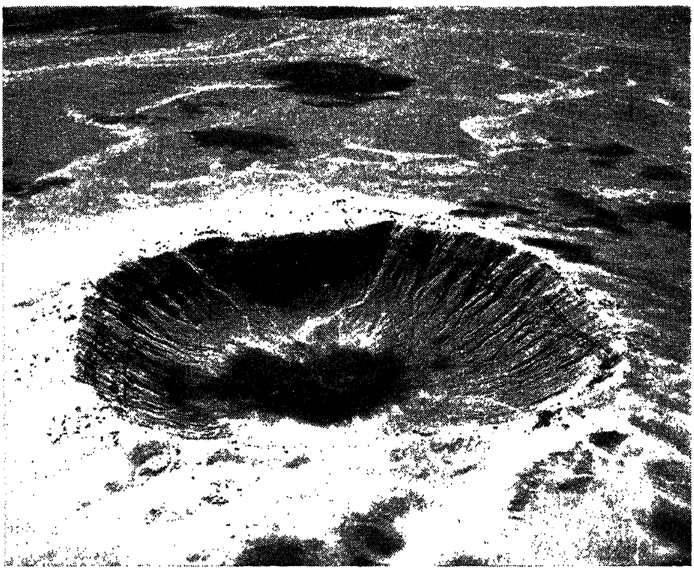


Fig. 214.—Aerial photograph of Meteor Crater, Arizona. (*A.I.M.E. Geophysical Prospecting,* Technical Paper, 1932.)

Structural Investigation of Meteor Crater

Meteor Crater (Figure 214) lies in the high plateau of northern Arizona.† It is a natural wonder whose origin and age have been discussed for many years. Data from geologic and topographic surveys have shown that in both geologic and topographic effects the crater is unusually symmetrical for so large a structural feature.

† J. J. Jakosky, C. H. Wilson, J. W. Daly, "Geophysical Examination of Meteor Crater, Arizona," *A.I.M.E. Geophysical Prospecting,* 1932, pp. 63-97.

The dips of the strata vary somewhat around the rim and the elevations of corresponding horizons also vary, mainly because of the effect of the numerous near-radial faults. These faults are rather symmetrically distributed around the crater and are very similar in character. Displacements of the beds by these faults may amount to 85 feet in a vertical direction. In addition to the faulting which now shows in the rim of the crater, the rim rocks have suffered considerable fracturing. The main geological features of the bottom of the crater are the extent and distribution of quaternary fill material, which consists essentially of the talus slopes extending from the crater floors up the inside slope of the rim, and the very fine sandy material covering the location of the playa lake and the lake beds below. The distribution of talus is also fairly uniform around the crater rim, being less extensive on the northwest side of the crater where, correspondingly, the fine fill material extends farther out toward the rim.

The chief purpose of the geophysical work was to determine the subsurface conditions in the crater and the possible occurrence of any large body of meteoritic material.

Electrical Survey.—The geoelectrical studies were conducted with a low frequency alternating current resistivity apparatus. One current electrode remained at a fixed position within the crater. The other current electrode and the two intermediate potential electrodes moved progressively outward as the depth of investigation was increased, the lines of measurement being radial with respect to the center of the crater.

A center point selected within the crater was used as a hub or reference point. Resistivity studies were made at stations distributed circumferentially around the crater as follows: (1) 1000 feet from the center point and entirely within the crater proper; (2) 1500 feet from the center on the inside slopes of the crater rim; (3) 2000 feet from the center and on the crater rim; (4) additional studies at points within and outside of the crater at several stations selected after the first section had been completed and a general idea obtained as to depth of bedrock in the crater. Spacing between stations was selected to give sufficient overlap to allow proper mapping of the subsurface structure. The orientations of the traverses were: (a) along lines radially from the center and (b) along lines (chord lines) at right angles to (a). On both radial and chord lines studies were made to effective depths of 1800 feet.

Brief Summary of Results

Due to the symmetry of the crater, the resistivity curves all exhibit similar characteristics. (Figure 215.) Final interpretation of the data was based chiefly upon direct curve correlation. From the geophysical

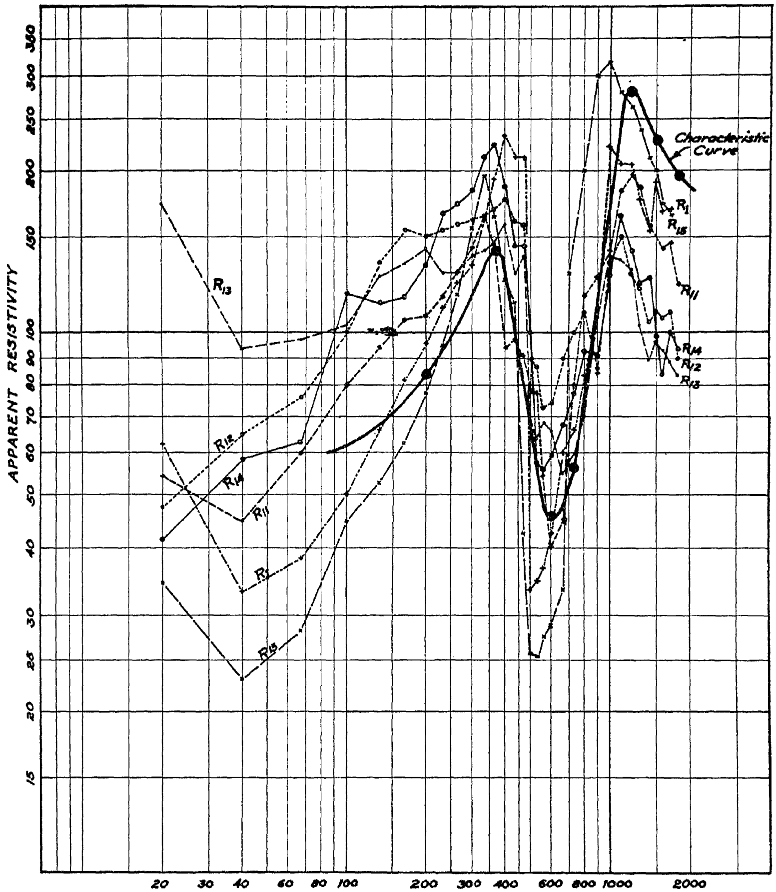


FIG. 215.—Calculated resistivity curves and characteristic curve, Meteor Crater, Arizona. (*A.I.M.E. Geophysical Prospecting,* Technical Paper, 1932.)

results a subsurface structural map was prepared showing probable thicknesses of fill material, bedrock contours, etc. This is shown by the cross sections of Figure 216.

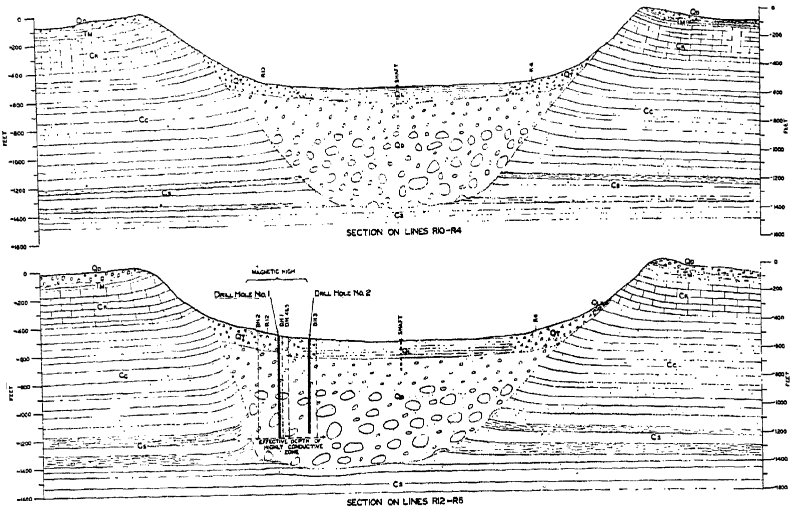


FIG. 216.—Probable geologic sections, Meteor Crater.

Q_D—Debris
Q_T—Landslides
Q_L—Lake beds
T_M—Moencopic sandstone
C_K—Kaibab limestone
C_C—Coconino sandstone
C_S—Supai formation

(*A.I.M.E. Geophysical Prospecting,* Technical Paper, 1932.)

The electrical survey gave indications of the presence of an area of higher conductivity in the southwest quadrant of the crater between the center and the rim. The main portion of the relatively conductive material lies at an effective depth of approximately 700 feet. A careful study of the original and altered materials found in the area indicates that this zone of higher conductivity is not due entirely to fill material or structural condition. The conclusions are that this area contains material of metallic character. The material is probably a fragmental zone having its greatest length in a general northeast-southwest direction. Subsequent to the geophysical work two holes were drilled to a depth of 600 feet and both encountered heavy zones of broken meteoritic iron from about 450 feet down. †

† C. H. Wilson, "Drilling Proves Existence of Meteoric Mass," *The Mining Journal,* April 30, 1932.

CHAPTER VI

ELECTRICAL METHODS: *Magnetometric and Inductive Methods*

MAGNETOMETRIC METHODS

Magnetometric methods measure directly the magnetic field associated with the flow of current in the subsurface. Various techniques are employed for determining the magnitude of this magnetic field created at the earth's surface by current supplied to the earth conductively. The data thus obtained may be interpreted by analysis based on well established physical laws, and the subsurface path of effective current flow determined.

PHYSICAL PRINCIPLES

Magnetic Field Associated with Electric Current.—It is well known that an electric current flowing in a conductor produces a magnetic field which radiates outwardly from the conductor in closed magnetic circuits. The lines of magnetic force surrounding an infinitely long, linear conductor are shown in Figure 217. The direction of the field is given by the familiar right hand rule: namely, if the thumb of the right hand indicates the direction in which the current is flowing, the lines of force circle the conductor in the direction of the fingers of the closed hand.

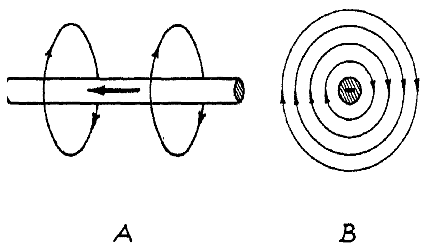


FIG. 217.—Lines of magnetic force due to current flowing in a linear conductor. A, side view; B, cross sectional view. (Minus sign indicates that current is flowing into the paper.)

Ampere's Law

The magnitude or strength of the magnetic field surrounding a current-carrying conductor may be obtained from Ampere's law. This law describes the magnetic field due to a conductor element of length dl at an external point P. Referring to Figure 218, the field at P is:

$$dH = \frac{i\,dl \sin\theta}{r^2} \qquad (1)$$

where

$dH =$ magnetic field
$I =$ current
$dl =$ length of current element
$r =$ distance between dl and P
$\theta =$ angle between dl and r

Furthermore, the magnetic field is perpendicular to the plane determined by r and dl and its direction is parallel to the direction of advance of a right-handed screw when rotated from the current vector I to the displacement vector r.

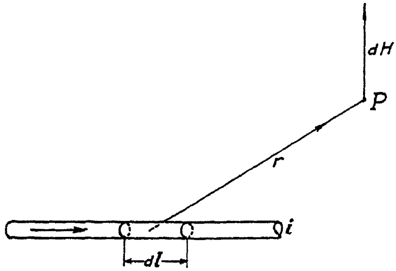


FIG. 218.—Sketch illustrating geometric relation between field dH at P and current I flowing in current element of length dl. (From Ampere's law dH is perpendicular to the plane formed by dl and r.)

Ampere's law governs the magnetic field of direct current and alternating current. If the current flowing in the conductor is an alternating current, the magnetic field surrounding the conductor is an alternating field which has the same frequency as the current, and is in phase with it.

Equation 1 may be used to determine the resultant or total magnetic field due to a current flowing in any conductor or group of conductors, provided the integration with respect to l and r can be carried out. In particular, Equation 1 may be applied, in a rather simple manner, for determining the magnetic field produced by a linear flow of current. Referring to Figure 219, it is desired to calculate the magnetic field H at a point P due to the current I flowing in the direction indicated. From Equation 1 the field due to an element of conductor of length dl is

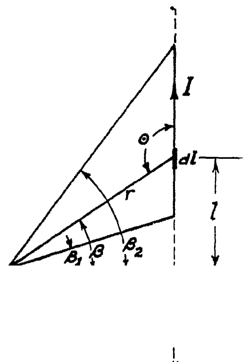


FIG. 219. — Sketch illustrating the geometric relations between the quantities I, dl, etc., which enter into the computation of the magnetic field at a point P.

$$dH = \frac{I\,dl\sin\theta}{}$$

An inspection of the figure shows that $\sin\theta = \cos\beta$. Hence,

$$dH = \frac{I\,dl\cos\beta}{r^2}$$

If x is the perpendicular distance from the point P to the conductor l,

$$r = x\sec\beta$$

$$l = x\tan\beta$$

$$dl = x\sec^2\beta\,d\beta$$

Substitution of these values of r and dl into the expression for dH yields

$$dH = \frac{I\,x\sec^2\beta\, d\beta\cos\beta}{x^2\sec^2\beta}$$

and

$$\cdot - \int \frac{I\cos\beta\, d\beta}{x}$$

On carrying out the integration, one obtains

$$H = \frac{I}{x}(\sin\beta_2 - \sin\beta_1 \tag{2}$$

If the wire is very long compared to x, β_2 approaches $+\pi/2$ and β_1 approaches $-\pi/2$. Equation 2 approaches the familiar form for the field about an infinite wire: namely,

$$x \tag{2a}$$

Equation 2a shows that the field surrounding a linear conductor may be represented by concentric circles. (Compare Figure 217B.)

Magnetic Field Produced at the Earth's Surface by Subsurface Current.—When current is conductively supplied to the earth a magnetic field will be set up, and a portion of the field will exist at the surface of the earth. Since the current distribution in the earth will be influenced by the geologic structure, the magnetic field set up by the current will likewise be influenced and measurements of this magnetic field or quantities which depend on this field give an indication of the subsurface geology.

For a given electrode spacing the greatest flow of current is along the path of greatest effective conductivity. As applied to mineral prospecting, the effective conductivity of a sulphide zone is much greater than that of the surrounding medium, and the mineralized zone may therefore be located by studying the magnetic field at the surface of the ground and by finding the path along which the current flow is greater. When applied to structural mapping, the path of effective current flow is calculated from the magnetic measurements and variations in the depth of this path or "marker bed" versus electrode spacing or position and is plotted for various points or stations in the area under investigation.

Theoretical Relationships

The calculation of the magnetic field at the surface for the case where the earth is *homogeneous* and *non-magnetic* can be carried out readily, provided the following assumptions are made: (1) the current penetrates the earth in all directions radially from the source; (2) the current enters the sink radially in all directions from the earth; (3) the

magnetic effects of the current leaving the source and the current entering the sink may be evaluated separately, and their separate effects combined ally; (4) the magnetic effect of the current leaving the source, ng the sink, may be computed by evaluating the effect (differential) due to a cone of solid angle dw and intergrating throughout the semi-infinite space occupied by the homogeneous earth; (5) the magnetic field due to the total current in an infinitesimal frustum of a cone is the same as that due to a current which flows along the axis and is equal in magnitude to the total current in the frustrum.

In Figure 220, the xy plane corresponds to the earth's surface and the origin of coordinates to the current source. I denotes the current leaving the source. x denotes the distance between the current source and a point P on the x axis.

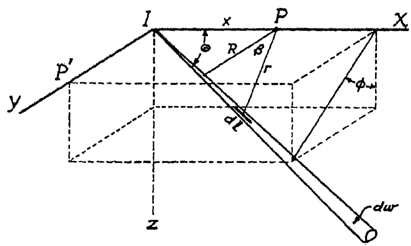


FIG. 220.—Sketch illustrating geometric quantities used in evaluating the field produced at a point P by a current I entering the earth at the origin of coordinates.

In order to determine the resultant magnetic field at P, it is convenient to calculate first the field produced at P by the current flowing in a solid angle dw. The axis l of the cone of revolution defining the solid angle makes an angle θ with the x axis. The perpendicular distance between the axis l of the cone and the point P is denoted by R. The current i passing through the cone is related to the current I leaving the source by the equation

Let dl denote the length of a differential element and r denote the distance between dl and the point P. Also, let (dl, r) denote the angle between dl and r. From Ampere's law, the magnitude of the magnetic field at P due to the current element dl is given by the equation

$$dH = i\,\frac{dl \sin (dl,r)}{r^2} = i\,\frac{dl \cos \beta}{r^2}$$

or

$$dH = \frac{I}{2\pi} \cdot \frac{dw\, dl \cos \beta}{r^2}$$

Also, the field dH is perpendicular to the plane passing through dl and r.

Due to the fact that the cones radiate from the source into the earth in all directions, the upward and downward vertical components of the fields produced by the cones cancel each other; that is, the resultant field

at any point on the earth's surface (xy plane) is parallel to the xy plane. Also, the resultant field is perpendicular to any radius through the current source; that is, at a point such as P the resultant field is perpendicular to x and parallel to the y axis.* Hence, the calculation of the total field at P reduces to a calculation of the y component.

The y component of the field due to the element dl is

$$dH_y = \frac{I}{2\pi}\frac{dw\, dl \cos\beta \cos\theta}{r^2}$$

where ϕ is the angle between the y axis and the perpendicular to the plane passing through dl and r.** The last equation may be written in a more convenient form by making use of the following relations:

$$dw = \sin\theta\, d\theta\, d\phi$$

$$r = R \sec\beta$$

$$R = x \sin\theta$$

$$l = R \tan\beta$$

$$dl = R \sec^2\beta \qquad R$$

Whence

$$dH_y = \frac{I}{2\pi x}\, d\theta \cos\phi\, d\phi \cos$$

or

$$H_y = \frac{I}{2\pi x}\int_0^{\pi} d\theta \int_{-\pi/2}^{\pi/2} \cos\phi\, d\phi \int_{\theta-\pi/2}^{\pi/2} \cos\beta\, d\beta$$

But

$$\int_{\theta-\pi/2}^{\pi/2} \cos\beta\, d\beta = 1 - \sin(\theta - \pi/2) = 1 + \cos\theta$$

Therefore,

$$H_y = \frac{I}{2\pi x}\int_0^{\pi} \qquad \int_{-\pi/2}^{\pi/2} \cos\phi\, d\phi$$

or

(3)

Equation 3 specifies the field produced at the surface of the earth by current flowing from a point source into homogeneous, non-magnetic

* At a point such as P', the field is parallel to the x axis.

** It is evident from elementary geometry that ϕ is also the angle between the plane passing through dl and r and the yz plane.

*** It is of some interest to point out that the same result would be obtained if the current flowed in a straight line vertically downward from the current source.

The field at P due to current flowing into a sink located on the x axis at a distance L from the source is $\frac{I}{L-x}$. This field has the same direction as the field produced by current leaving the source. Hence, the total magnetic field at the point P is:

$$\frac{I}{x} \qquad L-x \qquad\qquad\qquad -x \tag{4}$$

In this equation the value of H is given in gauss provided I is expressed in electromagnetic units (absolute amperes) and x and L in centimeters.

Equation 4 specifies the magnetic field at the surface due to current flow in homogeneous, non-magnetic earth. (This result does not take into account anomalous surface effects, magnetic effects produced by the return conductor connecting the current electrodes, or the natural magnetic field of the earth.)

If the area under investigation is not homogeneous, the magnetic field produced at the surface of the earth is not given by the simple formula derived above. When direct current or very low frequency energizing current is employed, the magnetic effects observed at the surface of the earth depend on the current paths, and these in turn depend on the relative resistivities of the materials constituting the subsurface. For example, if the subsurface consists of several layers having different resistivities, the effective depth of current penetration and the current paths will depend principally on the values of the resistivities and on the separation of the energizing electrodes. Thus, precisely as in the resistivity methods, variation of the electrode separation permits depth control.

The resolving power of the method however is greater than that of the resistivity methods. This increased resolving power is due largely to the following factors:

(1) The near-surface effects are minimized since a considerable portion of the electromagnetic field is due to the deeper current flow. In the resistivity methods, the only potentials that are effective or measured are those at the surface of the earth.

(2) Near-surface effects diminish, due to the increased flow of current to greater depths as the electrode spacing is increased.

(3) Because the electromagnetic field strength decreases inversely as the distance and the sine of the angle, the current flowing beneath the point of measurement has a much greater effect than current flowing to each side and farther removed from the instrument.

Theoretically, it is possible to calculate the magnetic effects produced by layered media in a manner similar to that used in the preceding section by taking into account the curvature of the current paths. Practically,

however, recourse is usually had to an empirical interpretative technique wherein observed electromagnetic anomalies are considered as diagnostic with respect to subsurface inhomogeneities. That is, interpretation is nearly always based on anomalous conditions as indicated by deviations of the field strength from the theoretically normal values. In this respect the electromagnetic data are interpreted by the same technique employed in interpretation of resistivity data. Curves are usually plotted showing the observed field strength per unit current versus: (1) electrode spacing or (2) traverse distance at a fixed energizing electrode spacing.

LOCATION OF SUBSURFACE CURRENT FLOW BY FIELD STRENGTH MEASUREMENTS

Methods for measuring the field created by current flowing through the earth between grounded electrodes are conveniently classified into two groups: namely, (1) methods using direct current energizing and (2) methods using alternating current energizing. The chief practical difference between these groups is the apparatus used to detect or measure the field strength. The usual detecting device in the first group (steady-state field) is a magnetometer (magnetometric suspension type, rotating coil type, etc.) which measures the magnetic field strength directly. The usual detector in the second group is a search coil which measures the field strength indirectly by means of the E.M.F. induced in it by the alternating field.

Field Strength Measurements Using D.C. Energizing.—The most direct method of studying the distribution of current is to employ a sensitive magnetometer for measuring the strength of the magnetic field created by a continuous direct current flowing through the subsurface. For shallow depths this method is rapid and reliable. For deeper work where weak fields must be measured, the application of the method is restricted due to short period variations in the earth's magnetic field which may be greater than the field created by the current flowing in the subsurface. (This factor will be discussed in more detail later.)

Energizing Apparatus

Current is applied to the ground between two electrodes connected with a suitable high wattage power supply. The separation of the two energizing electrodes may be held constant where lateral studies are to be made, or it may be varied to increasingly greater separations to obtain increasingly greater effective depths of penetration for vertical structural studies.

The general layout of equipment and field lines is illustrated in Figure 221.† The power supply must be capable of passing 10 to 20 amperes into the ground. Under usual surface conditions this will require a potential of 200 to 1000 volts. The most practical form of power supply comprises a heavy duty direct current generator and excitor driven through a power take-off from the motor of the truck housing the equipment. The field wires should be No. 14 stranded copper, or larger, and the electrodes should be of the multiple stake type well "wetted down" to minimize losses.

The area to be studied lies between the two grounded terminals or electrodes. The longer the legs of the "U," that is, the further away the portion of the cable parallel to the line of the electrodes, the less is the effect of the primary current flowing in the surface cable. The legs of the "U" preferably should be once or twice the depth to be worked, and

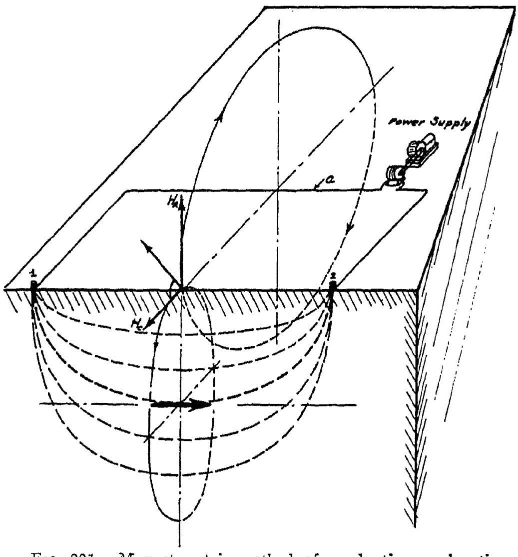


FIG. 221.—Magnetometric method of conductive exploration. H_A, field surrounding the surface wire *a*; H_C, field surrounding the subsurface flow of current; 1 and 2, energizing electrodes. (Courtesy of International Geophysics, Inc.)

measurements should not be made nearer the legs of the "U," or the electrodes, than the depth. The magnetometer is placed on the imaginary line connecting the two energizing electrodes 1 and 2. At this position, the field strength measuring apparatus is then subjected to two artificially created fields: (a) the essentially vertical magnetic field H_A created by the flow of current through the energizing wire *a*, and (b) the complex field (represented as a single field H_C) created by the flow of current in the subsurface.

† J. J. Jakosky, "Method and Apparatus for Determining Underground Structure," U. S. Patent 1 906,271, issued May 2, 1933.

Measuring Apparatus

The measuring apparatus is designed or oriented to respond primarily to the horizontal component H_C of the field.

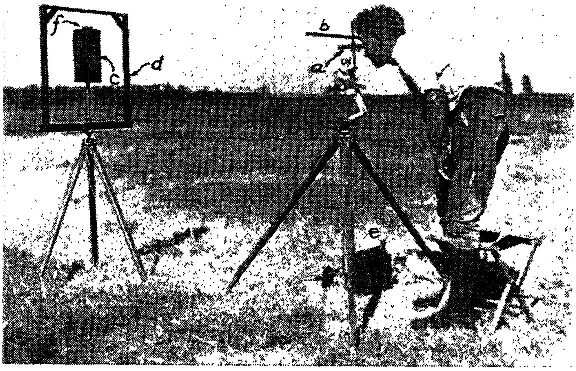


FIG. 222.—Early magnetometer for electromagnetic surveying. *a*, telescope; *b*, scale; *c*, magnetometer; *d*, neutralizing coil; *e*, battery for neutralizing field; *f*, torsion head.

Magnetometer (Suspension Type).—An early type of magnetometer employed for this work is illustrated in Figure 222. This magnetometer is of the suspension type with a torsion head at the top of the instrument for orientation. The initial and final readings (with energizing current on and off) are made by means of scale and telescope. The desired component of the earth's field is neutralized by means of current flowing through a coil surrounding the magnetometer. The strength of the resultant field is measured by one of the following procedures: (1) counting the time for a given number of oscillations of the suspended magnetic system (compare p. 67), (2) noting the scale deflection of the magnetic system.

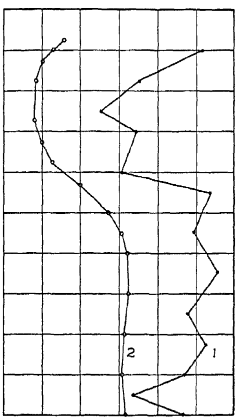


FIG. 223.—Magnetometric and resistivity curves to determine bedrock. (1) Wenner configuration resistivity measurements, (2) magnetometric measurements.

Field Procedure

In practice, the second procedure comprises the following steps: (1) After initial adjustment of the instrument, the magnetometer operator records the scale reading of the instrument. (2) The direct current power then is applied to the ground and the operator records the new reading of the instrument. (3) The power flow is interrupted and the operator again records the scale reading of the instrument. (4) The power is reversed and again passed into the ground and the scale reading recorded. (5) The power flow is again interrupted and the normal scale reading of the instrument recorded. The scale deflection is computed by taking the arithmetical difference between the averages for power-on and power-off. The observed magnetic field strength is equal to the product of the

scale deflection by the calibration constant.* Usually, the magnetic reading is expressed as the quotient of the field strength divided by the current used in energizing the ground at that set-up.

This reading is plotted versus electrode separation or traverse distance, as desired.

Figure 223 is a comparison of magnetometric and surface resistivity data obtained in an investigation of shallow subsurface structure. The elimination of near-surface effects no doubt accounts for the more consistent data obtained with the magnetometric method.

Rotating Coil Electromagnetometer.—For the deeper structural investigations with the magnetometer method it is necessary to employ a magnetometer having: (1) greater sensitivity, (2) much less response lag, and (3) means for compensating for earth field variations.

One type of system for compensating for the earth's magnetic variation utilizes a measuring system that records variations in the magnetic field instantaneously. In practice, the measurements are made by passing the current into the ground in a series of pulses and measuring the compensated earth's field during the off-current portion of the cycle and the total field (the compensated earth's field plus the electromagnetic field created by flow of the current) during the on-current portion of the cycle. The on and off portions of the cycle are made long enough to allow steady-state conditions to be reached during the measurement and short enough to allow the off-current portion of the cycle to act as a control in correcting for the earth's magnetic field variations. A generator supplies a series of intermittent current pulses to the two stake electrodes at a frequency of about one pulse per second. The current circuit is broken automatically by means of a high voltage relay operated by the energizing current. The output current is controlled by means of a commutator brush system.

The magnetic effect produced at the surface of the earth by the flow of the pulsating current through the ground is detected by a rotating coil assembly. (An early form of the equipment is shown diagrammatically in Figure 224.) The rotating (primary) coil is a copper ring which is driven by a shunt-wound motor or small, compressed air turbine. When the primary coil rotates, an E.M.F. is induced in it. This produces an alternating current, the magnitude of which depends on the resultant magnetic field, the rate of rotation, and the effective resistence of the ring. Because the resultant magnetic field is a varying field, the induced current also varies in intensity.

The current flowing in the rotating copper ring induces a potential in the multi-turn stationary coil. The value of this potential depends upon: (1) magnitude of induced current flowing in the rotating ring and (2) mutual inductance between the two coils for any instantaneous value of the current in the ring.** The mutual inductance will have a maximum value when the two coils are in the same plane and a minimum or zero value when the coils are at right angles. The orientation of the stationary

---

* The calibration constant depends primarily on the constants of the moving magnet and the torsion constant of the suspension fibre.

** The mutual inductance between the primary and secondary coil is equal to the magnetic flux linked with the secondary coil due to unit current in the primary coil. (Compare also p. 405.)

coil, therefore, governs the component measured. The terminals of the stationary coil are connected to a calibrated amplifier and a recording galvanometer. A second trace of the recording galvanometer is connected to a timing fork of known frequency in order to compute the speed of rotation of the ring, and hence the magnetic field strength.

A second stationary coil is provided for compensating for the entire earth's field or the component included in the measurements. Accurate control of current in the compensating coil is obtained by use of a potentiometer device connected across a storage battery.

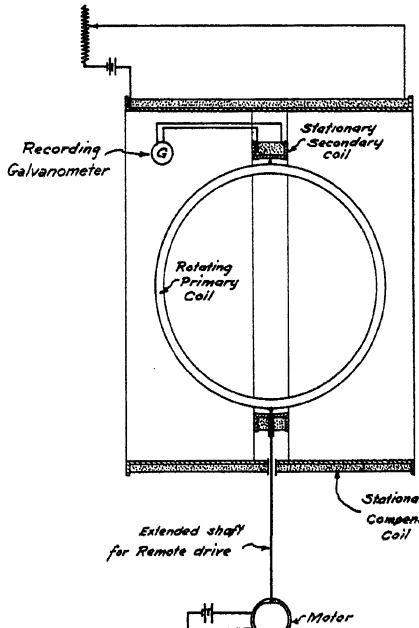


FIG. 224.—Diagrammatic representation of rotating coil electromagnetometer and neutralizing coil.

The use of this equipment in the field is similar to the use of the magnetometer previously described.

Field strength data are amenable to mathematical treatment and direct calculations may be made to determine the depth of effective current flow. That is, depth to an anomalous conductive zone may be calculated from the ratio of field strength to energizing current. Furthermore, a few readings at different electrode spacings suffice to supply substantially the same information as obtained from a whole series of resistivity readings at increasing electrode spacings. Changes in current penetration (which are indicative of structure) are determined from the variation of unit field strength versus electrode spacing.

Field Strength Measurements Using A.C. Energizing Means.—Current is applied to the ground between two electrodes connected with an alternating current power supply. The general layout of field lines is similar to that shown in Figure 221. However, the power supply in

this case may be much smaller and driven by a portable gasoline engine. An alternator with a capacity of 500 watts will furnish sufficient power for a 200 cycle supply, while approximately 1000 to 2000 watts should be provided for a 25 to 50 cycle supply. As previously discussed, the lower frequency is advantageous because of its better penetration.

Search Coil Measuring Apparatus

The term search coil, as used here, applies to a non-rotating coil comprising one or more turns of wire wound in any shape, usually circular or rectangular. The physical dimensions of the coil preferably are such as to allow it to be readily portable. The terminals of the coil are connected to a potential measuring device. This type of search coil can be used in magnetic field strength measurements only when the earth is energized with a varying current, e.g., an alternating current.

The operation of a stationary search coil depends on the fact that when an alternating current is caused to flow through the earth, an alternating magnetic field is produced whose magnitude and direction depend on: (1) strength of the energizing current, (2) orientation of the line joining the energizing electrodes, and (3) the subsurface materials and structure.

The alternating magnetic field induces an E.M.F. in the search coil, and data on the magnitude of the induced E.M.F. at a series of stations in a given area are diagnostic with respect to the paths of current flow in the subsurface. The magnitude of the induced E.M.F. is given by the equation:

$$E = -\frac{d\phi}{dt} \tag{5}$$

where ϕ is the normal component of the magnetic flux threading or cutting the coil.

Consider, for example, that an alternating current flows in the earth between two surface electrodes, with the surface conductor and power supply arranged as shown in Figure 221. Assume, as before, that the earth is homogeneous and non-magnetic. Under these conditions, the subsurface distribution of a very low frequency alternating current will be substantially the same as that of a direct current. The E.M.F. induced in a vertical coil or loop whose plane includes the line joining the energizing stake electrodes can be calculated readily, provided the area of the loop is small relative to the separation of the electrodes so that the magnetic field H may be considered as constant over the area embraced by the coil. Let A denote the area of the loop and let N denote the number of turns of wire in the loop.

Neglecting the field due to the surface or return conductor, the field at a point x on the line joining the two electrodes is given by Equation 4. That is,

This field is perpendicular to the line joining the electrodes. Hence, the magnetic flux cutting the loop is normal to the loop and is given by the relation

$$\phi = \mu NHA = \mu NIA \left(\frac{1}{x} + \frac{1}{L-x}, \right.$$

where μ is the effective magnetic permeability of the subsurface. If, as is frequently the case, μ may be set equal to unity, the expression for ϕ becomes:

$$x + \frac{1}{L-}.$$

The induced E.M.F. is given by Equation 5. That is,

$$x \quad L - x) \, dt$$

This equation gives the induced E.M.F. in electromagnetic units when I is expressed in electromagnetic units, A in square centimeters, x in centimeters, and L in centimeters.

In practical units, the last relation becomes

$$3.05 \quad NA \left(\frac{1}{x} \quad \frac{1}{L-x} \right) \frac{dI}{dt} \tag{6}$$

where E is given in volts, I in amperes, A in square feet, x in feet and L in feet.

Numerical Illustration.—A low frequency alternating current is applied to the ground by means of two stake electrodes separated by a distance of 100 feet. Suppose that the current has a maximum value of 10 amperes and a frequency of 5 cycles per second. That is,

$$I = 10 \cos 2\pi \cdot 5t$$

and

$$\frac{dI}{dt} = -100\pi \sin 10\pi t$$

Assume that the coil has 1000 turns and an area of 5 square feet and assume also that the coil is located midway between the stake electrodes.

Substitution of the assumed values into Equation 6 yields

$$E = -\frac{3.05}{10^8} \quad 1000 \cdot 5 \left(\frac{1}{50} + \frac{1}{50} \right) \left(- \right.$$

$$= .00192 \sin 10\pi t$$

$$E_{\max} = .00192 \text{ volts} = 1.92 \text{ millivolts}$$

Field Procedure.—Field strength measurements with search coils are conducted by measuring the potential generated in the coil at various locations in the area. The induced alternating potentials usually are measured by use of a calibrated constant gain amplifier and vacuum tube voltmeter. A fixed or systematic orientation of the coil, with respect to the energizing electrodes, must be employed throughout the measurements. The simple procedure usually employed is to maintain the coil so its plane is vertical and oriented parallel to the straight line joining the two energizing electrodes.

Limitations of A.C. Energizing Methods

From the preceding numerical example it will be seen that the voltage induced in a search coil is extremely small when a low frequency is employed for energizing the ground. If a higher frequency energizing current is employed, the induced voltage in the search coil will be proportionally increased, but the effective depth of penetration decreases because the subsurface distribution of current will be altered due to self inductance. That is, the alternating current flowing into the earth induces a voltage in the earth in such a direction as to oppose the flow of the current. The effect of this induced voltage is greatest at a depth where the normal current is small and hence the potential gradient is also small. This results in a crowding of the current towards the surface. For this reason it is impossible to detect inhomogeneities in the earth at any appreciable depth when using high frequency currents. In a two-layer structure, a considerable portion of the current never penetrates to the lower layer or inhomogeneity; hence, it can have no effect on the measurements.

Evjen† gives the current penetration for different frequencies as follows:

f	d	
(cycles/sec)	Km.	Ft.
0	∞	∞
1	1	3300
4	.5	1650
100	.25	825
1000	.025	83

where d is the asymptotic value of the depth of current penetration (the depth to which one-half of the total current penetrates) as the electrode spacing is increased.*

Thus, the indicated maximum frequency which could be employed when working to a depth of 3300 feet is one cycle per second, but it is impractical to obtain sufficient induced power at this frequency to use

† H. M. Evjen, "Electrical Methods in Geophysical Exploration," *Geologie en Mijnbouw* (Jaargang No. 1) pp. 2-8, January, 1939.

* The Evjen table affords only a general survey, because the penetration for a particular frequency is dependent upon the materials constituting the subsurface.

the method. On the other hand, a more useable frequency, such as one hundred cycles per second, will have an effective depth of penetration of only 825 feet. Hence, methods employing alternating current having an appreciable frequency are of little economic value except for shallow investigations.*

LOCATION OF CURRENT FLOW IN SHALLOW CONDUCTORS BY DIRECTIONAL SEARCH COILS

Instead of measuring the strength of a given component of the varying magnetic field associated with the flow of alternating current, the path of the subsurface current flow may be located by use of directional or direction-finding coils. This procedure utilizes a similar technique to that employed in radio direction finding. A directional search coil, with its necessary orientation and amplifying accessories, is readily portable and allows rapid manipulation for reconnaissance surveys.

Operating Principles of Directional Coils.—To obtain the most accurate determination of the direction of the resultant field, the per cent change in the magnitude of the E.M.F. for a small change in orientation angle must be as large as possible. This condition may be expressed by the relation

$$\frac{1}{E}\frac{dE}{d\theta} = \text{maximum}$$

where E is the induced E.M.F. and θ is the angle made by the direction of the magnetic field with the normal to the plane of the coil. But

$$dt$$

$$\frac{d}{dt} \qquad \cos\theta) = NA\,\frac{dH}{dt}$$

and

$$\frac{dE}{d\theta} \qquad dt$$

Hence,

$$dE$$

Evidently, $\frac{1}{E}\frac{dE}{d\theta}$ is a maximum for θ equal to 90°. That is, the direction of the resultant field is determined most accurately by rotating the coil until its plane is parallel to the magnetic field.

* This conclusion is quite at variance with the claims still being made by many advocates of high frequency methods. However, the work of numerous investigators during the past few years clearly indicates that the higher frequencies are unsuitable for the deeper subsurface investigations.

Figure 8 Curve

The magnitude of the induced voltage is often shown graphically by the so-called figure 8 curve. (Figure 225.)† This curve shows the relative value of the induced voltage in the loop as a function of the angle that the loop makes with the magnetic field for the case that the magnetic field is horizontal. When the coil is in the position AA' perpendicular to the direction of the field, the magnetic flux through the coil and consequently the induced voltage is a maximum. As the coil is rotated further, the flux through the coil decreases until at position DD' the coil is parallel to the field and the induced voltage is a minimum. If the rotation of the coil is continued the direction of the flux through the coil will reverse, thereby producing a reversal of phase of the induced voltage.

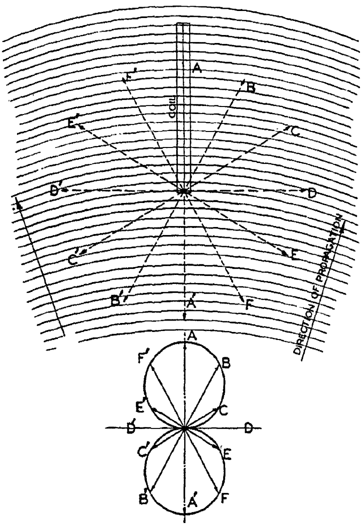


FIG. 225.—Plan view of a direction-finding coil in a uniform field. "Figure-eight curve." (*Proceedings, Institute of Radio Engineers.*)

Field Apparatus.‡—The complete direction-finding apparatus consists of a direction-finding coil, mounting head, tripod, amplifier (for audio-frequency range) or detector and amplifier set (for higher frequencies), and a pair of headphones or a vacuum tube voltmeter.

One type of direction-finding apparatus is illustrated in Figure 226. The mounting head on which the direction-finding coil is pivoted is provided with a sighting arrangement (similar to gun peep-sights) whereby the axis of rotation of the coil may be aligned quickly with the center of the energizing coil. A graduated vertical arc is attached to the pivoted plate holding the direction-finding coil. The vertical-angle index mark and level-bubble arc are attached to the movable arm, which is adjusted by a thumbscrew.

The head rotates on a vertical axis and the azimuth angle may be read on a graduated scale. The entire assembly is mounted on a ball-and-socket plate. The direction-finding coil is electrically connected to the vacuum-tube set-box which contains a detector (so arranged that it functions as an audio-frequency amplifying stage when using low-frequency coils) and a two-stage tuned transformer-coupled audio-frequency amplifier.

† J. J. Jakosky, "Electrical Prospecting," *Proc. Institute of Radio Engineers,* Vol. 16, No. 10, Oct. 1928.

‡ J. J. Jakosky, "Operating Principles of Inductive Geophysical Processes," *A.I.M.E. Geophysical Prospecting,* 1929, pp. 138-176.

The entire apparatus, consisting of heterodyne control dial, controls and batteries, is contained in a waterproof aluminum box 4½ by 4½ by 17 inches. A double-range voltmeter is placed behind a waterproof "port-hole" by means of which filament and plate voltages may be read. The control knobs are placed on a recessed panel for protection against mech-

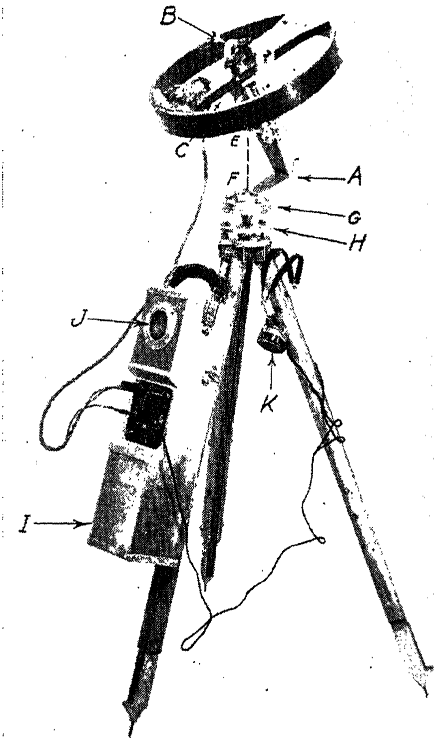


Fig. 226.—Electromagnetic direction-finding apparatus for high frequencies. *A*, mounting head; *B*, alignment sights; *C*, graduated arc for reading vertical angle; *D*, adjustment screw; *EF*, vertical rotation axis; *G*, azimuth scale; *H*, ball and socket plate; *I*, battery compartment; *J*, voltmeter; *K*, headphones. (*A.I.M.E. Geophysical Prospecting*, 1929.)

anical injury. Conventional type headphones are provided so that the operator can detect the point of minimum signal strength when determining the direction of the resultant field. The same mounting head and set-box are used for both low and high frequency work.

A schematic wiring diagram for high and low frequency apparatus is shown in Figure 227. When the high frequency coil is used, the detector

is of the oscillating type in order to provide a heterodyne signal for audibility and increased sensitivity. For low frequency work, the detector

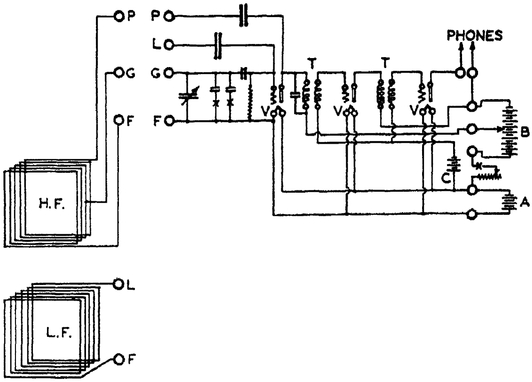


FIG. 227.—Wiring diagram for electromagnetic direction-finding apparatus.

tube functions merely as an audio-frequency amplifying tube. If desired, pentode tubes can be employed in the amplifying stages for greater amplification.

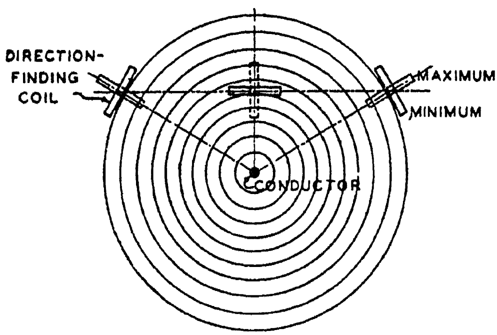


FIG. 228.—Field surrounding a simple conductor in a homogeneous medium. (*A.I.M.E. Geophysical Prospecting,* Technical Paper 134, 1928.)

Applicability of Directional Coils: Field Surrounding a Simple Conductor in a Homogeneous Medium.—The field surrounding a small diameter conductor in a homogeneous medium will travel outward from it in the form of concentric circles or envelopes, as depicted in Figure 228.† The speed of propagation will depend upon the conductivity of the medium surrounding the conductor; in air this speed will be that of

† J. J. Jakosky, "Fundamental Factors Underlying Electrical Methods of Geophysical Prospecting," *Eng. and Min. Journal,* Feb. 11-18, 1928.

light (300,000,000 meters per second), while in a highly conductive material such as sea water, the speed may be only a few meters per second. A direction-finding coil pivoted with its axis of revolution parallel to the conductor will give the maximum signal response when the coil is perpendicular to a tangent to the wave front. A minimum signal will be obtained when the coil is parallel to the tangent. This is illustrated for three positions in Figure 228. Under such conditions the direction-finding coil would become a simple means of locating a subsurface conductor by triangulation. In practice, it will be found that the field surrounding the conductor is distorted by: (1) irregular shape or configuration of the ore body or conductive zone; (2) inhomogeneity of the conductive medium surrounding the major zone of current flow; and (3) distortion of the wave front as the electromagnetic wave emerges from the ground.

Distortion of Wave Front

Because the velocity of propagation of an electromagnetic wave varies with the dielectric properties of the medium through which it passes, there is a resultant distortion of the wave front as it emerges from the ground. In practice, therefore, the wave front traveling outward from the current-carrying zone is not a true circle, and the conductor cannot be located by a simple triangulation process.

The velocity of an electromagnetic wave may be expressed in terms of its velocity in vacuum by the relationship:

where

V = velocity in medium of given properties

V_0 = velocity in vacuum (300,000,000 meters per second)

m = magnetic permeability of material

d = specific inductive capacity of material

K *and* K' = constants which vary with frequency, etc.

If this formula is applied to a conductor of infinite conductivity ($d = \infty$) the wave would have a limiting velocity equal to zero and its energy would be almost completely dissipated by generation of eddy currents. A small portion of the energy will be reradiated. This result indicates that a highly conductive ore body will absorb a considerable portion of the energy and reradiate (in the form of a secondary field) a smaller portion.

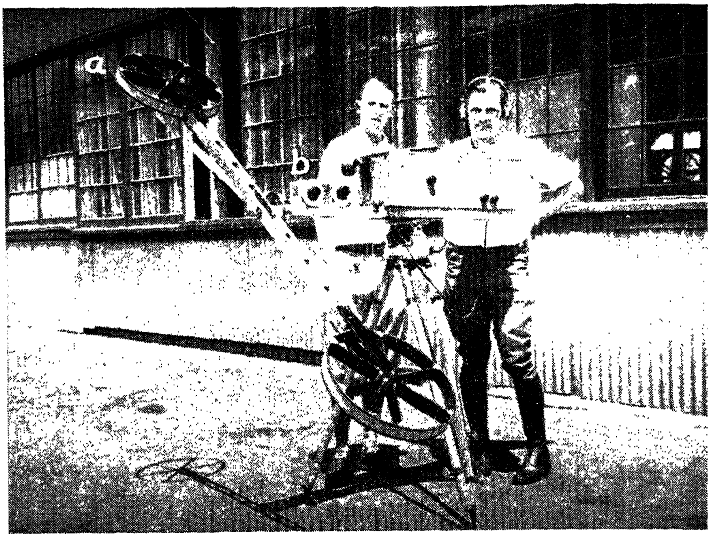


FIG. 229.—Apparatus for measuring angle of electromagnetic wave front. *a* and *a'*, two identical coils; *b*, shielded tuning condensers; *c*, shielded amplifier and phase compensator.

Experimental verification of the distortion of the wave front has been carried out by means of the apparatus illustrated in Figure 229. This apparatus comprised a shielded, antenna-compensated receiver and a double coil antenna rotatably mounted on a 6-foot arm. A graduated arc and vernier were attached to the arm for reading the position of the coils for zero signal input. The two coils had substantially identical electrical characteristics and were connected to a winding of a shielded differential transformer. The secondary of the transformer was connected to the input circuit of an amplifier and detector. In carrying out the measurements, the apparatus was so placed that the axis of rotation of the coil support was parallel to the axis of the conductor. The arm was then rotated until a minimum signal was heard, at which angle the two coils were symmetrically disposed with respect to the wave

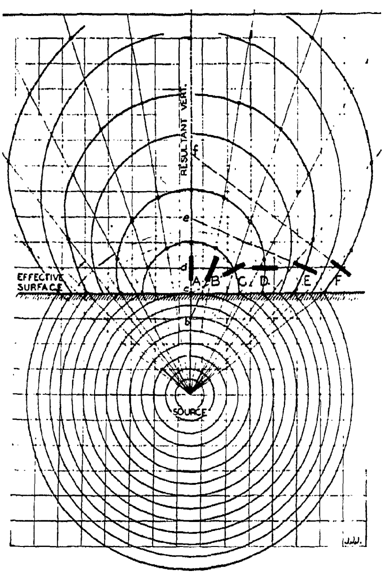


FIG. 230.—Distortion of electromagnetic waves in traveling from earth to air. (*Eng. and Min. Journal,* Feb. 11-18, 1928.)

Figure 230 is a schematic plot of the wave front for an electromagnetic wave emerging from average dry ground. Immediately above a highly conductive body, the arm connecting the two coils is downward and toward the conductor. As readings are taken to either side of the vertical, however, the direction is not toward the conductor, and empirical corrections must be made, as will be described later.

Contacting Ore Body.—In a modification of the electromagnetic process, developed at the University of Arizona,† one terminal of the current supply is connected with the ore body directly. The chief use of this method is in tracing out the extensions of known ore bodies. One energizing electrode is located to make contact with one portion of an

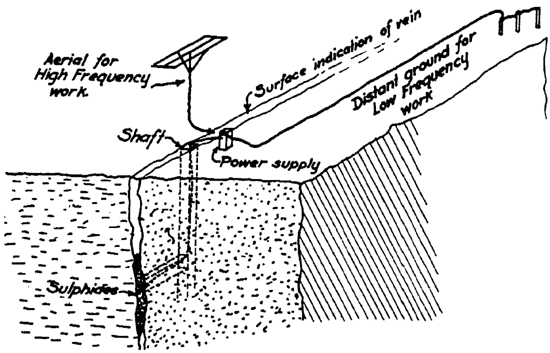


FIG. 231.—Diagram of connections for contacting method.

ore body (usually an outcropping or working face) and the other electrode is located at a considerable distance from the first contact. If the ore body is shallow, medium high frequencies may be employed for this work.

A schematic diagram of connections for this apparatus is shown in Figure 231. If radio frequencies are used, the ore body must be very shallow. The "antenna" should be of a proper size to allow efficient operation at or near its fundamental or a harmonic thereof.

A direction-finding apparatus similar to that illustrated in Figure 229 is employed for determining the location of the underground conductive zone. Both the dip (angle measured from the vertical) and the strike (azimuth angle) readings are made in determining the location of the body. When using radio frequencies, it will be found oftentimes that the strike readings are more reliable and give better indications of the presence of a conductive zone than the dip readings. When a vertical antenna is employed the strike readings normally should all be toward the antenna, unless an extension of the ore body exists, in which case there will be very definite deviations of the strike.

† D. G. Chilson, "Process of and Device for Locating Ore," U. S. Patent 1,491,900, issued April 29, 1924.

A simple modification of this method which permits its use for deeper lying ore bodies employs a medium frequency alternator for supplying the excitation energy. The equipotential lines are traced out with the aid of two probing electrodes connected with a pair of sensitive headphones or an audio-frequency amplifier and phones. The two probes are on the same equipotential line when no signal is heard in the headphones. Short exploring lines (100 feet or so) should be used to minimize inductive effects.

This same technique is employed for location of buried pipe lines. A contact is made with the pipe at a known location, and the other electrode placed as far away as practical, and in the probable direction of the pipe line. The concentration of current in the pipe will allow its location by use of suitable direction-finding coils.

INDUCTIVE METHODS

Inductive methods comprise those methods wherein the earth is energized inductively by alternating current, and measurements are made of parameters which are associated with the secondary magnetic field at the earth's surface produced by induced currents in the subsurface.

When an alternating current is supplied to an insulated loop or coil placed near the surface of the earth, the magnetic field set up by the coil (energizing coil) will cut into the earth and induce a complexity of varying voltages in the materials constituting the subsurface. These induced voltages will in turn set up currents in the earth with a resulting potential distribution at the surface and a redistribution of the magnetic field at the surface.

The effective electromagnetic field existing at the surface of the earth is the resultant of the primary field created by the energizing coil and the secondary fields due to the induced currents in the subsurface and in any other conductors in the vicinity of the energizing coil. Hence, any property of this electromagnetic field is diagnostic, theoretically at least, with respect to the subsurface distribution of current. The parameters most commonly measured are: contours of equal magnetic flux, phase of the electric or magnetic field, magnitude of vertical or horizontal component of the magnetic field, magnitude or direction of the resultant magnetic field.

The inductive methods are subject to an infinite variety of modifications as is attested by the diversity of inventions governing improvements on these methods. There are many advantages in the use of the inductive methods, chief of which are: (1) by use of electromagnetic means the ground may be energized and/or the subsurface distribution of current determined without the use of electrodes or other direct contact with

the ground, (2) at low frequencies the near-surface effects are minimized, and (3) inductive type methods are relatively rapid for reconnaissance work.

Detailed descriptions of the many modifications of these methods will not be given, but an attempt has been made to describe certain of the methods to illustrate general principles of operation and theory.

PHYSICAL PRINCIPLES

The present electromagnetic methods employing inductive energizing means are limited essentially to investigations at fairly shallow depths (of the order of a few hundred feet under favorable conditions). From a practical viewpoint, therefore, the inductive methods are chiefly useful for the location of ore bodies lying near the surface when covered with a thin overburden.

In general, the effect that an ore body will have on the measurements will depend on the size of the body, its relative electrical resistivity, the frequency of the energizing current, and the type and relative resistivity of the overburden.

The larger the effective length of the ore body the greater will be its effect on the magnetic field and consequently the easier will be its detection. The resistivity of the ore body relative to the surrounding earth is also important; the larger the ratio of resistivities the greater will be the effect of the ore body. Furthermore, it is important to realize that the relative resistivities may vary with the frequency used. (In particular, the resistivities of *disseminated* ores may depend markedly on the frequencies employed.)*

The frequency of the energizing current and the type of overburden largely govern the maximum depth at which an ore body can be detected. The theoretical formula for the E.M.F. induced in a small diameter ore body by an energizing coil is stated to be: †

$$E_0 = 2\pi \qquad {}^{-2\pi d\sqrt{\frac{f\mu}{}}} \tag{7}$$

where E_0 is the induced E.M.F.; f is the frequency of the current in the energizing coil; M the mutual inductance between the energizing coil and the conductor; I_1 the current in the energizing coil; μ the permeability and ρ the resistivity of the overburden.**

* A disseminated ore may be considered as composed of small, electrically conductive particles distributed in a matrix; as a rule this matrix is calcite, quartz, or similar material and has a high electrical resistance. A small electrostatic capacity exists between the conducting particles constituting the disseminated ore. From an electrical viewpoint, therefore, a disseminated ore may be considered as a resistance shunted by a capacity.

† A. B. Broughton Edge and Laby, *Geophysical Prospecting*, p. 293 (Cambr. Univ. Press, 1931).

** In applying this formula to the E.M.F. induced in an ore body, it is assumed that the material surrounding the ore body has the same values of ρ and μ as the overburden.

The electromotive force E_0 produces an alternating current I_0 in the ore body. The alternating magnetic field of the current I_0, in turn, induces in the search coil an electromotive force E_s whose magnitude depends on the current I_0, the frequency of the current I_0 which is the same as the frequency of the energizing circuit, the mutual inductance between the search coil and the ore body, etc. That is,

$$E_s = :$$

Hence, the net result is that the overburden exerts a *shielding effect* which varies approximately as the square of the frequency in the energizing circuit.*

The mutual inductance between the energizing coil and the conductor also is dependent upon the orientation of the coil with respect to the conductor. In methods where direction-finding coils are employed for detection of the subsurface conductive zone, the plane of the energizing coil is usually maintained in a vertical position and oriented so that it will be parallel to the probable direction of the subsurface conductive zone. Vertical coils of this type usually are circular or square and as large as may be conveniently handled in the field work, usually sixty to one-hundred square feet in area. Their small area is offset by use of many turns of wire, the number of turns depending upon the power and frequency of the energizing system. The coil should be tuned to the frequency, or a harmonic, of the energizing power.

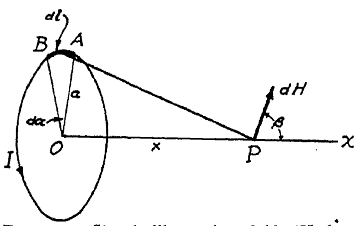


FIG. 232.—Sketch illustrating field dH due to the current element dl.

Field Due to a Circular Loop at a Point on the Axis of the Loop.— The field due to a circular circuit of one turn at a point P (Figure 232) on the axis of the circle at a distance x from the center may be calculated readily as follows: †

Consider first the field dH due to the current element AB. From Ampere's law (Equation 1),

$$dH = \frac{Idl \sin \theta}{\;}$$

* From a physical viewpoint, it is readily recognized that the shielding effect is due to the absorption of the magnetic energy by eddy currents in the overburden.

† L. Page and N. I. Adams, *Principles of Electricity*, p. 247 (Van Nostrand Company, 1931).

where $dl = AB$ and θ equals the angle between dl and r. This field is perpendicular to dl and r. Hence, $\sin\theta = 1$. Also, from the geometry of the figure, $dl = a\,d\alpha$. Hence,

From symmetry the resultant field is along the x axis and is equal to

$$H = \frac{Ia}{r^2}\Big[$$

or

$$H = \tag{8}$$

It is obvious that if the circular circuit has N turns, the last formula would become

$$H \quad 2\pi NIa^2$$

Mutual Inductance Between Parallel Coils.—The mutual inductance M between two electrical circuits is equal to the magnetic flux associated with one circuit due to unit current in the other. That is,

$$M = \frac{\text{magnetic flux in secondary circuit}}{\text{current in primary circuit}} = \frac{\text{magnetic flux in primary circuit}}{\text{current in secondary circuit}}$$

or

$$M = \frac{\phi}{I} \tag{9}$$

where ϕ denotes the magnetic flux in one circuit due to a current I in the other.

M is a constant for any two circuits and depends on the geometrical configurations and orientations of the circuits and the magnetic properties of the media surrounding them.

The approximate value of the mutual inductance between two parallel coaxial circles, one of which has a radius b small compared to the radius a of the other, is readily calculated. The field at the center P of the smaller coil due to the current I in the larger coil is given by Equation 8. That is,

$$H = \frac{2\pi Ia^2}{(a^2 + x^2)^{3/2}}$$

where $x = OP$. Because b is assumed to be small compared to a, the field H is approximately constant over the entire cross section of the smaller coil. Therefore, the magnetic flux through the smaller coil is

where μ is the magnetic permeability of the medium coils. But from Equation 9, the mutual inductance is netic flux of induction divided by the current. Hence,

$$M = \frac{2\mu a^2 b^2}{(a^2 + x^2)^{3/2}} \qquad (10)$$

If the coils have N_1 turns and N_2 turns respectively, the last equation becomes

$$(a^2 + x^2) \qquad (10a)$$

The mutual inductance M is an important quantity in the analysis of inductive processes bec e the magnitude of the induced E.M.F. in a secondary circuit due an alternating current in the primary circuit is directly proportional to M. That is,

$$d\phi$$

or

$$E = -l \frac{}{dt} \qquad (11)$$

The application of Equation 11 in inductive prospecting will be indicated in the following section.

General Characteristics of Induced Fields.—Certain general ideas relative to the type of magnetic field to be expected when currents are induced in conducting masses buried in the earth may be obtained from a consideration of the interaction between the two coils P and S shown in Figure 233.†

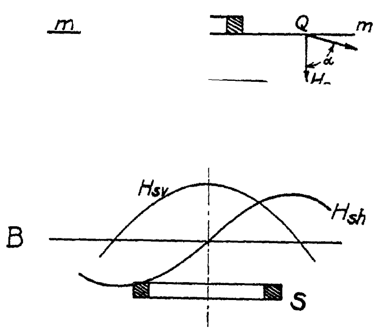


FIG. 233.—A, sketch illustrating resultant magnetic fields H_p and H_s due to the primary and secondary coils P and S. B, curve H_{sv} shows the general shape of the vertical component of the magnetic field due to the induced current in the secondary coil and curve H_{hs} shows the general shape of the horizontal component. (After Peters and Bardeen, Bulletin of the Univ. of Wisconsin, Engineering Experiment Station Series No. 71.)

An alternating current $I_p = I_{pm} \cos wt$ is sent through the primary circuit P. This current induces an E.M.F. in the secondary circuit whose magnitude is given by Equation 11; that is,

$$E = -M \frac{dI_p}{dt} = wMI_{pm} \sin wt$$

where M is the mutual inductance between the two circuits.

The electromotive force E causes a current I_s to flow in the secondary circuit. If the capacity

† L. J. Peters and J. Bardeen, "The Solution of Some Theoretical Problems which Arise in Electrical Methods of Geophysical Exploration" (Bulletin of the Univ. of Wisconsin, Engineering Experiment Station Series No. 71).

of the secondary circuit is small, the impedance is $R_s^2 + wL_s^2$, where R_s denotes the resistance and L_s the inductance of this circuit, and the current is:

$$I_s = \frac{w}{} \qquad \frac{\sin(wt - \beta)}{w^2 L_s^2}$$

where

$$\tan\beta = \qquad \frac{}{R_s}$$

The expression for I_s can be put into a more conventional form by making use of the trigonometric relation

$$\tan^{-1} \frac{wL_s}{} = \frac{\pi}{2} - \tan^{-1} \frac{R_s}{wL_s}$$

or

where

$$\tan\theta = \qquad (12)$$

On replacing β by its equivalent in terms of θ, we obtain

$$\sin(wt - \beta) = \sin\left(wt - \frac{\pi}{2} + \theta\right) = -\cos(wt + \theta)$$

and

$$\cos(w \qquad (13)$$

On comparing Equation 13 and the expression for the primary current, it is apparent that the time phase angle between the secondary and primary currents is $\pi + \theta$.

At any point such as Q on the plane mm (Figure 233), the magnetic flux density due to the secondary current is given by the relation

$$\sqrt{R_s^2 \cdot} \qquad \frac{pm}{2} \cos(wt + \theta) \qquad (14)$$

where f_s is a function of the geometry of the system, the location of the point Q, and the permeability of the medium surrounding the secondary coil. The magnetic flux density due to the primary current is given by the relation

$$H_p = f_p I_{pm} \cos wt \qquad (15)$$

where f_p is a function of the geometry of the system, etc.

At a particular instant of time, therefore, the fields at Q may be represented by vectors H_p and H_s as shown in Figure 233. These vectors are displaced in space by an angle α and in time phase by an angle $\pi + \theta$. Hence, *in general,* they will combine to give a rotating vector for the resultant field, and the tip of the rotating vector will trace an ellipse.*

In certain special cases, however, θ is approximately equal to zero so that the time phase angle between the secondary and primary fields is approximately 180°. For such cases, the major axis of the ellipse is much larger than the minor axis and the resultant field at any point may be represented without appreciable error by a vector which is constant in direction rather than by a rotating vector.

Consider, for example, a mineralized, highly conductive subsurface zone and assume that with respect to inductive phenomena this mineralized zone is equivalent to a coil. The inductance L_s and the resistance R_s of the mineralized zone will both be small, and it may be shown that $\tan\theta = R_s/wL_s$ approaches infinity; that is, θ is approximately equal to zero. The time phase angle between the primary and secondary fields is therefore approximately equal to 180°, and the direction of the resultant field is substantially constant or fixed. Thus, if a direction-finding loop were revolved about an axis parallel to this fixed direction, no signal would be heard in the headphones, because the field would always be parallel to the plane of the coil.

Usually, electrical prospecting methods which utilize inductive pick-up include appropriate electrical equipment either for evaluating the elliptical polarization properties of the magnetic field (e.g., double coil method) or for minimizing the effects of the elliptical polarization at the detector. However, some methods (e.g., Conklin horizontal loop method) involve balancing the effects of two coils without regard to the elliptical polarization properties of the resultant magnetic field. The latter methods will be discussed first.

HORIZONTAL LOOP METHODS

Conklin † was probably the first investigator to energize the ground successfully by passing alternating current through a large, horizontal, insulated loop or coil laid on the surface of the ground.

Considerable work has been done by subsequent investigators both in the United States and abroad, ‡ using modified equipment.

Magnetic Field Due to a Square Coil.—When current is passed through a large coil lying in a horizontal plane, the magnetic field produced

* The mathematical analysis proving that two fields which differ in time phase and "space phase" combine to produce an elliptically polarized field is given on p. 414.

† H. R. Conklin, "Method and Apparatus for Determining Subterraneous Conductors," U. S. Patent 1,241,197, issued Sept. 25, 1917.

‡ See, for example, Hans Lundberg, "Recent Results in Electrical Prospecting for Ore, *A.I.M.E. Geophysical Prospecting,* 1929, p. 87.

within the coil by this current will be vertical; i.e., perpendicular to the plane of the coil. The *magnitude* of the magnetic field may be evaluated as follows: Referring to Figure 234, which represents a square coil

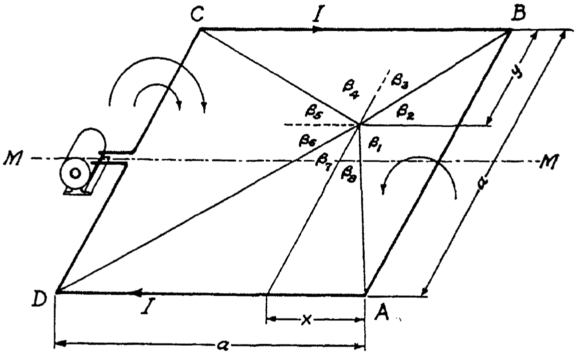


FIG. 234.—Sketch illustrating the geometric relations between the quantities I, a, etc., which enter into the computation of the magnetic field at a point having the coordinates x and y.

carrying a current I, it is desired to compute the magnetic field at a point situated at a distance x from the side AB and a distance y from the side BC. The field due to the side AB is given by Equation 2. That is,

$$= \frac{I}{x}(\sin\beta_2 - \sin$$

But

$$\sin\beta_2 =$$

and

$$\sin \qquad a - y$$

Hence

$$H_{AB} = \frac{I}{x}\left(\frac{y}{\sqrt{x^2+y^2}} + \frac{a-y}{\sqrt{(a-y)^2+x^2}}\right)$$

Similarly

$$H_{BC}$$

$$H_{CD} = \frac{I}{a-x}\left(\frac{a-y}{\sqrt{(a-y)^2+(a-x)^2}}\right.$$

$$+$$

If it is assumed that the current I flows in the direction of the arrow, the fields H_{AB} to H_{AD} in the figure are all directed into the paper (away from the reader). Hence, the resultant field at P is

$$H = H_{AB} + H \quad + H_{CD} + H_{DA}$$

On substituting the values for H_{AB}, $_{C}$, etc., and rearranging terms one obtains

$$H = I \left\{ \frac{\sqrt{x^2 + y^2}}{xy} + \frac{\sqrt{(a-y)^2 + x^2}}{x\,(a-y)} + \frac{\sqrt{(a-x)^2 + y^2}}{y\,(a-x)} \right.$$

$$\left. \frac{\qquad\qquad}{(a-x)\,(a-y)} \right\} \tag{16}$$

Equation 16 specifies the magnitude of the field at any point situated at a distance x from the side AB and a distance y from the side CB.

The value of the field at various points along a line parallel to DA ($y =$ constant) may be obtained by assigning a series of values to x. In particular, the value of H along the line MM ($y = a/2$) is obtained by substituting this value of y into Equation 16.

$$H_y \qquad\qquad x \qquad\qquad a - x \tag{16a}$$

A graph of Equation 16a is shown in Figure 235. It will be noticed

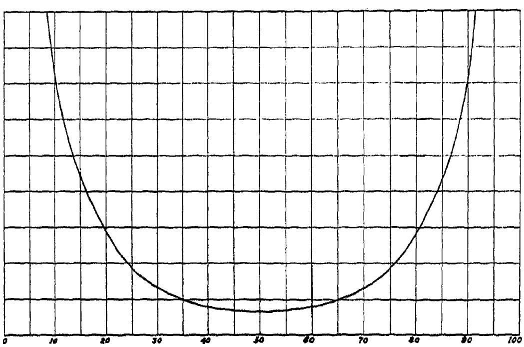


Fig. 235.—Plot showing variation of Ha/I at various distances along a profile inside a square coil of side a.

that H is substantially constant for various values of a between $0.25a$ and $0.75a$.

Field Operations.—The ground is energized by passing alternating current through a large single turn loop of insulated wire laid on the surface of the ground. Usually the current is generated by a gasoline-driven

alternator and has a magnitude of 5 to 20 amperes, and for shallow investigations, a frequency of 500 to 1000 cycles per second. The size of the ground loop varies with the nature of the problem, usually being from 1000 to 2500 feet on a side.

The observational technique consists in determining contours of equal magnetic flux anomaly. These isanomalic contours may be determined by using two search coils or loops which are opposingly connected. Each coil contains about 500 turns of fine wire wound on a four- to six-foot frame. The work can be carried out by the usual field procedure of mapping the location of points of equal magnetic strength, i.e., by leaving one coil at a station and then moving the second coil until a position is found where the induced potential between the two coils is a minimum; these two locations are marked and transferred to a plot of the area. The first coil is now moved along the traverse line beyond the second coil to the next null point of reading and the process repeated. In an alternate procedure, the second coil is moved to the position initially occupied by the first coil, and the first coil is moved forward to search for a new null point.

Precautions must be taken to have the two search coils as nearly identical electrically and magnetically as possible. The comparing circuit and amplifier should be well shielded and placed midway in the shielded two-circuit conductor connecting the two search coils. Proper electrical balance of the two coils can be readily checked during the progress of the survey by placing one coil exactly over the other. If their outputs are connected in opposition, no signal should be noted in the headphones or vacuum tube voltmeter.

The location of these points of equal magnetic flux anomaly is usually accomplished by keeping both coils in a horizontal position. In one field procedure, two operators are usually required, each one "wearing" a coil. The coil is suspended in a horizontal position by means of cross-straps which hang over the shoulders of the operator. The coil should be about waist high to allow the operator freedom in walking.

The method is relatively simple in application, and changes in the energizing current usually do not affect the comparisons. The chief limitation of this method is the fact that measurements are practically confined within the central portion of the area bounded by the large energizing ground loop. Within this area, the primary field (that created by the flow of energizing current through the coil) has its maximum value and the secondary field (that created by the flow of induced current) is superimposed on the primary field. Since the primary field is usually many thousands of times stronger than the secondary field associated with a subsurface conductive zone, it is often difficult (except when good conductive bodies lie close to the surface) to make reliable measurements which will show such small variations in the total or resultant field.

Other balancing techniques may be employed. If desired, the relative field strengths at two different points may be determined by keeping one coil horizontal and then tilting the other coil until the signal is at a minimum.† This technique necessitates that the tilted coil be the one located in the more intense field. Instead of tilting one coil, a variable potentiometer or a tap switch may be used to vary its output and the relative field strengths may be calculated from the relative settings of the potentiometer or the number of turns in the coil.

Absorption Method. — A modification of the electromagnetic method previously described employs a balance system ‡ which is essentially a differential transformer in which the inductance of the loop or exploring coil is measured by an impedance bridge or differential transformer. (Figure 236.) When a current of constant frequency is supplied by the alternator and proper balance obtained, a zero null or minimum signal will be heard in the headphones. If, now, the coil is moved over an area having a greater or smaller conductivity than that corresponding to the initial balance, the inductance of the coil will be changed resulting in an unbalanced condition, and a signal will be heard in the phone.

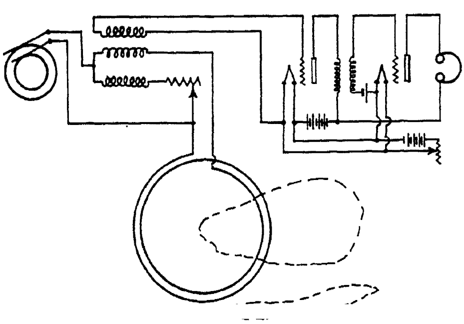


FIG. 236.—Diagram of connections for absorption method. (After Carlson and Hanson, U. S. Patent 1,325,554.)

This type of equipment, or a modification thereof, is oftentimes useful for the location of buried metal objects within a few feet of the surface. By making the coil approximately 5 feet in diameter and taking particular care to balance out capacity and other undesirable feed-back currents, a simple outfit can be made for the location of pipe lines, "buried treasures," etc., which occur within a few inches or a foot or two from the surface. In operation, the exploring coil is carried approximately 6 inches from the ground and the observer slowly walks over the area to be explored. The dimensions of the body to be located have a direct bearing on the depth at which they may be detected. As a general rule the conductive body should have at least two dimensions equal to or greater than the depth at which it is located from the coil. For instance, if the coil is held 6 inches above the surface of the ground and the body

† Karl Sundberg, "Method and Device for Detecting and Locating Ores in an Electromagnetic Way," U. S. Patent 1,678,489, issued July 24, 1928.

‡ W. L. Carlson and E. C. Hanson, "Means for Locating Ore Bodies by Audio-Frequency Currents," U. S. Patent 1,325,554, issued Dec. 23, 1919.

is buried one foot below the surface, the conducting body should have two dimensions $1\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

Method for Mapping Materials of Anomalous Magnetic Permeability.—Sundberg † has proposed a method for detecting magnetic materials which utilizes a magnetic field created by passing *direct* current through a coil or loop laid on the surface of the ground. This *constant* field, being vertically polarized, will cause a change in the intensity and direction of the magnetic field normally present in a magnetized subsurface body. By means of a sufficiently sensitive instrument for measuring magnetic field strength, the location and extent of the anomalous magnetic bodies may be determined.

It will be noted that fundamentally this is not a method of mapping the subsurface distribution of current, but primarily one of mapping materials of greater magnetic permeability than the surrounding earth.

Methods for Surveying an Area by Determining the Polarization Ellipse.—As indicated previously, the magnetic field at the surface of the earth is the resultant of the primary field due to the energizing coil and the secondary fields due to various induced currents.

To simplify the mathematical analysis the various secondary fields will be treated as one field. The vector representing this resultant secondary field and the vector representing the primary field will generally be displaced in space by some angle, α say, and in time phase by some other angle, for example $\pi + \theta$. (Compare p. 408.) Hence, the resultant of the primary and secondary fields is of the type known as an elliptically polarized field. The vector representation of an elliptically polarized field is a rotating vector whose tip periodically traces out an ellipse and whose length at any instant is proportional to the magnitude of the field at that instant.*

The production of a resultant elliptically polarized field by two fields which are out of phase may be shown mathematically as follows: Assume that at a point P the primary field is in the x direction and the secondary field is in the y direction.** Assume also that the two fields have the same frequency $(w/2\pi)$ and that they have a time phase difference of $\pi + \theta$.

† K. Sundberg, "Method and Apparatus for Magnetic Prospecting," U. S. Patent 1,748,659, issued Feb. 25, 1930.

* Consider for example, that the resultant magnetic field at any instant is given by the equation $H = H_0 \sin(wt - \phi)$ where H_0, w, and ϕ are constants and t denotes time. Assign a series of values to t; for example, $t = 0, \frac{\pi}{4w}, \frac{\pi}{2w}, \frac{3\pi}{4w}, \frac{\pi}{w}, \frac{5\pi}{4w}, \frac{3\pi}{2w}, \frac{7\pi}{4w}, \frac{2\pi}{w}$ and compute the corresponding values of H. Then draw vectors whose lengths are proportional to H and whose directions make angles, $-\phi$, $(\pi/4w - \phi)$, $(\pi/2w - \phi)$, etc., with some arbitrary axis. The envelope of the tips of these vectors will be found to be an ellipse.

** Note that this assumption corresponds to assuming that $\alpha = \pi/2$.

Corresponding to these assumptions, the primary and secondary fields are given by the relations:

$$X = X_0 \cos wt$$
$$Y = -\,Y_0 \cos (wt + \theta)$$

where X is the field component in the x direction and Y the component in the y direction, and X_0 and Y_0 the amplitudes, i.e., the maximum values of X and Y.

These two equations are the parametric equations of an ellipse as will be evident from the following analysis. On eliminating the parameter t between the two equations and simplifying, one obtains

$$\frac{Y^2}{Y_0^2 \sin^2 \theta} + \frac{2XY \cos \theta}{X_0 Y_0 \sin^2 \theta} + \frac{X^2}{X_0^2 \sin^2 \theta} = 1 \tag{17}$$

Equation 17 is of the form

where

$$A = 1/Y_0^2 \sin^2 \theta$$
$$B = \cos \theta / X_0 Y_0 \sin^2 \theta$$
$$C = 1/Y_0^2 \sin^2 \theta$$

Hence (17) is the equation of an ellipse.*

The analysis given in the above paragraphs treated a special case of two fields oriented in space at right angles to one another. However, it can be shown that irrespective of the number of secondary fields and irrespective of their amplitudes, phases and directions, the resultant of the primary and secondary fields may be represented by a single vector whose tip periodically traces out an ellipse.†

Determinations of the ratio of the major to the minor axis of the ellipse at various stations in a given area will frequently give valuable information on subsurface irregularities in conductivity. Such determinations are conveniently accomplished by the double coil method.

Double Coil Method

The double coil phase measuring system employs a large horizontal loop for energizing the earth. The Bieler-Watson detecting method‡ consists essentially of two coils permanently fixed at right angles to each

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* It is readily verified that Equation 17 degenerates into the equation of a straight line when $\theta = 0$. For on setting $\theta = 0$ in Equation 17 one obtains

$$\frac{}{X_0 Y_0} + \frac{X^2}{X_0^2}$$

(

and

which is the equation of a straight line.

† A. B. Broughton Edge and T. H. Laby, *Geophysical Prospecting*, p. 278 (Cambr. Univ. Press, 1931).

‡ E. S. Bieler and H. G. Watson, "Apparatus for Use in Discovering Ore Bodies," U. S. Patent 1,794,666, issued Dec. 12, 1927.

other and connected in opposition through an amplifier and telephones. The larger or vertical coil is wound with a fixed number of turns of fine wire and is connected in parallel with a condenser of suitable value for tuning the coil to the frequency of the alternator. The smaller or horizontal coil is tuned by the same condenser and is connected to a compound multipoint switch in such manner that its number of turns may be varied. The amplifier and headphones are connected in series with one of the coils.

To determine the resultant or total magnetic field at any station, the larger coil is mounted in an approximately vertical plane and oriented in the desired direction. The apparatus is then rocked backwards and forwards slowly and the number of turns in the horizontal coil is varied by

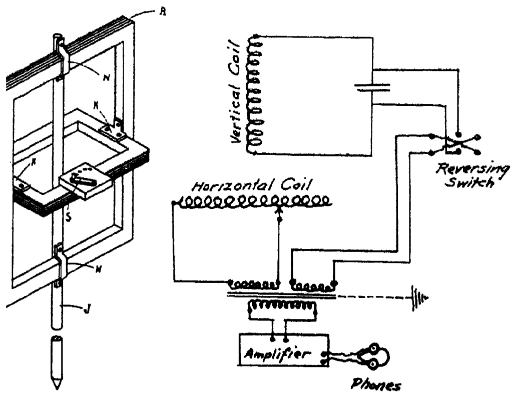

FIG. 237.—Double coil method for determining components of the elliptical electromagnetic field.

the multipoint switch until the null point or minimum signal is obtained when the double frame is vertical. The number of turns in the horizontal coil necessary to obtain this balance is noted.

If the axes of the coils are parallel to the axes of the ellipse, only one such reading is necessary, as will be evident from the following considerations:

Consider for simplicity that the horizontal coil is perpendicular to the major axis of the ellipse and the vertical coil is perpendicular to its minor axis. The induced E.M.F. of the horizontal coil (Figure 237) is altered by varying the number of turns until it balances the induced E.M.F. in the tuned vertical coil. (A reversing switch is provided for obtaining the opposed phase relationship.) The number of turns in the horizontal coil is a measure of the field threading the vertical coil. Thus, when the plane of the ellipse is parallel to that of the vertical coil, the number of turns in the secondary coil is proportional to the ratio of the minor to the major axis of the ellipse.

In practice, two separate readings are taken at each station, instead of spending the necessary time required in determining the plane of the ellipse. That is, after a balance has been obtained for one orientation, the apparatus is rotated through 90° about the vertical axis and another balance reading is made. The two readings (number of turns) are then compounded vectorially to obtain a resultant whose magnitude is proportional to the ratio of the minor to the major axis of the ellipse.

Electrical Characteristics of Apparatus.—The amplifier and coupling transformer should be well shielded electromagnetically and electrostatically. Undesirable coupling between circuits can be minimized most successfully by feeding the output of the horizontal and the vertical coils into separate peak-tuned input windings of the transformer. These windings are connected so their fluxes oppose and, when balanced, give zero induced potential in the secondary coil. An electrostatic shield should be provided between the two primary windings and the secondary winding.

The alternator supplying power for energizing the loop should be of good design to produce as pure a sine wave as possible in order to minimize harmonics. Also, an input filter should be provided in the amplifier to remove harmonics, because in circuits of this type proper null point balance can not be obtained unless the higher harmonics are well suppressed.

Field Procedure.—The initial step in the survey is the layout of the large horizontal loop for energizing the area under investigation. The loop should extend well beyond the limits of the area in order to minimize the effects of a non-uniform field near the loop itself. If terrain conditions permit, a square or rectangular loop is usually the most desirable. Inside the loop, a regular gridwork or pattern of observation points where the readings are to be made is surveyed. If a square loop is used, the observation points are preferably laid out by intersecting lines parallel to the sides of the loop. These points of observation are to be plotted on the map.

If the energizing loop is laid in flat country, the magnetic field within the loop (primary field) is substantially vertical at all of the observation points, provided these are not too close to the cable. The magnetic field due to induced currents, however, will usually have a horizontal component at the surface. Hence, the resultant magnetic field at the surface is usually elliptically polarized, and the ratio of the major to the minor axis at a series of stations will give a measure of the magnitude, direction, and relative phase of the secondary field. For example, if measurements are taken near a large conducting body, a large out-of-phase secondary field will occur and this will give rise to a relatively large minor axis in the representative ellipse. † On the other hand, if the secondary field is zero or if it lags or leads the primary field by 180°, the minor axis of the ellipse vanishes.

Double Coil Method for Measuring Phase and Amplitude

Hedstrom ‡ has developed the apparatus shown in Figure 238. The apparatus comprises two coils, each having approximately 1200 turns on a circular 3-foot frame, connected to an alternating current bridge curcuit. A three-stage amplifier and headphones are used for null determinations. The phase relationship between the fields affecting the two coils may be determined by changing the inductance or capacity in one leg of the bridge,

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† A. B. Broughton Edge and T. H. Laby, *loc. cit.*, p. 60,

‡ H. Hedstrom, "Phase Measurements in Electrical Prospecting," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 827, 1937.

while the ratio of field strengths may be determined by use of the conventional potentiometer method.

Interpretation is based upon phase differences and ratio of field strengths. The results usually are plotted as contours comprising equi-phase curves or equi-field-strength curves. (Figure 239.)

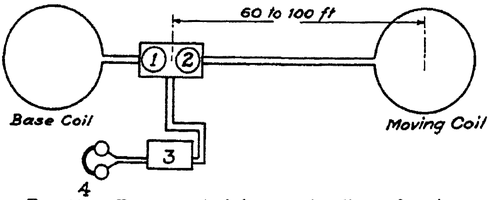

FIG. 238.—Turam method for mapping lines of equipotential, phase and amplitude. 1, amplitude control; 2, phase control; 3, audio-frequency amplifier; 4, headphones. (Hedstrom, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 827.)

METHODS WHEREIN THE EFFECTS OF THE PRIMARY FIELD ARE MINIMIZED

There are many methods by which the field from the surface energizing system may be minimized at the detecting coil. Theoretically, this can be accomplished with a conductive energizing system utilizing two grounded electrodes. For this work, a direction-finding coil is used, and its axis of rotation is on the extension of the imaginary line connecting the two current or energizing electrodes. When in this position very little of the primary field from the surface wire is picked up by the coil. On the other hand, the position of the coil allows it to pick up the secondary field from the subsurface conductive zone.

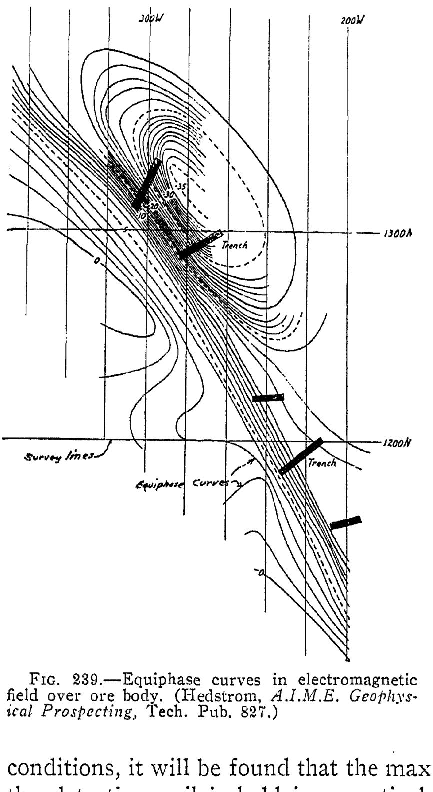

FIG. 239.—Equiphase curves in electromagnetic field over ore body. (Hedstrom, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 827.)

The terminals of the detector coil are connected to a detector and a two stage audio-frequency amplifier for the higher frequencies, or a three stage amplifier for the lower frequencies. The output of the amplifier is connected to a vacuum tube voltmeter. By keeping the amplification constant relatively simple field work will often show the presence of a shallow conductor. Under such conditions, it will be found that the maximum signal will be obtained when the detecting coil is held in a vertical position, and is located vertically

over the subsurface conductor. The method, however, is not generally recommended due to the difficulties encountered by phase-shift, and the unknown distribution of the primary current between electrodes.

Another method,† applicable to the higher frequencies only and hence of limited use, consists in setting up a primary high frequency electromagnetic field polarized about a linear axis. The direction of the secondary field produced by the induced current in a conductive body is determined at points on the linear axis. The direction-finding apparatus is similar to that described and shown in Figure 229. The energizing apparatus may be of the simple Hartley oscillator type. The method works fairly well at depths not exceeding 25 to 150 feet in dry desert countries where long vein conductors are encountered.

In another modification of this method, the surface wires are laid

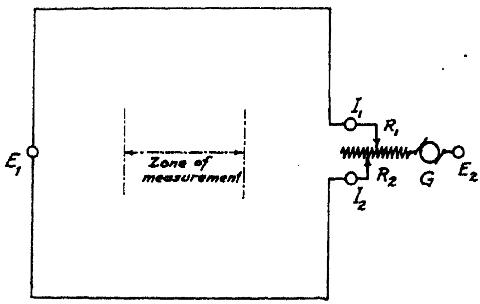

FIG. 240.—Conductive method of energizing the subsurface. E_1 and E_2, electrodes; R_1 and R_2, resistors for controlling current; G, generator; I_1 and I_2, line ammeters.

out as a divided conductor in the form of a rectangle or square as shown in Figure 240. If measurements are to be made along a line midway between the wires, the rheostats R_1 and R_2 are adjusted to give equal current flow in the two branches of the current. The primary fields from the two surface wires effectively neutralize each other, and the vector direction and magnitude measured by the detecting coil are chiefly due to the secondary field set up by the current flowing in the subsurface. Obviously, a disadvantage of the method is the necessity of laying the two surface lines. For general reconnaissance work only one line or "U" may be laid, although interpretation of the vector data is more difficult. This same divided conductor system for energizing the ground has also been employed successfully when magnetometric methods of measurement are used for measuring the magnetic field associated with the flow of direct current.

† J. J. ', "Method Apparatus for Locating Conductive Bodies," U. S. Patent 1,792,910, is b. 17, 1931.

METHODS EMPLOYING A VERTICAL COIL ENERGIZING SYSTEM

The most widely employed electromagnetic equipment utilizes a vertical coil for energizing the ground. The vertical coil is especially suitable for energizing conductive zones which are vertical or which have a steep dip, such as veins, etc. This type of equipment permits more rapid field work than the large horizontal loop type. Also, the results may be

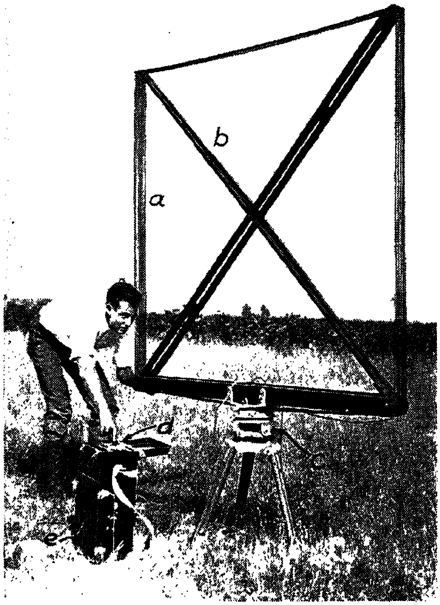

FIG. 241.—High frequency energizing system in operation: *a*, high frequency coil; *b*, folding coil support; *c*, tripod head allows rotation about vertical axis; *d*, high frequency oscillator; *e*, battery compartment.

interpreted more conveniently. The vertical type coils are usually energized with medium to high frequency alternating current because of their relatively small size.

Energizing Equipment.—A vertical type energizing system of the "high frequency" type is shown in Figure 241. This system operates at frequencies from 30 kc. (30,000 cycles per second) to 50 kc. The cross pieces for supporting the coil are each 10 feet in length, and when set up, the coil dimensions are approximately 7 feet by 7 feet. The coil is mounted on a tripod so that it may be leveled and oriented easily. Bakelite, or other insulating supports, must be provided for holding the turns

of the coil in place. (The number of turns depends on the spacing and frequency.) Formulas for proper design of this coil will be found in text books on radio and high frequency phenomena.†

The vacuum tube circuit and the batteries for both plate and filament supply are contained in a hard oak case. Shoulder straps and carrying handles are provided for convenient handling of the outfit. To provide

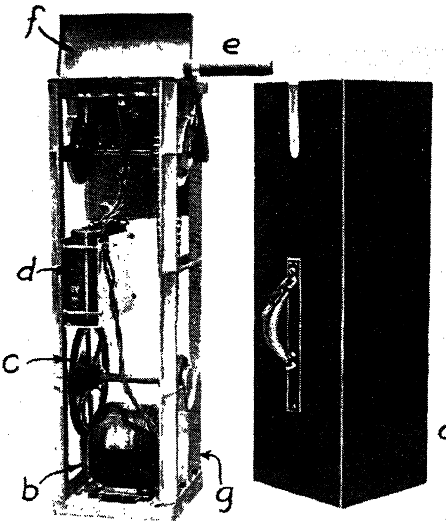

FIG. 242.—Alternator and its waterproof case. *a*, waterproof case; *b*, 500-cycle alternator; *c*, driving gear; *d*, dry cells for field of alternator; *e*, crank; *f*, waterproof lid covering control panel; *g*, cast aluminum frame.

the 450 volt plate supply, ten "B" batteries, of medium-duty type (45 volts each) are connected in series. No. 6 dry cell batteries are connected in series to provide the filament supply.

Power may also be obtained from a specially designed 500-cycle 75-watt hand-cranked alternator, an interior view of which is shown in Figure 242. Two handles are provided for cranking. These are connected to a gear driving the alternator. The alternator turns over at a speed of 4000 r.p.m., and the cranks rotate at a speed of approximately 100

† J. H. Morecroft, *Principles of Radio Communication* (John Wiley and Sons, 1927). F. E. Terman, "Radio Engineering" (McGraw-Hill, 1932).

r.p.m. Ball bearings which are provided throughout facilitate the cranking required to generate the necessary power. Approximately 50 watts input is required for operation of the high frequency apparatus. Direct current for the field of the alternator is supplied from three No. 6 dry cells. The entire apparatus is contained in a steel waterproof case.

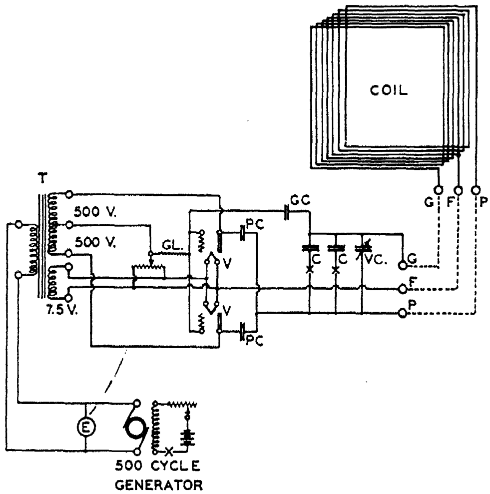

FIG. 243.—Wiring diagram for high frequency inductive equipment using 500-cycle generator for filament and plate supply. *C*, fixed condensers; *VC*, variable condensers; *T*, input transformer; *E*, voltmeter.

Figure 243 gives the wiring diagram for the oscillator when using alternating current for supplying power. The output of the two tubes shown is practically the same as would be obtained from one tube when direct current is employed for plate supply.

Medium Frequency System

Power for exciting the coil may be obtained from a suitable medium frequency (100-500 cycles) alternator or from a conventional vacuum tube oscillator. A rotating type alternator or a thyratron inverter tube † may also be used.

For lower frequencies, that is, under about 25 cycles per second, it is not practical to use either vacuum tube oscillators or rotating alternators for the generation of a usable amount of power with portable equipment. This is because of the large weight and bulk of the equipment necessary to handle these low frequencies, due chiefly to the large amount of iron required in the magnetic circuits. Where these extreme low frequencies are desired, it is necessary to use a suitable commutator for inverting a direct current supply. This type of equipment has been described earlier.

† K. Henney, *Electron Tubes in Industry* (McGraw-Hill).

Figure 244 shows a medium frequency energizing coil consisting of 50 turns of rubber-insulated stranded wire bound together into a single cable to facilitate handling. The flag seen in the illustration is placed at the electrical center of the coil and is used for proper alignment of the direction-finding coils. The energizing coil should be oriented substantially parallel to the strike of the vein system. Any convenient means may be employed for supporting the cable during operation. In the work illustrated, two 15-foot 3-inch x 2-inch hardwood supports were employed, properly guyed at each end. The coil had dimensions of approximately 15

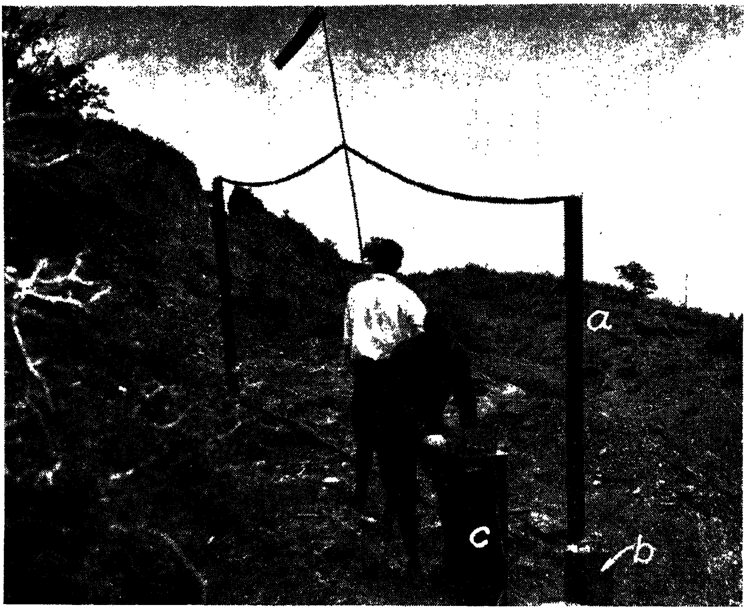

FIG. 244.—Vertical energizing coil in position. *a*, medium frequency energizing coil; *b*, impedance matching transformer with tuning condensers; *c*, hand-cranked alternator.

feet by 45 feet. An alternator of the type shown in Figure 242 may be used for exciting the coil where work is done at shallow depths. For deeper work, it is preferable to employ a gasoline engine-driven alternator of at least 1000 watts output.

Field Operations for Vertical Energizing Coil Method.—If the area under investigation is homogeneous, the electromagnetic field produced at the surface by the vertical coil will be polarized in a substantially horizontal direction, provided the other fields acting at the surface are negligible relative to that of the energizing coil.†. Hence, when a direction-finding coil is placed in such a position that its axis of rotation lies in the

† J. J. Jakosky, "Method and Apparatus for Locating Unknown Conductive Bodies," U. S. Patent 1,811,547, issued June 23, 1931.

plane of the energizing coil, the minimum signal will be obtained when the two coils are at right angles to each other, i.e., when the direction-finding coil is horizontal and the energizing coil is vertical. On the other hand, a maximum signal will be obtained when the direction-finding coil is in the same plane as the energizing coil.

Referring to Figure 245, it will be noticed that the axis of rotation for the direction-finding coil is horizontal *only* when the direction-finding coil is at the same elevation as the energizing coil. Initial setting up of the inductive energizing and direction-finding equipment therefore involves two operations: (1) proper alignment of the energizing coil so that its plane is always vertical and passes through the axis of rotation of the receiving coil, and (2) alignment of the direction-finding coil so that its axis of rotation passes through the center of the energizing coil. To allow this second step to be done accurately and quickly, alignment sights and a ball-socket joint are provided on the head of the direction-finding coil, as described previously.

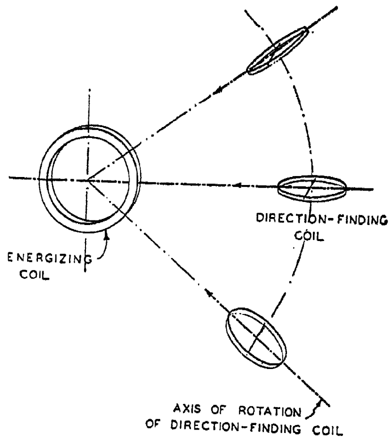

FIG. 245.—Sketch illustrating possible relative orientations of direction-finding coil and energizing coil for minimum signal. (*A.I.M.E. Geophysical Prospecting*, Technical Paper 134.)

When the equipment has been set up with the energizing coil vertical and "pointing toward" the direction-finding coil, the operator of the direction-finding coil knows that if he obtains any angle or dip other than zero (measuring from the vertical) and a strike not pointing toward the energizing coil, some disturbing influence is present. This disturbing influence may be another field caused by induced current flowing in an underground conductive mass.

Orientation of Search Coil Under Influence of Primary and Secondary Fields

If the direction-finding coil is cut by two electromagnetic fields which are in phase and of the same frequency, the position of the coil for maximum or minimum signal strength will be determined by a single resultant of the two fields. For example, if one field is horizontal and the other field is tilted so that it makes an angle of 60° with the horizontal, the direction-finding coil, under proper conditions, "points" somewhere between these two vectors, the exact direction depending on the relative strengths of the two fields and their phase relationship. If the fields were

of equal strength, and in phase, the resultant vector would lie equidistant between them.

The elementary conditions prevailing in actual operation are illustrated in Figure 246 which is a plan view of a conductor of considerable length and small diameter placed so as to be in the field of the energizing coil. The direction-finding coil now has two fields linking it.

At position *C* (lower right-hand part of the figure) the component fields would exert the following effects. Since the energizing coil is vertical, the primary magnetic field will tend to cause the direction-finding coil to give the loudest signal when it, too, is vertical. However, for proper in phase conditions, the field surrounding the subsurface conductor will tend

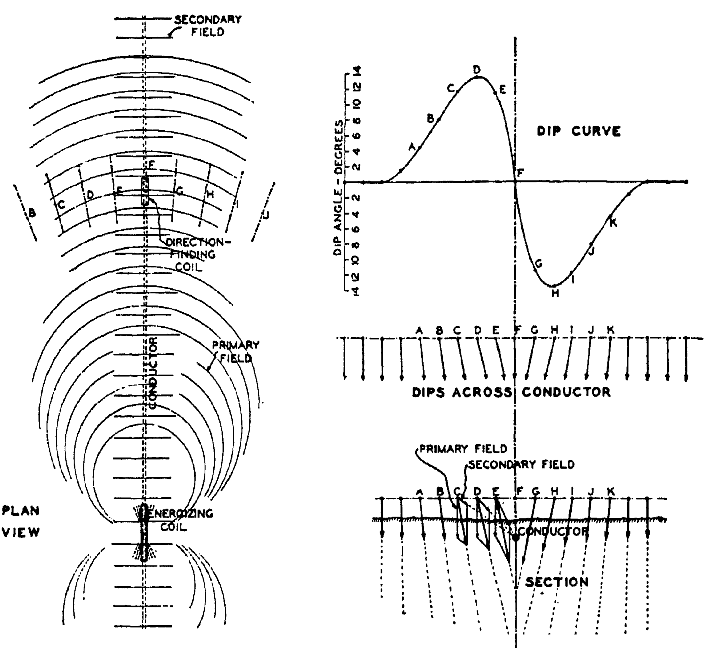

FIG. 246.—Plan view of a linear conductor situated in the magnetic field of the energizing coil. (*A.I.M.E. Geophysical Prospecting,* Tech. Paper 134.)

to produce the loudest signal when in the position shown by the resultant vector. If the direction-finding coil is moved to the position *F,* which is directly above the conductor, both the primary and the secondary fields will induce the loudest signal in the coil. As the coil is moved beyond the vertical position, the direction of the dip angle changes as shown by the vectors *G, H,* etc. Thus, it can be seen that as a traverse is taken across a conductor through which an induced current is flowing, a series of dips will be obtained on the direction-finding coil.

For purposes of illustration, assume that a series of readings is being taken on a circular traverse across the surface of the ground above the conductor, as shown in the plan view. As the distance from the conductor increases, the secondary field vectors are not sufficiently strong to give a noticeable deflection to the angle of the direction-finding coil. The resultant direction for all practical purposes is vertical, or a zero dip. As the direction-finding coil is moved along the traverse toward the conductor, the dip angle becomes increasingly larger until a maximum dip is reached, after which it decreases until a zero dip angle is obtained when vertically over the conductor. As the readings are continued beyond a point over the conductor, the same condition results, except that the dips are in the opposite direction. This is illustrated by the arrows in the diagram marked "dips across conductor," and by the dip curve.

The illustration shows a circular traverse for the direction-finding coil. The distance between the two coils is constant; hence, the primary field has a constant intensity, and the change in the resultant angle is due only to variation in intensity of the secondary field and its change in emergence angle. In practice, however, the traverses are taken along straight lines perpendicular to the conductor. (The relatively great distance between energizing and direction-finding coils allows this to be done without any appreciable error.)

When working over a conductor of considerable length and uniform depth, the ratio of primary to secondary field varies with the distance between the direction-finding and the energizing apparatus. This is usually due to the smaller attenuation of the induced current when traveling along the conductor as compared to the attenuation of the primary field. As a result, the secondary field vector may be relatively large at considerable distances from the energizer, while when very close to the energizer it may be completely masked by the strong primary field.

Since it is necessary that a certain minimum ratio exist between the field strengths of the primary and secondary fields in order that a readable dip angle may be obtained, it is evident that the deeper a conductive ore body lies, the longer must be its effective length for optimum operating conditions. After a given power input has been reached, increasing the power of the energizing system in order to impart more energy to the current induced in the conductor will not change conditions materially, inasmuch as the primary field is increased in proportion. At all times, however, enough power must be supplied to the earth to penetrate to the desired depth and induce currents of such magnitude that the magnetic fields of the induced currents travel through the overburden and reach the surface with sufficient intensity for producing a detectable effect on the direction-finding apparatus.

A high frequency energizer with an output of about 10 watts will be found to have an effective depth range of 25 to 50 feet in dry desert countries when working on favorable type conductors, i.e., a vein or other

elongated conductive mineralized zone. If the 500-cycle apparatus is used, a good practical rule, in ordinary work where the mineralized zones are imbedded in damp areas or under water level, has been to employ power supply of at least three watts per foot of depth; i.e., when working to a depth of 100 feet, the energizing coil should supply a power output of at least 300 to 350 watts. It should be noted that due to the low power factor of the coil, a high volt-ampere product may be necessary to obtain this power output, unless proper means (such as a resonating condenser) are employed for power-factor correction.

Interpretation of Data.—Owing to the distortion of the primary field or improper alignment of energizing and receiving equipment, it often happens that a small (usually less than 10 degrees) "dip" or improper strike direction is obtained. These are called phantom dips or strikes and are readily recognized by the experienced operator. Such dips are frequently obtained when the energizing and direction-finding coils are located on a ridge, in a narrow valley or canyon, or at the edge of a deep cut or precipice. Usually the greater the distance between the energizing and the direction-finding coils, the greater is the wave-front distortion. The lower the frequency employed, however, the less becomes the distortion both of the primary and the secondary fields, and the distortion may usually be neglected at frequencies of 1000 cycles or less.

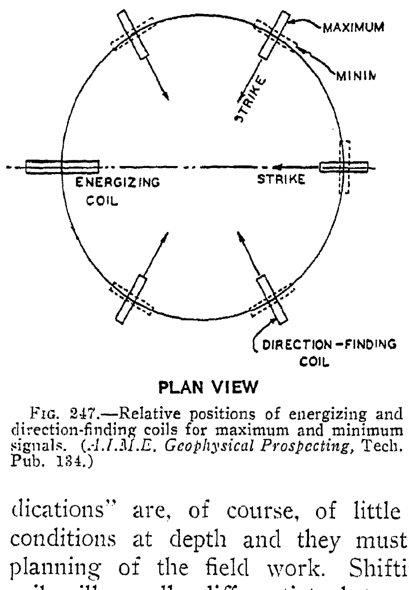

FIG. 247.—Relative positions of energizing and direction-finding coils for maximum and minimum signals. (*A.I.M.E. Geophysical Prospecting*, Tech. Pub. 134.)

Because this topographic effect increases when higher frequencies are employed, the use of radio frequencies will yield "indications" over many ridges, or valleys, and all electrical inhomogeneities existing close to the surface; such "indications" are, of course, of little value in indicating the structural conditions at depth and they must be carefully evaluated by proper planning of the field work. Shifting the position of the energizing coil will usually differentiate between topographic and subsurface conductor effects. Thus, in any area where "indications" are mapped, they should be checked by moving the energizer to a substantially different location and repeating the measurements. Although the high frequency methods are convenient and rapid for reconnaissance work, their "indications" must be checked by the low frequency or direct current methods having greater penetrating power, in order to differentiate between a surface effect and a subsurface conductive zone effect.

Strike Angle of Detector Coil

The azimuth angle or direction of the direction-finding coil when in a vertical plane is called the "strike" and represents the resultant direction of the fields cutting the direction-finding coil with reference to a horizontal plane or plan projection. The relative positions of the energizing coil and the direction-finding coil for maximum and minimum signals are shown in Figure 247. It will be noticed that the strike of the direction-finding coil is toward the energizing coil only when it lies in the plane of the energizing coil. When the direction-finding coil is not in the plane of the energizing coil, the directions for strike are as indicated. Thus, if the apparatus is mounted so as to "point" toward the direction-finding apparatus, the receiving operator knows that any strike other than toward the energizing equipment is caused by some distorting influence. This disturbing factor may be a distortion of the primary field wave front or a secondary field created by an induced current flowing in an underground conductor.

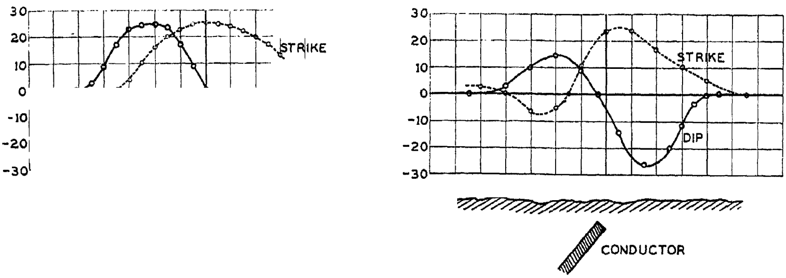

FIG. 248.—Dip and strike relationships over an experimental ore body.

Much information can be obtained by careful study of the dip and strike curves obtained in properly conducted field work. Because the conditions met with in field practice are varied and complex, care and experience are required in making final interpretations. Due to the greater distortion of the high frequencies, considerable caution must be employed in attempting to draw final conclusions from such work when not supported by supplemental data obtained from the application of other methods.

Figure 248 shows the dip and strike relationships over an experimental ore body which consisted of a conductive sheet (copper-covered wood panel) buried in moist sand. The left-hand portion of the figure shows the symmetrical dip angles obtained when the conductor is vertical. Placing the conductor at an angle of approximately 40° from the vertical gives the unsymmetrical dip curve shown in the right-hand portion of the illustration. The strike curve reaches its maximum value in the general vicinity of the point where zero dip is obtained. The relationship between the

position for maximum values of strike and minimum or zero dip angles is complicated because it depends on the direction of current flow in the conductor, the general distribution of current in the subsurface around the conductor, and the phase relationships between the current flow in the conductor and the primary field. However, because the dip curve passes through zero over the effective electrical axis of the conductor, it serves as a general means for determining the plan location of this axis. The general direction of dip of the conductor may also be predicted from the dip curve, because the smallest angle of dip will usually be obtained on the hanging-wall side. Various empirical methods have been developed for determining the depth of the effective conductive zone.†

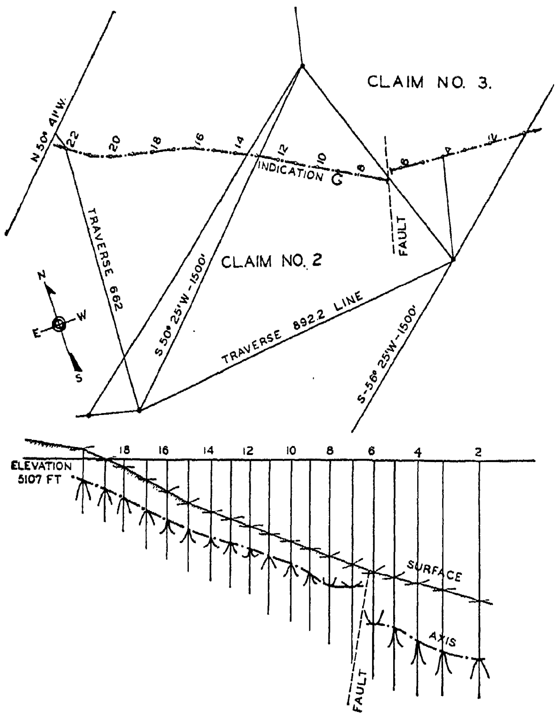

FIG. 249.—Geoelectrical map, Inyo County, California. (*I.R.E.*, Vol. 16, No. 10, Oct. 1928.)

SURVEYS USING VERTICAL COIL ENERGIZING AND RADIO FREQUENCIES

A portion of a map of a typical electrical survey is illustrated in Figure 249, which shows data obtained in a survey conducted in Inyo County, California. The high frequency "indications" in the plan view are tied-in

† J. J. Jakosky, "Electrical Prospecting," *Proc. Institute of Radio Engineers,*" Vol. 16, No. 10, October 1928, pp. 1305-1355.

by the usual surveying methods to known property corners, bench marks, and other features to allow later location of the indication should the electrical survey stakes be removed. The approximate depth of the conductor is obtained by plotting correction curves for traverses taken at intervals along the indication.† By drawing a curve through the indicated depths at each of these traverses, the indicated electrical axis of the conductor is located. The correction curves for each traverse are shown in the figure. The curves shown are drawn by imagining the traverses as being rotated 90 degrees to allow the curves to be plotted in the plane of the paper. The reversal of the index curves at the fault and the displacement of the conductor are very noticeable.

An interesting example of the continuity of a mineralized zone may be seen in Figure 250. This indication was located during a high frequency

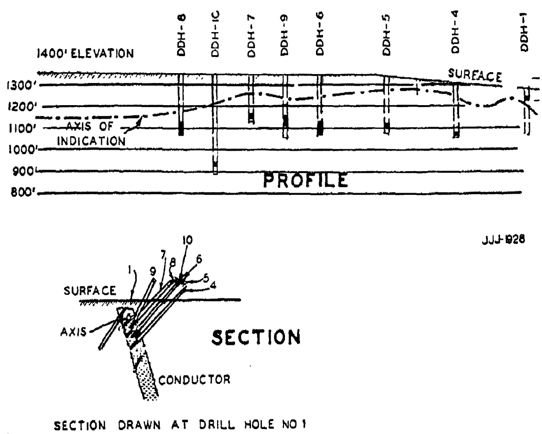

FIG. 250.—Drill results on a shallow high frequency indication. (*A.I.M.E. Geophysical Prospecting*, Tech. Pub. 134.)

electromagnetic survey of a property in Des Meloizes Township, Quebec, Canada. This indication was checked over a distance of 2000 feet between drill hole No. 1 and No. 8, and it is reported that the drill holes cut a wide zone of sulphide mineralization. The conductor is a dipping sheet vein. The "axis" of the conductive zone was determined by use of the correction curve. This axis lies close to the top of the conductor because of the poor penetrating power of the high frequencies and also because the near-surface flow of induced current has a much greater effect than that flowing in the lower portions.

† J. J. Jakosky, *loc. cit.*

ELECTRICAL METHODS

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CHAPTER VII

SEISMIC METHODS

BACKGROUND OF SEISMIC PROSPECTING

The fundamental principles utilized in seismic prospecting were derived from the study of earthquakes and, at first, progress in the application of these principles was slow. After 1924, however, the use of seismic methods for geophysical prospecting increased rapidly, and it will be evident from the following resume how wide an application the seismograph has found in recent years.

Geophysical activity is conveniently expressed in terms of steady employment of a crew for a year, i.e., in units of *crew years*. It is estimated that during 1938, reflection seismograph activity in the Gulf Coast area alone amounted to sixty crew years. From 1931 to 1935 seismic prospecting activity increased at the rate of 5 to 40 crew years per year. The peak of seismic activity was reached in 1938. The expenditures for reflection field work can be estimated on the basis of $6000 to $9000 per month per crew or $72,000 to $108,000 per crew year. During 1938, the Gulf Coast seismic operations represented a gross expenditure of about $6,500,000.

Relation Between Seismic Prospecting and Seismology.—Earthquakes are vibrations or sudden undulations in the earth's crust produced by the shearing of a mass of rock or by volcanic or other disruptive causes. Sudden displacements of great masses of rock create elastic vibrations (seismic waves) which travel away from the region of disturbance and are observed at permanent seismological stations located on the earth's surface at various locations. Several different types of seismic waves are recorded on an earthquake record at a seismological station, and from a determination of the times of arrival of the different types of seismic waves, the kind of motion—longitudinal, transverse, surface—the amplitude of the disturbance, and other factors, inferences may be drawn concerning the location of the source of the waves and the subsurface zones through which they have traveled.

In applied seismology, *artificial earthquakes* are generated by various mechanical or explosive means, such as a heavy mechanical blow at the surface of the earth or the explosion of a charge of dynamite usually buried a short distance beneath the surface of the earth. (An alternative mechanical method utilizes a shaking device whose operation depends on the rotation of a heavy eccentric mass around an axle, driven by a

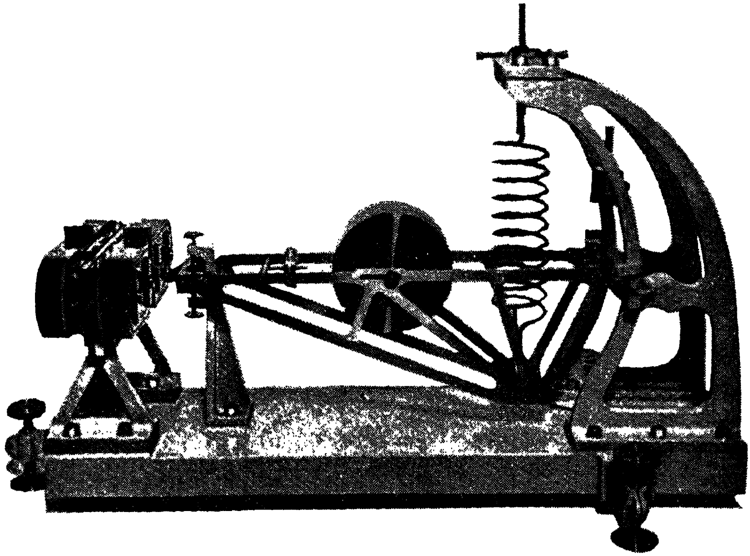

FIG. 251.—Critically damped vertical seismograph employing electrical recording. (Reproduced from Galitzin's *Seismometrie.*)

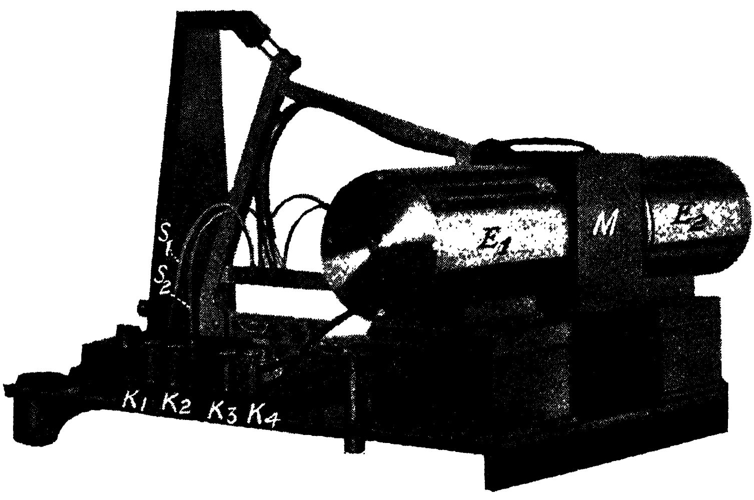

FIG. 252.—L. Grumach's seismometer. (Inductive type.) M = mass to which two coils are attached. (The coils vibrate freely in the field of the electromagnets E_1 and E_2, their axes being parallel to the direction of vibration.) S_1 and S_2 are silver strips, 0.02 mm. thick, which connect the binding posts K_1 and K_2 with the lead to the coils. K_3 and K_4 are the binding posts for the lead to the fixed electromagnet E_1. (Reproduced from Gutenberg's *Grundlagen der Erdbebenkunde*, Vol. 12, 1927. Published by Gebrüder Borntraeger, Berlin.)

motor.) The field technique includes measurements of the times of travel of the elastic waves from the point of disturbance to *several* seismometer stations located at the earth's surface. In favorable cases, knowledge of the travel-times and of the distances between the point of disturbance and the seismometer stations will yield useful information regarding the subsurface structure of the area.

Seismology

Early instruments are illustrated in Figures 251 and 252. These illustrations show typical earthquake seismographs (instruments for detecting and recording the arrival of earthquake waves).* Figure 251 shows a vertical seismograph which is critically damped by electromagnetic means and employs electrical recording. This seismograph, constructed by Galitzin in 1908, differs only in arrangement and size of parts from the typical seismograph employed in exploration at present.**

Most modern seismographs are designed to record on a moving strip of photographic paper or film the oscillatory currents generated by the relative motion of the seismometer housing and an inertia mass. The amplitude of the trace on the photographic record is ordinarily about 200 times greater than the corresponding amplitude of the earth's motion. In some installations, where electrical amplification is employed, the magnification may be as great as 100,000 times.

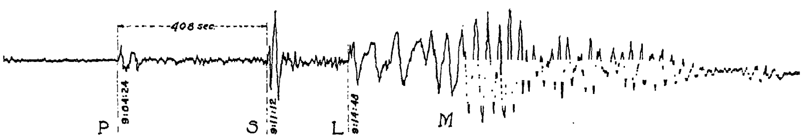

Fig. 253.—Schematic representation of an earthquake seismogram.

Figure 253 is a schematic representation of an earthquake record or seismogram. Some of the more prominent waves or "phases," as they are usually called, have been designated on the records by the letters *P*, *S*, *L*, and *M*. The various types of waves, i.e., the various "phases," are propagated with different velocities, depending upon their mode of vibration and the character of the medium through which they travel.

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* Descriptions of seismological instruments and records commonly contain the following technical terms: seismometer, seismograph, and seismogram. A *seismometer* is a device for measuring vibrations due to earthquakes. A *seismograph* is a seismometer equipped with an indicator (mechanical, optical, or electrical) for recording a quantity which depends on the vibration or relative motion of the instrument. The quantity recorded is usually a function of the displacement, velocity, or acceleration. A *seismogram* is the record, for example photographic record, made by a seismograph.

** A detailed description of the principal types of seismographs employed in seismological observatories is given in "Selection, Installation, and Operation of Seismographs" by H. E. McComb, U. S. Department of Commerce, *Special Pub. No. 206.*

In the primary or P waves, the vibrations of the earth particles are longitudinal, i.e., in the direction of propagation. In the secondary, or S waves, the vibrations are at right angles to the direction of propagation, and are termed transverse waves. Although the P and S waves traverse the same path, they arrive at a seismological station at different times, because the velocity of the longitudinal (P) wave is greater than that of the transverse (S) wave. The complex "surface" or L waves travel at a still slower speed; hence, in general, they arrive later than the P and S waves.

The earth is continually undergoing slight vibration as is seen in the extreme left-hand portion of Figure 253. The first sharp displacement of this wave-like line, at P, represents the beginning of a larger disturbance or vibration. The amplitude of the disturbance is small and the period of oscillation is fairly short. Also, the motion is very irregular indicating that additional impulses arrive at irregular intervals. The record shows that a few minutes after this first motion, at the point marked S, there begins another type of motion. This motion has a somewhat larger amplitude and about the same period as, or a greater period than, the first disturbance. Following the initial large amplitude the motion becomes quite irregular indicating, as before, the arrival of additional impulses.

A short time later, at the point marked L, a different type of motion begins. The amplitude of this motion is relatively large and the period relatively long. This portion of the disturbance is followed by a complex vibration M of even greater amplitude and somewhat shorter period. These two parts of the motion are referred to as the principal portion. In the final portion, the amplitude is greatly decreased and the period is somewhat lengthened.

The maximum displacement of the ground usually amounts to a fraction of a millimeter during light shocks. In destructive earthquakes, however, the maximum acceleration often exceeds two hundred millimeters per second per second and has been known to exceed 10,000 millimeters per second per second.*

Figure 254 shows the travel-times for the P, S, and L waves for various distances.** The S wave travels faster than the L wave and the apparent speed increases with distance, due to penetration of the wave into zones of higher velocity. Likewise, the P wave travels faster than

* It is of interest to contrast these displacements with the earth motion corresponding to a readily detectable reflection in the reflection shooting type of seismic prospecting. The amplitude of the motion at the seismometer due to the reflected wave may be a *few millionths* of a millimeter, while the acceleration may be a *few thousandths* of a millimeter per second per second. (Compare D. H. Gardner, "Measurement of Relative Ground Motion in Reflection Recording," *Geophysics,* vol. 3, No. 1, 1938, p. 40.)

** Essentially similar time-distance curves are obtained everywhere on the earth's surface. This fact attests to the uniformity of the earth's structure.

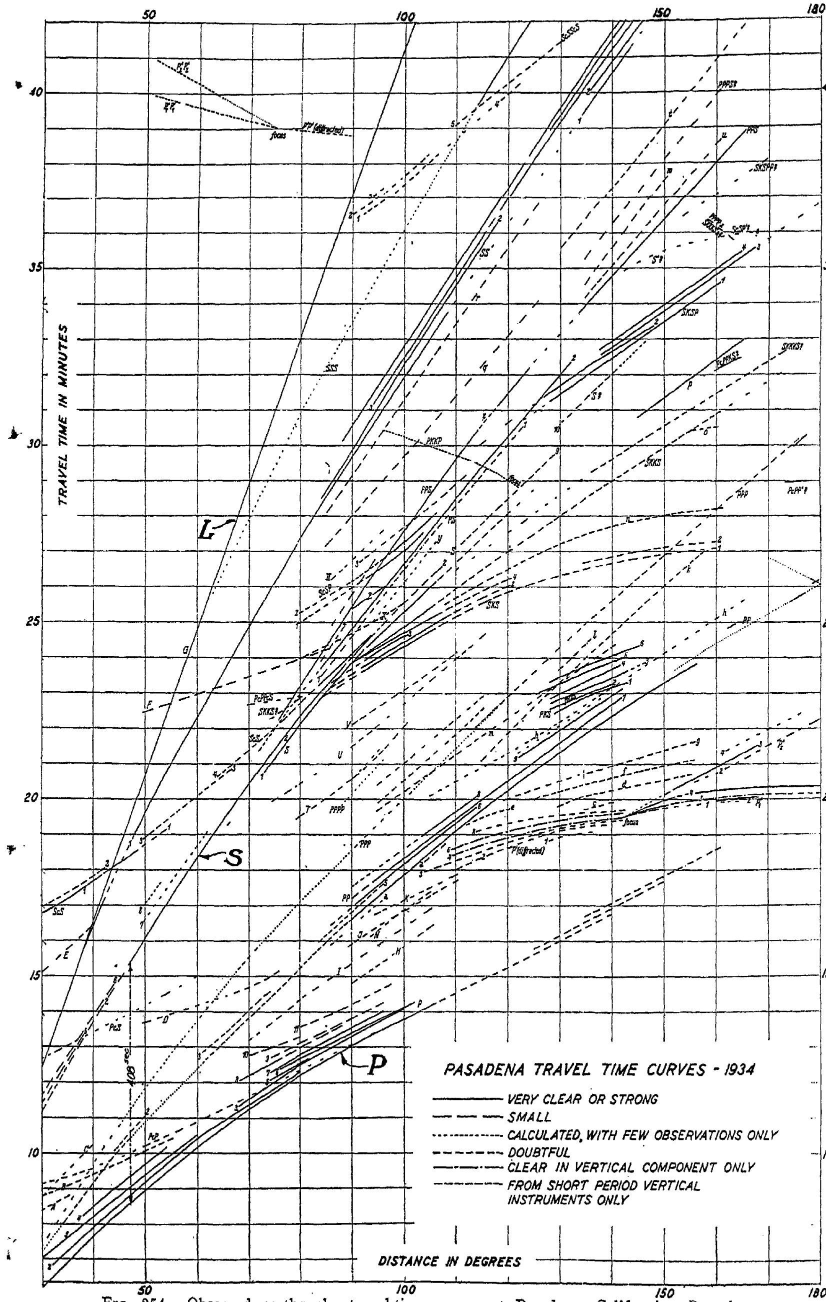

FIG. 254.—Observed earthquake travel-time curves at Pasadena, California. *P*, primary or longitudinal waves; *S*, secondary or transverse waves; *L*, complex surface waves. (Gutenberg and Richter, *Gerland's Beiträge zur Geophysik.*)

the S wave and its speed also increases with depth down to the core of the earth.

The method employed at seismological stations to determine the focus of an earthquake will be evident from Figures 253 and 254. In Figure 253, the P wave is recorded at 4 minutes and 24 seconds past 9 A.M., and the S wave at 11 minutes and 12 seconds past 9 A.M. The S wave, therefore, requires 6 minutes and 48 seconds longer than the P wave to reach the recording station. From Figure 254 it is seen that a difference of 6 minutes and 48 seconds (408 seconds) between the P and S curves corresponds approximately to 46.9 degrees or 5100 kilometers. Therefore, the earthquake must have occurred at about this distance from the Pasadena station at which the record shown in Figure 253 was obtained.

One of the most interesting applications of earthquake seismology is the investigation of the structure of the earth at great depths. Time-distance curves and tables derived from comprehensive and reliable data obtained at a great number of seismological stations have shown that over most of the earth the outer crust of the earth consists, essentially, of two layers, an upper granitic layer approximately seven and one-half miles thick and a lower basaltic layer approximately fifteen miles thick.† Below the basaltic layer, there is a layer some two thousand miles thick. Underneath the latter there lies a central core through which the primary waves pass with a velocity only two-thirds as great as the velocity just outside the core. The central core does not transmit the secondary waves, as is shown by their disappearance at large distances from the epicenter. (An *epicenter* is the point or area on the earth's surface vertically above the focus or point of origin of an earthquake.) This disappearance of the secondary waves has been taken to indicate that the central core is probably fluid or plastic.

A more detailed discussion of earthquake seismology will be found, for example, in B. Gutenberg's "Handbuch der Geophysik," Volume 4; G. W. Walker's "Modern Seismology"; or Galitzin's "Seismometrie."

Operating Principles of Seismic Prospecting.—General seismic prospecting makes use of artificially produced elastic waves which are the same as the longitudinal waves studied in earthquake seismology, except that they have a higher frequency of vibration. Travel-time curves are computed from prospect field seismograms in much the same way that travel-time curves are computed for earthquake waves. Characteristic features of seismic prospecting methods, however, are: (a) the location of the artificial "earthquake" and its instant of occurrence are known precisely; (b) far more sensitive instruments are used to detect the vibrations; and (c) more accurate time measurements must be made than are necessary in earthquake studies.

In seismology, the instrument used to receive and record the disturbances is called a seismograph. (Compare p. 445.) In seismic prospecting, however, it is customary to distinguish between the recording

† Charles Davison, Article on Earthquakes, *Encyclopaedia Britannica*, 14th Edition, 1929.

instrument and the instrument which receives the seismic energy and transforms it into some other type of energy that can be recorded.*

Two methods, described as reflection and refraction shooting, are employed in seismic prospecting.** In both methods a sudden, artificial disturbance is produced at the shot-point station—for example, by exploding a charge of dynamite. When the dynamite is exploded, the earth is set in motion like a miniature earthquake and the motion of the surface of the earth is detected and recorded at several seismometer stations located at known distances from the shot-point station. In addition to the observations of the times of arrival at various seismometer stations, an accurate record is obtained of the instant of explosion of the shots.

The basic difference between the two methods lies in the character (refracted or reflected) of the waves whose times of arrival are utilized. In the refraction method, travel-time data are obtained for those artificially produced elastic waves which have been *refracted* at boundaries separating media of different elastic constants or density in such manner that portions of the wave paths in the subsurface are approximately parallel to the refracting boundaries. In the reflection method, observations are made of the times of arrival of the artificially produced elastic waves which have been *reflected* from subsurface horizons.

In addition to measuring travel-times of different types of waves, the reflection and refraction methods differ in numerous practical and theoretical details. In the refraction method, only the times of the first motions at the seismometers generally are used; in the reflection method, on the other hand, the times of later motions are of chief importance.

By knowing the travel-times (intervals between occurrence of explosion and arrivals of the elastic waves at various seismometer stations) and the distances between the source of explosion and the seismometers, it is possible, in many cases, to calculate the depths and dips of the refracting boundaries (refraction shooting) or reflecting horizons (reflection shooting).

A modification of the refraction method, known as fan shooting, is used to find structures, e.g., salt domes having an elastic wave velocity

* There is no consistent terminology in seismic prospecting for the device which converts seismic energy into some other type of energy that can be recorded. The term seismometer will be employed throughout this chapter. This term is commonly used even though it is not strictly correct, because, as previously stated, a seismometer is a device for *measuring* seismic motion. Another common name is *geophone*. Because seismic waves are largely of frequencies below the phone range, this is a misnomer. Another common term, *pick-up*, has the disadvantage of not identifying the device as a seismic one. The same objection is raised to a fourth common term, *detector* or *receptor*. Various logical terms have been proposed but have met with little favor. Among these are *seismo-converter*, *seismic pick-up*, etc.

** The reflection method, however, is at present much more extensively employed than the refraction method.

markedly different from that of the surrounding materials. In this method, the travel-times for paths of equal lengths are compared.

Physical Principles.—A disturbance of equilibrium conditions produced by a *sudden stress* at any point in an unbounded elastic solid will cause two types of elastic waves to be propagated: (a) *longitudinal* or compressional waves, i.e., waves in which the particles vibrate in a direction parallel to the line of propagation, and (b) *transverse* or shear waves, i.e., waves in which the motion of the particles is at right angles to the direction of propagation. When the disturbance is an explosion—the usual case in seismic prospecting—the shear produced in the elastic medium is small compared to the change in volume; hence, in this case, most of the wave energy will be propagated in the form of longitudinal or compressional waves.

If the elastic solid is bounded, as is true of the earth, a third type of wave is produced. These waves travel along the surface and are called surface waves. Such waves are generated either by shear or change in volume close to the surface—this is the more important mode of origin—or when elastic waves reach the surface.

The sudden stress generally utilized in seismic prospecting is the explosion of a charge of dynamite. The explosion of a charge buried in the ground produces a strain in the walls of the cavity or hole in which the charge is placed due to the enormous pressure of the expanding gas. This strain is transmitted to the surrounding layers and propagated outward through the earth as elastic waves, chiefly longitudinal and surface waves. The longitudinal waves are classified conveniently into two types: (a) *direct* waves which travel in approximately straight lines from the shot-hole to the various seismometer stations and (b) waves which are *reflected* and *refracted* at various subsurface boundaries before reaching the seismometer stations. The *paths* traversed by the second type of longitudinal waves are determined by the wave velocities in the media constituting the subsurface and by the shapes of the boundaries between the media.

Distribution of Energy in Reflected and Refracted Waves

In an unbounded isotropic homogeneous medium, the energy radiates uniformly in all directions from the source, and the decrease in energy due to absorption depends only on the distance.* In the case of seismic waves traveling through the earth, however, the longitudinal waves spreading out from the shot-hole encounter boundaries separating media of different elastic constants or density, and the energy of the impinging waves is distributed among the several waves produced at these boundaries.

* The absorption of energy is due to internal friction. (Compare p. 457.)

When a longitudinal wave is incident on a boundary separating media of different elastic constants or density, the energy of this wave is generally distributed among four new waves, a longitudinal and a transverse wave in each of the media.

Formulas for computing the relative amounts of energy transferred to these four waves have been given by Knott,† and formulas for computing relative amplitudes have been given by Zoeppritz.‡

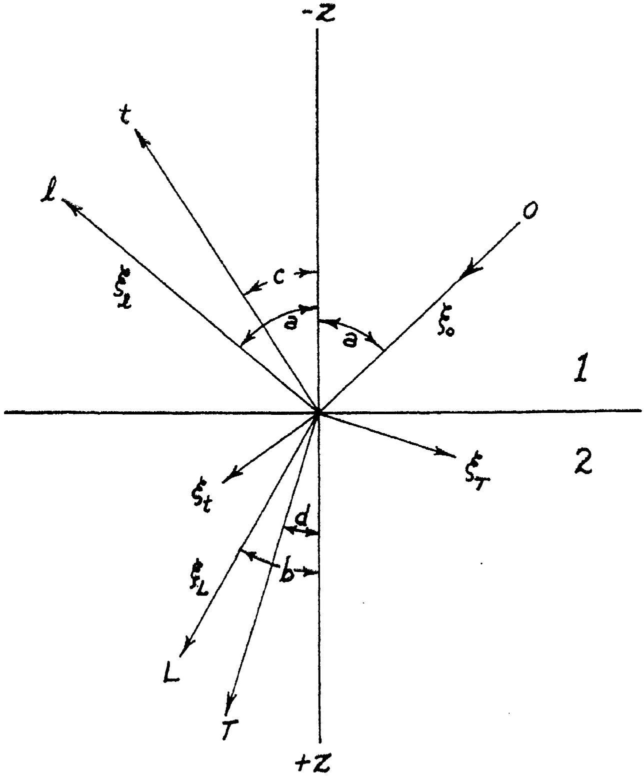

FIG. 255.—Sketch illustrating angles made by wave rays and displacements with the normal at the boundary between media 1 and 2.

Both the energy and the amplitude equations are derived on the assumption that the velocities in the two layers differ by finite amounts so that the boundary corresponds to a substantially abrupt discontinuity.*

† C. G. Knott, "Reflexion and Refraction of Elastic Waves, with Seismological Applications," *Philosophical Magazine,* Vol. 48, July, 1899, p. 64.

‡ K. Zoeppritz, "Ueber Erdbebenwellen VIIb," *Nachrichten der Königlichen Gesellschaft der Wissenschaften zu Göttingen,* mathematische-physikalische Klasse, 1919, p. 57.

* It is important to note that a continuous gradual change, such as that produced by increase of pressure with depth, generally will not produce detectable reflections.

mp ***the Waves.***—The densities of the two media will be denoted by . and d_2 and the following notation will be used for the different waves.

Wave	*Index*	*Velocity*	*Medium*
Incident longitudinal .	o		1
Reflected longitudinal .	l		1
Reflected transverse ..	t	v_1	1
Refracted longitudinal	L		2
Refracted transverse ..	T	v_2	2

The positive axis of z is in the plane of the paper and is parallel to the normal at the boundary directed from medium 1 to medium 2, and the positive axis of x is in the plane of the paper and is directed toward the left. (Figure 255.) The positive direction of displacement of the waves is assumed to be the direction from medium 1 towards medium 2.

The angles made by the rays with the normal are related by the equation:

$$\sin a : \sin b : \sin c : \sin d = V_1 : V_2 : v_1 : v_2$$

When the motion is a simple harmonic one, the displacement ξ at a time τ can be expressed by an equation of the form:

$$\xi = Me^{pi\,[\tau - (x \sin a + z \cos a)/V]}$$

where M is the amplitude, $(x \sin a + z \cos a)/V$ is the phase lag, p is equal to 2π times the frequency and i as usual denotes the square root of minus 1. The phase lag is obtained by substituting an appropriate value for the "initial" time θ in the equation of the wave front:

$$x \sin a + z \cos a = V\theta$$

The resultant displacements ξ of the several waves and their component displacements u and w in the x and z directions respectively may be written as follows, provided it is assumed that the incident longitudinal wave and the four waves produced at the boundary between media 1 and 2 are plane waves in the plane of the paper. (Figure 255):

$$\xi_0 = M_0 e^{i \quad - (x \sin a + z \cos} \qquad u_0 = \xi_0 \sin a$$

$$w_0 = \xi_0 \cos a$$

$$\xi_l = M_l e^{pi\,[\tau - (x \sin a - z \cos a)/V_1]} \qquad u_l = -\xi_l \sin a$$

$$w_l = +\xi_l \cos a$$

$$\xi_t = M_t e^{pi\,[\tau - (x \sin c - z \cos c)/v_1]} \qquad u_t = \xi_t \cos c$$

$$w_t = \xi_t \sin c$$

$$= M_L e^{pi\,[\tau - (x \sin b + z \cos} \qquad u_L = \xi_L \sin b$$

$$w_L = \xi_L \cos b$$

$$= M_T e^{pi\,[\tau - (x \sin d + z \cos d)/v_2]} \qquad u_T = -\xi_T \cos d$$

$$w_T = \xi_T \sin d$$

The component displacements given by the system of equations (1) must satisfy two equations of the form

$$(V^2 - v^2)\frac{\partial \nabla}{\partial x} + v^2 \nabla^2 u = \frac{\partial^2 u}{\partial t^2}$$

where

$$\nabla = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \text{ and } \nabla^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}$$

In addition, the following boundary conditions must be satisfied: The sum of the components of the displacement on both sides of the boundary must be the same and the sum of the normal and the tangential components of stress must be the same. That is,

$$\sum (\overline{u_1}) = \sum (\overline{u_2}) \tag{2}$$

$$\sum (\overline{w_1}) = \sum (\overline{w_2}) \tag{2a}$$

$$\sum d_1 v_1^2 \left(\overline{\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}}\right) = \sum d_2 v_2^2 \left(\overline{\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x}}\right) \tag{3}$$

$$\sum \left\{ d_1 V_1^2 \overline{\nabla}_1 - 2d_1 v_1^2 \left(\overline{\frac{\partial u_1}{\partial x}}\right)\right\} = \sum \left\{ d_2 V_2^2 \overline{\nabla}_2 - 2d_2 v_2^2 \left(\overline{\frac{\partial u_2}{\partial x}}\right)\right\} \tag{3a}$$

where the bar above the various components and operators denotes values taken at the boundary.

On substituting the values given by the system of equations (1) into the boundary conditions, one obtains

$$\overline{u_0 + u_l + u_t} = \overline{u_L + u_T}$$

$$\overline{w_0 + w_l + w_t} = \overline{w_L + w_T}$$

$$d_1 v_1^2 \left\{ \left(\overline{\frac{\partial u_0}{\partial z} + \frac{\partial w_0}{\partial x}}\right) + \left(\overline{\frac{\partial u_l}{\partial z} + \frac{\partial w_l}{\partial x}}\right) + \left(\overline{\frac{\partial u_t}{\partial z} + \frac{\partial w_t}{\partial x}}\right)\right\} = d_2 v_2^2 \left\{ \left(\overline{\frac{\partial u_L}{\partial z} + \frac{\partial w_L}{\partial x}}\right) + \left(\overline{\frac{\partial u_T}{\partial z} + \frac{\partial w_T}{\partial x}}\right)\right\}$$

$$d_1 V_1^2 \left\{ \left(\overline{\frac{\partial u_0}{\partial x} + \frac{\partial w_0}{\partial z}}\right) + \left(\overline{\frac{\partial u_l}{\partial x} + \frac{\partial w_l}{\partial z}}\right) + \left(\overline{\frac{\partial u_t}{\partial x} + \frac{\partial w_t}{\partial z}}\right)\right\} -$$

$$2d_1 v_1^2 \left(\overline{\frac{\partial u_0}{\partial x} + \frac{\partial u_l}{\partial x} + \frac{\partial u_t}{\partial x}}\right) = d_2 V_2^2 \left\{ \left(\overline{\frac{\partial u_L}{\partial x} + \frac{\partial w_L}{\partial z}}\right) + \left(\overline{\frac{\partial u_T}{\partial x} + \frac{\partial w_T}{\partial z}}\right)\right\}$$

$$- 2d_2 v_2^2 \left(\overline{\frac{\partial u_L}{\partial x} + \frac{\partial u_T}{\partial x}}\right)$$

At the instant when the longitudinal wave impinges on the boundary, the refracted waves at 0 have displacements in the direction from medium 1 to medium 2. Hence, the displacements are taken as positive. The reflected waves at 0, however,

have displacements in the direction from medium 2 to medium 1. This is the negative direction and therefore the resultant displacements and the component displacements are taken as negative. The boundary conditions (2) and (2a) can thus be written in the form:

$$M_0 \sin a + M_l \sin a - M_t \cos c = M_L \sin b - M_T \cos d$$

$$M_0 \cos a - M_l \cos a - M_t \sin c = M_L \cos b + M_T \sin d$$

The boundary condition (3) becomes:

$$d_1 v_1^2 \left\{ \left(-\frac{\cos a}{V_1} M_0 \sin a - \frac{\sin a}{V_1} M_0 \cos a \right) + \left(\frac{\cos a}{V_1} M_L \sin a + \frac{\sin a}{V_1} M_L \cos a \right) \right.$$

$$\left. + \left(-\frac{\cos c}{v_1} M_t \cos c + \frac{\sin c}{v_1} M_t \sin c \right) \right\} = d_2 v_2^2 \left\{ \left(-\frac{\cos b}{V_2} M_L \sin b \right. \right.$$

$$\left. \left. \frac{\sin b}{V_2} M_L \cos b \right) + \left(\frac{\cos d}{v_2} M_T \cos d - \frac{\sin d}{v_2} M_T \sin d \right) \right\}$$

On simplifying, one obtains

$$-\frac{d_1 v_1^2 M_0 \sin 2a}{V_1} + \frac{d_1 v_1^2 M_l \sin 2a}{V} - \frac{d_1 v_1^2 M_t \cos 2c}{} \qquad \frac{d_2 v_2^2 M_L \sin 2b}{V_2}$$

$$\frac{d_2 v_2^2 M_T \cos 2d}{v_2}$$

$$-M_0 \sin 2a + M_l \sin 2a - \frac{V_1}{v_1} M_t \cos 2c = -\frac{d_2}{d_1} \frac{v_2^2}{v_1^2} \frac{V_1}{V_2} M_L \sin 2b +$$

$$\frac{d_2}{} \frac{v_2^2}{} \frac{V_1}{} M_T \cos 2d$$

On setting,

$$\frac{V_1}{v_1} = F; \quad \frac{d_2}{d_1} \frac{v_2^2}{v_1^2} \frac{V_1}{V_2} = G; \quad \frac{d_2}{d_1} \frac{v_2^2}{v_1^2} \frac{V_1}{v_2} = J$$

the boundary condition (3) becomes:

$$-M_0 \sin 2a + M_l \sin 2a - FM_t \cos 2c = -GM_L \sin 2b + JM_T \cos 2d$$

The boundary condition (3a) can be written in the form:

$$d_1 V_1^2 \left\{ \left(-\frac{\sin a}{V_1} M_0 \sin a - \frac{\cos a}{V_1} M_0 \cos a \right) + \left(-\frac{\sin a}{V_1} M_l \sin a - \frac{\cos a}{V_1} M_l \cos a \right) \right.$$

$$\left. + \left(\frac{\sin c}{v_1} M_t \cos c - \frac{\cos c}{v_1} M_t \sin c \right) \right\} - 2 d_1 v_1^2 \left(-\frac{\sin a}{V_1} M_0 \sin a - \frac{\sin a}{V_1} M_l \sin a + \right.$$

$$\left. \frac{\sin c}{v_1} M_t \cos c \right) = d_2 V_2^2 \left\{ \left(-\frac{\sin b}{V_2} M_L \sin b - \frac{\cos b}{V_2} M_L \cos b \right) + \right.$$

$$\left. \left(\frac{\sin d}{v_2} M_T \cos d - \frac{\cos d}{v_2} M_T \sin d \right) \right\} - 2 d_2 v_2^2 \left\{ -\frac{\sin b}{V_2} M_L \sin b + \frac{\sin d}{v_2} M_T \cos d \right\}$$

On simplifying in several steps, one obtains

$$-d_1 V_1 M_0 - d_1 V_1 M_l + 2d_1 \frac{v_1^2}{V_1} M_0 \sin^2 a + 2d_1 \frac{v_1}{V} M_l \sin^2 a$$

$$-2d_1 v_1 M_t \sin c \cos c = - \quad {}_2 M_L \quad {}_2 \frac{v_2^2}{V_2} M_L \sin^2 b - 2d_2 v_2 M_T \sin d \cos d$$

$$-d_1 V_1 M_0 \left(1 - 2\frac{v_1^2}{V_1^2}\sin^2 a\right) - d_1 V_1 M_l \left(1 - 2\frac{v_1^2}{V_2^2}\sin^2 a\right) - d_1 v_1 M_t \sin 2c$$

$$= -d_2 V_2 M_L \left(1 - 2\frac{v_2^2}{V_2^2}\sin^2 b\right) \quad {}_2 V_2 M_T \sin$$

$$-d_1 V_1 M_0 (1 - 2\sin^2 c) - d_1 V_1 M_l (1 - 2\sin^2 c) - d_1 v_1 M_t \sin 2c$$
$$= -d_2 V_2 M_L (1 - 2\sin^2 d) - d_2 v_2 M_T \sin 2d$$

$$-d_1 V_1 M_0 \cos 2c - d_1 V_1 M_l \cos 2c - d_1 v_1 M_t \sin 2c$$
$$= -d_2 V_2 M_L \cos 2d - d_2 v_2 M_T \sin 2d$$

$$-M_0 \cos 2c - M_l \cos 2c - HM_t \sin 2c = -IM_L \cos 2d - KM_T \sin 2d$$

where

$$I = \frac{d_2}{d_1}\frac{V_2}{V_1} \qquad\qquad K = \frac{d_2}{\cdot}\frac{v_2}{\cdot_1}$$

Summarizing the results just obtained, the four boundary conditions are:

$$\begin{aligned}
M_0 \sin a + M_l \sin a - M_t \cos c - M_L \sin b + M_T \cos d &= 0\\
M_0 \cos a - M_l \cos a - M_t \sin c - M_L \cos b - M_T \sin d &= 0\\
-M_0 \sin 2a + M_l \sin 2a - FM_t \cos 2c + GM_L \sin 2b - JM_T \cos 2d &= 0\\
-M_0 \cos 2c - M_l \cos 2c - HM_t \sin 2c + IM_L \cos 2d + KM_T \sin 2d &= 0
\end{aligned} \tag{4}$$

The 4 boundary conditions and the relation previously stated between the angles and the velocities ($\sin a : \sin b : \sin c : \sin d = V_1 : V_2 : v_1 : v_2$) allow us to calculate the ratios of the amplitudes of the 4 waves produced at the boundary to the amplitude of the incident longitudinal wave for any angle of incidence a.

When the distances between the point of disturbance and the seismometer stations are small relative to the depths of the subsurface boundaries, the reflected and refracted waves strike the surface of the earth almost vertically. In this case, all the angles in the system (4) are approximately zero, and it follows from the second and fourth equations of this system that

$$\frac{\quad}{M_0} = \frac{\quad}{I+1}, \quad \frac{M_L}{M_0} = \frac{2}{I+1} \qquad \text{where } I =$$

If, for example, the densities in the two media are equal ($d_2 = d_1$) and if $V_2 = 2V_1$, $I = 2$ and $\frac{M_l}{M_0} = \frac{1}{3}$.

Energy of the Waves.†—The formulas for the energies as given by Knott utilize the following abbreviations:

C = cotangent of angle of incidence of longitudinal wave in layer 1.
C' = cotangent of angle of refraction of longitudinal wave in layer 2.
γ = cotangent of angle of reflection of transverse wave in layer 1.
γ' = cotangent of angle of refraction of transverse wave in layer 2.
n = modulus of rigidity of layer 1.
n' = modulus of rigidity of layer 2.
A = energy factor of incident longitudinal wave.
A_1 = energy factor of reflected longitudinal wave.
A' = energy factor of refracted longitudinal wave.
B_1 = energy factor of reflected transverse wave.
B' = energy factor of refracted transverse wave.

$$X = A + A_1 \qquad Y = A - A_1 \tag{5}$$

Knott showed that the quantities just defined are related by the following ions:

$$\left.\begin{aligned} B_1 + CY &= B' + C'A' \\ \gamma B_1 + X &= -\gamma' B' + A' \\ -2\gamma B_1 + (\gamma^2 - 1)\, X &= 2\,\frac{n'}{n}\,\gamma' B' + \frac{n'}{n}\,(\gamma'^2 - 1)\, A' \\ (\gamma^2 - 1)\, B_1 - 2CY &= \frac{n'}{n}\,(\gamma'^2 - 1)\, B' - 2\,\frac{n'}{n}\, C'A' \end{aligned}\right\} \tag{6}$$

The systems of equations (5) and (6) determine the values of A_1, A', B_1, and B' as a function of A, C, C', γ, γ', n, and n'.

If, as before, the densities in the layers are denoted by d_1 and d_2, the energy equation may be written in the form:

$$Cd_1A^2 = Cd_1A_1^2 + \gamma d_1B_1^2 + C'd_2A'^2 + \gamma' d_2B'^2$$

where the left-hand member represents the energy of the incident longitudinal wave and the right-hand member the energies of the reflected and refracted waves. Furthermore, if the energy of the incident longitudinal wave be denoted by E_0, the energy of the refracted longitudinal wave by E_L, and the energy of the reflected longitudinal wave by E_l,

$$\frac{E_L}{E_0} = \frac{C'd_2A'^2}{Cd_1A^2} \quad \text{and} \quad \frac{E_l}{E_0} = \frac{Cd_1A_1^2}{Cd_1A^2} = \frac{A_1^2}{A^2} \tag{7}$$

Obviously, if the ratios $\frac{A'}{A}$ and $\frac{A_1}{A}$ of the energy factors and the ratio $\frac{d_1}{d_2}$ of the densities are known, Equations 7 may be used to obtain the ratio of the energy of the refracted wave to that of the incident wave $\left(\frac{E_L}{E_0}\right)$ and the ratio of the energy of the reflected wave to that of the incident wave $\left(\frac{E_l}{E_0}\right)$. But for a given value of the angle of incidence,

† This section is taken largely from a paper by B. Gutenberg, H. O. Wood, and J. P. Buwalda, "Experiments Testing Seismograph Methods," *Bulletin of the Seismological Society of America*, Vol. 22, 1922, pp. 185-246.

the ratio of the energy factors can be expressed as a function of the velocities and the moduli of rigidity by solving the systems of equations (5) and (6). Furthermore, if the ratio of the velocities and the ratio of the densities are specified, the ratio of the moduli of rigidity are determined. (Compare p. 459, Equation 9.) Hence, for a particular angle of incidence, the only data necessary to determine the quotients $\frac{E_L}{E_0}$ and $\frac{E_t}{E_0}$ are (1) the ratio of the velocities and (2) the ratio of the densities.

In particular, if the densities of the two layers are the same and if the velocities have the ratios

$$V_1 : V_2 : v_1 : v_2 = 1.82 : 2.05 : 1.00 : 1.31$$

the following results are obtained.

The reflected longitudinal wave receives 4 per cent of the energy of the incident wave when the latter arrives at the surface normally ($a = 0$). As the angle of incidence increases, the energy of the reflected longitudinal wave at first decreases until at a value equal to approximately 15° it reaches a minimum value of 0.2 per cent of the incident energy. Thereafter the per cent energy is small until $a = 60°$. For values of a greater than 62½° (the *critical angle* for the longitudinal wave), the reflected longitudinal wave receives more than two-thirds of the energy.* For values of a less than the critical angle 62½°, the refracted longitudinal wave carries by far the greater part of the energy. (The transverse wave, largely because of the minuteness of the energy which it represents in field operations, is not generally considered in seismic prospecting and will be neglected in the present treatment.)

If it is assumed as before that the longitudinal wave strikes the surface almost vertically, it may be shown that the systems of equations (6) and (7) become

$$\frac{A_1}{A} = \frac{I-1}{I+1} \qquad \frac{A'}{A} = \frac{V_2}{V_1}\frac{2}{(I+1)}$$

and

$$\frac{E_t}{E_0} = \frac{(I-1)^2}{(I+1)^2} \qquad \frac{E_L}{E_0} = \frac{4I}{(I+1)^2}$$

Also, if $I = 2$,

$$\frac{E_t}{E_0} = \frac{1}{9}$$

On comparing this result with that obtained for the amplitudes on p. 455, one verifies the well-known fact that the energy is proportional to the square of the amplitude.

Effect of Medium on Transmission

As the wave travels from the point of disturbance, it will, in general, undergo a variety of changes. (1) The reaction of the wave at an interface between two media has already been given by the equations of Knott and Zoeppritz. (2) Internal friction of the medium will produce loss of energy. This absorption of energy increases with frequency. Hence, the distribution with respect to frequency of the energy carried by the wave alters as the wave advances in such a manner that the proportion of the low frequencies increases with distance. (The useful waves in seismic prospecting have a frequency of about 30 to 100 c.p.s. [cycles

* The critical angle is defined by the relation: $a_{critical} \equiv \sin^{-1}\frac{V_1}{V_2}$

per second]. The corresponding range of wave lengths in a zone of 6,000 ft./sec. velocity would be 200 ft. to 60 ft.) (3) *Dispersion* or variation of velocity with frequency is extremely small and has not been detected with surety either in seismic geophysics or in seismology. (Such a phenomenon would be extremely detrimental to seismic geophysics, because all measurements and computations associate a given time-distance relation with each recorded impulse.) (4) *Scattering* or the reflection of energy from particles whose linear dimensions are an appreciable fraction of the wave length is produced, for example, by large boulders. Scattering will cause a greater loss of energy for the higher frequency waves and will produce a general background of disturbance. (5) Another effect is sometimes labeled *diffraction*. This phenomenon is related to the effects produced by subsurface lenses which occur in the path of the waves. The boundaries of such lenses act as sources of new waves. (Diffraction may be said to be a scattering effect wherein the dimensions of the reacting bodies are at least of the magnitude of the wave length of the impinging wave.) The wave motions due to these lenses that are recorded on the seismograms confuse the records, because they are superimposed on the wave motions associated with boundaries separating media of different elastic constants or density.

Velocities of Elastic Waves and Elastic Constants.—The elastic wave velocities of any medium depend on the elastic constants and density of that medium.* In the case of a homogeneous isotropic solid, the elastic properties are described by two moduli of elasticity. The two moduli most frequently used are: the compressibility k and rigidity n, or Lamé's constants λ and μ, or Young's modulus E and Poisson's ratio σ.† These sets of moduli are not independent of each other. Thus

$$\overline{3}\ n = \overline{(1+\sigma)\,(1-}$$

$$9kn$$

$$\lambda + \mu \qquad 3k + n \qquad (8)$$

$$n = \mu = \qquad \frac{3k - 2n}{6k + 2n}$$

A recent comprehensive investigation of the elastic constants was the research program sponsored by the Harvard University Committee for Geophysical Research.‡

* The expression elastic constants of a medium refers to the strains produced in the medium by the application of stresses.

† Compare A. B. Broughton Edge and T. H. Laby, *Geophysical Prospecting*, pp. 330-331 (Cambr. Univ. Press, 1931).

‡ L. Don Leet, *Practical Seismology and Seismic Prospecting* (Appleton Century, 1938) p. 94.

The investigations included static and dynamic laboratory measurements of elastic constants and field measurements of the wave velocities in rock bodies from which the laboratory specimens were taken. †

The velocities of longitudinal and transverse waves in an isotropic homogeneous solid are related to the elastic constants and the density by the equations: ‡

$$V = \sqrt{\frac{E(1-\sigma)}{d(1+\sigma)(1-2\sigma)}} = \sqrt{\frac{\lambda + 2\mu}{d}} = \sqrt{\frac{k + 4/3\,n}{d}} \tag{9}$$

$$v = \sqrt{\frac{E}{d}\left[\frac{1}{2(1+\sigma)}\right]} = \sqrt{\frac{\mu}{d}} = \sqrt{\frac{n}{d}} \tag{9a}$$

where d denotes the density, V the velocity of the longitudinal wave, and v the velocity of the transverse wave.

The elastic constant E varies over a wide range while σ is very nearly ¼ for materials having good elastic properties.* Hence, the velocities of the elastic waves depend almost entirely upon the ratio of the Young's modulus to the density of the material. Also, it is evident from Equations 9 and 9a that the velocity of the longitudinal wave is greater than that of the transverse wave.**

The velocities of longitudinal waves in various materials found in the earth's crust are summarized in Table 15.*** The table shows that the value of the velocity for a given material varies over a considerable range. This occurs because the composition, porosity, and water content affect the wave velocity appreciably. § In addition, the experimental results show that the velocity varies markedly with the depth of the formation.

† W. A. Zisman, "Young's Modulus and Poisson's Ratio with Reference to Geophysical Applications," *Proc. Nat. Acad. Sci.*, 19, pp. 653-665, 1933; "Compressibility and Anistropy of Rocks at and near the Earth's Surface," *Proc. Nat. Acad. Sci.*, 19, pp. 666-679, 1933; "An Improved Apparatus for the Measurement of Poisson's Ratio," *Rev. of Sci. Inst.*, 4, pp. 342-344, 1933; "Comparison of the Statically and Seismologically Determined Elastic Constants of Rocks," *Proc. Nat. Acad. Sci.*, 19, pp. 680-686, 1933.

John M. Ide, "Some Dynamic Methods for the Determination of Young's Modulus," *Rev. of Sci. Inst.*, 6, pp. 296-298, 1935; "The Elastic Properties of Rocks: A Correlation of Theory and Experiment," *Proc. Nat. Acad. Sci.*, 22, pp. 482-496, 1936; "Comparison of Statically and Dynamically Determined Young's Modulus of Rocks," *Proc. Nat. Acad. Sci.*, 22, pp. 81-92, 1936.

L. D. Leet, "Velocity of Elastic Waves in Granite and Norite," *Physics*, 4, pp. 375-385, 1933.

F. Birch and R. R. Law, "The Measurement of Compressibility at High Temperature and High Pressures," *Bull. Geol. Soc. America*, 46, pp. 1219-1250, 1935.

F. Birch and R. B. Dow, "Compressibility of Glasses and Rocks at High Temperatures and Pressures," *Bull. Geol. Soc. America*, 47, pp. 1235-1255, 1936.

‡ Broughton Edge and Laby, *loc. cit.*

* A great many deposits found in strata close to the surface of the earth are poorly consolidated. Hence, their elastic properties are not very good and their σ value may be much less than ¼.

** For the rocks composing the earth's crust, this ratio is found to be of the order of 1.6.

*** A table giving the longitudinal and transverse wave velocities, densities, and Young's moduli for laboratory specimens is given by A. Sieberg, *Geologische, physikalische und angewandte Erdbebenkunde* (Gustav Fischer, Jena, 1923), p. 171.

§ Gutenberg, Wood, Buwalda, *loc. cit.*, p. 213.

The variations in velocity which are most commonly encountered may be summarized as follows: (1) Within a single layer of homogeneous material, the velocity generally increases slowly with depth. (2) The velocity is usually greater in deeper layers, although many exceptions to this rule exist. (3) In many regions, the increase of velocity with depth is approximately linear.

Seismic Wave Paths.—For simplicity consider that the waves originate at a point source located in a homogeneous isotropic medium. The elastic disturbances which travel outwardly from this source are spheres with the source as center. (Figure 256.) Any one of these spheres

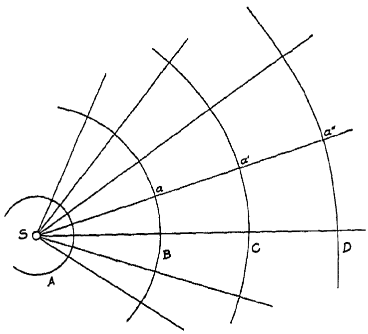

FIG. 256.—Sketch illustrating the propagation of elastic disturbances in an elastic medium.

is a wave front, i.e., a surface such that all points on it are vibrating in phase. Obviously, at large distances from the source, small portions of the wave front will approximate a plane. (Compare wave front *D* of Figure 256.)

The normal to the wave front at any instant is called the *wave ray*, or the *ray*. Rays, for example, *aa'* and *aa'a"* of Figure 256 are generally used as a convenient tool for describing the paths of elastic waves. A ray, such as *aa'a"*. . . , has a number of important properties, chief of which is that the travel time along such a path from the source to any point on the path is the "least" time in which energy of the type considered can travel from the source to the point.*

Reflection and Refraction of Rays

As is well known, the basic law of reflection states (a) that the incident ray and the reflected ray lie in the same plane and (b) that the angle of incidence (angle made by the incident ray with the normal at the boundary) equals the angle of reflection (angle made by the reflected ray with the normal).

* It should be pointed out, however, that the entire energy reaching a given point need not travel along the wave ray.

TABLE 15

APPROXIMATE RANGE OF VELOCITIES OF LONGITUDINAL WAVES FOR REPRESENTATIVE MATERIALS FOUND IN THE EARTH'S CRUST

A. *Classification According to Material*

Material	*Velocity** *Ft./Sec.*	*M./Sec.*
Weathered surface material	1,000— 2,000	305— 610
Gravel, rubble, or sand (dry)	1,500— 3,000	468— 915
Sand (wet)	2,000— 6,000	610— 1,830
Clay	3,000— 9,000	915— 2,750
Water (depending on temperature and salt content)	4,700— 5,500	1,430— 1,680
Sea water	4,800— 5,000	1,460— 1,530
Sandstone	6,000—13,000	1,830— 3,970
Shale	9,000—14,000	2,750— 4,270
Chalk	6,000—13,000	1,830— 3,970
Limestone	7,000—20,000	2,140— 6,100
Salt	14,000—17,000	4,270— 5,190
Granite	15,000—19,000	4,580— 5,800
Metamorphic rocks	10,000—23,000	3,050— 7,020

B. *Classification According to Geologic Age*

Age	*Type of Rock*	*Velocity* *Ft./Sec.*	*M./Sec.*
Quaternary	Sediments (various degrees of consolidation)	1,000— 7,500	305— 2,290
Tertiary	Consolidated Sediments	5,000—14,000	1,530— 4,270
Mesozoic	Consolidated Sediments	6,000—19,500	1,830— 5,950
Paleozoic	Consolidated Sediments	6,500—19,500	1,980— 5,950
Archeozoic	Various	12,500—23,000	3,810— 7,020

C. *Classification According to Depth* †

	0—2000 ft. (0—600 M.) Ft./Sec.	2000—3000 ft. (600—900 M.) Ft./Sec.	3000—4000 ft (900—1200 M.) Ft./Sec.
Devonian	13,300	13,400	13,500
Pennsylvanian	9,500	11,200	11,700
Permian	8,500	10,000	
Cretaceous	7,400	9,300	10,700
Eocene	7,100	9,000	10,100
Pleistocene-to-Oligocene	6,500	7,200	8,100

* The higher values in a given range are usually obtained at depth.

† Data from B. B. Weatherby and L. Y. Faust, *Bull. Amer. Assoc. Petrol. Geologists,* 19 (1935) 1.

The basic law of refraction (Snell's law) states (a) that the incident ray and the refracted ray lie in the same plane and (b) that the sine of the angle of incidence in the first medium is to the sine of the angle of refraction in the second medium as the velocity in the first medium is to the velocity in the second medium.

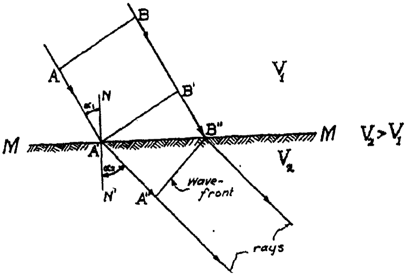

FIG. 257.—Sketch illustrating the refraction of a plane wave $A'B'$ at a boundary MM'.

The law of refraction may be proved directly by use of ray paths.† Referring to Figure 257, MM' is a horizontal boundary separating two homogeneous media in which the elastic wave velocities are V_1 and V_2, respectively. $A'B'$ is a trace of a plane wave front which is perpendicular to the plane of the paper. α_1 is the angle between the ray AA' and the normal NN' to the boundary, and α_2 is the corresponding angle in the second medium. Let t be the time required for the wave to travel from B' to B''. Then, the distance $\overline{B'B''}$ equals tV_1. In this same interval of time the wave will travel a distance $\overline{A'A''}$ in the second medium where A'' is the point corresponding to A' on the wave front at this later time. The distance $\overline{A'A''}$ is equal to the product of t and the velocity in the second medium; that is,

$$\overline{A'A''} =$$

Hence

$$\frac{\overline{A'A''}}{V_2} = \frac{\overline{B'B''}}{V_1}$$

Elementary trigonometry shows that

$$\overline{A'A''} = \overline{A'B''} \sin \alpha_2$$

and

$$\overline{B'B''} = \overline{A'B''} \sin \alpha_1$$

so that

$$\frac{\sin \alpha_2}{\sin \alpha_1} = \frac{V_2}{V_1} \quad \text{or} \quad \frac{\sin \alpha_1}{\quad} = \frac{\sin \alpha_2}{\quad}$$

which is the relation desired.

† A proof of the laws of reflection and refraction utilizing *Huyghen's construction* may be found in any standard text on optics. See, for example, R. A. Houston, *A Treatise on Light*, 5th Edition, pp. 127-128 (Longmans, Green and Co., London) 1928.

Basic Equations of Seismic Wave Paths

Referring to Figure 258, let the velocities in the successive horizontal layers be V_1, V_2 . . . respectively, and the corresponding thicknesses h_1, h_2 . . . If the wave path in the kth layer makes an angle α_k with the normal to the boundary, the angles of incidence satisfy the relation:

$$p = \frac{\sin \alpha_1}{V_1} = \cdots\cdots = \frac{\sin \alpha_k}{V_k} = \frac{\sin \alpha_n}{V_n} \tag{10}$$

where p is a parameter which is constant for any one ray and specifies the particular ray under consideration.*

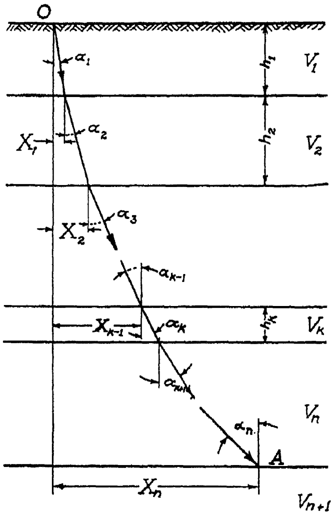

FIG. 258. — Downward path of a refracted ray through n horizontal layers.

The segment of the ray or wave path in the kth layer has a length $h_k/\cos \alpha_k$ and a horizontal projection $h_k \tan \alpha_k$. The time required for the wave to traverse the segment in the kth layer is thus $h_k/V_k \cos \alpha_k$.

The total time for the wave to travel from 0 to a point A on the boundary between the nth and the $n + 1$th layers is

$$t_n = \qquad \cos \alpha_k \tag{11}$$

The horizontal displacement from 0 to A is

$$X_n = \sum_{k=1}^{n} h_k \tan \alpha_k \tag{12}$$

* Evidently, Equation 10 is a generalization of Snell's law.

Furthermore, since $\frac{\sin a_k}{V_k} = p$,

$$\cos a_k = \sqrt{1 - p^2},$$

and

$$\tan a_k = \frac{pV_k}{\quad}$$

Hence

$$t_n = \sum^{n} \cdot \tag{13}$$

and

$$\tag{14}$$

Limiting Case: Continuously Varying Velocity.—The limiting case occurs when the thickness of the beds decreases to zero and the number of beds increases without limit. Assume that the velocity is a continuous function of the depth h which may be written in the form: $V = V(h)$.

On substituting this value of V into the expressions for the time and the horizontal displacement (Equations 13 and 14) and passing to the limiting case of n equal to infinity, one obtains †

$$= \int_0 \frac{dh}{V(h)\sqrt{1-p^2V^2(h)}} \tag{15}$$

$$= \int_0 \frac{pV(h)\,dh}{\sqrt{1-p^2V^2(h)}} \tag{16}$$

Equations 15 and 16 are the governing basic ray equations when the velocity is a continuous function of the depth.

The radius of curvature of the rays ρ may be obtained by substituting appropriate values in the well-known formula

$$\overline{dh^2}$$

$\frac{dX}{dh}$ is obtained by differentiating the right-hand member of Equation 16 with respect to h. That is,

$$\frac{dX}{dh} = \frac{pV(h)}{\sqrt{1 - p^2V^2(h)}}$$

†, M. M. Slotnick, "On Seismic Computation, with Application I," *Geophysics*, Vol.

and

$$\frac{d^2X}{dh^2} = \frac{p}{[1 - p^2V^2(h)]^{3/2}} \frac{dV(h}{dh}$$

Hence

$$\rho = \frac{\left[1 + \frac{p^2V^2(h)}{1 - p^2V^2(h)}\right]^{3/2}}{\frac{p}{[1 - p^2V^2(h)]^{3/2}} \frac{dV}{dh}}$$

or

$$1$$

where ρ is the radius of curvature of the rays.

Critical Angle of Incidence

Consider now the case of two strata separated by a horizontal boundary at a depth h. (Figure 259.) Let the velocity of the elastic wave in the upper stratum be V_1 and that in the lower stratum be V_2 and assume that V_2 is greater than V_1 by a finite amount.

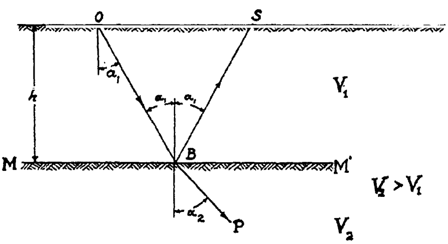

FIG. 259.—Sketch illustrating refraction and reflection of rays at a horizontal boundary.

In the general case, a ray starting at the source 0 and reaching the boundary at the point B will produce a reflected ray BS and a refracted ray BP. The ray BS which is reflected back into the first medium makes an angle α_1 with the normal. The ray BP which is refracted into the second medium makes an angle α_2 with the normal such that α_2 is related to α_1 by Snell's law (Equation 9); that is,

$$\sin \alpha_1 = \frac{V_1}{V_2} \sin \alpha_2$$

In certain cases, namely, when α_1 exceeds a certain critical value, no refracted ray will be produced. This will be evident from the following consideration. For values of α_1 such that $\sin \alpha_1$ is greater than $\frac{V_1}{V_2}$, Equation 9 cannot be satisfied, and hence no refracted ray such as BS can be produced because $\sin \alpha_2$ would have to be greater than 1. (Physically, this means that the incident ray $0B$ is totally reflected for all values of α_1 greater than the critical angle $\sin^{-1} \frac{V_1}{V_2}$.)

The particular case of interest in refraction work occurs when α_1 *approaches the critical value* $\sin^{-1}\dfrac{V_1}{V_2}$. In this case, α_2 is almost 90° and the refracted ray travels approximately parallel to the boundary.

Travel-Time Curves for a Subsurface Section Consisting of Two Horizontal Layers.—Figure 260 represents a subsurface section consisting of two horizontal strata which are homogeneous and isotropic and have longitudinal wave velocities V_1 and V_2 respectively. Assume that the velocities V_1 and V_2 increase *slightly* with depth. Assume also that V_1 and V_2 differ by a finite amount so that the boundary NN' corresponds to an abrupt discontinuity in velocity. 0 represents the *shot-point** and S_k a seismometer station located at a distance x_k from 0.

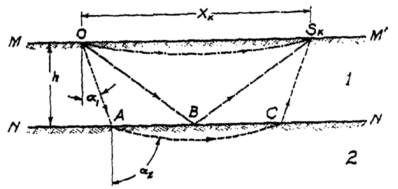

FIG. 260.—Ray paths in a subsurface section consisting of two horizontal strata. (After Gutenberg, Wood and Buwalda, *Bulletin of the Seismological Society of America.*)

Following the explosion at 0, surface waves and three types of longitudinal waves (direct, reflected, and refracted) are received at S_k provided x_k is sufficiently great. The surface wave travels along the surface. The direct wave traverses the path $0S_k$, the reflected wave the path $0BS_k$, and the refracted wave the path $0ACS_k$. The travel-time curves for the three longitudinal waves may be determined as follows:

The length of the curved path $0S_k$ is approximately equal to x_k, because the curvature of the path is due solely to the slight increase of velocity with depth. Hence, the *travel-time* T_k required by the direct wave to traverse the distance $0S_k = x_k$ is equal to the quotient of the horizontal distance x_k divided by the velocity V_1. That is,

The equation of the *travel-time curve* of the direct wave may be obtained from the last equation by replacing x_k by x and T_k by T, where x denotes the distance between the shot-point 0 and any one of several seismometers

* In seismic prospecting the charge usually is imbedded at some depth from the surface. Hence, it generally is necessary to differentiate between the actual location of the charge, i.e., the bottom of the shot-hole, and the shot-point which is a point on the surface vertically above the charge. In the present illustration, however, the charge is assumed to be located at the surface. Hence, the shot-point and the bottom case.

located in a straight line through 0 and T denotes the travel-time over the distance x. Thus, the equation of the travel-time curve is

$$T = \frac{x}{V} \tag{17}$$

The curve is therefore a straight line which passes through the origin and has a slope of magnitude $\frac{1}{V_1}$. (Figure 261.)

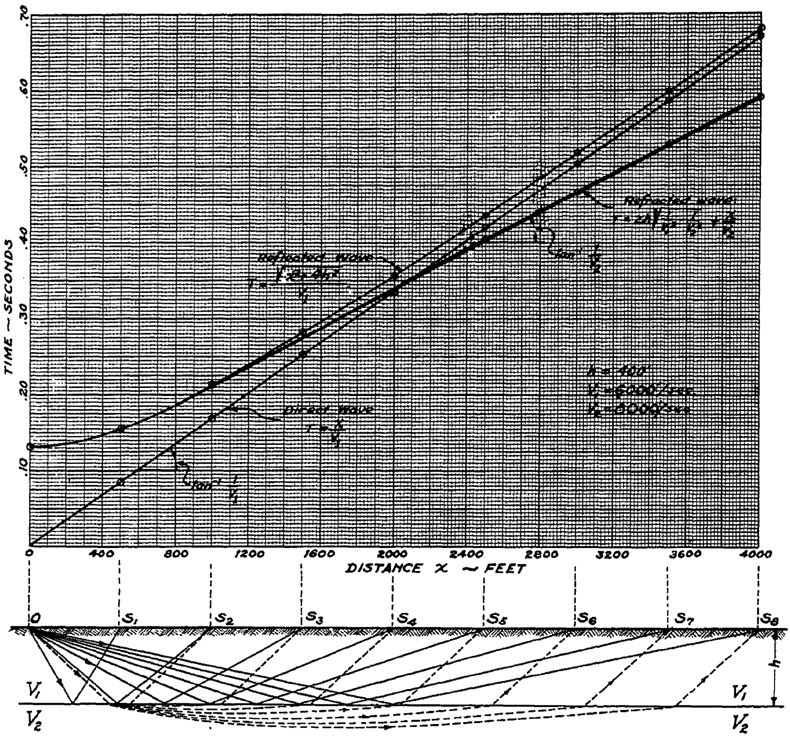

FIG. 261.—Travel-time curves for a subsurface section consisting of two horizontal strata.

It is evident from the geometry of Figure 260 that the travel-time curve of the reflected wave is:

$$T = \tag{18}$$

The curve is therefore a rectangular hyperbola. (Figure 261.)

The travel-time of the wave which is refracted near the critical angle, and therefore travels in the lower stratum along a path which is approxi-

mately parallel to the boundary, is obtained by adding the travel-times in the upper and lower strata. The travel-times in the two strata are $\frac{0A + CS_k}{V_1}$ and $\frac{AC}{V_2}$, respectively. It is evident from Figure 260 that $0A = CS_k = \frac{h}{\cos\alpha_1}$; also, $AC = x_k - 2h\tan\alpha_1$. Hence, the travel-time for the path

$$2h \qquad x_k - 2h\tan\alpha_1$$

The equation of the *travel-time curve* is

$$\cos\alpha_1 + \frac{x - 2h\tan}{V_2}$$

where T and x denote travel-time and horizontal distance respectively.

The last equation may be simplified by replacing the trigonometric functions by their equivalents in terms of the velocities. It follows from the critical condition $\left(\sin\alpha_1 = \frac{V_1}{V_2}\right)$ that

$$\cos\alpha_1 = \sqrt{1 - \left(\frac{V_1}{V_2}\right)^2}$$

and

$$\tan\alpha_1 = \frac{}{V_2\cos}$$

Hence

$$T = 2h \qquad \frac{1}{V_2^2} + \frac{x}{V_2} \tag{19}$$

The travel-time curve for the refracted wave is therefore a straight line having a slope of magnitude $\frac{1}{V_2}$. (Figure 261.)

Some disagreement exists among investigators employing seismic methods as to whether the refracted wave travels along the slightly curved path shown in Figure 260 or along the interface itself (straight line path).† A vertical gradient in velocity would produce a curved path. However, it is not necessary to assume a vertical velocity gradient below the upper layer to explain the emergence of a refracted wave at the surface. Even though no such gradient existed, the refracted wave would be

† See O. v. Schmidt, "Ueber Kopfwellen in der Seismik," *Zeitschrift für Geophysik XV*, 1939, p. 141; "Ueber Knallwellenausbreitung in Flussigkeiten und festern Körpern," *Zeitschrift für technische Physik, 12*, 1938, p. 554.

detected at the surface. The wave may be transmitted by the propagation of a strain set up at the lower boundary of the upper layer. See, for example, G. Joos and J. Teltow, "Zur Deutung der Knallwellenausbreitung an der Trennschicht zweier Medien," *Physikalische Zeitschrift,* vol. 40, 1939, pp. 289-293.

The Low Velocity Layer.—In practically all areas, a layer occurs at the surface of the earth which is unconsolidated, and often heterogeneous in character, and transmits waves of low velocity. The thickness of the layer may vary from almost zero to several hundred feet; however, the most common thickness is from about 25 feet to 100 feet.

In literature on seismic prospecting, the term "weathered" is frequently applied to this layer although this is a misnomer in that the layer may not necessarily exhibit a weathering condition. Furthermore, the term "weathered layer" as used by seismic investigators differs in meaning from the well established term "zone of weathering" used by geologists. The terms, low velocity, aerated and unconsolidated layer, are also used.

Of importance to seismic prospecting is the low velocity property of this layer; most commonly between 1500 and 2500 ft./sec., in contrast to velocities of about 6,000 ft./sec. and more in the deeper layers. The term "low velocity layer," therefore, has found increasing favor and will be used here.* The low velocity layer may consist of more than a single layer, and it is customary to define this layer simply as one having a velocity less than about 5,000 ft./sec.—or less than any other velocity when occasion arises.

Special attention must always be directed to the low velocity layer in seismic prospecting because the usual heterogeneous character of this layer would lead to inaccurate results if the anomalous velocity in this layer were not taken into account. The layer plays but a small part in the subsurface section despite its relatively large effect on travel-time data and must be taken into account before the data can be analyzed.

The low velocity layer also plays a prominent part in seismic prospecting in that explosives generally must be fired below the low velocity layer to produce efficient as well as effective seismic disturbance.

REFLECTION METHOD

The characteristic feature of the reflection method is the measurement of the travel-times of longitudinal waves which have been reflected at subsurface boundaries separating media of different elastic wave velocities. From measurements of reflection times it is usually possible to determine the depths and dips of reflecting horizons and the velocities of the seismic waves.

* To avoid excessive repetition of the terms *velocity of the low velocity layer* and *vertical travel-time in the low velocity layer,* these terms will be designated by V_u and t_u respectively, and the symbols will be used.

The subscript u may stand for *unconsolidated* or for *upper layer. Consolidated* is here used in the sense of unaerated; that is, a condition in which large interstices, if any, between grains are filled by water or by cementation.

Basis of Reflection Method.—The elementary consideration of reflection in the case of two horizontal homogeneous and isotropic strata, page 467, will be reopened for extension to the case of sloping strata.

When the ray paths are straight lines, they are handled conveniently by utilizing the concept of an image source.† In Figure 262 the image of

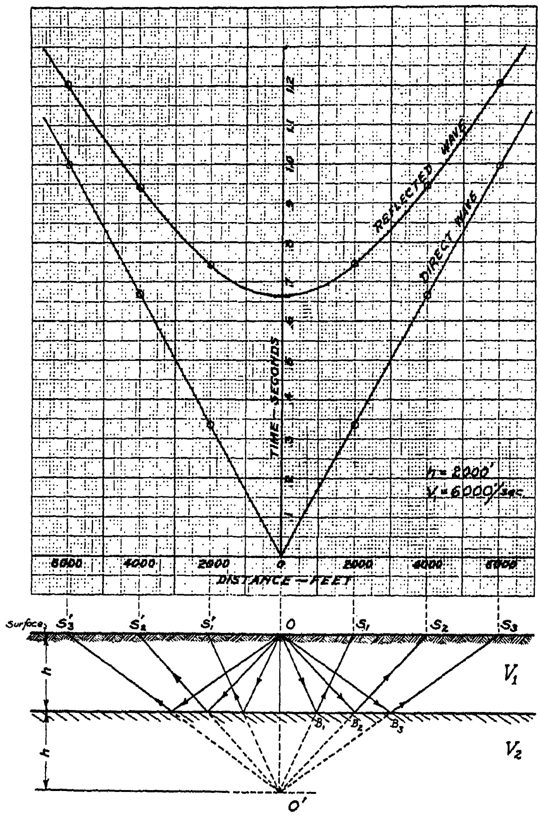

FIG. 262.—Ray paths and travel-time curve for rays reflected from a horizontal surface.

the shot-point 0 is located at 0′. The ray path $0B_kS_k$, where S_k is the position of any seismometer, is equivalent to the path $0'B_kS_k$, which is straight and in a medium of constant velocity V_1. The response at the seismometers is entirely unchanged by the substitution of an image shot-point.

In the case of a sloping interface between the two strata (Figure 263), the image shot-point is located at 0′, where $00' = 2b$. The movement of the wave emanating from 0′ is indicated by a succession of arcs of circles

whose origin is at $0'$. Each arc designates the position of a particular phase of the wave at a particular time, and therefore each arc constitutes a portion of a *wave front*.

If the seismometers having the same subscript (Figure 263) are located symmetrically with respect to 0, it is evident that due to tilting of the reflecting horizon, the distance $0B_3S_3$ is greater than $0B_3'S_3'$ so that

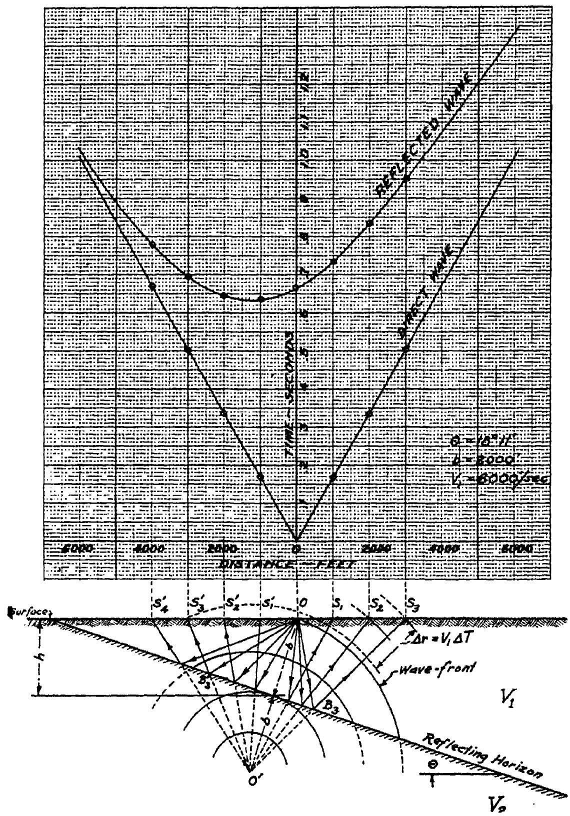

FIG. 263.—Ray paths and travel-time curve for rays reflected from a sloping surface.

when a given wave front reaches the point S_3', the wave front is still short of point S_3 by a distance Δr. This distance is therefore the difference, ΔT, of the reflection times at points S_3 and S_3' multiplied by the wave velocity of the upper stratum; i.e.,

$$\Delta r = V_1 \Delta T$$

The difference ΔT of the reflection times at two seismometers symmetrically disposed with respect to the shot-point is frequently termed

the "move-out" or "step-out" for reasons which will be apparent when viewing the field reflection record. ΔT is a measure of dipping of strata when depth and velocity are known.* It is further evident that when the seismometers are located symmetrically with respect to the shot-point, ΔT increases both with degree of dip and *spread length*.**

The travel-time is calculated conveniently by introducing images both of the shot-point 0 and the seismometer station S. In Figure 264, MM represents a trace of the earth's surface, NN a trace of the upper surface of the dipping formation, and $M'M'$ the image of MM in NN. As before, 0 is the shot-point and S the seismometer station. The perpendiculars to the lower formation from 0 and S will be called b and b' respectively. $0'$ and S' are the images of 0 and S in NN; their positions are fixed by extending the perpendiculars b and b' as shown. $0''$ is a point vertically above $0'$. The point of reflection B is the intersection of the diagonals $0S'$ and $0'S$; thus, the path of the reflected wave is $0BS$. Designate the distance $0S$ by x, and the angle made by NN with the horizontal, i.e., the dip, by θ. Evidently,

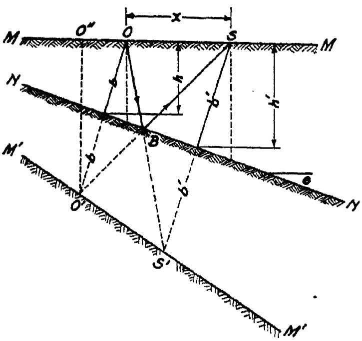

FIG. 264.—Sketch for computing travel-time of rays reflected from an inclined layer NN.

$$b' = b + x \sin\theta$$

$$00'' = 2b \sin\theta$$

$$0'0'' = 2b \cos\theta$$

The distance $0B$ equals $0'B$ so that $0BS$ equals $0'BS$. $0'BS$ is the hypotenuse of the right triangle $0''0'S$; hence

$$(0'BS)^2 = (0'0'')^2 + (0''0S)^2 = 4b^2 + x^2 + 4xb \sin\theta$$

Thus the length of the reflected ray path is

$$0BS = 0'BS = \quad + x^2 + \quad \sin\theta$$

Let V_1 denote the velocity in the upper bed, then the travel-time T to a seismometer located down-dip from the shot-point is:

$$T = \frac{{}^2 + x^2 + xb \sin\theta}{V_1} \tag{20}$$

* Throughout this treatment, the term ΔT will be used to designate differences of reflection time on traces of the same record, irrespective of the disposition with respect to the shot-point of the seismometers producing the traces.

** The term *spread* refers to the disposition of seismometers and shot-point on the surface. The *spread length* is the distance between the end seismometers of a spread.

If the seismometer is located up-dip from the shot-point, the angle θ entering into Equation 20 is replaced by $-\theta$. Hence, the travel-time T' to a seismometer located up-dip from the shot-point is:

$$T' = \frac{\sqrt{4b^2 + x^2 - 4xb \sin\theta}}{V} \tag{20a}$$

Since three unknowns (V_1, b, and θ) appear in Equations 20 and 20a, a set of three reflection times from three seismometers located at different distances from the shot-point would suffice to determine the dip, depth, and velocity for a reflecting bed.* In the general practical procedure, however, it is not customary to evaluate the velocity with each dip-depth determination. Instead, the velocity is investigated separately, as discussed in a later section.

The difference in the reflection times to the two seismometers located on either side of 0 is:

$$1 \qquad \Big(\quad + \quad 4b^2 + x^2 - \sqrt{1 -} \qquad \sin$$

A tabulation of the ΔT values corresponding to a fixed value of the dip θ and various values of the depth h is given in Table 16.

The table is computed for various values of the vertical distance h between the surface of the earth and the point of incidence of the normal ray at the reflecting horizon. It is evident from Figure 264 that h and b are related by the equation $h = b \cos\theta$. The velocity values approximate those encountered in certain parts of California and the Gulf Coast region. The table shows the *order of magnitude* to be expected in observed time differences ΔT.

TABLE 16

VALUES OF ΔT COMPUTED FROM EQUATION 21

x	$h = 1{,}000$ ft. $b = 1{,}015.43$ ft. $V = 6{,}250$ ft./sec.	$h = 5{,}000$ ft. $b = 5{,}077.15$ ft. $V = 7{,}250$ ft./sec.	$h = 10{,}000$ ft. $b = 10{,}154.30$ ft. $V = 8{,}250$ ft./sec.	$h = 15{,}000$ ft. $b = 15{,}321.45$ ft. $V = 9{,}250$ ft./sec.
100 ft.	$\Delta T = .0056$ sec.	.0048 sec.	.0042 sec.	.0037 sec.
500	.0270	.0240	.0210	.0188
1,000	.0499	.0477	.0420	.0374
2,000	.0795	.0940	.0838	.0749

Legend:

dip $= \theta = 10°$
$h = b \cos\theta$

* This is true because the spread was assumed in the direction of maximum dip. Otherwise four reflection times would be required.

The Recorded Reflection

When the subsurface under investigation consists of more than two strata, the penetration of the rays to the deeper strata must be considered. At each interface the radiating wave is divested of energy, a part of which appears as a reflected longitudinal wave and is detected by the seismometer on the ground surface. The seismometer is therefore activated by a series of reflected waves from successively deeper reflections.*

It might be expected that superimposition of other energies (surface waves, direct waves, transverse waves, refracted waves, sustained strata vibrations, reverberations, and disturbances not attributable to the shot) on the reflected energy might outweigh the effect of the reflected wave and thereby prohibit reproducible records. However, the non-useful energy produced by the shot can be avoided to a large degree by present-day instrumental and field techniques. Interfering energy from sources other than the shot-point is generally maintained below a "noise-level" so that confusion with reflected energy occurs only in the case of a disturbance of unexpectedly large amplitude, and this is eliminated conclusively by comparison of two or more records secured from the same shot-point. Seismic records, therefore, are generally reproducible as far as energy from the shot is concerned in that successive shots under essentially similar conditions in the shot-hole will yield substantially similar records.

However, if a single seismometer were utilized to determine movement of the ground following a shot, it would generally be quite difficult to distinguish with surety the reflected waves from the spurious energy recorded on the seismograms. The identification of reflections is made possible by determining the motion of the ground at more than one station. (For purposes of reliability and to facilitate identification and correlation of reflections, the motions of several seismometers are recorded simultaneously on the same record.)

A segment of a reflection record secured in a good reflecting region is shown in Figure 265.** Each trace is due to a separate station, the ten

* Secondary reflections may occur and show up on the record. Secondary reflections may be due to waves which have undergone multiple reflection before striking the seismometers. Generally, multiple reflection, or reverberation, between two strata is very seldom detectable because the energy imparted to a seismometer by a wave which has undergone multiple reflection is small relative to the other energy which the seismometer is receiving at the same time. Another class of secondary reflections, however, is sometimes noticeable: namely, that corresponding to reflection of direct waves from the base of the low velocity layer or from the ground surface.

** A reluctance type of electromagnetic seismometer which responds to velocity of ground movement was used. The frequency discrimination of seismometer and amplifier units produces an overall response which differs from the actual ground movement.

stations being disposed in a straight line symmetrically with respect to the shot-point. Lines crossing the record are time lines recorded at intervals of 0.01 seconds. (An erratic low velocity layer produced a distortion of the reflection "line-up" which is similar for all reflections, e.g., a lag of 0.005 seconds between the 9th and 10th traces.)

In a study of records, similar wave forms which appear in a permissible pattern will be identified as reflections. Prominent reflections are picked, or marked, at corresponding points or phases* down the traces, as shown in Figure 265.

FIG. 265.—Typical dip record. ΔT is the difference in reflection times to the two end seismometers. Note 0.005 sec. lag between bottom two traces due to change of conditions in low velocity layer. (Courtesy of Western Geophysical Company.)

From a knowledge of the average velocity in the region under prospect and the disposition of the seismometers with respect to the shot-point, the interpreter of the field records can determine the range of difference of times which may be expected for reflections from beds at any depth and dip. The time-distance relation for reflection indicates the nature of the patterns or so-called "line-ups" which are possible for reflections. For a *split spread,* i.e., seismometers located in line with and on opposite sides of the shot-point, the pattern or "line-up" is of the form indicated in Figure 265; with increasing depth the "line-up" approaches a straight line, i.e., curves drawn through troughs or peaks corresponding to the same reflection on the various traces approximate straight lines. Actually,

* The term "phase" is used here in the conventional sense of any particular time in the history of a wave, rather than in the more limited meaning, p. 445, of the onset of a new type of wave on earthquake seismograms.

it is seldom necessary to refer to velocity or spread length to recognize the reflections because the difference in reflection times between successive traces usually is small.

The evaluation of the quality of a recorded reflection involves the following factors: (1) The *consistency of wave form* for every trace of a record and the consistency of the difference in reflection times for different phases of the wave are the primary factors by which reflections are identified and by which accuracy of dip determination is gauged. (2) The wave form or *character* is judged by the degree to which it approaches the response to a single impulse of steep wave front to a seismometer. The presence of all phases of a wave without interference is some assurance that a reflected wave and not spurious energy is involved. (3) The *relative amplitude* of a reflection with respect to that of the waves on neighboring portions of the record determines the prominence of a reflection and is some gauge of its reliability.

Judgment of relative reliability of reflections as based upon the considerations outlined above differs widely in practice. Sometimes a scheme of "grading" the reflections is used, indicated by *G* (good), *F* (fair), *P* (poor), with more detailed grading when warranted. (See Figure 265.)

It is usually found that the definiteness of reflections is dependent on the length of the spread. This is not surprising because one expects that as seismometer stations are separated over greater distances, variations in the character of the reflecting bed and overlying strata will produce dissimilarities in the character of the reflection across the record.

Although the character of a reflection furnishes no definite information regarding the composition of the constituent layers of the subsurface, it is often observed that certain strata yield a similar and distinctive character of reflection throughout certain areas. In these cases, therefore, it is possible to trace and map specific geologic strata *without* dip information. Such mapping is done by *character correlation,* i.e., picking corresponding phases of those reflections which have substantially similar wave forms on records obtained at two or more separated seismometer spreads.

However, due to the limited number of identifying characteristics in any reflection and to the smallness of change in character capable of confusing the phases of a reflection, a positive correlation by character—and one which is accurate in phase—is impractical in many regions, unless two or more reflections from neighboring strata support the correlation. In such correlation, consideration is given to the possibility of stratigraphic changes which may cause a change of interval (ΔT) between records

with a disappearance of some correlating reflections and an appearance of others. (Examples of correlation are shown in Figures 288, 289, and 336.)

In actuality, the picking of reflections across a record constitutes a correlation across that record. If the gaps between successive seismometers of a spread are not excessive, the small values of ΔT (intervals of time between corresponding phases of two traces on one record) permit accurate correlation of any reflection which is clearly indicated on the traces. One type of correlation shooting therefore consists in employing a continuous line of seismometer spreads wherein the gaps between seismometers, and particularly the gaps between successive reflection points on a reflecting bed, are kept sufficiently short that correlation is positive. This method requires a great number of spreads in order to maintain adequate daily coverage of prospect and is relatively expensive.

Differentiation Between Dip Shooting and Correlation Shooting

The correlation method of mapping is a powerful tool in those regions where reliable correlations are possible. However, in numerous other areas where reflections can be obtained, correlation is doubtful or impossible due to lack of persistent reflecting horizons, great extent of stratigraphic change, and general lenticular character of the subsurface. (A considerable part of California and the Gulf Coast exhibits this condition.) When such areas are explored by seismic methods, recourse usually is made to reflection dip shooting.*

Correlation usually is good in most of Oklahoma, West Texas, and Kansas. In these areas, the strata dip gently and the dip is seldom greater than two degrees. On the other hand, gentle dipping of strata is not always a sufficient condition for reliable correlation, so that in some cases dip shooting is used in areas of gentle dipping strata. Where dips much in excess of two degrees are expected, dip shooting is generally utilized, assisted by correlation when possible.

Correlation shooting may not necessarily be the same as dip shooting. Correlation shooting requires a disposition of seismometers which will permit the identification of reflections above the "spurious energy," a condition which is usually lenient since reflections are prominent in regions where correlation shooting is possible. The most efficient spread, although not always used, is one in which the seismometers are located on a straight line leading in one direction away from the shot-point, *the*

---

* Refraction methods may also be possible in some cases but in general are used only when the reflection method is inapplicable.

uni-directional spread. Such a spread affords the greatest difference between pattern or "line-up" due to reflections and that due to surface and refracted waves. A total spread between seismometers of 200 to 500 feet is usually sufficient.*

Requirements for dip shooting are more severe. A 500-foot spread length is generally much too short. The actual value of the spread length to be used in any area depends on the values observed for ΔT in that area, because the value of ΔT must be appreciably greater than the magnitude of probable error.** (See also the tabulation of ΔT versus spread length for a typical case, page 473.)

In correlation shooting, the identification of the same phase of a reflection is of paramount importance; in dip shooting, the ΔT values corresponding to the reflections under investigation are of paramount importance. As far as type of spread is concerned, in correlation shooting the uni-directional spread is favored, while in dip shooting more use is made of split spreads and continuous profiling.

The dip spread is applicable for correlation shooting, even though short correlation spreads are inapplicable for dip shooting. That this condition is fortunate may be inferred from previous considerations; that is, generally where correlation shooting is used, the dip is so small that accurate dip data could not be obtained without prohibitively long spreads. However, in regions where dip shooting is utilized, correlations quite frequently appear and are plotted. If dip data contradict correlation data, whether dip or correlation or both are to be used rests on the following consideration: Because the reflection time interval for dip determination is obtained on a single record, it is inherently more accurate than reflection time intervals which are recorded on separate records at different shot-points. However, when correlations are carried over shot-points separated by large distances, the difference of reflection time is increased, and this permits increased accuracy. The balance of advantage must also be governed by the reliability of correlations and the possibility that if correlation is carried too far, small intermediate structural features may be overlooked.

* The total spread between seismometers depends on the structural aspects of the area under observation. When the dips of the strata are small, spreads can be fairly long, say 100 feet between seismometers, but when large dips and rapid lateral changes are encountered, it is necessary to cut down the distance between seismometers to 50 and even 20 feet to get reliable correlation between traces and between different records.

** The accuracy of the travel-time values is of the order of 0.001 sec. at its best and therefore values of ΔT of larger magnitude than this are required. Important inherent deterrents to an accuracy in the ΔT value greater than 0.001 sec. are: general irregularities in the subsurface such as lenticular structure or lateral stratigraphic variation; erratic characteristics of the reflecting horizon; inaccuracies in the correction for the low velocity layer; etc.

The computation and mapping involved in dip shooting are presented in the following section. The computation of data in correlation shooting is essentially a special case in which dip is small and is treated in the succeeding section.

Dip Shooting.*—The basic ray equations were developed on pages 463 and 464 for the case of a velocity function varying in the vertical direction. Successive applications of Snell's law of reflection led to a relation applicable at any given point on a ray; namely,

$$\frac{\sin \alpha}{V} \tag{10}$$

where α is the angle between the ray and the vertical at that point; V is the velocity; and p, the *ray constant,* is a parameter specifying a particular ray. Also, the time of travel from the ground surface to a point A is given by Equation 15; that is,

$$t = \int_0^h \frac{dh}{V\sqrt{1-p^2V^2}}. \tag{15}$$

and the horizontal displacement between the shot-point and the point A is

$$X_n = p \int V dh \tag{16}$$

If the point A is located on a reflecting horizon dipping at an angle θ and if θ equals α at that point, the ray will be reflected back along its original path. (Figure 266.) Such a ray which strikes a reflecting bed normally is termed a *normal ray* and the reflection time for a normal ray is specified by twice the time given by Equation 15.

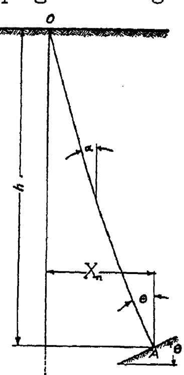

FIG. 266. — Curved ray path.

Except for some specific velocity functions, the integrals of Equations 15 and 16 cannot be evaluated explicitly. Instead, the equations are treated in the manner in which they were derived; that is, one considers the subsurface section as composed of several layers, each of constant velocity, and determines the ray path by applying Snell's law at each of the successive interfaces.** The ray is specified by the ray constant p. This constant may be expressed in terms of recorded data as shown in the following paragraphs.

* Pages 479 to 509, pertaining to Dip Shooting, have been prepared largely by M. B. Widess, in collaboration with H. Salvatori, and with the assistance of N. A. Haskell. A condensed analysis of Dip Shooting is given by Widess and Haskell in *Geophysics,* Vol. 5, No. 2, April, 1940.

** Approximate methods of transforming time data to space coordinates by using straight rays are common and are discussed under "Approximate Computation Methods," p. 495. These and other approximate methods involve approximate evaluation of the integrals of Equations 15 and 16.

The dip of a bed is derived from the difference in the times of arrival of a reflected wave at two separated seismometer stations. Consider two seismometer stations S_1 and S_2 located at the same elevation and separated by a distance s, s being sufficiently small that the reflected wave front striking the seismometers may be considered a plane. (Figure 267.) Assume, also, that the wave front is perpendicular to the plane of the paper. For this case, rays in the neighborhood of S_1 and S_2 are approximately straight; also, they are parallel to the plane of the paper.

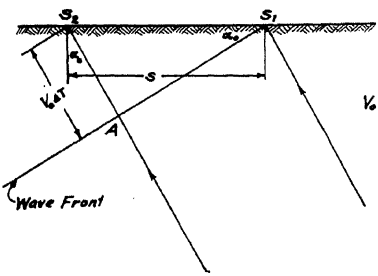

FIG. 267.—Plane wave front.

When the wave front strikes S_1, it is still a distance $\overline{AS_2}$ from S_2 Therefore the difference in the reflection times (ΔT) is given by the equation

$$\Delta T = \frac{\overline{AS_2}}{V_0}$$

where V_0 is the velocity of the medium in the vicinity of the spread.

The distance AS_2 can be expressed directly in terms of spread length s and the angle of arrival of the wave front α_0 by the relation

$$\overline{AS_2} = s \sin$$

Hence,

$$\sin \qquad (22)^*$$

On combining Equations 22 and 10, one obtains

$$(23)$$

Equation 23 expresses the ray constant in terms of observed quantities: namely, reflection time difference ΔT and spread length s. Reflection time and horizontal displacement equations are thus complete when reflection time difference, reflection time, and velocity are specified.

* It should be noted that Equation 22 defines the maximum ΔT which can be recorded due to dip of a bed; i.e., ΔT cannot exceed the value s/V_0 obtained when the ray travels parallel to the ground surface. (When records show a value of ΔT greater than this figure, the excess is due to differences in travel-times in the low velocity zone at the seismometers, or else the traces show interference between waves rather than a single wave.)

The dip of the bed is obtained from Equation 10. That is,

$$\sin a_h = \sin \theta = \sin \iota \cdot V_h$$

where V_h is the velocity at depth h. On combining the last relation and Equation 22, we obtain

$$= \frac{\Delta T}{s} V_h \qquad (24)^*$$

Equation 24 together with Equations 15 and 16 complete the set required for conversion of field data, s, T, and ΔT, into space data, θ, h, and x, when the velocity is specified. In general, the conversion is obtained through charts derived by direct use of these equations or approximate versions of these equations. (See pages 493 and 494.)

Applied Reflection Time and ΔT Equations **

The normal ray discussed in the previous section is one which returns along its original path. Hence, its exact travel-time usually cannot be obtained, because a seismometer generally cannot be placed in the immediate neighborhood of a shot-point without being subjected to an intolerable amount of undesirable disturbance. However, by use of appropriate spreads, one can obtain the approximate value of this travel-time, as will be evident from the following considerations.

If a symmetric split spread is used, an approximate value for reflection time along the normal ray may be obtained by averaging the times recorded by the two seismometer stations nearest the shot-point (one on either side of the shot-point). The rays striking these stations approximate normal rays and sometimes it is sufficiently accurate to postulate that they are normal rays. However, in many cases, use of the travel-times of non-normal rays is not sufficiently accurate and a correction must be applied.

This correction may be obtained by analyzing the wave fronts in the neighborhood of the seismometer spread.

If the reflections occur at shallow beds or if long spreads are used, it is necessary to take into account the fact that the wave front in the neighborhood of the seismometers is not a plane but a curve. It will be assumed as an approximation that the wave front is an arc of a circle whose radius equals the complete ray path from shot-point to reflection

* Equation 24 shows that for constant θ, i.e., parallel strata, ΔT will generally decrease farther down the record, because the velocity increases with the depth. When this is not the case, either a change in dip of the beds or an abnormal change in V or both may be responsible.

** This treatment was obtained from an analysis by N. A. Haskell.

point and back to the recording seismometer. This condition is satisfied exactly if the medium has a constant elastic wave velocity and is satisfied approximately for almost all those practical cases where the curved ray path is considered.*

In Figure 268, 0 and S, respectively, represent a shot-point and a seismometer station located at the same elevation and separated by a distance x. The reflecting horizon is a plane normal to the plane of the paper and dips as shown at an angle θ. The curvature of the normal and reflected rays indicates that the velocity is not constant with depth, although it is assumed that the velocity does not vary laterally. Let the curve W represent a circular wave front of the reflected wave at the instant it reaches the shot-point. The wave front impinges upon the spread at an angle α_0 at the shot-point 0. Let the center of curvature of the wave front be at C; that is, let C be the point from which the wave appears to diverge. For simplicity of derivation, it will be assumed that a constant velocity V_0 persists from the top of the section to the depth of the point A so that rays are straight in the immediate neighborhood of the spread.

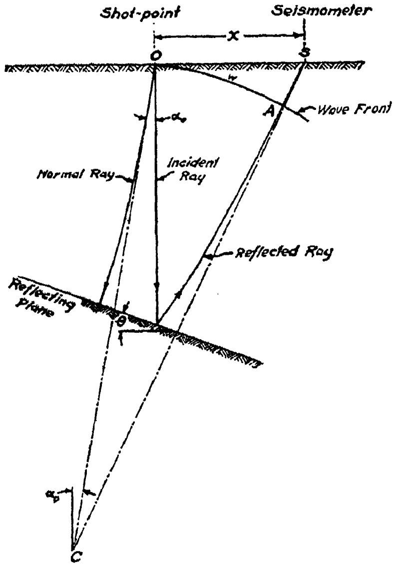

FIG. 268.—Wave front traveling toward seismometer.

It will be the purpose of the first derivation to determine the difference ΔT_x between the reflection time T_x recorded at the seismometer and the reflection time T_0 along the normal ray. Since the wave front is a curve of constant reflection time, the travel-time from C to 0 is the same as that from C to A. Hence,

$$\Delta T_x = T_x - T_0 = \frac{\overline{AS}}{V_0}$$

Denote $\overline{CO}$, the radius of curvature of the wave front when the wave strikes point 0, by ρ. In the new notation,

$$\Delta T = \overline{CS} - \rho$$

* When a divergent wave passes from one medium to another whose elastic wave velocity is greater than the first, the radius of curvature of the wave is decreased compared with the value it would possess if no change of velocity were present. Thus, in general, on the downward course (increasing velocity) the radius of curvature increases less rapidly than the total distance traveled. On the return course after reflection, the effect is reversed, and the radius of curvature increases more rapidly than the distance traveled. The two effects will thus tend to neutralize each other.

In the triangle OCS,

$$\sin \alpha_0 \tag{25}$$

On introducing the power series expansion

and simplifying, Equation 25 becomes

$$\frac{\overline{CS}}{\rho} = 1 + \frac{x}{\rho}\sin\alpha_0 + \tfrac{1}{2}\left(\frac{x}{\rho}\right)^2\cos^2\alpha_0 - \tfrac{1}{2}\left(\frac{x}{\rho}\right)^3\sin\alpha_0\cos^2$$
$$\cdot \text{ terms of order } \left(\frac{x}{\rho}\right)^4 \text{and higher.}$$

Hence, the expression $\Delta T_x = \dfrac{\overline{CS}-\rho}{V_0}$ becomes:

$$\Delta T_x = \frac{\rho}{V_0}\left\{\frac{x}{\rho}\sin\alpha_0 + \left(\frac{x}{\rho}\right)^2\cos^2\alpha_0 - \ldots \sin\alpha_0\cos^2\alpha_0\right.$$
$$\left. + \text{ terms of order } \left(\frac{x}{\rho}\right)^4 \text{and higher.}\right\} \tag{26}$$

Equation 26 is a basic ΔT equation. It can be extended to any spread and to the comparison of any two rays simply by giving the proper value to x corresponding to the two rays. (When the distance x is on the up-dip side of the shot-point, a negative sign must be attached to x.)

If the times, T_x and T_{-x}, to the nearest seismometers in a symmetric split spread are averaged to approach the case of a normal ray, the difference between this average and T_0 is given by:

$$\frac{T_x + T_{-x}}{2} - T_0 = \frac{\rho}{V_0}\left\{\left(\frac{x}{\rho}\right)^2\cos^2\alpha_0 + \text{terms of order}\left(\frac{x}{\rho}\right)^4\text{and higher}\right\} \tag{27}$$

Terms of the order $\left(\dfrac{x}{\rho}\right)^4$ and higher may be neglected, leaving a single term on the right-hand side. This quantity must therefore be subtracted from the average of T_x and T_{-x} to obtain the time T_0 along the normal ray.

The accurate expression for the difference in reflection times to the end seismometers of a symmetrical split spread of total length s is obtained

by substituting $+s/2$ and $-s/2$ separately into Equation 26 and taking the difference; i.e.,

$$\Delta T_s = \frac{\rho}{V_0}\left\{\frac{s}{\rho}\sin\alpha_0 - \tfrac{1}{8}\left(\frac{s}{\rho}\right)^3 \sin\alpha_0\cos^2\alpha_0 + \text{terms of order } \left(\frac{s}{\rho}\right)^5\right.$$

and higher.

$$= s \quad \overline{V_0}\left\{ 1 - \tfrac{1}{8}\left(\frac{s}{\rho}\right)^2 \cos^2\alpha_0 + \text{terms of order } \left(\frac{s}{\ }\right)^4\right.$$

and higher.$\}$

Again ignoring terms of order $\left(\frac{s}{\rho}\right)^4$ and higher, and introducing a factor γ,

$$-\tfrac{1}{8}\left(\frac{s}{\rho}\right)^2 \cos^2\alpha_0 \qquad (28)$$

one obtains*

$$(29)$$

On comparing this equation with Equation 22 one observes that the term γ is a *spread correction term* which corrects for curvature of wave front when the spread is long.

The dip of the bed is given by

$$\sin\theta = \frac{1}{\gamma}\,\frac{\Delta T}{s}\,V_h \qquad (30)$$

The evaluation of ρ for use in the correction terms cannot be carried out accurately without considerable labor. However, great accuracy is not required since ρ appears in a relatively small term of the time and ΔT equations. It is generally sufficient therefore to use the product of the approximate reflection time for the ray arriving at the seismometer under consideration and the so-called average velocity to the corresponding reflecting point; that is, $\rho \cong T_x\overline{V_h}$. The so-called average or equivalent velocity $\overline{V_h}$ is defined by the ratio of depth to the vertical travel-time to that depth, or

$$V dt$$

$$: \overline{\quad} \int_0^h \frac{dh}{V(h)}$$

* The assumption of constant velocity to the depth A, Figure 268, is not essential to the accuracy of this final equation for the case of those velocity functions which are in general use. The velocity V_0 should be the velocity at the top of the section. The proof of this can be deduced by utilizing the symmetry of the spread.

Alternative Derivation of a Dip Equation

An alternative method † for deriving a dip equation will be evident from Figure 269. The section is assumed to be parallel to the direction of maximum dip. Also, it is assumed that the velocity in the upper medium is substantially constant and equal to V. As before, 0 denotes the shot-point and S a seismometer station. θ is the angle of dip; b is the perpendicular to the lower formation from 0; h is the vertical depth of the lower formation below 0. I, as usual, is the image of the shot-point in the lower formation. I' is a point of the surface vertically above I.

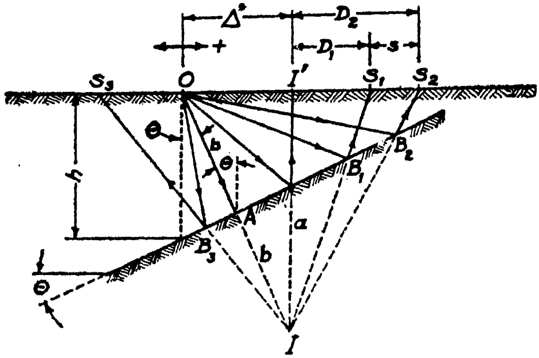

FIG. 269.—Sketch of a subsurface section showing ray paths. (Gutenberg, *Beiträge zur angewandten Geophysik.*)

The distance II' will be denoted by a; the distances $0I'$, $I'S_1$ and $I'S_2$ will be denoted by Δ^*, D_1, and D_2, respectively, and the distance between the seismometers S_1 and S_2 will be denoted by s.

It is apparent from the figure that

Also

$$\Delta^* = 2h \cos\theta \sin\theta$$

$$b = h \cos\theta \tag{31}$$

$$0BS = IBS = \cdot$$

where D is equal to D_1, D_2, etc., as the case may be.

The travel-time t_1 along the path $0B_1S_1 = IB_1S_1$ is:

Similarly, the travel-time t_2 along the path $0B_2S_2 = IB_2S_2$ is:

$$t - \sqrt{a^2 + D_2^2}$$

On eliminating a from these equations, one obtains

$$V^2 (t_2^2 - t_1^2) = D_2^2 - D_1^2 = (D_2 + D_1)\ (D_2 - . \tag{32}$$

But

† B. Gutenberg, "On Some Problems Concerning the Seismic Field Methods," *Beiträge zur angewandten Geophysik,* Vol. 6, part 2, 1936, pp. 125-140. (Published by V. Conrad, Wien, ;sberger, Freiburg : ᴰ- u. H. Reich, Berlin. Akademische Verlagsgesellschaft m. b.

and

$$D_2 + D_1 = 0S_2 + 0S_1 - 2\Delta^* = 2\Delta_m - 2\Delta^*$$

where Δ_m = average distance of the seismometers from the shot-point. Δ_m is assumed to be *negative* in the direction of dip.

On rewriting Equation 32 in terms of s, Δ_m, and Δ^*, and factoring the left-hand member, one obtains

Set

t_m $\frac{1}{2}(t_2 + t_1)$ = average travel-time to the two seismometers

and

$t_d = (t_2 - t_1)$ = difference in travel-times to the two seismometers

then

$$V^2 (t_2 + t_1)(t_2 - t_1) = 2V^2 t_d t_m = 2s(\Delta_m - \Delta^*)$$

$$\Delta^* = \Delta_m - \cdot$$

But from Equation 31,

$$\Delta^* = 2h \cos\theta \sin\theta = 2b \sin\theta$$

Hence,

(33)

Furthermore, along the path $0A0$,

$$b = \tfrac{1}{2} t_0 V$$

where t_0 = reflection time along normal ray.

Hence, Equation 33 may be written in the form:

$$\cdot\ \overline{t_0V} - \overline{s}\ \overline{t_0}$$

In most cases, the term $\frac{t_m}{t_0}$ can be set equal to 1 without appreciable error. Hence,

$$| = \frac{\Delta_m}{t_0V} - \frac{Vt_d}{s} \text{ (approx.)} \tag{34}$$

Evidently, if the velocity V is known, the dip θ can be calculated from Equation 34, provided the dip is not too large. (t_d is to be taken as positive if the seismometer closer to the shot-point records the reflection earlier than the more distant seismometer.)

If the dip is large, Equation 34 will not yield an accurate value, especially if Δ_m is large. Furthermore, the point of reflection B at

which the dip is found, will be far removed from the shot-point, and this introduces an error if the dip changes with distance, as is not unusual.

Treatment of Three-Dimensional Problems

Equivalent Time Horizons and Equivalent Rays. — The three-dimensional structure may be studied with relative ease by introducing the concept of *equivalent time horizons.* Consider a reflecting horizon whose slope at all points is continuous and assume that from every point on the horizon a normal ray is drawn to the ground surface. If the velocity varies with depth only and if the extent of folding of the horizon is limited to such a degree that no rays cross, a one to one correspondence is established by the rays between points on the reflecting horizon and points on the ground surface. (Figure 270.) (That is, the reflecting point is associated with a surface point midway between the shot-point 0 and the station S.) Designate by T the reflection time along a normal ray from any point in the medium to the reflecting horizon and back to that point. T is a function of depth and lateral position in the medium. The function is continuous, and at the ground surface it gives the reflection time which would be recorded if the actual rays were normal rays.

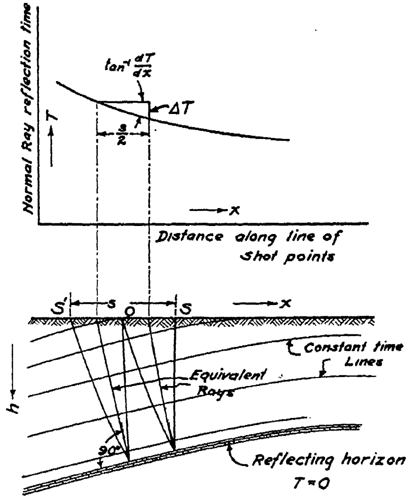

FIG. 270.—Reflection time along equivalent or normal rays.

When the distance from seismometer to shot-point is short compared to the distance from shot-point to reflecting bed, the travel-time along the ray path from shot-point 0 to seismometer S is very approximately equal to the travel-time along the *equivalent ray*: namely, that normal ray which strikes the ground surface at a point midway between shot-point and seismometer. For short symmetrical split spreads, therefore, the ray system to the end seismometers may be replaced by two equivalent rays each impinging on the surface at a distance of $s/4$ from the shot-point, where s is the length of total spread.

When seismometers and shot-points lie on a line, not necessarily along maximum dip,* the recorded reflection times would be given by an arrival-time curve such as shown in Figure 270, where each *observed*

* Compare following section entitled "Oblique Orientation of Spread."

reflection time is plotted against a value of x, x being the distance from an arbitrary point on the line between 0 and a point midway between the shot-point and the seismometer station at which that reflection is recorded. Furthermore, the slope dT/dX at any point on this travel-time curve is equal approximately to the difference ΔT in the reflection times to two seismometers such as S' and S divided by half the spread length; or

$$\Delta T = \frac{s}{2}\frac{dT}{dx} \text{ (approximately)} \tag{35}$$

An equivalent time horizon, from which reflection time gradients in any direction can be determined directly, may be obtained by plotting the values of the reflection time along the normal rays at the points of intersection of the rays with the ground surface and drawing a smooth curve through the plotted points.

Oblique Orientation of Spread.—The relations between reflection time data and space coordinates developed in the previous sections apply strictly only to spreads which are in line with the direction of maximum dip of the reflecting bed. In general, however, because the direction is seldom known before the shooting, the spread will be found oriented obliquely with respect to the reflecting bed. The resulting problem of dealing with components of dip as related to the ΔT obtained with spreads oriented obliquely is an involved one when exact solutions are desired. Generally, the problem is simplified by proceeding with computations on the assumption, usually justified, that to a sufficient approximation the component of dip of the reflected bed in the direction of the spread is related to the ΔT recorded at that spread by the equations already derived for spreads along line of maximum dip.

A more exact solution can be obtained as follows: At a given point on the ground surface, a spread in the direction of maximum gradient of reflection time will be oriented along maximum dip, while a spread in the direction of zero gradient or constant reflection time will be oriented along strike. Furthermore, because the gradient of any scalar quantity is a vector, the component of the time gradient in any direction is obtained by the usual procedure for resolving vectors. Hence, if the maximum gradient of the reflection time at a point P on the ground surface is $(dT/dx)_{P\,\max}$, and is directed, for example, due north, the magnitude of the gradient at a bearing ϕ is:

$$\left.\frac{dT}{dx}\right)_{P,\,\mathrm{n}} \quad \cos$$

The vector property of the time gradient is applied in the following manner. Draw two lines radiating from point 0, each in the direction in which the time gradient was determined from dip reflection shooting. (Figure 271.) Choose a convenient unit of length and mark off lengths

equal to the magnitude of the corresponding time gradient. Strike off normals to the vectors at their tips. The intersection of these normals establishes the point of the maximum gradient vector; that is, the length of the vector delimited by the point 0 and the intersection of the normals is proportional to the magnitude of the maximum gradient and its direction is parallel to that of the maximum gradient. The magnitude and direction of *maximum* dip is thus obtained from the determination of values of ΔT in two directions.

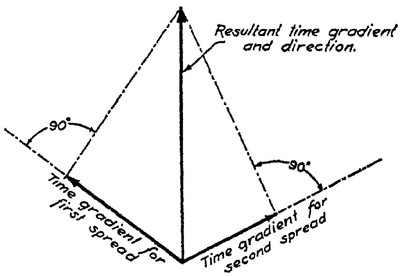

FIG. 271.—Composition for maximum reflection time gradient.

The most effective relative orientation of spreads for determination of maximum dip is one in which the spreads are at right angles to each other. In this case, the magnitude of the maximum gradient is equal to the square root of the sum of the squares of the component gradients, and its direction makes an angle with the direction of one of the spreads which is proportional to the inverse tangent, or cotangent, of the ratio of the component gradients.

After the magnitude and direction of the maximum gradient have been obtained, the gradient is expressed in terms of ΔT by means of Equation 35. The ΔT thus found is then inserted in Equation 24 in order to determine the magnitude of dip of the reflecting horizon.

It is customary to locate shot-points along a straight line ("shot-point line") in order to gain approximately a vertical cross-sectional view of the geologic strata. When *cross dip,* i.e., dip normal to the shot-point line, is small or constant, a close approach to a vertical section may be obtained. Because dip data along a line may be treated more readily than data in scattered directions, stress is sometimes given to *line spreads,* and *cross spreads* at some of the shot-points on the shot-point line are omitted.

Rigorous interpretation of dips over a prospect can be achieved by time gradients. In dip shooting, time gradients alone are determined and the process of preparing an equivalent time horizon is similar to that of preparing a contour map from a knowledge of dip only; that is, an equivalent time horizon is obtained by running time phantoms incorporating the time gradients. The time maps would be converted to space maps by measuring maximum time gradients and applying the equation (24) already established for maximum dip.

In general, however, the data from seismograms are converted into space coordinates long before a space map is drawn up in order that direct

space information on a prospect may be furnished during the course of the survey.

The usual procedure is to utilize the time gradients determined along the line spread as though they were maximum gradients and to plot space coordinates computed from these time gradients on a vertical section through the shot-point line. The degree of approximation implicit in a vertical section of this sort depends on the magnitude of cross dip and its change along the section.

From geometric considerations, it is apparent that because the tangent of dip is a gradient of depth, the tangent of maximum dip of a sloping plane is related to the tangents of the component dips in exactly the manner that the maximum time gradient is related to its components. Hence, the approximation introduced by using time gradients in the direction of the spread to determine the component of dip in that direction is subject to an error of the same degree as that by which the maximum dip as a function of ΔT *for a given reflection time* deviates from a linear function.

In these considerations it must be recalled that the spread has been assumed sufficiently short that equivalent rays could be employed. When this is not the case the spread correction factor,* page 484, is to be considered.

Computation Chart

For routine computation of space coordinates, it is generally out of the question to expect that ray path equations be evaluated for every set of reflection time and ΔT data obtained from the records. Consequently, computation charts are drawn up which permit direct determination of space coordinates for a *specified* velocity function. (In all cases it is assumed for purposes of these computations that the seismometer spread is along the line of maximum dip.)

The most obvious computation method is achieved by the so-called *wave front chart* which depicts rays and wave fronts radiating from a shot-point. A rectilinear background scale corresponds to depth and horizontal displacement of the reflection point. Lines of constant time are indicated by wave fronts, while lines of constant reflection time difference (for a given spread length) are indicated by rays.**

A variation of the wave front chart is shown in Figure 272, where the depth and horizontal displacement curves are drawn on a rectilinear scale corresponding to reflection time T and time difference ΔT. In addition to this chart, a separate set of curves (Figure 273) is used to supply the value of dip.

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* It might be noted, however, that the correction factor will be greater than indicated on page 484 when a cross dip is present, since the curvature of wave front at the spread will be greater in the vertical plane through the spread than in a vertical plane through the line of maximum dip.

** The spread correction factor γ must be applied when ratio of spread length to depth of the reflecting horizon is not small.

In drawing up the computation chart, it is generally necessary to approximate the true velocity, because an accurate determination of velocity is possible only if wells have been drilled and seismometers can be descended to those depths to which the velocity is to be ascertained. (Compare page 527.) Furthermore, in some regions the velocity has an appreciable lateral variation. Because it is impractical to develop computation charts for every velocity encountered and because velocity is seldom known accurately, except for a few specific points, the practical procedure has generally been to use a single computation chart for a given region and to correct maps as additional and more precise velocity information is obtained.

The actual observed well velocity is rarely used to specify the velocity function over the entire range of depth because the velocity function is usually erratic and it is extremely laborious to develop charts for any but relatively smooth or simple velocity functions. A velocity function which is sometimes used because of its simplicity, ease of computation, and approximate compliance with some geological sections is given by the linear equation

$$V = V_0 + ah \tag{36}$$

where V_0 and a are constants.

When this value is substituted into Equation 15, the following expression is obtained for the total travel-time $T = 2t_n$:

$$= 2\int_0^h \qquad dh$$

To integrate this equation, set

$$z = p(V_0 + ah), \quad dz = ap\,dh$$

When $h = 0$, $z = pV_0 = \sin a_0$ and when $h = h$, $z = p(V_0 + ah) = pV(h)$ $= \sin a_n = \sin$

Hence,*

$$T = \frac{2}{a}\int_{\sin a_0}^{\sin\theta} \frac{dz}{z\sqrt{1-z^2}} = -\frac{2}{a}\left[\log_e\left(\frac{1+\sqrt{1-z^2}}{z}\right)\right]_{\sin a_0}^{\sin\theta}$$

$$= \frac{2}{a}\left[\log_e \quad \frac{1+\sqrt{1-\sin^2 a_0}}{\sin a_0}\right) - \log_e\left(\frac{1+\sqrt{1-\sin^2\theta}}{\sin\theta}\right)\right]$$

$$\frac{\sin\theta}{\sin a_0}\,\frac{1+\cos a_0}{1+\cos\theta} = \frac{1}{a}\log_e \frac{\sin^2\theta}{\sin^2 a_0}\,\frac{(1+\cos a_0)^2}{(1+\cos\theta)^2}$$

or

$$= \frac{1}{a}\log_e \frac{(1-\cos\theta)\,(1+\cos a_0)}{(1-\cos a_0)\,(1+\cos\theta)} \tag{37}$$

* The value of this integral will be found in any standard table of integrals. (See, for example, B. O. Peirce, *A Short Table of Integrals,* Third Revised Edition (Ginn and Co.) Formula 129.)

By making use of the relation:

$$\frac{\sin\theta}{\sin\alpha_0} = \frac{V}{V_0}$$

the travel-time equation may also be written in the form

$$T = \frac{2}{a}\log_e \quad {}_0 \quad \frac{\cos\alpha_0}{1+\cos\theta} \tag{37a}$$

Equation 37 is the travel-time equation when the velocity is a linear function of the depth.

The corresponding horizontal displacement equation is obtained by substituting $(V_0 + ah)$ for $V(h)$ in Equation 16. That is,

$$X = X_n \qquad \frac{p\,(V_0 + ah)\,dh}{}$$

The integration may be carried out as before by setting

Then

$$-\frac{1}{ap}\left[\sqrt{1-z^2}\right. \qquad = \frac{1}{ap}(\cos\alpha_0 - \cos\theta)$$

The last relation may be put in a more useful form by expressing ap in terms of θ, α_0 and h as follows:

$$V = \frac{\sin\theta}{p} \text{ and } V_0 = \frac{\sin\alpha_0}{p}$$

On substituting these values of V and V_0 into the expression $V = V_0 + ah$, one obtains

$$\frac{\sin\theta}{p} - \frac{\sin\alpha_0}{p} \qquad ah$$

or

$$ap = \frac{\sin\theta - \sin\alpha_0}{h}$$

Hence

$$X = h\,\frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0} \tag{38}$$

Equation 38 is the horizontal displacement equation when the velocity is a linear function of the depth.

Thus, when the velocity is assumed to be a linear function of the depth, the complete set of equations used in evaluating the computation chart is:

$$V = V_0 + ah \tag{36}$$

$$= s \frac{\sin\theta}{V_h} \tag{30}$$

$$T = \cdot \quad \cdot \log_e \frac{(1-\cos\theta)\ (1+\cos\alpha_0)}{(1-\cos\alpha_0)\ (1+\cos\theta)} = \frac{2}{a} 1 \quad \frac{V}{V_0}\ \frac{1+\cos\alpha_0}{1+\cos\theta} \tag{37}$$

$$X = h\ \frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0} \tag{38}$$

$$\sin\alpha_0 = \frac{1}{\gamma}\frac{\Delta T}{s} V_0 \tag{29}$$

A typical set of constants applicable to some regions in California and the Gulf Coast is $V_0 = 6{,}000$ and $a = 0.5$. Hence, for these regions, the velocity function may be written in the form:

$$V = 6{,}000 + 0.5h$$

The charts, Figure 272 and Figure 273, are based on this function. The charts show the computed values of ΔT, T and X obtained by setting $s = 1{,}000$ ft. (γ is set equal to 1 for depths in excess of 2,000 feet, and for shallow depths, γ is computed from Equation 28.)

The ray paths for the linear velocity function (36) are arcs of circles, as is evident from the following relations.

$$X = \frac{1}{ap}(\cos\alpha_0 - \cos\theta)$$

$$(apX - \cos\alpha_0)^2 = \cos^2\theta$$

But

$$\sin\theta = pV = p(V_0 + ah)$$

and

$$\cos\theta = \sqrt{1 - p^2(l}$$

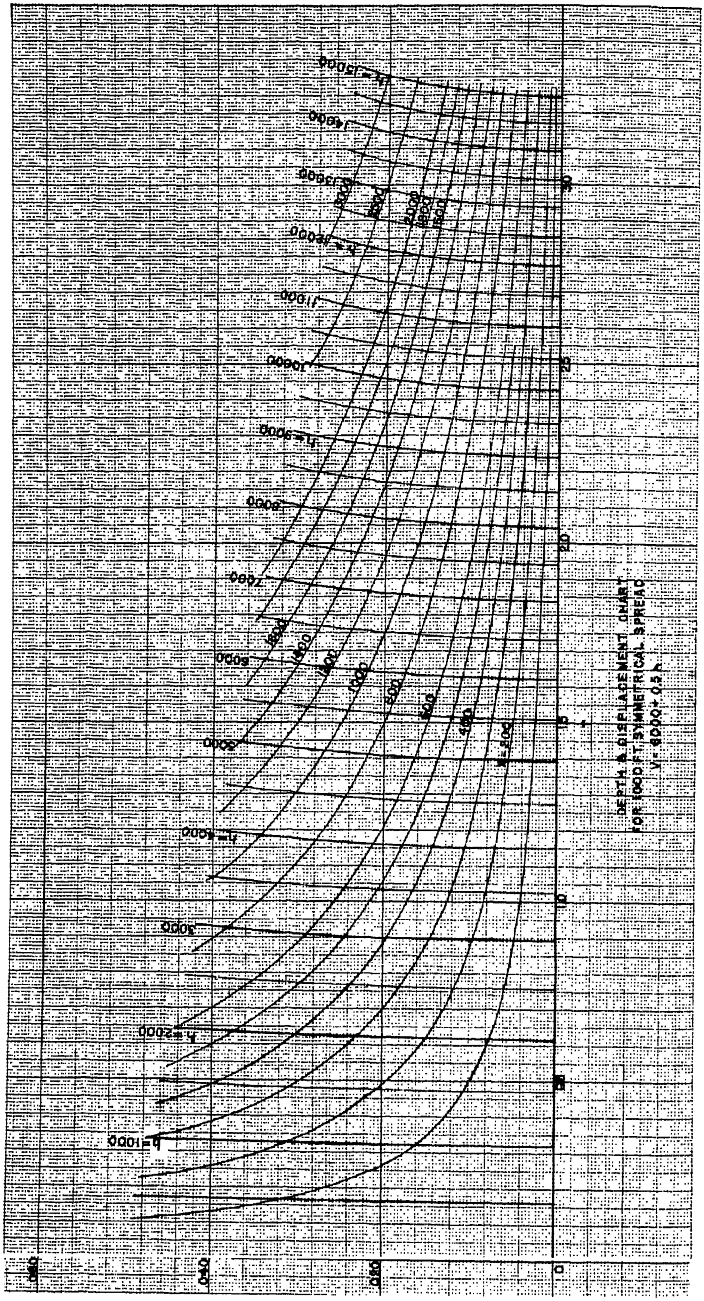
Hence

$$(apX - \cos\alpha_0)^2 = 1 - p^2(V_0 + ah)^2$$

or

$$(ap)^2$$

The last equation is the equation of a circle. The radius of the circle is $\frac{1}{ap}$ and the coordinates of the center of the circle are $\frac{(1-pV_0)^{1/2}}{ap}$ and $-\frac{V_0}{a}$. The wave paths, therefore, are arcs of circles whose centers lie on a fixed line parallel to the x-axis. (Compare Figure 295.)


DEPTH & DISPLACEMENT CHART
FOR 6000 FT. SYMMETRICAL SPREAD
V= 6000+ 0.5 h

ent chart

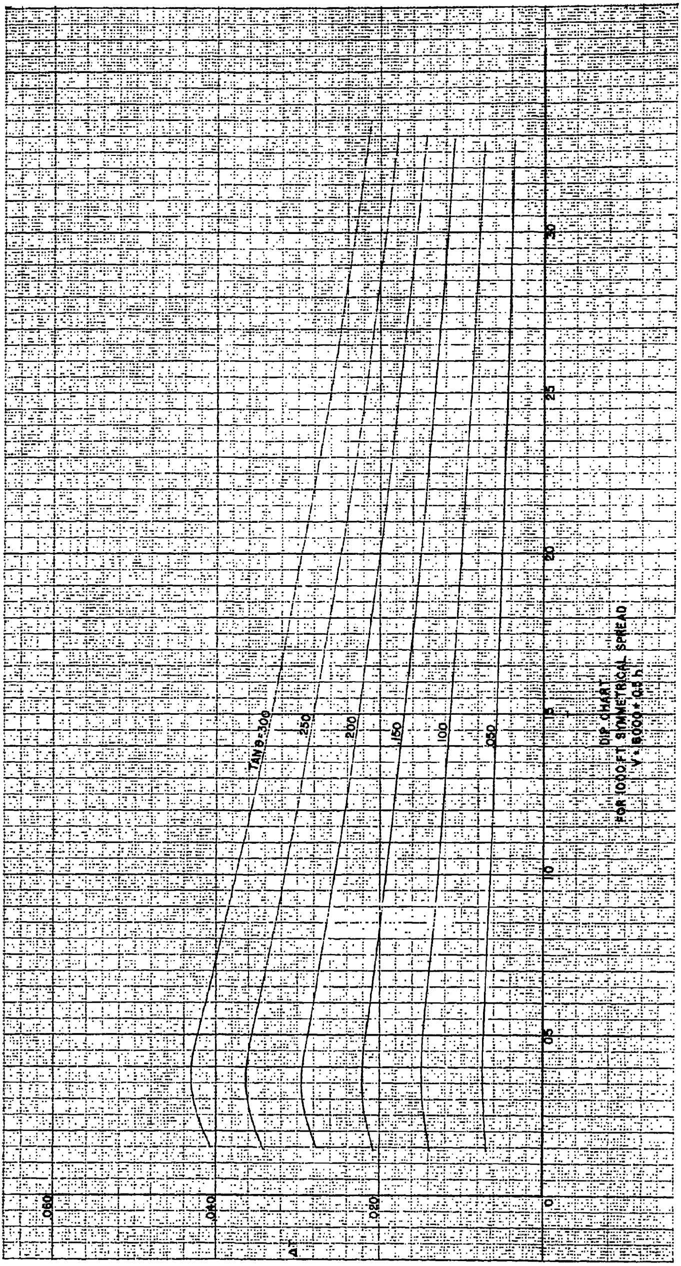


FIG. 273.—Dip chart for 1,000-foot symmetrical spread and $V = 6{,}000 + 0.5h$.

TABLE 17
VALUES USED IN DRAWING CHARTS OF FIGURES 272 AND 273

$\tan\theta = 0.00$

$$T_r = \frac{2h}{V} = \frac{2h}{\dfrac{0.5h}{\log\left(1+\dfrac{h}{12{,}000}\right)}} = 4\log\left(1+\frac{h}{12{,}000}\right)$$

$\tan\theta = 0.05$ $\quad V = 6{,}000 + 0.5h \quad \cos\theta = .99875$

$$1-\gamma = \tfrac{1}{8}\left(\frac{1000\cos\alpha_0}{V T_r}\right)^2$$

$h\times10^{-3}$ ft.	T_r	$h\times10^{-3}$ ft.	V ft/sec.	$V/V_0 = V/6000$	$\Delta T = \frac{1000\sin\theta}{V} = \frac{49.94}{V}$	$\sin\alpha_0 = \frac{V_0\Delta T}{1000} = 6\Delta T$	$\cos\alpha_0$	$T = 4\log\frac{V}{V_0}\frac{1+\cos\alpha_0}{1+\cos\theta}$	$X = h\frac{\cos\alpha_0-\cos\theta}{\sin\theta-\sin\alpha_0}$	$\Delta T(1-\gamma)$	$\Delta T_{corr.} = \Delta T-\Delta T(1-\gamma)$
.5M	.1610	0.5	6250	1.04166	.007990	.04794	.99885	.1634	25	.00100	.0070
1.0	.3202	1.0	6500	1.08333	.007683	.04609	.99893	.3205	47	.00024	.0074
1.5	.4711	1.5	6750	1.12500	.007398	.04439	.99901	.4716	70	.00010	.0073
2.0	.6155	2.0	7000	1.16666	.007134	.04280	.99909	.6172	95	.00006	.0071
2.5	.7570	2.5	7250	1.20833	.006888	.04133	.99915	.7576	116		= ΔT
3.0	.8926	3.0	7500	1.25000	.006658	.03994	.99921	.8936	138		↓
3.5	1.0238	3.5	7750	1.29166	.006443	.03866	.99925	1.0248	155		
4.0	1.1507	4.0	8000	1.33333	.006242	.03745	.99930	1.1516	176		
4.5	1.2738	4.5	8250	1.37500	.006053	.03632	.99934	1.2752	195		
5.0	1.3932	5.0	8500	1.41666	.005875	.03525	.99938	1.3944	214		
5.5	1.5092	5.5	8750	1.45833	.005707	.03424	.99941	1.5104	231		
6.0	1.6219	6.0	9000	1.50000	.005548	.03329	.99945	1.6232	252		
6.5	1.7314	6.5	9250	1.54166	.005398	.03239	.99948	1.7328	270		
7.0	1.8381	7.0	9500	1.58333	.005256	.03154	.99951	1.8396	289		
7.5	1.9420	7.5	9750	1.62500	.005122	.03073	.99952	1.9436	301		
8.0	2.0432	8.0	10000	1.66666	.004994	.02996	.99955	2.0488	320		
8.5	2.1420	8.5	10250	1.70833	.004872	.02923	.99957	2.1436	339		
9.0	2.2385	9.0	10500	1.75000	.004756	.02854	.99959	2.2400	353		
9.5	2.3324	9.5	10750	1.79166	.004645	.02787	.99961	2.3340	370		
10.0	2.4245	10.0	11000	1.83333	.004540	.02724	.99963	2.4264	388		
10.5	2.5134	10.5	11250	1.87500	.004439	.02663	.99964	2.5160	401		
11.0	2.6022	11.0	11500	1.91666	.004342	.02605	.99966	2.6040	419		
11.5	2.6883	11.5	11750	1.95833	.004250	.02550	.99967	2.6900	433		
12.0	2.7726	12.0	12000	2.00000	.004161	.02497	.99969	2.7744	452		
12.5	2.8550	12.5	12250	2.04166	.004076	.02446	.99970	2.8568	466		
13.0	2.9358	13.0	12500	2.08333	.003995	.02397	.99972	2.9376	486		
13.5	3.0151	13.5	12750	2.12500	.003917	.02350	.99972	3.0108	495		
14.0	3.0926	14.0	13000	2.16666	.003841	.02305	.99974	3.0948	515		
14.5	3.1689	14.5	13250	2.20833	.003769	.02261	.99974	3.1708	525		
15.0	3.2437	15.0	13500	2.25000	.003699	.02219	.99976	3.2456	546		

TABLE 17 (Continued)

$V = 6000 + 0.5h$			$\tan\theta = 0.1000$, $\sin\theta = 0.09950$				$\cos\theta = .99504$			$\tan\theta = 0.150$, $\sin\theta = 0.14834$				$\cos\theta = 0.98894$		
$h \times 10^{-3}$ ft.	V ft/sec.	V/V_0	$\Delta T = \frac{1000 \sin\theta}{V} = \frac{99.50}{V}$	$\sin\alpha_0 = 6\Delta T$	$\cos\alpha_0$	$T = 4 \log \frac{V}{V_0} \frac{1+\cos\alpha_0}{1+\cos\theta}$	$X = h \frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0}$	$\Delta T(1-\gamma)$	$\Delta T_{corr.}$	$\Delta T = \frac{1000 \sin\theta}{V} = \frac{148.34}{V}$	$\sin\alpha_0 = 6\Delta T$	$\cos\alpha_0$	$T = 4 \log \frac{V}{V_0} \frac{1+\cos\alpha_0}{1+\cos\theta}$	$X = h \frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0}$	$\Delta T(1-\gamma)$	$\Delta T_{corr.}$
0.5	6250	1.04166	.015920	.09552	.99542	1640	−9500	.00190	.0140	.023734	.14240	.98981	.1652	73	.00275	.0210
1.0	6500	1.08333	.015307	.09184	.99578	3216	+97	.00048	.0148	.022822	.13693	.99058	.3236	144	.00069	.0221
2.0	7000	1.16666	.014214	.08477	.99640	.6192	185	.00011	.0142	.021191	.12715	.99188	.6224	277	.00017	.0210
3.0	7500	1.25000	.013266	.07960	.99683	.8960	270		$= \Delta T \downarrow$	.019779	.11867	.99293	.9004	403	.00007	.0197
4.0	8000	1.33333	.012437	.07462	.99721	1.1548	349			.018543	.11126	.99379	1.1604	523		$= \Delta T \downarrow$
5.0	8500	1.41666	.011705	.07023	.99752	1.3980	424			.017452	.10471	.99450	1.4044	637		
6.0	9000	1.50000	.011055	.06633	.99780	1.6272	499			.016482	.09889	.995095	1.6344	747		
7.0	9500	1.58333	.010473	.06284	.99803	1.8436	571			.015615	.09369	.995605	1.8516	854		
8.0	10000	1.66666	.009950	.05970	.99822	2.0496	639			.014834	.08900	.99603	2.0576	956		
9.0	10500	1.75000	.009476	.05686	.99838	2.2452	705			.014128	.08477	.99640	2.2536	1056		
10.0	11000	1.83333	.009045	.05427	.99853	2.4316	772			.013485	.08091	.99672	2.4400	1154		
11.0	11500	1.91666	.008652	.05191	.99865	2.6096	834			.012899	.07739	.99700	2.6184	1250		
12.0	12000	2.00000	.008292	.04975	.99876	2.7800	897			.012362	.07417	.99725	2.7892	1344		
13.0	12500	2.08333	.007960	.04776	.99886	2.9436	959			.011867	.07120	.99746	2.9528	1436		
14.0	13000	2.16666	.007654	.04592	.99894	3.1004	1019			.011411	.06847	.99765	3.1104	1527		
15.0	13500	2.25000	.007370	.04422	.99902	3.2524	1060			.010988	.06593	.99782	3.2616	1616		

TABLE 17 (Continued)

$\tan\theta = 0.20$ $\sin\theta = 0.19612$ $\cos\theta = 0.98058$

$h \times 10^{-3}$ ft.	V ft./sec.	V/V_0	$\Delta T = \frac{1000 \sin\theta}{V} = \frac{196.12}{V}$	$\sin\alpha_0 = 6\Delta T$	$\cos\alpha_0$	$T = 4 \log \frac{V}{V_0} \frac{1+\cos\alpha_0}{1+\cos\theta}$	$X = h \frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0}$	$\Delta T(1-\gamma)$	$\Delta T_{corr.}$
0.5	6250	1.04166	.031379	.18827	.98211	1664	97	.00353	.0279
1.0	6500	1.08333	.030172	.18103	.98348	3260	192	.00088	.0293
2.0	7000	1.16666	.028017	.16810	.98577	6272	370	.00020	.0278
3.0	7500	1.25000	.026149	.15689	.98762	9068	538	.00009	.0261
4.0	8000	1.33333	.024515	.14709	.98912	11648	697		= ΔT
5.0	8500	1.41666	.023073	.13844	.99037	14128	849		↓
6.0	9000	1.50000	.021791	.13075	.99142	16436	995		
7.0	9500	1.58333	.020644	.12386	.99230	18616	1135		
8.0	10000	1.66666	.019612	.11767	.99305	20684	1272		
9.0	10500	1.75000	.018678	.11207	.99370	22648	1405		
10.0	11000	1.83333	.017829	.10697	.99426	24520	1534		
11.0	11500	1.91666	.017054	.10232	.99475	26308	1662		
12.0	12000	2.00000	.016343	.09806	.99518	28020	1797		
13.0	12500	2.08333	.015690	.09414	.99556	29660	1910		
14.0	13000	2.16666	.015086	.09052	.99589	31236	2030		
15.0	13500	2.25000	.014520	.08712	.99619	32752	2148		

$\tan\theta = 0.250$ $\sin\theta = 0.24254$ $\cos\theta = 0.97014$

$h \times 10^{-3}$ ft.	$\Delta T = \frac{1000 \sin\theta}{V} = \frac{242.54}{V}$	$\sin\alpha_0 = 6\Delta T$	$\cos\alpha_0$	$T = 4 \log \frac{V}{V_0} \frac{1+\cos\alpha_0}{1+\cos\theta}$	$X = h \frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0}$	$\Delta T(1-\gamma)$	$\Delta T_{corr.}$
0.5	.038806	.23284	.97252	1680	123	.00420	.0346
1.0	.037314	.22388	.97462	3292	240	.00102	.0363
2.0	.034649	.20789	.97815	6328	462	.00025	.0344
3.0	.032339	.19403	.98100	9144	672	.00011	.0322
4.0	.030318	.18191	.98332	11772	870	.00006	.0303
5.0	.028534	.17120	.98524	14236	1058		= ΔT ↓
6.0	.026949	.16169	.98684	16556	1239		
7.0	.025531	.15319	.98819	18748	1414		
8.0	.024254	.14552	.98936	20820	1585		
9.0	.023099	.13859	.99035	22792	1750		
10.0	.022049	.13229	.99122	24672	1912		
11.0	.021090	.12654	.99196	26464	2069		
12.0	.020212	.12127	.99262	28180	2224		
13.0	.019403	.11642	.99320	29824	2377		
14.0	.018657	.11194	.99372	31404	2528		
15.0	.017966	.10780	.99417	32920	2675		

$\tan\theta = 0.300$ $\sin\theta = 0.28735$ $\cos\theta = 0.95782$

$h \times 10^{-3}$ ft.	V ft./sec.	V/V_0	$\Delta T = \frac{1000 \sin\theta}{V} = \frac{287.35}{V}$	$\sin\alpha_0 = 6\Delta T$	$\cos\alpha_0$	$T = 4 \log \frac{V}{V_0} \frac{1+\cos\alpha_0}{1+\cos\theta}$	$X = h \frac{\cos\alpha_0 - \cos\theta}{\sin\theta - \sin\alpha_0}$	$\Delta T(1-\gamma)$	$\Delta T_{corr.}$
0.5	6250	1.04166	.045976	.27586	.96120	1700	147	.00415	.0411
1.0	6500	1.08333	.044208	.26525	.96418	3332	288	.00109	.0431
2.0	7000	1.16666	.041050	.24630	.96919	6396	554	.00028	.0408
3.0	7500	1.25000	.038313	.22988	.97322	9240	804	.00011	.0382
4.0	8000	1.33333	.035919	.21551	.97650	11888	1040	.00007	.0358
5.0	8500	1.41666	.033806	.20284	.97921	14368	1265		= ΔT ↓
6.0	9000	1.50000	.031928	.19157	.98148	16700	1482		
7.0	9500	1.58333	.030247	.18148	.98339	18900	1691		
8.0	10000	1.66666	.028735	.17241	.98503	20984	1894		
9.0	10500	1.75000	.027367	.16420	.98643	22964	2091		
10.0	11000	1.83333	.026123	.15674	.98764	24848	2283		
11.0	11500	1.91666	.024987	.14992	.98870	26648	2471		
12.0	12000	2.00000	.023946	.14368	.98962	28372	2656		
13.0	12500	2.08333	.022988	.13793	.99044	30020	2838		
14.0	13000	2.16666	.022104	.13262	.99117	31604	3018		
15.0	13500	2.25000	.021285	.12771	.99181	33124	3194		

Approximate Computation Methods

The phenomenon of refraction requires that rays must in general suffer deviation from a straight line when the medium is variable in velocity, the degree of deviation being governed by Snell's law. However, a rigorous computation method which accounts for curvature of ray path becomes extremely laborious except when certain velocity functions such as the linear equation $V = V_0 + ah$ is used. Consequently, various approximation methods are employed wherein the actual curved ray path is replaced by a straight ray path.

The *average velocity approximation* method assumes that the velocity of the medium above any bed is constant and equal to the true average velocity to that bed. The ray paths for this approximation are therefore straight. The main and also serious inaccuracy in this method lies in the error in dip computation. The true dip equation is:

$$\sin\theta = \frac{1}{\gamma}\,\frac{\Delta T}{s}\,V_h \tag{30}$$

For a given observed ΔT, the average velocity approximation method substitutes in this equation an average velocity for the actual velocity at the bed, an approximation which may be too extreme in some cases.

To avoid this error another type of approximation, the *modified straight path approximation,* is sometimes used. This method adopts Equation 30 and still uses straight ray paths. The computed dip, at least, is correct. The first fundamental ray equation may be written in the form:

$$T = \frac{dh}{V\cos\alpha}$$

For small dips, $\cos\alpha$ is nearly unity and varies only slowly along a given ray. Hence, the approximation is made of imparting to $\cos\alpha$ its final value: namely, $\cos\theta$ at depth h. The ray equation becomes

$$T = \frac{2}{\cos\theta}\int_0^h \frac{dh}{V}.$$

If one now makes use of the relation (compare page 484)

$$\overline{V} = \frac{h}{\int_0^h \frac{dh}{V}}$$

the expression for the reflection time T becomes:

$$T = \frac{2}{\cos\theta}\,\frac{h}{\overline{V}} \text{ (true for } \theta \text{ small)} \tag{39}$$

where $\overline{V}$ is the average vertical velocity to the depth h. The value of the depth obtained by this method is not as good as that obtained by the average velocity approximation method, because the final value of α, namely θ, is used rather than an intermediate value of α.

For the modified straight path approximation the horizontal displacement equation for straight rays is

$$X = h \tan \theta \tag{40}$$

This again is not as accurate a value as that obtained from the average velocity approximation, because in the fundamental equation

$$X = \int_0^h \tan \alpha \, dh$$

$\tan \alpha$ is given its final value rather than an intermediate one.

The simplicity of the straight path equations permits convenient manipulation of results and ready evaluation of computation charts, and the charts thus obtained usually are sufficiently accurate for regions of relatively gentle relief. Where the dip is important the modified straight path approximation may be used, and where location of reflection point is important the average velocity approximation may be used.

As might be expected, these two methods of approximation are not the only ones in use.

Time Corrections for Low Velocity Layer

For the sake of simplicity, the treatment of reflection data has thus far neglected the low velocity or so-called "weathered" unconsolidated zone, which generally appears as the upper layer in subsurface sections.* Actually, it is customary to refer all computations of depth and reflection time to the base of the low velocity layer. This is done because the low velocity zone plays no part in the consolidated subsection which is of paramount interest in reflection prospecting.**

For a shot at depth H, the time to be deducted from the observed reflection time to eliminate the effect of the low velocity layer and to refer the time to the base of the low velocity layer is:

$$t_u = \frac{U_0 - H}{[illegible]} \qquad \text{for } H < U_0$$

$$t_u = \frac{U_s}{V_{u,s}} \quad \frac{H - U_0}{V_c} \qquad \text{for } H > U_0 \tag{41}$$

where H is the depth of the shot

U_0 is the thickness of the low velocity layer at the shot-point

U_s is the thickness of the low velocity layer at the seismometer

$V_{u,0}$ is the velocity of the low velocity layer at the shot-point

$V_{u,s}$ is the velocity of the low velocity layer at the seismometer

V_c is the velocity of the consolidated section immediately below the low velocity layer

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* See p. 469 for description of low velocity zone.

** To avoid a reference base for computations which base is rendered erratic by variation in the lower surface of the low velocity zone, the method of referring computations to a plane is sometimes used. The reference plane is drawn through an average depth of the low velocity zone along a prospect. Reflection times and elevations are then computed with respect to this plane.

To obtain a mean *datum point* between shot-point and seismometer, the elevation of the base of the low velocity layer to be used is:

$$\cdot E_s \tag{42}$$

where E_0 = elevation of shot-point above mean datum
E_s = elevation of seismometer above mean datum

The problem of determining the *travel-time* in the low velocity layer is one for which the refraction method is well adapted and for which the reflection method is inapplicable. A method essentially the same as that already described for a two-layer section, page 466 is used. However, the *velocity* in the low velocity layer cannot be determined accurately from the slope of the direct wave travel-time curve for short distances, because the velocity in the low velocity layer is not constant in the vertical direction.

The most accurate direct determination of the average velocity of the low velocity zone is obtained by so-called *up-hole shooting*: several light shots are discharged at varying depths in a shot-hole and the first-break times are recorded with the aid of a single seismometer placed at the shot-point.* The depth versus first-break time curve will indicate the thickness of the low velocity zone and the travel-time in this zone. (Figure 274.) (The thickness of the low velocity zone may also be obtained by electrical resistivity measurements. The electrical method is especially advantageous in that the thickness can be determined accurately at several points with relative rapidity.)

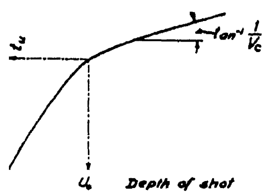

FIG. 274.—Low velocity layer diagram from up-hole shooting. t_u = vertical travel-time through low velocity layer; U_0 = thickness of low velocity layer.

The use of this type of curve would dispose of the low velocity layer problem were it not for the fact that the low velocity layer is seldom uniform, even over the length of a spread. A difference of several milliseconds in the vertical travel-time in the low velocity zone is sometimes present between seismometers separated by only a few feet. It is impractical to drill holes below every seismometer in order to secure complete low velocity zone data and therefore some approximation must be made. As stated in the last paragraph, the vertical travel-time but not the velocity in the low velocity layer can be obtained accurately by the refraction method. However, because the depth U_0 of the low velocity layer and the average vertical velocity V_u in this layer may both vary from point to point along the surface, an estimate of the variation of each must

* *Up-hole shooting* or *coming up the hole* at a new location is frequently employed to determine the best depth for shooting.

depend on trend of data obtained from scattered up-hole shooting in a given region.

Figure 275 shows the ray paths for reflected waves and the paths for first-break or first arrival-times. The shot usually is placed below the low

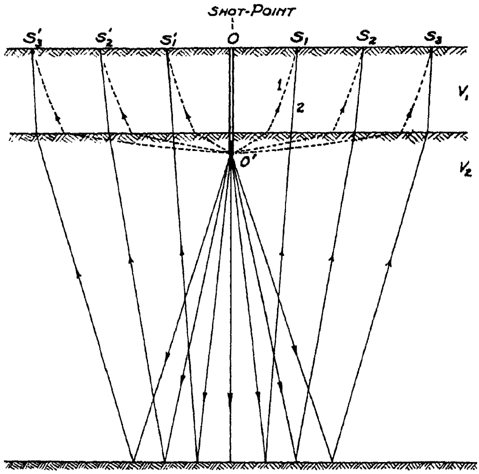

FIG. 275.—Ray paths for reflected and refracted rays. 1, first arrival ray paths; 2, reflected ray paths.

velocity layer. For the present investigation, it will be assumed that the shot is placed at a short distance below the base of the low velocity zone and that the ray paths for first arrival waves are essentially horizontal in that part of the paths which is in the consolidated zone, i.e., below the low velocity layer. Figure 276 shows the corresponding low velocity zone refraction diagram, where first arrival-times are plotted versus distance from shot-point. The slope of the best-fitting straight line through these points is the reciprocal of the velocity V_c in the top of the consolidated section.

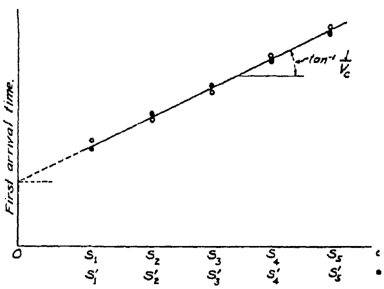

FIG. 276.—Low velocity layer refraction diagram.

Treating the velocity V_u in the low velocity layer as though it were constant in the vertical direction, the angle of the first arrival ray at the surface is given approximately by the critical angle equation

$$\sin \theta_k = \qquad V_c$$

where $V_{u,k}$ is the average vertical velocity between the base of the low

velocity layer and the kth seismometer. The first arrival-time to the kth seismometer is derived in the same manner as was Equation 19. That is,

$$- \frac{\overline{0S}_k - U_k \tan \theta_k}{\quad} \;.+$$

where $\overline{0S}_k$ is the horizontal distance from the shot-point to the kth seismometer. But from the critical angle relation,

$$\cos \theta_k = \sqrt{1 -} \qquad \text{and} \;\; \tan \theta_k = \; V_c \cos$$

$$V_c \qquad V_{u,k} \tag{43}$$

Referring to the approximately vertical direction assumed by those rays in the low velocity layer which are of importance, it is desired to determine the vertical travel-time $t_{u,k}$ through the low velocity layer, where

$$t_{u,k} \equiv \frac{U_k}{V_{u,k}} \tag{44}$$

On solving Equation 43 for U_k, one obtains

$$U_k = \frac{\left(t_k - \dfrac{\overline{0S}_k}{V_c} \right) V_{u,k}}{\sqrt{1 - \left(\dfrac{V_{u,k}}{V_c} \right)^2}}$$

Hence

$$t_{u,k} = \frac{t_k - \dfrac{\overline{0S}_k}{V_c}}{\sqrt{1 - \left(\dfrac{V_{u,k}}{V_c} \right)^2}} \tag{45}$$

Because, in general, the effect of $V_{u,k}$ in this equation is only about 5% or less in the practical case, it is sufficient to assume that $V_{u,k}$ may be replaced by the V_u found from up-hole shooting at the shot-point.

It is seldom felt justified to expend the effort in up-hole shooting at every shot-point in a prospect for low velocity layer data alone. Instead, a seismometer is left at the shot-point in routine set-ups and is used to record the shot-hole time. (See fifth trace of Figure 277.) This *shot-point seismometer* acts as a rough gauge of the change in V_u from point to point and is used in conjunction with the low velocity layer refraction diagram.

The correction for variation in velocity or thickness of low velocity zone is particularly important when the difference in reflection times due to dip of reflecting strata is to be determined. This time difference ΔT is itself a comparatively small quantity; hence, it frequently is of the

same order of magnitude as the variations in the quantity t_u corresponding to the various seismometers of the same spread. The difference in travel-times through the low velocity zone to any two seismometers (Δt_u) is obtained by substracting two terms of the form given by Equation 45. To avoid introducing the velocity V_c in the evaluation of Δt_u, the distances from the two seismometers to the shot-point are made equal; also,

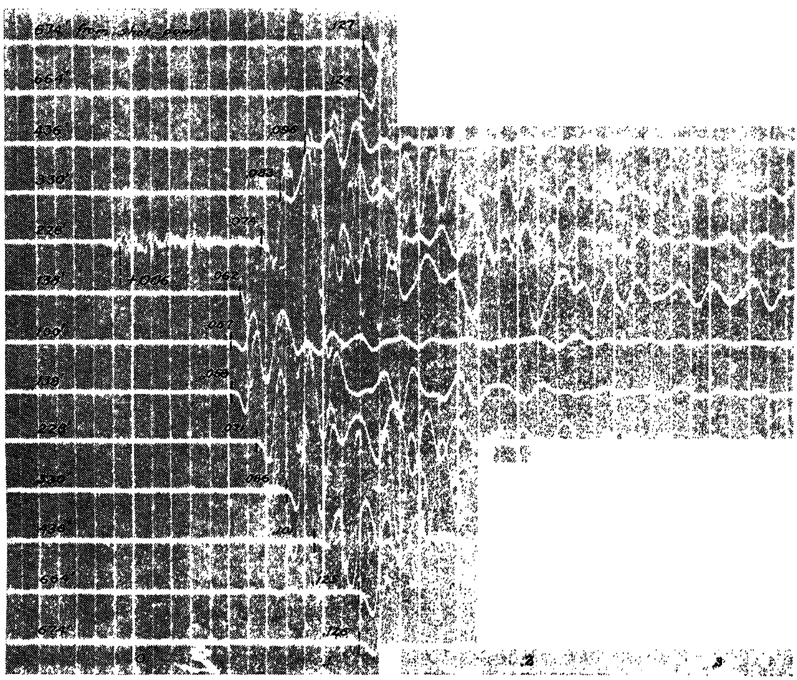

FIG. 277.—Low velocity layer refraction record. (Courtesy of Western Geophysical Co.)

the radical term can easily be reduced to unity within the general degree of accuracy.* Hence, the differential low velocity layer correction is

where t_k and t_{-k} are the first arrival-times at the seismometer stations "k" and "$-k$" respectively. The first arrival-times of a record are thus used directly to obtain the correction which must be applied to the dip time difference ΔT to compensate for variations in the low velocity layer.

* Setting the radical term equal to unity is equivalent to assuming that the critical angle is 90° or that the waves travel through the low velocity zone in a vertical direction.

An example of a low velocity zone refraction record is shown in Figure 277. This record illustrates a type which is especially designed for an accurate determination of the ΔT correction for extreme points of the spread. At each end of the spread two seismometers separated by only a few feet are used to obtain the first arrival-times. This form is sometimes used when more than one seismometer feeds the end traces of a dip record* and the condition of the low velocity layer at each seismometer is to be determined. The record shows that the magnitude of the velocity V_c is approximately 8200 ft./sec. (Compare Figure 278.)

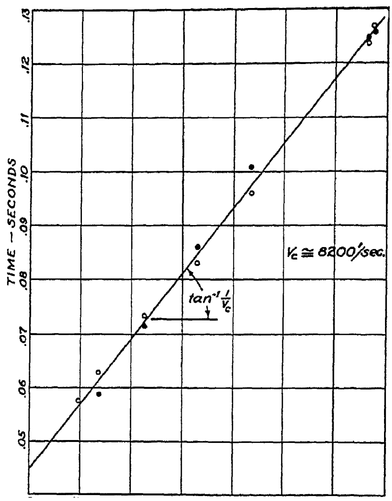

FIG. 278.—Refraction diagram corresponding to Figure 277. Open circles correspond to arrival-times at seismometers located on one side of the shot-point and solid circles correspond to arrival-times at seismometers located on the other side of the shot-point. The irregularity of the low velocity layer is indicated by the scattered character of the points.

It will be recalled that relations between dip time difference ΔT and the magnitude of dip were obtained on the basis of seismometer stations at the same elevation. Consequently, when the base of the low velocity layer is at different elevations at the seismometer stations, one must apply an elevation correction, ΔT_{El}, to the dip time difference ΔT. This correction, ΔT_{El}, is equal to the difference of elevation divided by the velocity V_c in the consolidated zone. The problem again arises as to variation of thickness H of the low velocity layer over the length of the spread and may be handled by scattered up-hole shooting, as already mentioned.

The treatment of the low velocity layer has presumed a constant value for the velocity V_c in the consolidated zone, the assumption being based precisely upon the consolidated nature of the lower zone. Although generally uniform, V_c does vary over the length of a spread in some regions. A test of the constancy of this velocity may be attained by employing so-called checking shot-points placed on a line normal to and passing through the center of the spread, as will be evident from the following considerations: The difference Δt_u of first arrival-times to symmetrically located seismometer stations depends on the low velocity layer conditions alone only if the velocity V_c in the consolidated zone is constant; hence, a test of the constancy of V_c consists in observing how constant the difference Δt_u remains for successive shot-points placed in the prescribed manner, preferably on opposite sides of the first shot-point.

* See section on Multiple Detectors.

The pattern or "line-up" of a reflection will be modified by the low velocity layer from that which would be expected from purely geometric considerations of reflected rays. The "line-up" in fact is largely determined by the low velocity zone in those cases where split spreads are used for mapping of deep beds.

Furthermore, the reflection pattern must be examined not only with a view to identifying reflections but also with a view to reading reflection time differences accurately on the records. This is the case because the evaluation of the reflection time difference is usually based on the trend of several traces adjacent the end traces, as well as the end traces themselves.

Cross Sections and Maps

The data on dip and depth of strata are generally shown on cross sections or profiles which depict strata information gained from spreads and shot-points oriented in a straight line.

One form of section is shown in Figure 279 which shows the computed reflecting beds picked up by line spreads. Rays emanating from the shot-point are indicated only at the beds in order to avoid confusion. Plotted depths are referred to sea level. The thickness of the low velocity layer is indicated.

The cross section represents a true vertical section only when the shot-point line is in the direction of maximum dip of the reflecting beds. (Compare p. 488.) However, because the approximation generally is adequate for gentle dips, the cross section is used as a vertical section for structures of gentle and gradual relief, even though the shot-point may be oriented obliquely with respect to direction of maximum dip.

To obtain a picture of the degree of relief corresponding to a horizon at a particular depth, a starting point is chosen on a section and a *phantom* or *traverse* from this point is drawn paralleling dips. The phantom is extended into other sections at intersecting or *tie-points* until the region is covered. (When warranted by the accuracy of the survey the data on cross dips may be used in phantom running, particularly at tie-points.) After a phantom is carried around any complete loop, it will be generally found that a different datum from the original is obtained on the closing tie of the loop. Such a condition is, of course, brought about by inaccuracy of the data used in drawing the closed traverse.* Evidently the *error of closure* of a series of loops must be eliminated before mapping is possible.

The common methods of adjustment for closure are the linear correction, which is based on a constant depth adjustment per unit length, and the correction proportional to dip. Sometimes the method of least squares is used. A prospect may be restudied when errors of closure

* An exception appears when a fault of varying displacement, of the scissor type, crosses the traverse.

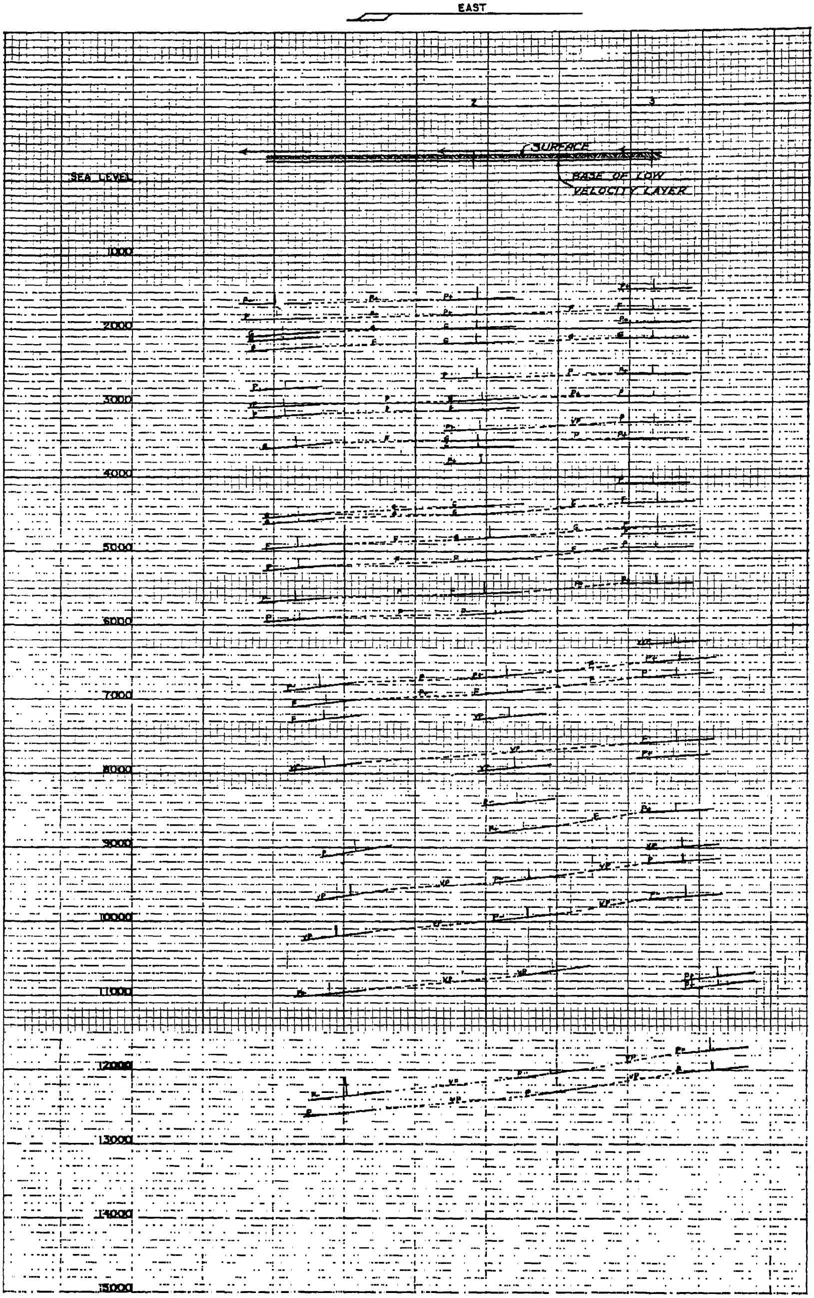

FIG. 279.—Dip section obtained from the records shown in Figure 280.

are determined, and often in any case, closure adjustments are weighed on the basis of weakest links in the traverse.

After the adjusted traverse data are recorded on a base map, a plot is made of the direction and magnitude of maximum dip obtained from those shot-points where cross dip was determined, and contours are then drawn.

The completed map in some cases requires a correction for a lateral velocity variation because routine computation generally is based on a velocity function which varies only with depth. The customary correction method for velocity error multiplies the depth h below the low velocity base by the ratio $\frac{\overline{V}_t}{\overline{V}_h}$, where $\overline{V}_t$ is the actual average velocity in the vertical direction and $\overline{V}_h$ is the originally employed average velocity* in the vertical direction for a time T equal to $\frac{2h}{\overline{V}_h}$. The correction, although customary, is approximate because the dip of the structure is not considered in the correction of data.

Illustration of Dip Shooting

Some of the individual operations in treating dip records will be presented for one of the many methods which are in use.

The three field records, Figure 280, were secured in the San Joaquin valley, California. The shot-points were situated in a straight line and separated by approximately one-half mile. A variation of split spreads, each spread having a total length of 1,000 feet and symmetrically oriented in the direction of the shot-point line, was used. The low velocity layer data were derived from other records so that no effort was made to obtain sharp first-arrival breaks on the records shown. The low velocity layer data for the first record are summarized in Table 18.

TABLE 18

LOW VELOCITY ZONE CORRECTION FOR RECORD 1 OF FIGURE 280

Mean elevation	$E_m =$	342
Mean thickness of low velocity zone	$U_m =$	80
Mean elevation of base of low velocity zone	$E_u = E_m - U_m =$	262
Depth of shot	$H =$	108
Thickness of low velocity zone at shot-point	$U_0 =$	80
Thickness of low velocity zone at seismometers used in picking reflection time	$U_k =$	80
Average velocity of low velocity zone	$V_{u,k} =$	2100
Velocity at top of consolidated zone	$V_c =$	5860
Low velocity zone time correction	$t_u' = \frac{U_s}{V_{u,s}} - \frac{H - U_0}{V_c} =$	.033
Low velocity zone correction of ΔT	$\Delta t_u =$	$-.002$
Elevation correction of ΔT	$\Delta t_{El} =$	.000
Total ΔT correction	$\Delta t_{El} + \Delta t_u =$	$-.002$

* See p. 484 for definition of average velocity as used here.

The reflections on these records are indicated by marked patterns. The times appearing at the center of the record are the average times for the center two traces while the figures below these times are the ΔT's or "move-outs" determined by the differences between the bottom trace times and the top trace times. (A positive ΔT or "move-out" is indicative of a dip in the direction of the spread.) The figures appearing at the bottom of the record are the final computed depths below sea level and are copied from computation sheets. The adjacent letters are the grades of the reflections. (See page 476.) It will be noticed that the three records reveal good correlation and consequently, when possible, the reflections were picked at corresponding phase points throughout the three records.

The computation sheet, Table 19, was completed using the charts of Figures 272 and 273. The true velocity was reasonably well approximated by the linear equation used in filling out the charts. A correction for error in velocity would be applied to the map.

The computed data are shown sectioned in Figure 279. Correlations between reflections are indicated by dotted lines and the reliability is designated by grades which appear on these dotted lines. If the correlations had not been sufficiently reliable, a phantom paralleling the dips would have been run through the section.

Curvature of the Reflecting Horizon

Ray path and dip equations have been treated thus far on the basis of a plane reflecting horizon. The effect of curvature in the reflecting horizon, however, can be significant.*

It will be recalled that the effect of spread length, i.e., the spread length correction, was evaluated by using curved wave fronts. (Compare page 481.) Furthermore, the spread was assumed sufficiently short that the wave front striking the spread approximated an arc of a circle of radius ρ. The same method of attack will be used here. Specifically, it will be assumed that the reflected rays impinging on the end seismometers of the spread originate in a portion of the reflecting horizon which is sufficiently small that it may be regarded as approximately spherical in shape.

It is a well-known law of geometrical optics that when a wave is reflected from a curved surface, the radii of curvature, ρ_i and ρ_r, of the incident and reflected wave fronts are related to the radius of curvature of the reflecting surface, ρ_s, by the equation

$$\frac{1}{\rho_i} \quad \frac{1}{\rho_r} \quad \frac{2}{\rho_s}$$

In using the equation, convex upward curvature will be reckoned positive and convex downward curvature negative. Let the path length from

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* For example, the reflecting bed could be so severely distorted that for a given shot-point the spread would be actuated by several reflecting portions scattered over the bed, a condition of multiple reflections.

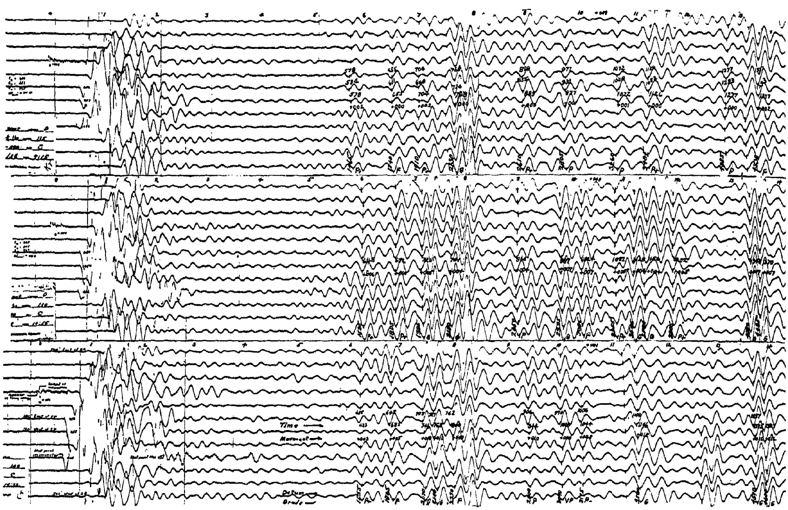


FIG. 280.—Records from three collinea

rtesy of Western Geophysical Co.)

TABLE 19
COMPUTATION FORM

Shot-point No. 1 Spread 500-500 Direction West Elev. 342

T = recorded reflection time; ΔT = recorded moveout; T − t_u' = reflection time corrected to base of low velocity zone; ΔT − Δt_u + ΔT_{el} = moveout corrected for low velocity zone and elevation; Tan θ = tangent of dip angle; h = depth of reflecting point below base of low velocity zone; Datum = depth of reflecting point with respect to sea level; X = horizontal displacement of reflecting point

t_u' = 033 E_u = 260 $\Delta t_u + \Delta T_{el}$ = −.002 E_m = 342

Kejl No.	T (sec)	ΔT (ms)	T − t_u'	ΔT + Δt_u + ΔT_{el}	Tan θ	h	Datum	X	θ	Grade
1	623	+003	590	+001	+.007	1915	1655	75	½° W	P+
2	695	+005	662	+003	+.021	2125	1865	90	1°	F
3	746	+010	713	+008	+.057	2330	2070	125	3½°	G
4	765	+010	732	+008	+.058	2400	2140	130	3½°	G
5	800	+010	767	+008	+.058	2530	2270	140	3½°	F
6	944	+010	911	+008	+.060	3060	2800	170	3½°	P
7	1008	+008	975	+006	+.047	3360	3040	174	2½°	VP
8	1044	+009	1011	+007	+.054	3435	3175	165	3°	P
9	1146	+014	1113	+012	+.096	3840	3580	265	5½°	G
10	1375	+012	1342	+010	+.063	4770	4510	245	4½°	G
11	1397	+012	1364	+010	+.084	4860	4600	350	5°	G
12	1475	+011	1442	+009	+.078	5195	4935	[illegible]	4½°	F
13	1541	+011	1508	+009	+.080	5480	5220	360	4½°	P
14	1640	+009	1607	+007	+.063	5920	5660	375	3½°	P−
15	1697	+010	1664	+008	+.073	6170	5910	375	4°	P
16	1899	+014	1866	+012	+.117	7085	6825	660	6½°	P−
17	1948	+015	1915	+013	+.128	7315	7055	745	7½°	F
18	1990	+014	1957	+012	+.118	7525	7265	720	6½°	F
19	2118	+013	2085	+011	+.112	8165	7905	730	6½°	VP
20	2348	+017	2315	+015	+.162	9315	9055	1115	9°	P
21	2455	+015	2422	+013	+.144	9900	9640	1100	8°	VP
22	2549	+013	2516	+011	+.125	10455	10195	990	7°	VP
23	2681	+010	2648	+008	+.093	11230	10970	800	5½°	P+
24	2715	+011	2682	+009	+.112	12615	12355	1170	6½°	P+
25	2951	+010	2915	+008	+.100	12845	12585	910	5½°	P

Shot-point No. 2 Spread 500-500 Direction West Elev. = 347

t_u' = 035 E_u = 265 $\Delta t_u + \Delta T_{el}$ = −.001 E_m = 347

Kejl No.	T	ΔT	T − t_u'	ΔT + Δt_u + ΔT_{el}	Tan θ	h	Datum	X	θ	Grade
1	611	+006	576	+005	.015	1860	1595	60	2° W	P+
2	672	+005	637	+004	[illegible]	2070	1805	55	1½°	P+
3	721	+005	686	+004	.038	2250	1985	60	1½°	G
4	781	+004	746	+003	.022	2455	2190	56	1½°	G
5	802	+004	770	+003	.022	2915	2650	60	1½°	P
6	987	+007	952	+006	.066	3220	2955	135	2½°	G
7	1024	+007	989	+006	.046	3350	3085	140	2½°	F
8	1092	+005	1057	+004	.031	3620	3355	105	2°	P+
9	1128	+004	1093	+003	.024	3770	3505	80	1½°	G
10	1150	+004	1115	+003	[illegible]	3855	3590	80	1½°	G
11	1202	+005	1167	+004	.032	4070	3805	115	2°	P+
12	1347	+007	1312	+006	.050	4655	4390	200	3°	G
13	1370	+007	1335	+006	.050	4750	4485	200	3°	G
14	1450	+008	1415	+007	.060	5075	4810	225	3½°	G
15	1517	+006	1482	+005	.044	5375	5110	195	2½°	P
16	1619	+005	1584	+004	[illegible]	5825	5560	170	2°	F
17	1680	+008	1645	+007	[illegible]	6090	5825	320	3½°	P−
18	1864	+010	1829	+009	.087	6935	6670	485	5°	P+
19	1911	+010	1876	+009	.081	7160	6895	500	5°	F
20	1981	+010	1946	+009	.084	7495	7230	530	5°	VP
21	2121	+010	2086	+009	.081	8185	7920	600	5°	VP
22	2215	+010	2180	+009	.094	8655	8390	640	5½°	P−
23	2286	+011	2251	+010	.100	9015	8750	740	6°	P+
24	2410	+011	2375	+010	.103	9685	9420	820	6°	P−
25	2503	+010	2468	+009	.100	10200	9935	795	5½°	P−
26	2636	+013	2603	+012	.117	10925	10660	1040	8°	VP
27	2869	+011	2834	+010	.112	12305	12040	1110	7°	P−
28	2912	+012	2877	+011	.137	12350	12215	1260	8°	P

Shot-point No. 3 Spread 500-500 Direction-West Elev. = 353

t_u' = 039 E_u = 265 $\Delta t_u + \Delta T_{el}$ = +.001 E_m = 353

Kejl No.	T	ΔT	T − t_u'	ΔT + Δt_u + ΔT_{el}	Tan θ	h	Datum	X	θ	Grade
1	578	+002	539	+003	.021	1730	1665	[illegible]	1° W	P+
2	655	+000	616	+001	.007	2005	1940	[illegible]	[illegible]	F
3	704	+003	665	+003	.021	2175	1910	[illegible]	1°	P+
4	765	+000	727	+001	.007	2395	2330	[illegible]	½°	G
5	899	+000	860	+001	.013	2875	2810	[illegible]	½°	P+
6	977	+001	938	+002	.015	3170	3105	[illegible]	1°	P
7	1072	+001	1033	+002	.016	3525	3460	[illegible]	1°	P
8	1126	+000	1087	+001	.008	3750	3685	[illegible]	½°	P+
9	1277	+000	1238	+001	.008	4350	4285	[illegible]	½°	P
10	1327	+002	1288	+003	.025	4545	4480	[illegible]	1½°	F
11	1418	+002	1379	+003	.025	4935	4870	[illegible]	1½°	F
12	1434	+002	[illegible]	+003	.036	5025	4960	[illegible]	1½°	F
13	1482	+001	1443	+002	.017	5210	5145	[illegible]	1°	P
14	1492	+001	1453	+002	.018	5490	5425	[illegible]	1°	P+
15	1731	+006	1692	+007	.065	6490	6425	[illegible]	3½°	VVP
16	1823	+007	1784	+008	.075	6725	6660	[illegible]	4½°	P+
17	1866	+006	1827	+007	.067	6930	6865	[illegible]	4°	P
18	2049	+005	2010	+006	.060	7615	7550	[illegible]	3½°	F
19	2080	+004	2041	+005	.050	8020	7955	[illegible]	3°	P+
20	2336	+004	2297	+005	.052	8765	8700	[illegible]	3°	P+
21	2351	+005	2312	+006	.064	9035	8970	[illegible]	3½°	VP
22	2367	+005	2328	+006	.064	9145	9080	[illegible]	3½°	P
23	2453	+005	2414	+006	.066	9720	9655	[illegible]	4°	P−
24	2449	+009	2410	+010	.115	10395	10330	[illegible]	6½°	P+
25	2570	+009	2531	+010	.116	11115	11050	[illegible]	6½°	P+
26	2520	+007	2481	+008	.096	12000	11935	[illegible]	5½°	P+
27	2561	+007	2522	+008	.097	12260	12195	[illegible]	5½°	P

the origin to the point of reflection be $l/2$, then the radius, ρ_i is $-l/2$. (As in the previous treatment, the effect of refraction on curvature of the wave front is neglected.) The reflected wave starts back with a radius of curvature

$$\rho_r = \frac{1}{2 \quad 1} = \qquad \rho_s l$$

The radius of curvature of the reflected wave when it arrives back at the origin is ρ, where $\rho = \frac{l}{2} + \rho_r$

or

$$\rho = \frac{1 + 2\,\frac{\rho_s}{\,}}{2} \tag{46}$$

The effect of curvature of the reflecting surface amounts to changing the curvature of the arriving wave front according to this equation. Because the effect of the ρ term on computation equations is small, the change due to curvature of the reflecting horizon will, in general, be even smaller, provided that the spread is not excessive and that the condition of multiple reflections does not obtain.

Variations in Dip Shooting.—Dip shooting is flexible and permits of a multitude of variations depending upon geologic section, operating conditions, and economic considerations.

Continuous Seismic Profiling

A technique which has recently gained widespread application is the method of "continuous profiling." In this method the seismometer stations are spaced uniformly along the entire length of a prospect line, being

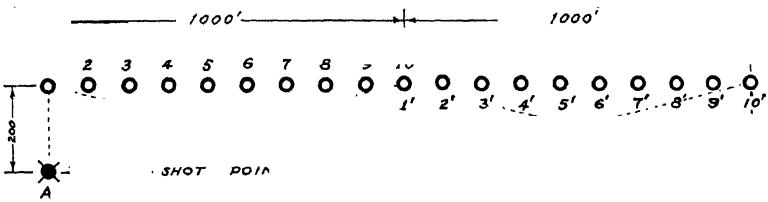


FIG. 281.—Spread for continuous profiling. (Salvatori, *Geophysics*.)

offset a constant distance from the line of shot-points. Shot-points are evenly spaced and intercept a constant number of seismometers. Figure 281 shows a specific arrangement where ten seismometers are used.*

* See "Mapping Faults by the Reflection Method," Henry Salvatori, *Geophysics* 2, No. 4, October, 1937.

Alternative arrangements are referred to on p. 514.

From shot-point A, shots are fired to give the first record which comprises traces from seismometer stations 1 through 10; for the second record, the seismometers are left in place and a shot is fired at shot-point B. The seismometers are then deposited in stations 1′ through 10′ and shot-points B and C are used in succession. Thereafter the entire procedure is repeated for the next interval which starts at C, and so on.

A section showing the paths of the reflected rays corresponding to the record for shot-point A is shown in Figure 282. Reflecting points on the

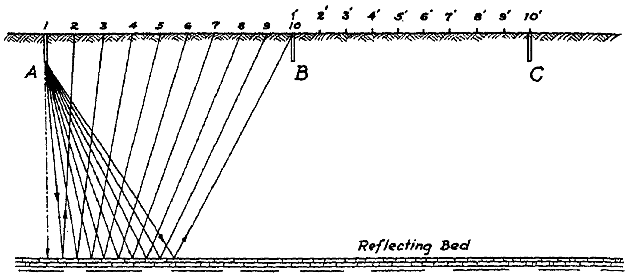


FIG. 282.—Reflected ray paths in continuous profiling; seismometers at 1 . . . 10 and 1′. . . 10′; shot-points at A, B, C.

reflecting bed are spaced evenly at a distance equal to approximately half the seismometer separation, and, from the sequence of shots and spreads, it is evident that the entire length of reflecting bed is covered to this detail. Moreover, the ray path traversed from shot-point A to seismometer 10 has virtually the same reflecting point as the ray path from shot-point B to seismometer 1. These ray paths would therefore lead to the same reflection times were it not for the difference in the thickness of the low velocity layer at the two seismometers and the difference in the depth of shots. Because these differences can be determined, a positive correlation can be effected from the first to the second record. Furthermore, when seismometers are transferred to the opposite side of shot-point B for the third record, the single seismometer opposite shot-point B is undisturbed. Consequently, the traces corresponding to this seismometer furnish another positive correlation from the second to the third records.

Instead of making a correction on one of the records for the difference in travel-times for the ray paths A-10 and B-1, use may be made of an overall correction ΔT_1 without regard to the various sources that contribute to the correction. Thus,

$$t_{A-10} = t_{B-1} + \Delta T_1$$

Next, one adds the same correction to t_{B-0} and t_{B-2}' and averages these values.

The next step is the tie-in between the two profiles, shot from B in both directions. On the two corresponding records one finds two values for t_{B-0} and t_{B-2}'. In general, the corresponding values differ by a few thousandths of a second, probably due to repeated shooting in the same hole with consequent changes in elastic properties of the rocks at the bottom of the hole and to instrumental errors. These times are

corrected by a value Δt_2 to make the average time for the two seismometers on both profiles the same. Now one adds the same correction to t_{B-10}' and repeats the whole process for the next shot-point. In this way it is possible to correlate the records with little more than an occasional check on the shot-hole time (time that it takes the wave to travel from the bottom of the shot-hole to the surface). The times are then translated into terms of depths with the aid of data obtained in well-shooting.

Other methods of continuous profiling are sometimes used. The term itself applies only to a "continuous coverage" on a reflecting bed. Several advantages are apparent for the method of continuous profiling. First, the reflection points are uniformly spaced, thereby permitting most efficient correlation as well as fault investigation. (Compare p. 516.) Second, the common tie-in from record to record eliminates errors due to differences in shot-hole depth or in the timing system, thereby allowing positive correlations. Third, by use of up-hole shooting, good corrections for the low velocity layer in dip determination may be obtained. Fourth, the long distance from the shot-point to the extreme seismometer of the accompanying spread may permit adequate velocity determination. (Compare p. 527.)

Several disadvantages of the method are often cited to offset these advantages. First, the close spacing of shot-points and seismometers greatly increases the expense of the work. Second, the method is rather inflexible, requiring continuity in seismometer line despite obstructions which may be encountered. Third, the common tie-in from record to record increases the tendency of the computer to force correlation, which may lead to error.

In continuous profiling it is theoretically impossible to differentiate between correlation data and strictly dip, or ΔT, data because the series of records placed side by side in proper sequence may be viewed simply as one record and the corresponding actual ray paths may be treated by equivalent ray paths. Reflections can be picked not only over the traces constituting a single record but over those traces of other records for which the reflection seems reliable.

The continuous profiling method outlined above is sometimes modified in such a way that symmetrical spreads are involved—an arrangement which requires double the number of shot-points for the same spread length. Here the entire length from A to C (Figure 281) is covered by the available set of seismometers and only half the seismometers are transferred for successive spread changes.* Each shot-point then need be used but once. This method of overlapping seismometers is gaining favor because the dip is more readily computed for symmetrical spreads than for uni-directional spreads.

* Referring to Figure 281, the first shot-point would be B, the second C, and so on. The disposition of the 10 seismometers supplying the traces for the first record would be symmetrical about B and would cover the distance between A and C. An odd number of seismometers and traces, however, is best adapted for this spread in order that one seismometer appear opposite the shot-point.

Continuous profiling by symmetrical spreads as described in the last paragraph is sometimes modified to achieve a compromise between quality of correlation and speed of progress in mapping a prospect. Instead of a complete overlap, shot-points are separated by a distance greater than half the spread length and the seismometer stations nearest a shot-point are used only in conjunction with that shot-point. The common tie-in between records is lost but the gap on the reflecting bed between successive spreads may be kept sufficiently short that correlation is reliable.

Fault Mapping †

The most common method of investigating faults by seismic prospecting is a *negative* one in that fault areas are generally first detected on a shot-point line when an area of poor reflections is encountered. If parallel lines also reveal similar gaps on the cross sections and if these gaps can be aligned, the fault may be delineated. An examination must be made for assurance that it is not an anomalous cause such as unfavorable surface conditions which is responsible for the poor results.

Reflections from a fault plane are helpful when available. Usually, however, reflections are not recorded from a fault plane for reasons which will be evident from the following considerations. Wave energy is reflected from a fault plane only at points where two beds of different elastic constants or density come into sharp contact. Generally, however, the displacement of the beds at the fault is relatively small, and there are many portions of the fault plane along which either the same beds or different beds of approximately similar characteristics are in contact. Hence, even when the plane of faulting is very sharp, it will rarely act as a good reflector of wave energy over an appreciable section.

A correlation showing a vertical displacement is the best kind of evidence when supported, of course, by general dip information in the region, but such evidence is not often found. Sometimes reflections indicating the fault drag are detected.

Correlations do, however, supply important negative evidence, particularly when the records have been obtained by the continuous profiling method. If correlations for several reflections cease roughly on a vertical or sloping line, as depicted on a cross section, and continue again at a certain distance beyond this line, strong evidence of a fault zone in that interval is obtained, the dip of the zone roughly indicating the hade in the plane of the cross section. This method, though strikingly effective on paper, cannot always be depended upon because of its negative character; also, correlations must be reliable and persist for several strata, and the zone of fracture must not be too extensive.*

† Henry Salvatori, "Mapping Faults by the Reflection Method," *Geophysics*, Vol. 2, No. 4, October, 1937, pp. 342-356.

* The methods of refraction shooting and continuous electrical profiling often offer a more convenient method of locating shallow faults.

Mapping of Structure by Angular Divergence or Interval Change Method

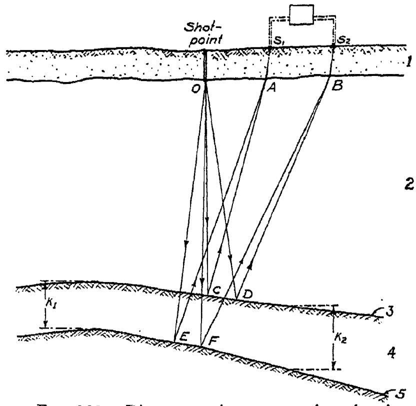


FIG. 283.—Diagrammatic cross section showing rays reflected from various reflecting horizons located above an anticlinal structure. (After McCollum, U. S. Patent 2,118,441.)

This method is based on the premise that a change of interval between two or more reflections recorded over an area is an indication of structure in that the geological section is normally thinner over the crest of a structure than in the surrounding area.†

The principles utilized in the method are best explained by referring to Figures 283, 284, and 285. Figure 283 shows a schematic geological cross section through an anticlinal structure and the ray paths of rays which originate at a shot-point 0 and are reflected from the surfaces of layers 3 and 5 to the seismometers S_1 and S_2.* The low velocity or aerated layer 1 is variable in thickness and in physical characteristics. Layer 2 consists of consolidated rock having elastic properties which are different from those of layer 1. Layer 3 is rock having elastic properties which are different from those of layer 2. Layer 4 has the same general character as layer 2, and layer 5 has the same general character as layer 3. Layer 6 (not shown) is rock which has a porosity such that petroleum may be trapped within it.

The shapes of beds 3 and 5 illustrate a geological condition usually realized: viz., the deeper beds exhibit more closure or steeper dip. Such beds are said to diverge from each other "off the structural feature." Thus, K_1 is the interval "on the structure" and K_2 is the interval at a point "off the structure."

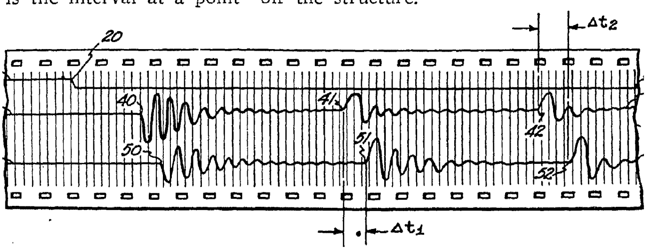


FIG. 284.—Theoretical seismogram showing traces produced by oscillographs connected with seismometers S_1 and S_2 of Figure 283. (McCollum, U. S. Patent 2,118,441.)

The times of arrival of the rays traversing the various paths are shown on the seismogram. (Figure 284.) Trace 23 is the trace produced by the oscillograph connected to seismometer S_1. Point 40 on trace 23 represents the instant of arrival at S_1 of the ray traveling along $0AS_1$; point 41 represents the instant of arrival of the ray traveling along path $0CAS_1$; point 42 represents the instant of arrival of the ray traveling along path $0EAS_1$.

† E. V. McCollum, "Method of Making Geological Explorations," U. S. Patent 2,118,441. Issued May 24, 1938.

E. V. McCollum and L. F. Athy, "Geophysical Method of Determining Geological Structures." U. S. Patent 2,118,442. Issued May 24, 1938.

E. V. McCollum and G. C. McGhee, "Method of Making Dip Determinations of Geological Strata," U. S. Patent 2,001,429. Issued May 14, 1935.

* In this analysis it will be assumed that ray paths can be approximated with sufficient accuracy by straight lines, an assumption justified only for relatively gentle dips.

Point 20 on trace 25 represents the instant of explosion.

Trace 24 is the trace of the oscillograph connected to seismometer S_2. Point 50 represents the instant of arrival of the ray traveling along the path $0BS_2$. Point 51 represents the instant of arrival of the ray traveling along the path $0DBS_2$. Point 52 represents the instant of arrival of the ray traveling along the path $0FBS_2$.

The observed time intervals used for mapping the structure are ΔT_1 which is the difference between points 41 and 51 on traces 23 and 24 and ΔT_2 which is the difference between points 42 and 52 on traces 23 and 24. It will be noted that these time intervals are independent of time of origin represented at point 20.

The *normal* value of $(\Delta T_2 - \Delta T_1)$ for a given pair of reflecting horizons may be obtained by averaging a large number of observed values of $(\Delta T_2 - \Delta T_1)$ obtained at random over the area being explored.

In all portions of an area in which there is no angular divergence or in which the geological formations are parallel, the value of $(\Delta T_2 - \Delta T_1)$ is a constant for all seismograms for which the same size spreads were used. If the quantity $(\Delta T_2 - \Delta T_1)$

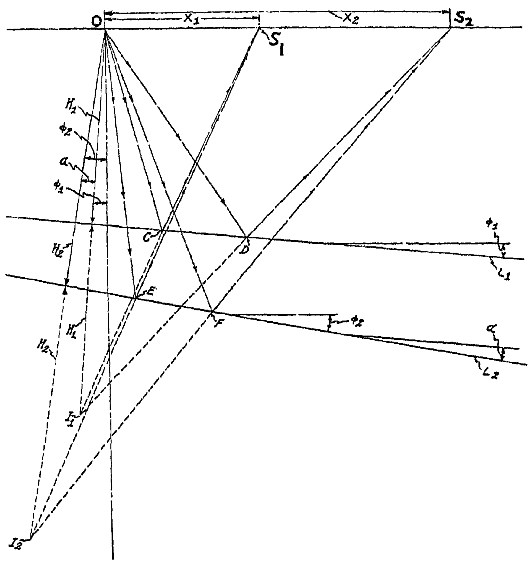


FIG. 285.—Diagrammatic sketch of ray paths corresponding to Figure 283. (McCollum, U. S. Patent 2,118,441.)

is not constant, it is indicative of angular divergence. In particular, if the observed $(\Delta T_2 - \Delta T_1)$ is greater than the normal value for the area, the direction in which the more distant seismometer is positioned from the shot-point is the direction of angular divergence from the shot-point. If the observed $(\Delta T_2 - \Delta T_1)$ is less than the normal value for the area, the direction of angular divergence is reversed.

Evidently, it is possible to plot arrows with lengths proportional to the divergence on maps and thus determine the differences in intervals.

Moreover, it is possible to evaluate the angle of divergence α between layer 1 and layer 2 in terms of measurable quantities. To calculate α, it is convenient to use the constructions shown in Figure 285. x_1 equals the distance $0S_1$ and x_2 equals the distance $0S_2$. L_1 corresponds to layer 3 and L_2 to layer 5. H_1 is the perpendicular distance from 0 to plane L_1 and H_2 is the perpendicular distance from 0 to plane L_2. ϕ_1 is the dip of layer L_1 and ϕ_2 the dip of layer L_2.

I_1 is the image of the shot-point 0 in plane L_1 and I_2 is the image of 0 in the plane L_2.

In triangle $0I_1S_1$ the angle S_10I_1 is equal to $90 + \phi_1$, the distance $0I_1 = 2H_1$, and the distance $I_1S_1 = 0CS_1 = V_1T_1$. (The velocity is assumed to vary with depth only and V_1 is defined as the "equivalent" or "average" vertical velocity to depth H_1. T_1 is the time interval between point 41 and 20 on the seismogram.) From the cosine law

$$V_1^2T_1^2 = x_1^2 + 4H_1^2 + 4H_1x_1 \sin \phi_1 \tag{47}$$

Similarly, from triangle $0I_1S_2$

$$V_1^2T_2^2 = x_2^2 + 4H_1^2 + 4H_1x_2 \sin \phi_1 \tag{48}$$

and from triangles $0I_2S_1$ and $0I_2S_2$

$$V_2^2T_3^2 = x_1^2 + 4H_2^2 + 4H_2x_1 \sin \phi_2 \tag{49}$$

$$V_2^2T_4^2 = x_2^2 + 4H_2^2 + 4H_2x_2 \sin \phi_2 \tag{50}$$

In the above equations, the quantities x_1 and x_2 are known. The quantities T_2, T_3, and T_4 are obtained from the seismogram as was T_1. (Figure 284.) The quantity V_2 is an average velocity in the material above bed 5 (L_2). Both V_1 and V_2 may be determined by methods outlined in the section on Velocity Shooting. The unknown quantities, therefore, are ϕ_1, ϕ_2, H_1 and H_2.

It is evident from Figure 285 that

$$\phi_2 - \phi_1 = \alpha \tag{51}$$

Elimination of H_1 between Equations 47 and 48 gives

$$(V_1^2T_2^2 - x_2^2 \cos^2 \quad)^{1/2} - \quad {}_2 \sin \phi_1 = (V_1^2T_1^2 - x_1^2 \cos^2 \quad \quad {}_1 \sin \phi_1$$

For the gentle dips usually treated, it is generally sufficiently accurate to replace $\cos^2 \phi_1$ by 1. On rearranging terms, the last equation becomes:

On expanding the expressions within the parentheses on the right-hand side of this equation and neglecting all terms after the second, the equation becomes

$$(x_2 - x_1) \sin \phi_1 = V_1T_2 - V_1T_1 - \quad \frac{x_2^2}{V_1T_2} + \tfrac{1}{2} \frac{x_1^2}{V_1T_1}$$

However,

Also, it can be assumed without appreciable error that V_1T_2 is equal to V_1T_1. Hence,

$$_1 = \frac{V_1\Delta T_1}{\quad} - \frac{x_2 + x_1}{2V \ T} \tag{52}$$

Similarly, elimination of H_2 between Equations 49 and 50 leads to

$$\sin \phi_2 = \frac{V_2\Delta T_2}{x_2 - x_1} - \frac{x_2 + x_1}{2V_2T_3}$$

Furthermore, for small angles $\sin \phi_1$ and $\sin \phi_2$ are approximately equal to ϕ_1 and ϕ_2 respectively. Hence,

$$\sin \phi_2 - \sin \phi_1 = \phi_2 - \phi_1 = \alpha = \quad \frac{\quad}{x_2 - x_1} \quad \frac{x_1}{\quad}\left(\frac{1}{V_1T_1} - \frac{1}{V_2T_3}\right) \tag{54}$$

Equation 54 expresses α as a function of measurable quantities and is therefore the relation sought. It is important to note that the values of ΔT_2 and ΔT_1 have opposite signs in Equation 54. Hence, errors due to the low velocity layer are substantially avoided.

Figure 286 is a map showing a structural divergence over an actual salt dome in Louisiana. The figures on the dip arrows indicate the amount of divergence in feet per mile in the direction of the arrow. The length of each arrow represents the spacing of 50-foot contours at the position of the arrow. The divergence was observed between beds located at approximately 3000 feet and 7000 feet; that is, the divergence was observed over an interval of approximately 4000 feet. It is evident from the map that the crest of the structure coincides closely with the center of convergence.

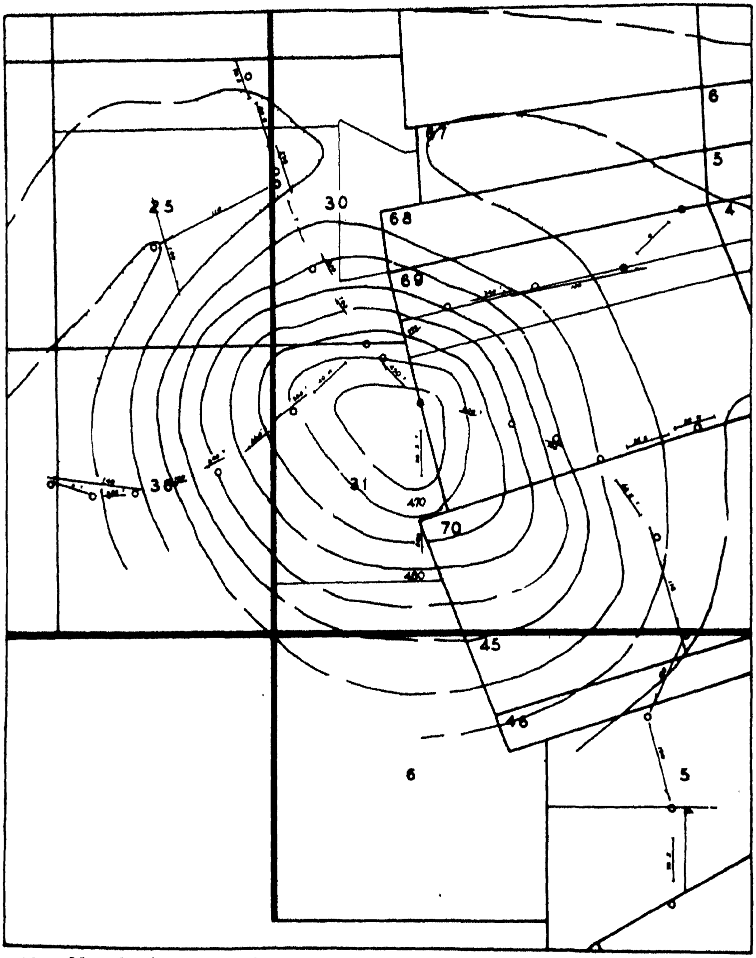


FIG. 286.—Map showing structural divergence over a salt dome. (Courtesy Continental Oil Co.)

Correlation Shooting.—Correlation shooting is distinguished from dip shooting in that in the former no effort is made to determine the difference ΔT in reflection times.* Because spreads in correlation shooting

* Though correlation may be used in conjunction with dip shooting, the term "correlation shooting" is generally used in the above sense.

are determined only from the point of view of identifying reflections, they can be short.* Furthermore, relatively few seismometers are needed, and speed of operation is high. When, however, continuous profiling is utilized, speed is sacrificed in favor of positive correlations.

Generally, in regions where correlation shooting is performed, the dips are of such small magnitude that computations of original data are based on no dip at all,** a basis which will be pursued in the following treatment. Often, the distance from shot-point to seismometer is sufficiently short that for all but the very shallow beds, the depth of the beds is computed simply by multiplying the average vertical velocity by half the reflection time (corrected for low velocity layer).

When the distance from shot-point to seismometer is to be taken into account, a computation chart based upon simple velocity functions can be developed fairly readily. The case of a linear velocity function will be presented here. † Consider the velocity function

$$V = V_0 + ah \qquad (36)$$

On returning to the fundamental equations, p. 464, and evaluating the integrals, one obtains

$$(55)$$

$$\frac{2}{a} \qquad [pV_0] - \sec h^{-1} \qquad (56)^{***}$$

where x denotes the *horizontal distance as measured along the surface* between shot-point and seismometer and T denotes, as usual, the travel-time of the reflected ray.

---

* The distance between seismometers should not be too long because changes in stratigraphy over a long profile are liable to cause poor line-ups which might introduce doubts in certain cases that a reflection is observed. However, one should not try to improve the appearance of the record by exaggerated shortening of the spread, because it is always possible to produce a certain line-up of phases by this means, irrespective of the origin of the waves recorded on the seismograms. However, rapid lateral changes in stratigraphy and steeply dipping beds make it necessary to compromise in certain regions (e.g., in certain regions of California).

** An accurate method of treating correlation data when dips are appreciable is to obtain reflection times for a particular reflecting bed and map these times at the shot-points. The reflection time map is then converted to space coordinates in the manner previously described.

† M. M. Slotnick, "On Seismic Computations, with Applications," *Geophysics*, Vol. 1, No. 1, January, 1936, pp. 9-22.

*** This expression for T differs from that given on p. 491 only in that the logarithmic function has been replaced by its equivalent hyperbolic function: viz.,

$$\log_e \left(\frac{1}{\ } \qquad = \sec h^{-1} z$$

(See, for example, B. O. Peirce, *A Short Table of Integrals*, Third Revised Edition (Ginn and Co.) p. 120.)

It is convenient to introduce three dimensionless variables defined by the relations:

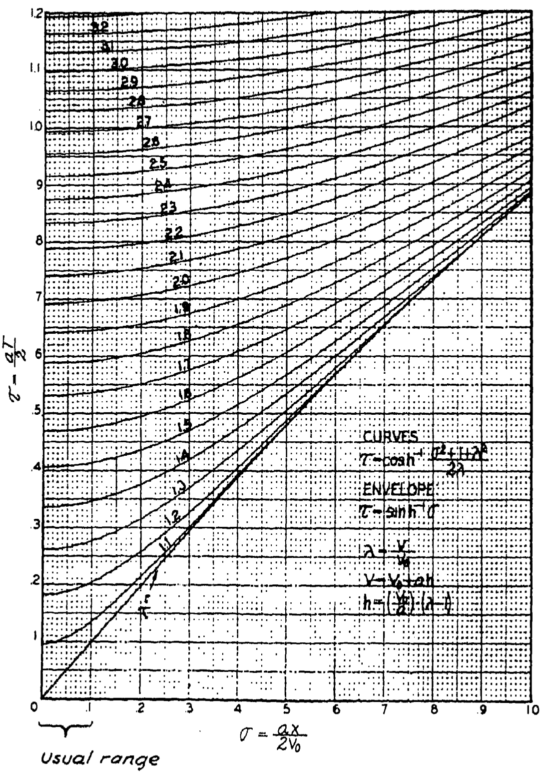
$$\frac{aT}{2} \tag{57}$$

$$\frac{ax}{2V_0} \tag{57a}$$

$$V_0 + ah \tag{57b}$$

On writing the parametric Equations 55 and 56 in terms of the new variables, τ, σ, and λ, and eliminating the parameter p, one obtains the convenient relation

$$\cdot = \cos h^{-1} \quad 2\lambda \tag{58}$$



287.— Plots of τ versus σ for various values of λ· (Slotnick, *Geophysics.*)

This equation permits the construction of a time-depth chart which is applicable for any distance x and for any linear velocity. The chart is plotted, for example, with $\tau = \frac{aT}{2}$ as ordinate and $\sigma = \frac{ax}{2V_0}$ as abscissa, while lines of constant λ are drawn across the chart. (Figure 287.)

The use of the chart (Figure 287) for depth calculations may be summarized as follows: (1) It is assumed that the values of a and V_0 are known from a refraction profile or other data. (2) τ and σ are computed by substituting a, V_0, and the observed travel-time T for a seismometer located at a distance x into Equations 57 and 57a. (3) λ is read off the curve at the point having the coordinates τ and σ computed from Equations 57 and 57a. (4) From Equation 57b

$$\lambda = \frac{V_0 + ah}{V_0}$$

or

The last equation expresses the depth h in terms of V_0, a and λ. Thus, by using the chart to evaluate λ, the depth may be computed accurately provided V_0 and a are known and provided the hypothesis of the linearity of the velocity equation is accurately satisfied in the region under investigation.*

Reflection picking for correlation shooting differs slightly from that for dip shooting in that, because correlation and not dip determination is involved, stress is placed on wave character and relative amplitude rather than consistency in ΔT. Considerable care must be taken to see that corresponding phases of a reflection are picked from record to record. A great deal of experience is required to develop the technique of correlation, as identification of phases is sometimes obscure and difficult.

The thickness of the low velocity zone and the travel-time in this zone are ascertained in essentially the same manner already described, except, of course, that there is no Δt_u correction. In the typical correlation prospects of relatively flat strata, small variations in relief are more important than in normal dip prospects; hence, corrections for the low velocity zone must be examined with particular care. This sometimes takes the form of up-hole shooting at every shot-point and the picking of reflection times on every trace, each trace time being corrected for low velocity effect, and the average net reflection time used for computation.

In correlation shooting, the computation sheet for each shot-point is generally simple or entirely missing since the dip factor is not introduced.

* A discussion of the use of charts when the velocity-depth function has an exponential form will be found in the Slotnick article referred to above.

Often the cross section is not used; instead, tabulations of reflection times and depths for each persistent reflecting stratum are maintained separately. These depths are mapped directly, the data being associated directly with the shot-point.

Methods of continuous profiling which have already been treated under dip shooting may in part be used with correlation shooting and their discussion will not be repeated at this time.

Illustrations of Correlation Shooting

Two examples of field correlation records are shown. These examples are not typical in that they constitute better records than obtained in a normal run, and they are used only to illustrate the method of correlation.

Figure 288 shows three consecutive records secured in the general vicinity of Greenville, Texas, the shot-points being separated by about 1,200 feet for relatively close control. An asymmetric split spread was used, extending 150 feet in one direction and 285 feet in the other. Every trace of the records is picked for reflection times and these are averaged for each reflection to determine the depth of the reflecting horizon. The travel-times in the low velocity zone are obtained from separate records (not shown) and are also averaged, and a slight correction term is applied to compensate for effect on reflection time of distance between shot-point and seismometer. Computed depths with respect to sea level are marked at the bottom of the records. Correlation grades are also indicated at the bottom. The horizon label letters are shown at the top of the records.

It will be noted that not all reflections are picked. This procedure is customary in correlation shooting where only those beds which persist are of value, particularly when a sufficient number of such beds are present. The characteristics of each reflection are carefully studied, particularly the sequence of phases between neighboring reflections. The presence of these phases in succeeding records is the type of correlation evidence sought.

The second set of correlation records, Figure 289, was secured in the vicinity of St. John, Kansas. A symmetric split spread of total length equal to 270 feet was used. The low velocity zone correction was determined only for the two nearest seismometer stations and, correspondingly, reflection time was picked only on the two center traces. Depths below sea level and correlation grades are given at the bottom of the record and the horizon label at the top.

These records show the possible quality of correlation carried across long distances as the two shot-points were separated by over three miles. The correlating bed labeled A stands isolated and yet can be correlated with certainty due to persistent and distinctive character. This reflection is from the anhydrite layer. Reflections labeled T_1 and T_2 represent the Topeka strata, which again reveal excellent persistency in character.

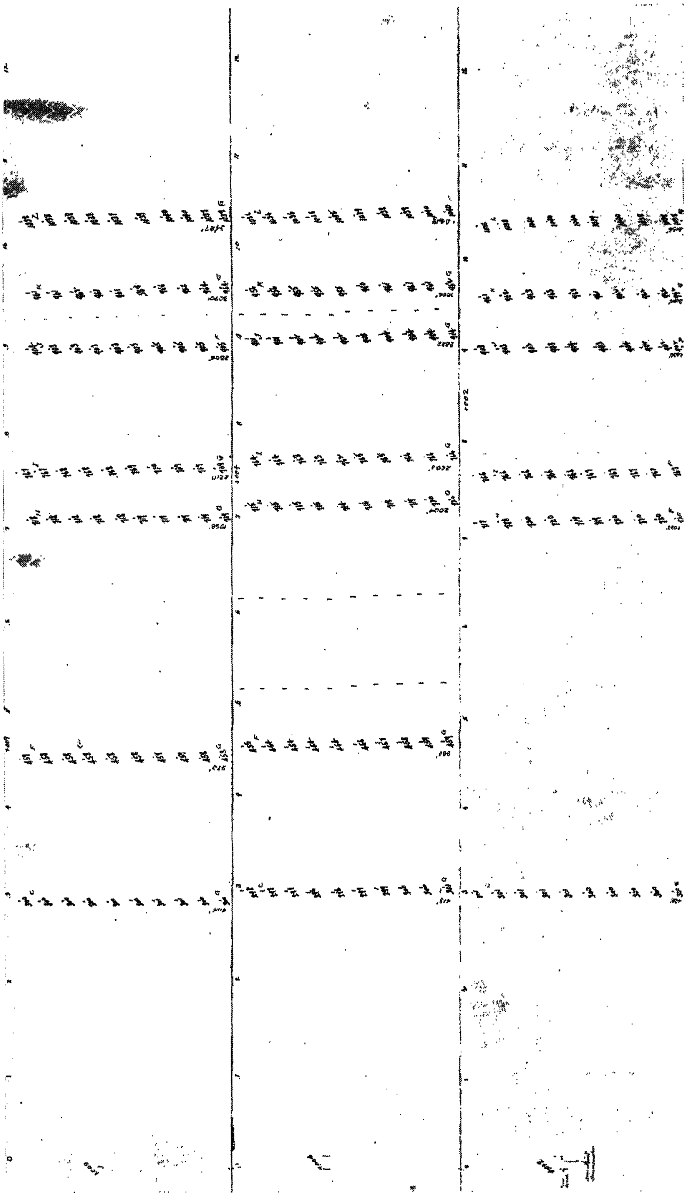


FIG. 288.—Correlation records secured near Greenville, Texas. (Courtesy of Western Geophysical Co.)

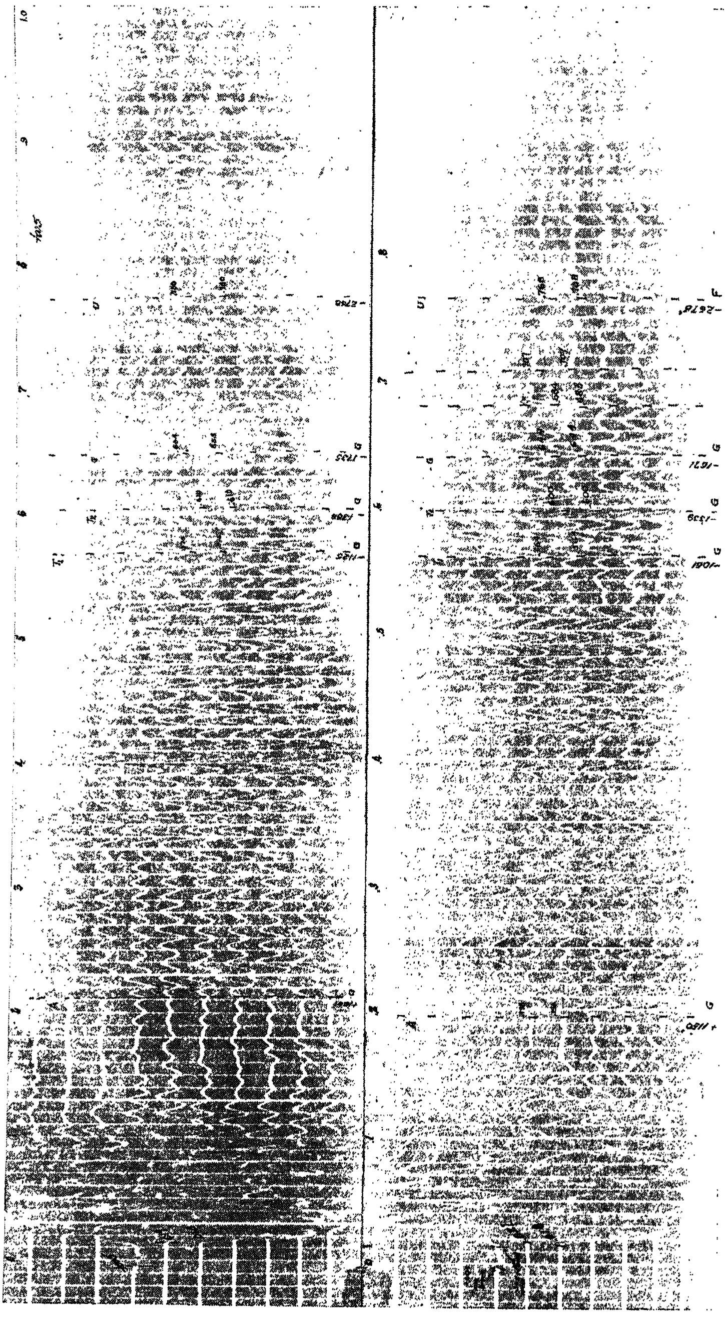


FIG. 289.—Correlation records secured near St. John, Kansas. (Courtesy of Western Geophysical Co.)

Velocity Shooting.—Velocity can be determined in two general ways by the use of seismometer spreads: namely, by refraction and by reflection methods. Because the evaluation of velocity is an integral part of refraction shooting, that method of determining velocity is covered in a subsequent section and will not be considered here. The usual limitation, namely, that only relatively shallow depths can be investigated accurately by the refraction method, is the reason that the reflection rather than the refraction method generally is used for velocity measurements when seismometer spreads are employed.

It was observed in the treatment of reflected ray paths through a section consisting of two layers (p. 467) that velocity could be determined directly from reflection times recorded at the end seismometers of a spread. However, this method requires a large spread length. A step in refinement is to utilize reflection times recorded on each trace, to plot the times against distance between shot-point and seismometer, and to obtain an average value of velocity from the slope of the best fitting curve through the data.

Consider, Figure 290, a simplified section showing reflection ray paths from shot-points A and B to seismometers 1 and 10, the spread lying between these two seismometers. Separate records are of course obtained for each shot-point. It is assumed that the reflecting bed and the base of the low velocity layer are horizontal, and that the velocity V of the medium included between the low velocity layer and the reflecting bed is substantially constant.

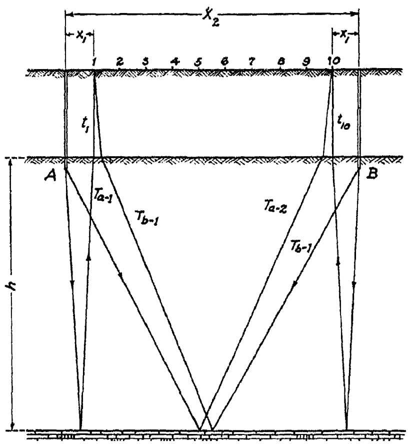


FIG. 290.—Determining average velocity by reflection method.

The time of travel for the first ray corrected to the base of the low velocity layer is given by Equation 18; that is

$$T_{a-1} - t_1 =$$

where T_{a-1} is the time recorded at seismometer number 1 from shot-point A and t_1 is a correction time factor (equal approximately to the vertical travel-time through the low velocity layer below seismometer 1 less the time of travel from the shot to the base of the low velocity zone). Evidently this equation may also be written in the form:

(59)

If, therefore, one were to plot † the square of the quantity reflection time $T_{a\text{-}k}$ less the correction time factor t_k versus the square of the distance x_k from shot-point to seismometer, the resulting points should fall on a straight line having a slope $1/V^2$ and an intercept $4h^2/V^2$. (Here, as usual, k applies to the kth seismometer.)

This procedure is followed: the distance x is measured from that shot-point for which the record was obtained, the data from both shot-points being plotted on the same graph. The effect of low velocity travel-times, which would be prominent if only one shot-point were used, is largely cancelled when the double arrangement of shot-points is used. Similarly, the effect of the dip of the stratum and of the base of the low velocity layer in the direction of the spread is largely cancelled. Nevertheless, for accuracy of final data it is important that, so far as is feasible, an area be chosen for velocity shooting in which these strata, particularly the low velocity stratum, are substantially horizontal.

To obtain useful accuracy it is often necessary to employ such extremely long spreads that they cannot be reached by the usual cables or covered adequately by the usual number of seismometers. In this event, spreads are shot in tandem, i.e., the length to be covered is traversed in steps, wherein successive spreads are begun where previous ones stopped, and one trace is left in common in order that accurate tie-in between records be available. For such long spreads it is necessary first to determine whether some of the reflecting horizons persist over the full length of the spread.

The reflection method itself gives way to a far more accurate method, namely, to direct velocity determination obtained from well shooting, when wells are available. The latter method is described in the following section.

Well Shooting

In this method, a seismometer is lowered into the well by an insulated conducting cable. ‡ First arrival-times are measured for waves from shots which are fired from a shot-point located at the surface and near the well. This method utilizes direct waves and measures depths directly, thereby securing accurate velocity data.

The well seismometer is of special construction, being designed to withstand the high pressures encountered at the bottom of deep wells. Cylindrical and of small diameter, the seismometer is built to minimize the risk that it will be mired in the settled mud at the bottom of the hole and to permit entrance in narrow casing. The cable supporting the seismometer must be strong, well insulated electrically, and accurately measurable in length during operation. Fortunately, such cables are available in the equipment of electrical logging firms. The length of cable under tension is accurately known and a continuous reading of depth of seismometer is supplied by the cable trucks.

† See also C. H. Green, "Velocity Determinations by Means of Reflection Profiles," *Geophysics*, Vol. 3, No. 4, p. 295, October, 1938.

‡ See also H. M. Rutherford, "Reflection Methods in Seismic Prospecting," *A.I.M.E. Geophysical Prospecting*, 1934, pp. 395-396.

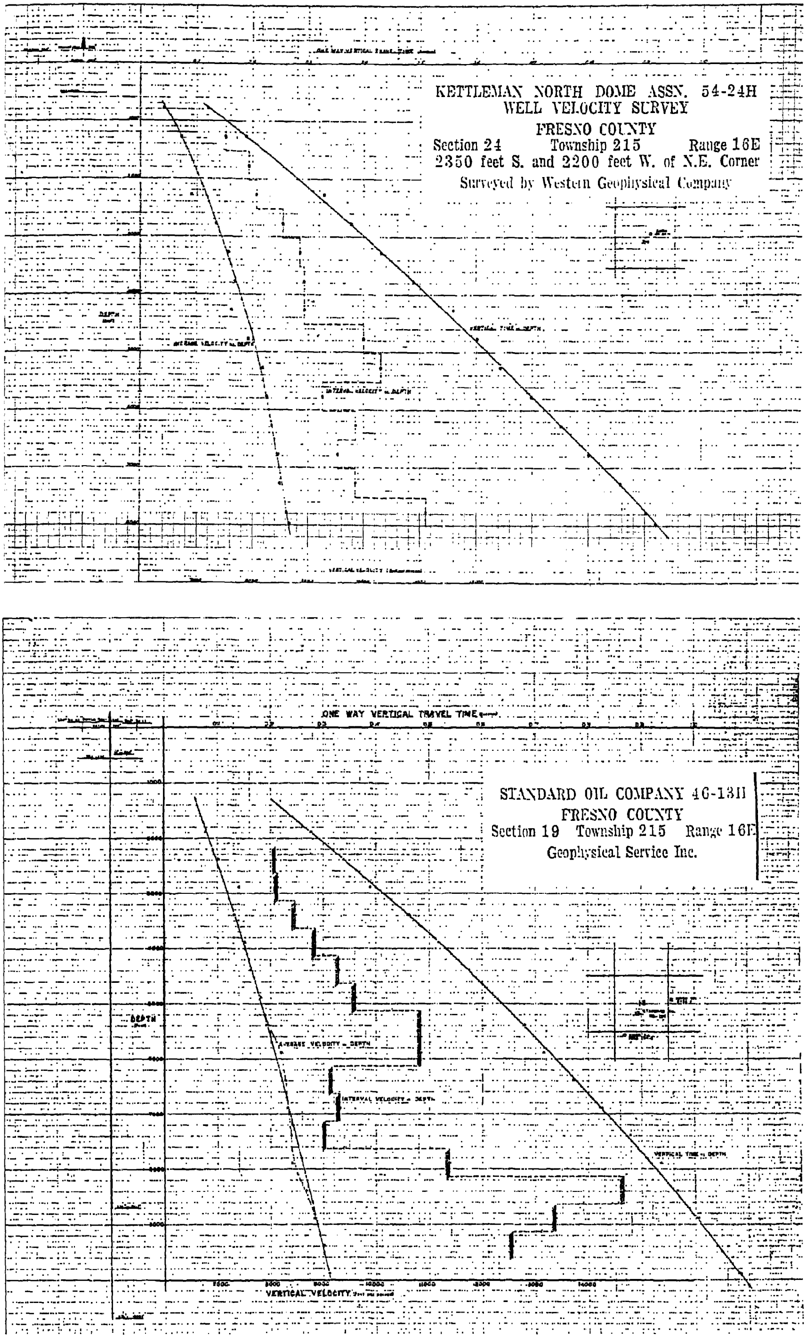


FIG. 291.—Results of well shooting of two wells about one mile apart, San Joaquin Valley, Calif.

Data on the low velocity zone are obtained at the well by up-hole shooting. A surface seismometer at the shot-point serves as a check on depth of shot and a distant surface seismometer is used to check time-break during well velocity determination.

Figure 291 shows the results obtained on shooting two wells in the San Joaquin Valley, California.* The interval velocity is decidedly erratic. The presence of low velocity strata is confirmed by the two wells and a correlation of these strata is noted. The interval velocities were determined from the difference of time over a depth difference of 500 feet. A considerably more accurate determination of interval velocity would be possible if the two times at the ends of the interval could be determined simultaneously. Multiple seismometers separated by a constant interval of cable are designed to realize this accuracy.† (When no special seismometers are available for well shooting but an abandoned well can be located in the area under investigation, it is possible to obtain information about the increase of the velocity with depth by shooting small charges—for shallow depths a cap is sufficient—at different depths and using a seismometer very close to the well as a detector.)

It is sometimes desired to determine the velocity V at any depth by the use of a curve such as Figure 295 which shows the average velocity $\overline{V}$ as a function of depth. The velocity V is assumed to be a continuous function of the depth. By the definition of average velocity (the average with respect to time is used throughout this chapter), one has

$$\overline{V} \quad \frac{h}{\int_0^h \frac{dh}{V}}$$

On differentiation, one obtains

$$\frac{d\overline{V}}{dh} \qquad\qquad \overline{V}$$

or

$$\frac{\overline{V^2}}{\overline{V} - h\frac{d\overline{V}}{dh}}$$

The velocity at any depth is thus obtained from a knowledge of the average velocity to that depth and the rate of change of the average velocity at that depth.

---

* These results cannot be taken as typical of the Valley. It will be noted also that the velocity $V = 6{,}000 + 0.5h$ which was attributed to many regions of California differs from the values shown here. The variation of velocities between these two wells only one mile apart indicates the need for frequent velocity adjustments, if the seismic mapping is to represent a particular prospect accurately.

† H. Salvatori, U. S. Patent 2,137,985. Issued July 9, 1937.

REFRACTION METHOD

In the refraction method of seismic prospecting, measurements are made of the times of arrival of waves which have been refracted at subsurface boundaries so that portions of the wave paths in the subsurface media are approximately parallel to the boundaries. In favorable cases, data on the travel-times of the refracted longitudinal waves may be used to map the boundaries separating the media of different elastic wave velocities.

Subsurface Section Consisting of Two Horizontal Layers.—The path of a refracted wave through a section consisting of two horizontal layers has already been described. (Figures 261 and 262.) A seismogram showing the arrival of refracted waves P, reflected waves R, and surface waves L, is given in Figure 292.

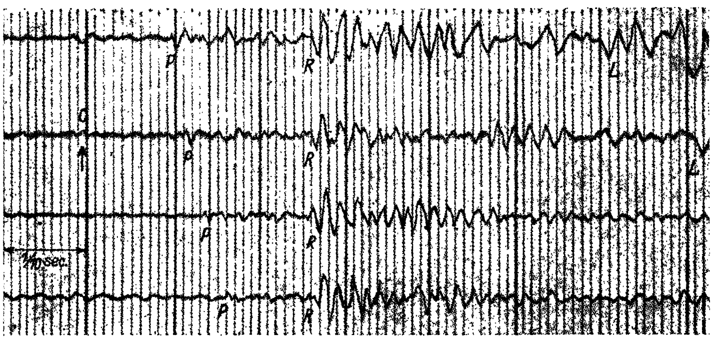


FIG. 292.—Seismogram showing the arrival of refracted waves P, reflected waves R, and surface waves L. The shot was 1/8 pound of 60% dynamite and was exploded at a depth of 3 feet. (Reproduced from B. Gutenberg, *Beiträge zur angewandten Geophysik*, Vol. 6, No. 2, 1936.)

The travel-time T for this path is given by Equation 19, that is,

$$T = 2h\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}} + \frac{x}{V_2} \tag{19}$$

where h denotes the depth to the boundary; V_1 the velocity in the upper stratum; V_2 the velocity in the lower stratum; and x the horizontal distance between the shot-point and a seismometer station.

If the velocity V_2 is a constant, it may be determined by measuring the slope of the straight line travel-time curve given by Equation 19 and computing its reciprocal. (Compare Figure 261.) Likewise, if the velocity V_1 is a constant, it may be evaluated from the slope of the straight line travel-curve passing through the origin, i.e., the travel-time curve corresponding to the direct wave.

e travel-time
intercept the
given by the equation:

$$\overline{\frac{1}{} - \frac{1}{V_n^2}} \qquad (60)$$

the intercept T_0 and the velocities V_1 and V_2 are known, the
h of the upper stratum may be computed from the intercept
formula.

The depth may also be computed from the coordinates x_c and T_c of the point of intersection of the travel-time curves corresponding to the direct wave and the refracted wave. To obtain the values x_c and T_c which satisfy the equations of both travel-time curves: namely, $T = \frac{x}{V_1}$ and $= 2h\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}} + \frac{x}{V_2}$, one solves these two equations simultaneously as follows:

$$\overline{V_2 + V_1}\ \frac{1}{1}; \quad h = \frac{x_c}{2} \quad \overline{V_2 - V_1}$$

n **Horizontal Strata.**—Referring to Figure 293, the velocities in the successive layers are $V_1, \ldots . V_{n+1}$ respectively and the thicknesses of the layers above the $(n+1)$th are: $h_1, \ldots h_n$. It was shown in a previous section that the angles made by the rays with the normals at the boundaries satisfy the equations:

$$p = \frac{\sin a_1}{V_1} = \ldots . = \frac{\sin a_k}{V_k} = \ldots . = \frac{\sin a_n}{V_n} \qquad (10)$$

where p is a parameter. It was also shown that the time required by the wave in traveling from the shot-point 0 to the point A is

Furthermore, the horizontal distance from 0 to A is

$$X_n = \sum_{k=1}^{n} h_k \tan a_k \qquad (12)$$

The case of interest in refraction prospecting is that wherein the angle a_{n+1} has a value of 90° so that

$$\frac{1}{V_{n+1}}$$

and

The total time from 0 to a point S at a distance x on the surface is obviously

$$T - 2t + \frac{x - 2X_n}{V_{n+1}} - \quad x \quad + 2\sum_{k=1}^{n} h_k \left(\frac{1}{V_k \cos a_k} - \frac{\tan a_k}{V_{n+1}}\right)$$

$$x \qquad V_k \cos a_k$$

or

$$T = \cdot \qquad h_k \cos a_k \qquad (61)$$

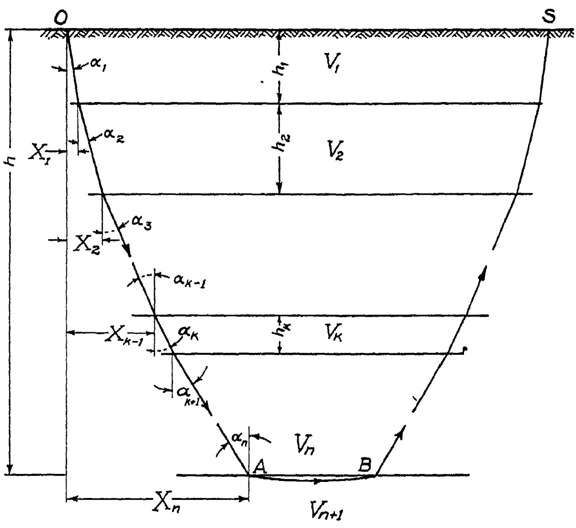


FIG. 293.—Path of a refracted ray through n horizontal layers.

It is evident that the travel-time curve will consist of as many segments as there are layers of different velocity, each segment corresponding to a wave path along an interface. Each segment is a straight line, and the reciprocal of the slope of each segment equals the velocity in the corresponding layer. Moreover, it is possible either from the time-intercepts or the critical distances to compute the depths to the layers.

The thicknesses of the layers may be determined successively by solving for h_n; that is, since

$$T = \frac{x}{V_{n+1}} + 2\sum_{k=1}^{n-1} \frac{h_k \cos \alpha_k}{V_k} + \frac{2h_n \cos \alpha_n}{V_n}$$

$$h_n = \frac{V_n}{\cos \alpha_n} \left\{ \tau_{n+1} - \sum_{k=1}^{n-1} \frac{h_k \cos \alpha_k}{V_k} \right\} \tag{62}$$

where

$$\tau_{n+1} = \frac{1}{2}\left(T - \frac{x}{V_{n+1}}\right) \tag{63}$$

is one-half the intercept time of the $(n+1)$th segment of the travel-time curve.

Numerical Illustration

Suppose the velocities and intercept-times shown in the following table have been determined from a travel-time curve.*

Layer n	V_n (ft./sec.)	T_{0n} (sec.)**	(sec.)
1	5710	0	0
2	6550	.102	.051
3	7490	.252	.126
4	8320	.418	.209
5	9170	.576	.288

Let $\alpha_{k,n}$ be the inclination in the kth layer of a ray which meets the interface between the $(n\text{-}1)$th and the nth layer at the critical angle so that $\sin \alpha_{k,n} = V_k/V_n$. With the velocities given above, $\sin \alpha_{k,n}$ and $\cos \alpha_{k,n}$ have the following values:

k \ n	1	2	3	4	5
1	1.0000	.8718	.7623	.6863	.6227
2		1.0000	.8745	.7873	.7143
3			1.0000	.9002	.8168
4				1.0000	.9073
5					1.0000

$\cos \alpha_{k,n}$

k \ n	1	2	3	4	5
1	.0000	.4899	.6472	.7273	.7825
2		.0000	.4851	.6166	.6998
3			.0000	.4355	.5769
4				.0000	.4205
5					.0000

* In layer 1 the refracted wave and the direct wave are identical.

** T_{0n} is the time at which the ray reaches the nth segment.

Successive computations of the layer thicknesses proceed in the manner indicated in the table. The travel-time curve for the five-layer section is shown in Figure 294.

n	k	h_k	$\frac{h_k}{V_k}$	$\frac{h_k}{V_k} \cos \alpha_{k,\,n+1}$	$\sum_{k=1}^{n-1} \frac{h_k}{V_k} \cos \alpha_{k,\,n+1}$	$\tau_{n+1} - \sum_{1}^{n-1}$	$\frac{V_n}{\cos \alpha_{n,\,n-1}}$	h_n	TOTAL DEPTH $= \Sigma h_n$
1	...	.....	.....	.....	.....	.051	11,655	594	594
2	1	594	.104	.067	.067	.059	13,502	797	1391
3	1	594	.104	.076	.....	.....	....	.....	....
	2	797	.122	.075	.151	.058	17,199	998	2389
4	1	594	.104	.081	.....	.....	....	.....	....
	2	797	.122	.085	.....	.....	....	.....	....
	3	998	.133	.077	.243	.045	19,786	890	3279

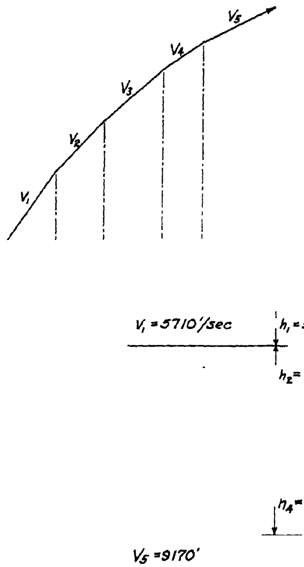


FIG. 294.—Travel-time curve for a subsurface section consisting of 5 horizontal layers.

Vertical Section in Which the Velocity Increases Continuously with Depth.—When the velocity increases continuously with depth, the path of the refracted ray has the form shown in Figure 295. Due to continued refraction, the angle made by the ray with the normal at any point along its path becomes increasingly greater until at some depth H the angle becomes equal to 90°, where the ray path becomes horizontal. Thereafter, the effect produced by the continuous variation of velocity is reversed and the ray gradually swings upward and returns to the surface along a path RS which is symmetrical to the path $0R$.

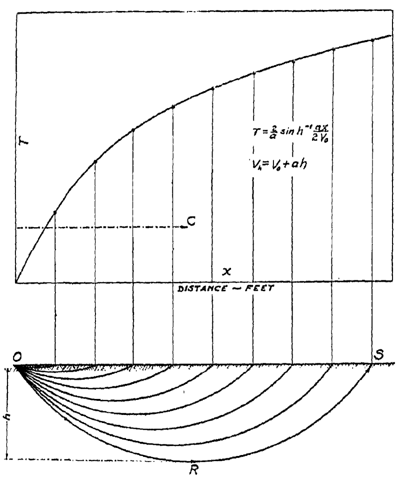


FIG. 295.—Travel-time curve for a formation in which the velocity increases linearly with depth. (Dotted line C is the locus of the centers of curvature of the ray paths.)

To obtain the travel-time over the path $0RS$, assume that the velocity function may be written in the form $V = V(h)$ where $V(h)$ is continuous. The travel-time T and the horizontal distance x between the shot-point and a seismometer station may be obtained directly from the basic ray equations 15 and 16; that is,

$$`= 2t_n = 2\int_0^H \frac{dh}{V(h)\sqrt{1-p^2V^2(h)}} \tag{64}$$

and

$$\frac{) \, dh}{-p^2V^2(h)} \tag{65}$$

For the case under discussion, the parameter $p = \dfrac{\sin \alpha_k}{V_k}$ has the following important properties

1. The maximum depth H is that at which $\alpha_k = \alpha_H = 90°$. Hence,

$$p = \frac{\sin 90°}{V_H} = \frac{1}{V_H} \tag{66}$$

That is, the parameter p is equal to the reciprocal of the velocity of the medium at the point of maximum penetration of the refracted ray.

2. The parameter p is equal to the slope of the travel-time curve corresponding to the path $0RS$. This may be proved as follows: The slope of the travel-time curve at any point is $\dfrac{dT}{dx}$. But $\dfrac{dT}{dx} = \dfrac{\frac{dT}{dp}}{\frac{dx}{dp}}$. Hence, the slope may be evaluated by differentiating Equations 64 and 65 with respect to the parameter p and forming the quotient.

$$\frac{dT}{dp} = 2\int_0^H \frac{pV^3(h)}{} \cdot dh = : \qquad dh$$

$$dh$$

$$= 2 \qquad dh$$

Hence,

$$\frac{dT}{dx} : \qquad \frac{\frac{V(h)}{}\quad dh}{\frac{V(h)}{}} \tag{67}$$

3. The parameter p is equal to the quotient of the sine of the angle of emergence divided by the velocity at the surface of the medium. That is,

$$p = \frac{\sin \alpha_0}{}$$

where a_0 is the angle of emergence of the refracted wave and V_0 is the velocity at the surface of the medium. (Compare p. 463.)

When the velocity is a linear function ($V=V_0+ah$), the expression for the travel-time becomes

$$T = 2\int_0^H \frac{dh}{(V_0+ah)\sqrt{1-p^2(V_0+ah)^2}}$$

or

$$T = \frac{2}{a}\int_{pV_0}^{1} \frac{dz}{z\sqrt{1-z^2}} \qquad (68)^*$$

where z is defined by the equation: $z = p\,(V_0+ah)$. On indicated integration the expression for T becomes

$$T = -\frac{2}{a}\log_e\left[\frac{1+\sqrt{1-z^2}}{z}\right]_{pV_0}^{1} = -\frac{2}{a}\,[\sec h^{-1}\,.\,]_{pV}^{1}$$

$$= \frac{2}{a} \qquad (69)$$

Similarly,

$$x = \frac{2}{ap}\int_{pV_0}^{1}\frac{z\,dz}{\sqrt{1-z^2}} \qquad \frac{2}{ap}\left[\sqrt{1-z^2}\right]_{pV_0}^{1}$$

$$(70)$$

Equations 69 and 70 are the parametric equations of the travel-time curve. The usual form of the equation of the travel-time curve may be obtained by eliminating p from equations 69 and 70. From Equation 69,

$$= \frac{1}{V_0}\sec h\ aT$$

Hence,

$$x = \qquad \tan h\ aT$$

or

$$x = \frac{2V_0}{a}\sin h\frac{aT}{2} \qquad (71)$$

Thus, the travel-time curve is smooth and concave downward. (Figure 295.)

---

* The expression for the travel-time given here differs from that given on p. 491 only in the value assigned to the upper limit in the integration. The value for the upper limit used here follows directly from Equation 66.

Sloping Interfaces Between Strata.—While the cases of horizontal interfaces are useful in formulating ideas concerning seismic wave paths, horizontal interfaces are seldom encountered in practice. Consider, therefore, a simple two-layer problem in which the velocity in the upper layer is and the velocity in the lower layer is V_2 and assume that V_2 and V_1 differ by a finite amount. Referring to Figure 296, the *vertical depth below the shot-point* 0 is h, and the boundary dips *down* to the *right* so as to form an angle θ with the horizontal. (It is assumed that the strike of the layer is perpendicular to the plane of the figure.) x is, as usual, the distance between the shot-point and a seismometer station. $0'$ is a point

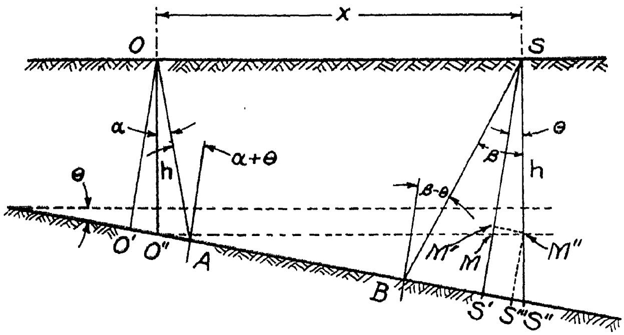


FIG. 296.—Seismic ray paths through an inclined layer.

on the bed from which a normal to the bed passes through 0; S' is the corresponding point below S; A is the point where the ray strikes the bed; and B the point where the ray leaves the bed. The angle α is the angle between the ray and the vertical at 0; β is the corresponding angle at S.

The travel-time for the path $0ABS$ is

$$\frac{0A + BS}{\;} + \frac{AB}{\;} \qquad (72)$$

In triangle $00''A$,

$$\frac{0A}{h} = \frac{\sin (90 + \theta)}{\sin [90 - (\alpha + \theta}$$

or

$$0A = h \frac{\cos \theta}{\cos (\alpha + \theta)}$$

In triangle BSS''

$$\frac{BS}{SS''} = \frac{\sin (90 - \theta)}{\sin [90 - (\beta - \theta)]}$$

or

$$BS = SS'' \cdot \frac{\cos \theta}{\cos (\beta - \theta)}$$

But

$$S'' = SM'' + M''S'' = h + x \tan \theta$$

Hence

$$BS = (h + x \tan \theta) \frac{\cos \theta}{\cos (\beta - \theta)}$$

Also,

$$AB = 0''S''' - 0''A - BS' - S'S$$

and

$$= x \cos \theta$$

$$0''A = \frac{\quad \sin \alpha}{\cos (\alpha + \theta)}$$

$$BS' = \quad \sin (\beta - \theta) = \quad \tan \quad \frac{\cos \quad - \theta)}{\cos \quad - \theta)}$$

Hence,

$$x \cos \theta - \frac{h \sin \alpha}{\cos (\alpha + \theta)} - (h + x \tan \theta) \frac{\cos \theta \sin (\beta - \theta)}{\cos (\beta - \theta)} - h \sin \theta$$

If one substitutes the values of $0A$, BS and AB into Equation 72 and makes use of the relation $(\alpha + \theta) = (\beta - \theta)$ and collects similar terms, Equation 72 becomes

$$\frac{2h \cos \theta + x \sin \theta}{V_1 \cos (\alpha + \theta)} + \frac{x \cos \theta}{V_2} - \quad h \quad \sin \theta \cos$$

$$\cos \theta \sin \quad \theta)] \quad \frac{x \sin \theta \sin (\alpha + \theta)}{V_2 \cos (\alpha + \theta)}$$

On making use of the trigonometric identities

$$\cos (\alpha + \theta) = \cos \alpha \cos \theta - \sin \alpha \sin \theta$$

$$\sin (\alpha + \theta) = \sin \alpha \cos \theta + \cos \alpha \sin \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

the expression for the travel-time T may be written in the form:

$$T = \frac{2h \cos \theta + x \sin \theta}{V_1 \cos (\alpha + \theta)} - \frac{2h \cos \theta \sin (\alpha + \theta)}{V_2 \cos (\alpha + \theta)} + \frac{x \cos \theta}{V_2} - \frac{x \sin \theta \sin (\alpha + \theta)}{V_2 \cos (\alpha + \theta)}$$

Also, since $(\alpha + \theta)$ is the critical angle,

$$\cos (\alpha + \theta) =$$

Hence,

$$T = \quad 2h\cos\theta \qquad \frac{1}{V_1} - \frac{V_1}{V_2^2}\Big) + \qquad x\sin\theta \qquad \frac{}{V_1} - \frac{V_1}{V_2^2} \qquad x\cos$$

or

$$T = 2h\cos\theta \qquad \frac{1}{V^2} - \frac{1}{V^2}\Big) \tag{73}$$

Equation 73 is the equation of a straight line.

The travel-time curve therefore consists of two straight line segments, one corresponding to the direct wave and the other to the refracted wave. The segment corresponding to the direct wave passes through the origin and has a slope $\frac{1}{V_1}$. The segment corresponding to the refracted wave has a slope $\cos \quad \sin\theta\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}}$ and a time intercept

$$\tau_0 = 2h\cos\theta\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}}$$

If the seismometer is *up dip* from the shot-point, θ must be replaced by $-\theta$. In the usual procedure two shot-points are used, one at the up dip end of the profile and the other at the down dip end. The *vertical distance to the boundary at the down dip shot-point* will be denoted by h' and the distance between the shot-points will be denoted by L. It is evident from Figure 296 that $h' = SS'' = h + L\tan\theta$.

The travel-time curve for up dip shooting consists of two straight line segments. The first segment has a slope $1/V_1$ and passes through the point $x = L$. The second segment has a slope of magnitude $(\cos\theta/V_2 - \sin\theta\sqrt{1/V_1^2 - 1/V_2^2})$ and a time intercept with the line $x = L$ of magnitude $\tau_L = 2h'\cos\theta\sqrt{1/V_1^2 - 1/V_2^2}$.

In terms of the new variables (h' and L), the travel-time equations for down dip shooting and up dip shooting are:

$$T =$$

and

respectively.

If h' is
id that for T. T
) dip shooting is the same a
, the values of V_1 obtained at the two shot-points may
same and therefore the check times for the reversed
entical. In this case, the travel-time curves are adjusted

V_2 and θ may be obtained from the slopes of the line segments corresponding to the refracted paths. That is, V_2 and θ may be obtained from the relations

$$\frac{1}{'} = \text{slope}_{\text{(down dip)}} = \frac{c}{V_2} \tag{74}$$

$$\frac{1}{{}_2''} = \text{slope}_{\text{(up dip)}} = \frac{\cos\theta}{V_2} \qquad \overline{V_1^2} \tag{74a}$$

The values of V_2' and V_2'' (*apparent velocities*) and V_1 are obtained from the field curve directly or from the adjusted field curve, depending on whether V_1 is the same at the two shot-points or not.

A more practical method makes use of the relationship that the sine of the angle of emergence is equal to the product of the velocity in the surface layer by the slope of the travel-time curve. This relationship may be derived from Equation 74 by making use of the relation: $\frac{V_1}{V_2} = \sin\,(\alpha + \theta)$. Thus

$$\text{slope}_{\text{(down dip)}} = \frac{1}{V_1}\,[\cos\theta\sin\,(\alpha+\theta) + \sin\theta\cos\,(\alpha+\theta)]$$

But

Hence

$$\text{slope}_{\text{(down dip)}} = \frac{1}{V_1}\sin\beta$$

Similarly

$$\text{slope}_{\text{(up dip)}} = \frac{1}{V}\sin\alpha$$

The equations just derived permit the calculation of β and α and so of θ [which is equal to $\frac{1}{2}(\beta-\alpha)$]. Also, V_2 may be obtained from the relation: $\frac{V_1}{V_2} = \sin\,(\alpha+\theta)$. In practice, a good approximation is $V_2 =$

Evidently, when the values of θ, V_1, and V_2 are known, the depth h may be calculated from the intercept formula

$$\tau_0 = 2h \cos \theta \sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}}$$

and h' may be calculated from the relation

$$h' = h + L \tan \theta$$

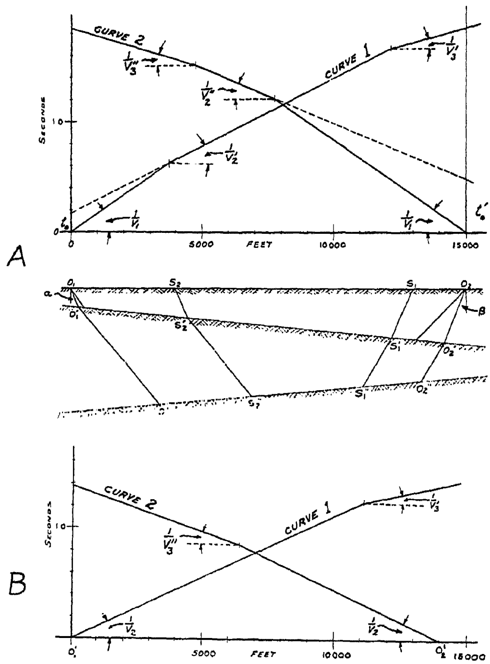


FIG. 297.—*A*, travel-time curves corresponding to a geological structure which consists of two inclined layers; *B*, computed curves obtained when the three layer case is "reduced" to a two layer case. Curve 1 is the travel-time curve for down dip shooting (shot-point at 0_1); Curve 2 is the travel-time curve for up dip shooting (shot-point at 0_2). (After W. M. Rust, Jr.)

For successively deeper layers the calculation can proceed in a similar way; that is, formulas may be derived for the depth, dips, and velocities for the multi-layer, dipping cases by methods analogous to those already described. Another procedure favored by many investigators is to reduce the problem at each step to the two layer case. Figure 297 shows how this may be done. The dip and depth of the first interface and the velocities V_1 and V_2 are determined by the methods outlined above.*

* The numerical computations are indicated on p. 544.

From the slopes $\frac{1}{V_3'}$ and $\frac{1}{V_3''}$ of the third segments of the travel-time curves and the fact that the slopes are equal to the sines of the emergence angles divided by V_1, the emergence angles α and β can be found. Choose any two points S_1 and S_2 between the two shot-points. B

... $0_1'0_2'$. $0_1'S_1'$ and $0_2'S_2'$ can be found.

along the seg... $S_2''S_2'$ are readily determined. For example, the ... $0_10_1'$ $0_1''S_1''S_1'S_1$ is read from the travel-time ... $0_1''S_1''S_1'$ is obtained by subtracting the times along the ... and $S_1'S_1$ from the total time. This gives two points on ... of the second segments of the new travel-time curve (Figu... so that these two segments may be drawn. The first segment ... to direct waves in the second layer with velocity V_2 ... ht lines with slope $1/V_2$ through $0_1'$ and $0_2'$ respectively. The depth, ... and velocity are now computed as before. (It should be ... "depths" determined by this procedure are directed along the perpendiculars to the second boundary at $0_1'$ and $0_2'$.) Obviously, this process can be continued for any number of layers.

The discussion given above is restricted to ideal cases only because space does not permit coverage of field records. The solution of special cases must depend primarily on the diligence of the computer and can be learned only through experience. Usually each experienced operator eventually develops his own computing technique.

Numerical Illustration

The values of the apparent velocities obtained from the travel-time curve are:

$$' = 8250 \text{ ft./sec.}$$

$$' = 10{,}000 \text{ ft./sec.}$$

Hence,

$$\sin \alpha = \cdot$$

$$\sin \beta = \cdot$$

$$\sin^{-} \quad V_2$$

$$\frac{V_1}{V_2} \qquad = 9000 \text{ ft./sec.} \qquad \frac{1}{V_2} =$$

$$- \alpha) = 4°55' \qquad \cos \theta = .996$$

Extending the segments back to the shot-points gives

$$\tau_0 = .174 \qquad\qquad \tau_0' = .490$$

so that

$$: = 700 \text{ ft.}; \quad h' = \cdot \qquad\qquad = 2000 \text{ ft.}$$

From the third sections

$$V_3' = 15{,}500 \text{ ft./sec.}$$

$$V_3'' = 14{,}600 \text{ ft./sec.}$$

$$\alpha_1 = 22°45'$$

$$\sin = \frac{V_1}{V_3''} = .411 \qquad\qquad \beta_1 = 24°15'$$

The times along the segments $0_10_1'$, S_2S_2', S_1S_1' and $0_20_2'$ may be computed either trigonometrically or graphically by using the values of V_1, h, h', α_1, and β_1 given above. The times are found to be 0.117, 0.191, 0.306 and 0.341 respectively. By subtracting the times along $0_10_1'$ and S_1S_1' from the time 1.698 along $0_10_1'S_1'S_1$ the time 1.275 is obtained at the distance 12,000 feet as measured along $0_1'S'_1$. The calculations of the times along $0_1'0_2'$ and $0_1''0_2''$ are carried out in the same way, and the solution proceeds as already outlined.

Limitations of Outlined Calculations

By the methods just outlined, the thicknesses, dips, and velocities of the subsurface strata can be calculated when these quantities satisfy the assumptions upon which the methods of calculation are based. Strata with velocities lower than the velocities of the overlying strata or strata which are too thin do not show up in the travel-time curves. For example, refracted waves traversing the structures *a, b, c,* and *d* (Figure 298B) have the same travel-time curve (Figure 298A) even though the geological conditions are radically different. *d,* only, can be differentiated from the others by use of a refraction shot in the reversed direction. (Figure 298B, (*c*) shows the case of an intermediate thin layer. A layer of this thickness would, in general, be detected.)

An interesting example of the use of refracted waves other than first arrivals is the recent work of Ewing, Crary and Rutherford † on the Atlantic Coastal Plane. By using the indications of the arrival of waves refracted through beds that were not thick enough to give first arrivals, they were able to increase the accuracy of their work and to discriminate between cases corresponding to (*a*) and (*b*) of Figure 298B.

† *Bulletin of the Geological Society of America,* Vol. 48 (1937) pp. 753-802.

Stated briefly, the assumptions on which refraction calculations are based are:

1. The velocities in successive strata increase as the depth increases.
2. The materials of the strata are such that the velocities in any direction are the same, i.e., the velocities are constant throughout each stratum.
3. The strata are sufficiently thick.
4. The boundaries between the strata are planes.

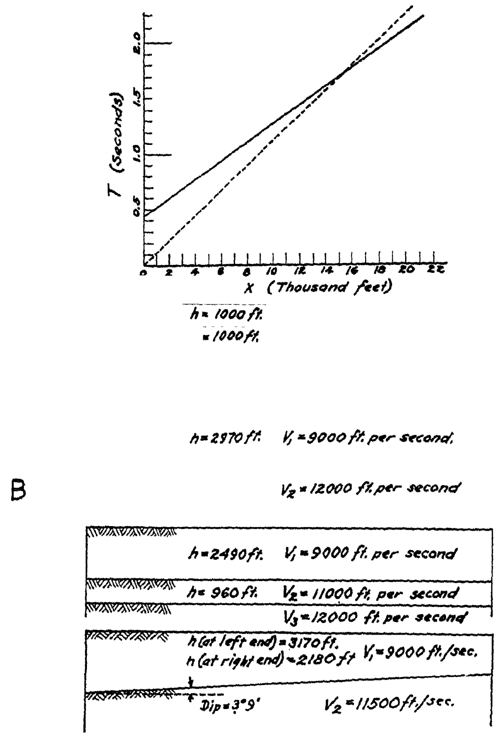


FIG. 298.—The same travel-time curve (portion A) is obtained for refracted waves traversing the structures a, b, c, d (portion B). (After W. M. Rust, Jr.)

If these conditions are satisfied, the travel-time curve will consist of straight line segments having successively decreasing slopes. In practice, these conditions are rarely, if ever, satisfied. Generally the velocity increases with the depth through a single layer. Also, loose layers frequently exist under dense layers, and the velocities in the loose layers are less than the velocities in the dense layers.

Mapping Subsurface Structure from Determinations of "Delay Times."—The methods described in this section constitute a means

for mapping the surface of a *marker horizon,* which exhibits gentle relief and a definite velocity increase over that of the overlying stratum.*

Referring to Figure 299, it is assumed that the actual ray path of the refracted wave which traverses layer 5 is $0A'B'S$, where the portion of the ray path in layer 5 is here assumed to follow the interface. In addition, to facilitate calculations, it is assumed that the actual ray path $0A'B'S$ may be replaced by a hypothetical path $0ABS$ defined as that path which the waves would follow if all the beds in the neighborhood of the baths $0A'$ and $B'S$ were horizontal. Calculations based on this assumption will have only a small error if the dips are less than 10°.

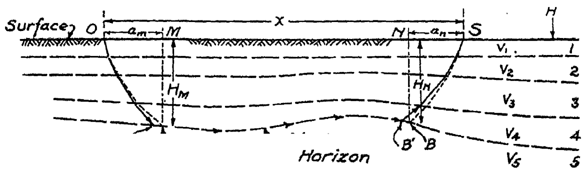


FIG. 299.—Ray path of wave following marker horizon. (After Gardner, *Geophysics.*)

Let

$$a_m = 0M = \text{offset distance corresponding to } 0$$

$$a_n = NS = \text{offset distance corresponding to } S$$

The travel-time T between 0 and S is:

$$T = T_{0A'B'S} = T_{0ABS} \text{ (approximately)}$$

or

$$T = t_{0A} + t_{AB} + t_{BS}$$

where t_{0A}, t_{AB} and t_{BS} denote the travel-times along the paths $0A$, AB and BS respectively.

Assume that the velocity V_5 in the layer whose surface constitutes the marker horizon is constant. Then

and the expression for the travel-time T may be written in the form

(75)

* With the exception of some minor changes, this section follows L. W. Gardner, "An Areal Plan of Mapping Subsurface Structure by Refraction Shooting," *Geophysics,* Vol. IV, No. 4, Oct., 1939, pp. 247-259.

Introduce the quantities t_m' and t_n' defined by the relations:

$$t_m' \equiv t_{0A} - \frac{a_m}{V_5} = \text{delay time at } M$$

$$t_n' \equiv t_{BS} - \frac{a_n}{V} = \text{delay time at } N$$

Also, set

$$b \equiv T - \frac{x}{V_5} = \text{intercept time}$$

The intercept time may also be expressed in the form:

$$b = t_m' + t_n' \tag{76}$$

Equation 76 is the basic relation of subsequent deductions. Since T, x, and V_5 are determinable from the observed data, the intercept time b may be regarded as an observed quantity.*

For a given velocity distribution in the geologic section, the delay times t_m' and t_n' are dependent only on the depths H_m and H_n respectively. Furthermore, if the velocity distribution is known, it is theoretically possible to deduce a relationship between the delay time t' and the depth H and to portray this relationship graphically. Thus, if the delay times can be determined from observed data, the corresponding depths may be read from the graph.

The relationships of delay times to depths and to offset distances may be obtained from the geometry of the hypothetical path. In Figure 300, α is the angle formed by the hypothetical wave path with the vertical at a point in the depth interval ΔH; Δa denotes an increment of offset distance; Δs is the path length in the interval ΔH. The increment of delay time corresponding to Δa will be denoted by $\Delta t'$. From the geometry of the figure,

$$\Delta H = \Delta s \cos \alpha, \qquad \Delta a = \Delta H \tan \alpha$$

and from Snell's law

$$V$$

On combining these relations with the definition of delay time

one obtains

$$\frac{\Delta H \cos \alpha}{V}$$

The total delay time to a given depth H of the marker horizon is

* To achieve accuracy in the final results, it is of course necessary that the value of the observed travel-time be corrected for the low velocity or so-called "weathered" layer.

and the depth of the horizon is

$$H = \sum \Delta H \tag{78}$$

Equations 77 and 78 constitute a set of parametric equations from which the delay time t' corresponding to a depth H of a marker horizon can be obtained provided the quantity ΔH is known as a function of V.

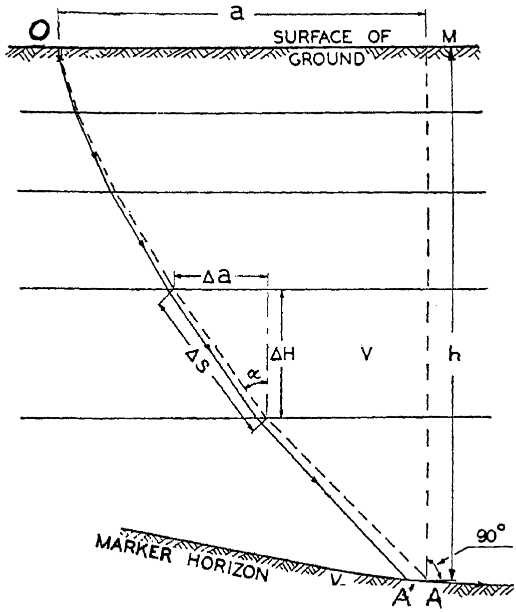


Fig. 300.—Portion of actual ray path and hypothetical ray path. (After Gardner, *Geophysics*.)

Also, the total offset distance $a = \Sigma\, \Delta a$ may be written in the form

$$a = \sum \Delta H \tan \alpha \tag{79}$$

A. B.

Fig. 301.—*A*, general type of depth vs. delay time graph; *B*, general type of offset distance vs. delay time graph. (Gardner, *Geophysics*.)

'7 are a set of parametric equations from
a corresponding to a total delay time t'

can be obtained.

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ie general types of
time graphs frequent
change in the slope
increase of velocity with depth.)

Intersecting Spreads

The methods to b

0_1S_1
posit
time
be approximately equal.

The expressions for the intercept times are

$$-t_m' + t_n'$$

Hence, (80)

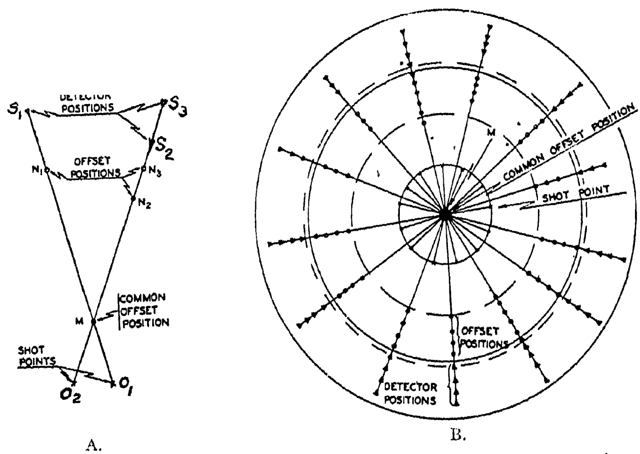


FIG. 302.—*A*, element of ring shooting arrangement; *B*, complete ring shooting arrangement. (Gardner, *Geophysics*.)

Evidently, Equation 80 can be used to calculate the difference in delay times from the *observed* intersect times b_2 and b_1. The corresponding difference in depths may be obtained from the depth vs. delay time graph.

Also, a seismometer at S_3 will have a common offset position at M with the seismometer at S_2; hence,

and the corresponding depth distance may be obtained, as before, by using the depth vs. delay time graph.

Furthermore, if a number of seismometers are placed on a relatively short line which passes through O_2 and is orientated in an azimuth with respect to O_2S_2, the offset positions associated with the common shot-point will be approximately coincident.

An extension of this arrangement which comprises a number of short spreads is shown in Figure 302B. The offset positions M associated with the ends of the various radial spreads lie in a small annular area. Undulations of the relative depths below the annular area can be determined by computing the delay times from the observed intercept times (Equation 80), and making use of the depth vs. delay time graph. Evidently, by arranging a number of such rings in a manner such that one or more control points from adjacent rings overlap, it is possible to combine all the data into a map showing relative depths over a large area. A triangular arrangement can also be used.

Application of Refraction Method to Salt Dome Exploration.—The velocity of seismic waves in salt domes is very much greater than that in the sedimentary materials surrounding the domes. Thus, a wave which has traveled in part through a salt dome will have a much greater average velocity than a wave traveling the same distance in a nearby area where no salt dome is present. This fact was established in 1924 and led to wide-spread use of refraction fan shooting.

Fan Shooting

In fan shooting, the seismometers are located at approximately equal distances from a shot-point. The distance is usually determined by shooting an auxiliary profile; it must be sufficiently great to insure that the waves penetrate to the desired depth. In the Gulf Coast, it is customary to use the empirical figure of one-fifth for the ratio of penetration to shot distance.*

* This relationship is similar to that prevailing in electrical work where the effective depth of current penetration is assumed to vary from about 1/3 to 1/9 of the electrode separation.

The seismometers are so positioned with reference to the shot-point that they cover any desired portion of the 360° surrounding the shot-point. Figure 303 illustrates a common arrangement of seismometers for covering a 135° sector. After the seismometers have been positioned at known and preferably equal distances from the shot-point, a charge is exploded, and the travel-times taken by the waves to reach the various seismometers are recorded. If the waves which have traveled through any particular sector have a higher velocity than the waves which have traveled through the other sectors, a high velocity plug is indicated in that sector.

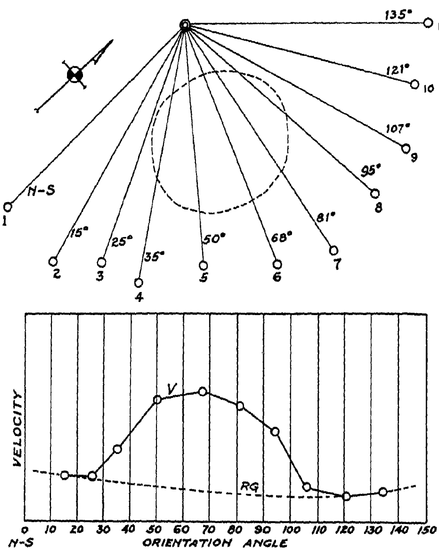


FIG. 303.—Diagrammatic sketch of fan shooting. Seismometers are placed at ends of lines 1, 2, 3, . . . 11 which radiate from the shot-point O; V, mean velocity; RG, probable regional velocity.

The interpretation of the results of refraction fan shooting is extremely simple, because the time differences observed are rather great. It is necessary only to plot the deviation of the travel times from the normal values. (Where it is not possible or convenient to locate all seismometers at the same distance from the shot-point, a short profile covering the interval from the shortest distance to the longest distance is shot in order to establish the normal travel-times for the various distances.)

Figure 303 shows the shot-point at 0 and seismometers at the ends of radii 1 to 11. The lower portion of the figure schematically indicates the variation of velocity with orientation angle. (In an alternative pre-

sentation, the deviation of the travel-time from the normal value would be plotted along each radius, zero deviation being plotted on an arbitrary circle with the center at 0.)

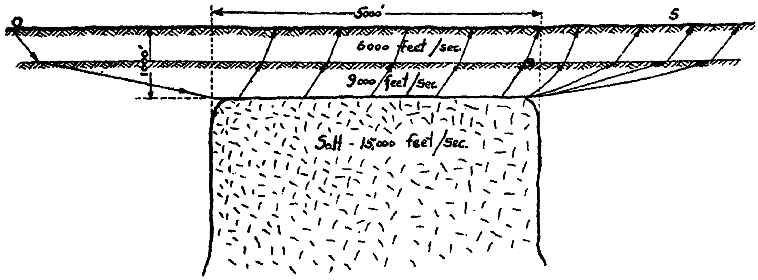


FIG. 304.—Theoretical ray paths through a cross section of a salt dome.

By the use of a number of strategically located shot-points and seismometer stations, a large area can be explored fairly rapidly. If the results indicate the presence of a shallow salt dome, the position and outline of the dome can be located more accurately by additional shots approximately at right angles to the first fan.

Figure 304 shows the theoretical ray paths through a salt dome that is located 1000 feet below the surface and has a cross sectional extent of 5000 feet.

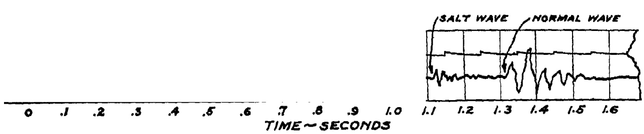


FIG. 305.—Schematic seismogram for a refraction shot across a salt dome.

Figure 305 shows a theoretical seismogram corresponding to a seismometer S located a distance of 10,000 feet from the shot-point 0. It will be observed that at the time 1.12 there is a small disturbance which corresponds to the wave traveling through the salt. The amplitude of the salt wave disturbance diminishes rapidly. At the time 1.31 a second disturbance starts. This disturbance corresponds to the normal wave through the material surrounding the dome. The difference 0.19 seconds is the deviation of the travel-time from the normal value for that distance and is called the *lead*.

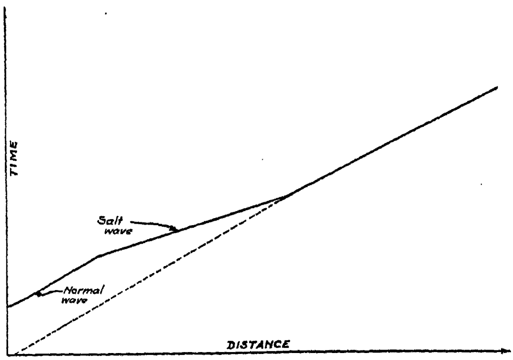


FIG. 306.—Travel-time curve for salt dome shown in Figure 304

Figure 306 shows a schematic travel-time curve corresponding to a traverse over the dome. A portion of the lower branch of the travel-time curve corresponds to wave energy which has traveled through the salt; the remainder of the lower branch and the upper branch corresponds to energy which has traveled through the surrounding material.

A large number of comparatively shallow salt domes has been located by fan shooting. At the present time, it is believed that the numerous fan surveys carried out in the Gulf Coast have probably disclosed practically all the shallow salt domes in that region.

In addition to discovering new salt domes, fan shooting in some cases has outlined successfully regions underlain by large structural features. *

Profiling Salt Dome Boundaries

Bore Hole Method.—A method for use in bore holes has been proposed by McCollum. § The principles of the method will be evident from Figure 307 wherein 0 represents the shot-point, 7 the recorder, and 5 the seismometer. The seismometer S is adjusted to various positions in the bore hole as, for example, d_2, d_3 or d_6, by means of a hoisting winch and cable. Consider that the seismometer is stationed at d_6. The wave traversing the path $0kf_6d_6$ will reach the point d_6 in the shortest possible time. This wave will actuate the seismometer S and its exact time of arrival will be recorded by 7. Let the time of travel over the path $0kf_6d_6$ be T.

Evidently,

$$T = t_1 + t_2 + t_3$$

where t_1 is the time to traverse the path $0k$; t_2 is the time to traverse

* The theoretical basis of this type of refraction shooting has been discussed by Barton † and by Roman.‡.

† D. C. Barton, "The Seismic Method of Mapping Geologic Structure," *A.I.M.E. Geophysical Prospecting,* 1929, pp. 590-598.

‡ Irwin Roman, "Analysis of Seismic Profiles," *A.I.M.E. Geophysical Prospecting,* 1934, pp. 493-527.

§ Burton McCollum, "Seismic Method of Profiling Geologic Formations," U. S. Patent 1,923,107. Issued August 22, 1933.

kf_6, and t_3 is the time to traverse f_6d_6. The last equation may also be written in the form

where V_s is the velocity along the path kf_6 and V_6 is the velocity along the path f_6d_6.

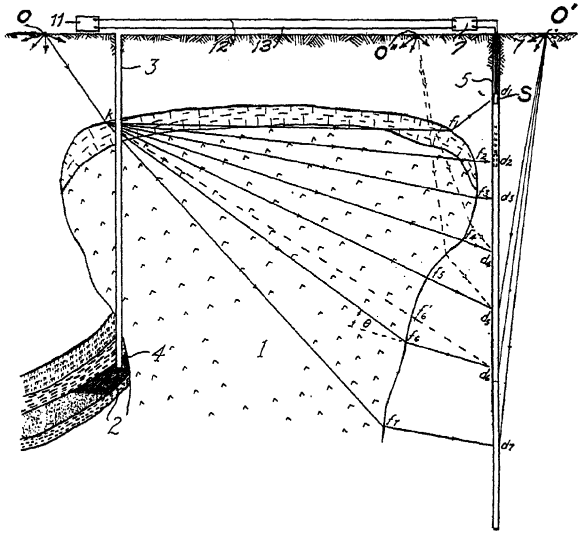


FIG. 307.—Schematic drawing illustrating a method for profiling the boundary surface of a subsurface geological structure. (After McCollum, U. S. Patent 1,923,107.)

Here t_1 and V_s may be determined from reflection travel-time data using the normal arrangement wherein both seismometer and shot-point are located at the surface of the earth.† The velocity V_6 will depend on the path kd_6; that is, the velocities along the various paths f_5d_5, f_6d_6, etc. will, in general, be different. However, a mean value of V_5 can be determined by obtaining the travel-times T_5, T_6 and T_7 between a shot-point located at a point 0′ on the surface near the bore hole and a seismometer positioned at d_5, d_6 and d_7 respectively.*

The position of the point k can be determined from reflection travel-time data. The position of the point d_6 is, of course, known.

Hence, it is possible to determine the paths fd_6 and kf_6; that is, it is possible to determine the position of the point f_6 on the flank of the dome. The positions of the points f_4, f_5, etc., may be determined in a similar way. Thus, the method is capable of profiling the boundary surface of the subsurface geological structure.

† Burton McCollum, U. S. Patents 1,724,495 and 1,724,720.

* The velocity thus determined is an average or equivalent velocity.

FIELD OPERATIONS IN SEISMIC PROSPECTING

Early Refraction Shooting.—Refraction shooting was the pioneer method and it is to be expected that the early refraction apparatus and procedure were less refined than those now employed.

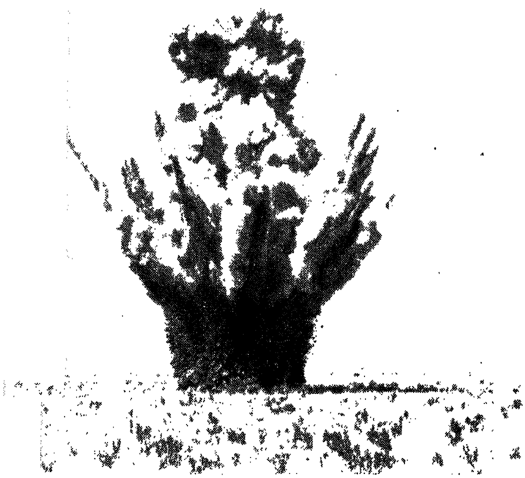


FIG. 308.—Early refraction shot.

In the early days of refraction seismic work, the shooting was conducted on a time schedule. The clocks of the shooters and recorders were synchronized daily, and at definite prearranged times the shooter exploded the charge. Just prior to the shot-times, the various operators, each of whom had a set of seismometers, amplifiers, a recorder, and developing equipment, started the recorders and then allowed them to run for a definite length of time. After getting his record, the operator developed it and then moved to the next station, where the recorder was again set-up for the next scheduled shot. The shooter often remained in one location all day. If for any reason his first shot could not be fired on schedule, he fired it fifteen minutes later. The instant of the shot was recorded at the point of observation by an electric signal which was transmitted from the shot-point by means of a wire.

Later, radios were introduced, each operator having a receiver and the shooter a transmitter. The shooter sent a message just before firing the shot, and the instant of explosion (shot-time) was transmitted by radio.

Still later, the operator and the shooter used two-way radio communication to make sure that all instruments were in working condition before the heavy charge was fired.

The quantity of dynamite in the shot depended upon the distance; often charges of several thousand pounds were used. Originally, the shots were placed on the surface of the ground, or, after the first shot, in the crater made by preceding shots. Later the dynamite was put in holes as deep as 200 feet. A photograph of an early refraction shot is shown in Figure 308. (The resulting crater was over 20 feet deep.)

Modern Refraction and Reflection Operations.—From an operational viewpoint, refraction field procedure differs from reflection field procedure in numerous respects: viz., the refracted waves are of primary interest rather than the reflected waves; the size of charge (amount of dynamite) is usually much greater; the seismometer spread length and the distance of the spread from the shot-point are greater; usually shallow depths only are investigated; and so forth.

However, the refraction and reflection field operations and equipment also have several similarities. Thus the exploration crew and the general division of duties of the crew members are similar; the field trucks used in the two types of prospecting are similar, etc. Hence, in the general description given here, refraction and reflection operations will be distinguished specifically only in the discussion of types of spread and in the methods of improving detection of reflections.

The Exploration Crew

A typical seismic party comprises a party chief, assistant party chief, permit man, surveyors, a recording truck staff, shooters, drillers, and computers. The general division of duties is given in Table 20.

The party chief* is responsible for all details of the work and arranges the schedule so that the work is done in the most efficient manner. After the party chief and the geologists to whom he is responsible have selected the areas to be explored, the permit man secures permission from the owners and lessees for the necessary work on the land. The permit man also makes agreements regarding the condition in which the land and fences must be left. Sometimes the final decision as to exact location of shot-holes and seismometer stations is not made until after permits are obtained. The surveyors then locate these points and mark them with stakes; the stakes are numbered and their location plotted on maps. The points are now ready for the drillers who must drill the holes to depths that have been determined from experience in the area or in similar areas. Next, the shooting and recording operations take place, preferably soon after the holes are drilled.

* In most organizations the party chief is in charge of geophysical interpretation and prospect mapping as well as field operations. In other organizations these two general functions are divided between two persons.

TABLE 20

GENERAL DIVISION OF DUTIES OF MEMBERS OF SEISMIC CREW

Party Chief	Field supervision of all operations. Interpretation of field data. Contact man between geophysical and geological departments.
Assistant Chief	Assistant to Party Chief and next in charge.
Computer	Computation of results and plotting.
Operator	Responsible for recording truck operation and in charge of crew while on field.
Assistant Operator	Assistant to operator.
Shooter	Responsible for explosives, loading, and shooting operations.
Assistant Shooter	Assistant to shooter.
Driller	Responsible for drilling of shot-points and for drilling equipment.
Assistant Driller	Assistant to driller.
Laborers	General field operations such as digging of shallow holes for seismometers, handling of cables. Helpers to drillers and shooters.
Permit Man	Right of way and shooting permits.
Surveyor	Location of seismometers and shot-holes.
Rodman	Assistant to surveyor.

The basic seismic operations are performed by two units: namely, the shooting unit and the recording unit. The procedure in the field is managed by the operator who is also responsible for the recording equipment. The recording unit consists of the recording truck and three or more men, at least one of whom, besides the operator, is sufficiently familiar with the equipment to assist in its maintenance. The shooting unit usually consists of two or more men and one truck. The head of the unit, the shooter, is responsible for his equipment, for efficient loading of the holes, and for the handling of explosives by his unit according to the legal and safety regulations.

After the seismometers of the recording truck unit are disposed and circuits checked, the recording equipment is ready for operation. By this time the shooter generally has loaded the hole with a charge at a depth already designated by the operator. From his controls in the recording truck, the operator communicates with the shooter by telephone or radio. The shooter repeats the depth and magnitude of charge and states his readiness to fire. The final order to fire is given by the operator, who then sets the recording equipment into operation when the shooter signals back.* After the records have been developed, the operator carefully studies them in order to direct the shooter regarding depth and pounds or sticks of

* Sometimes the operator is provided with means to fire the shot, and patents have been granted for a device to make the shot-instant coincide with one of the time marks on the record. However, the practice of shooting by the observer is rightly criticised as dangerous for the shooting crew and chance passers-by at the shot-point.

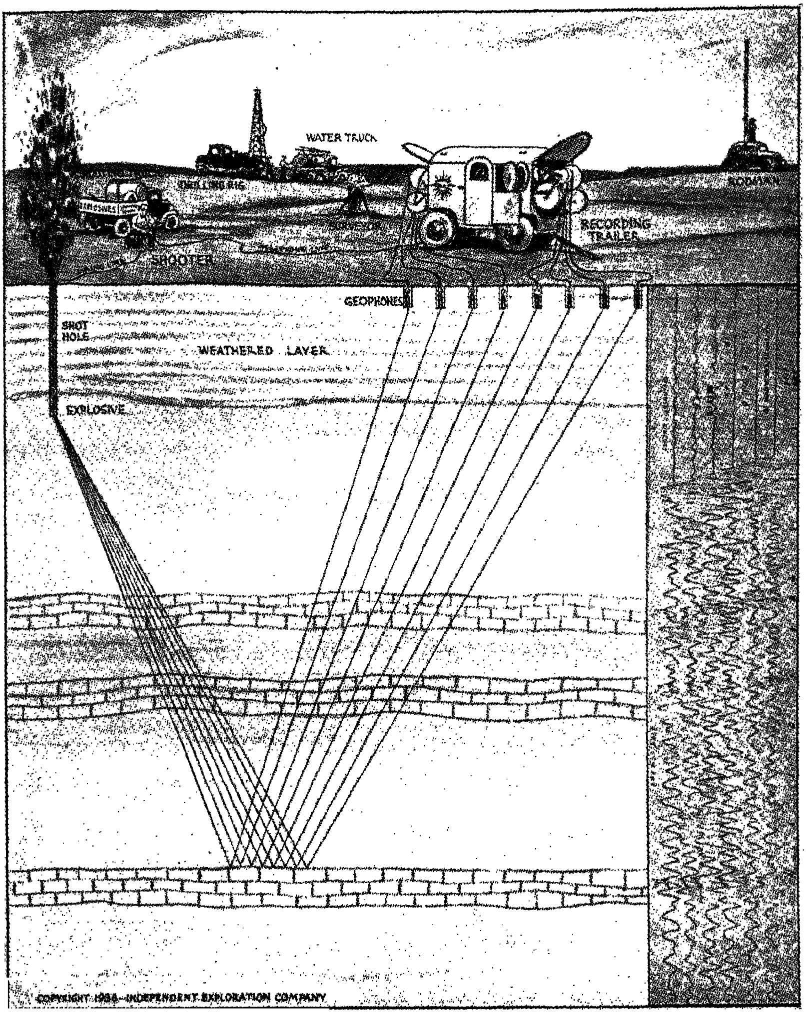


FIG. 309.—Reflection party in the field. Illustrating the path of the reflection and the record. (Courtesy of the Independent Exploration Company.)

explosive to be used in the shots to follow. Upon completion of the shooting, the hole is suitably filled to prevent damage to crops and injury to man and cattle. Sometimes when casing is used, it is pulled later by the drilling crew. When the holes are deep, it is cheaper and more efficient to have a special crew with suitable equipment to transport, pull, and lay down casing at the shot-points.

The day's records are taken to the party headquarters and usually are computed on the following day. The first step in the interpretation of the records is to mark off 0.1 sec. lines and to determine "weathering" or low velocity layer data. The records are then picked for refractions or reflections. Next, the data are computed and plotted—generally on cross sections from which mapping proceeds. Subsequent field operations are outlined by the geologists in collaboration with the party chief from studies of the current data.

A schematic layout of a reflection crew in operation, together with the path of the reflection and the record, is shown in Figure 309. The record indicates the shot-instants and the onsets of the reflections at the three reflecting horizons. For a refraction crew, the paths of the refracted rays and the refraction record would be shown; otherwise the arrangement is much the same except for distances between seismometers and between operator and shot-hole.

Field Trucks

The mobile equipment required for effective seismic operations is largely dependent on local conditions. Generally, the automotive equipment will comprise: recording or instrument truck, shooting truck, drill truck, water truck for drilling operations, surveyor's car, permit man's car, and party chief's car. The applicability and relative merits of standard tire equipment, dual or tandem conversion wheels, caterpillar treads, or large size marsh tires depend on the region surveyed. Even under favorable field conditions it is desirable, from the viewpoint of good mobility and minimum periods of delay, to equip the heavier trucks with four-wheel drives or dual tires. (When field conditions are difficult, it is necessary to equip the heavier trucks with winches.)

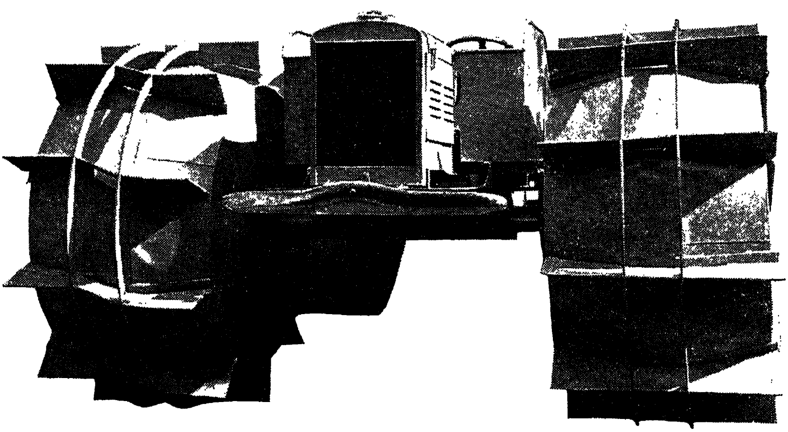
Special types of transportation are often necessary in areas of difficult terrain. In certain sections of the bayous and swamps of the Louisiana Gulf Coast, it is convenient to travel by means of so-called "marsh-buggies." Marsh buggies are usually trucks with special wheels that provide a large supporting surface. (Figures 310 and 311.) †

In some portions of the Louisiana Gulf Coast swamps, the network of streams is sufficiently close to permit the field men to reach much of the area in small power cruisers or boats. In this case the procedure is identical with the procedure on land, with the exception that the equipment is mounted in boats instead of in trucks. Special precautions must be exercised in the design of equipment in order to prevent damage due to exposure; for example, the seismometers are placed in water-tight cases.

Frequently, however, the swamp is cut up by streams which are too small to permit the use of boats and too wide to be crossed by marsh buggies. In order to avoid the necessity of transporting the amplifiers

† Harry F. Simons, "Marsh Buggy for Use in Gulf Coast Fields," *Oil and Gas Journal*, Feb. 22, 1940, p. 71.

FIG. 310.—Seismic recording truck for use in marsh and swamp areas. (Courtesy of Continental Oil Company.)



G. 311.—Marsh buggy. (Courtesy of Western Geophysical Company.)

and recorders across difficult marsh by foot, a method of "remote control" is employed wherein the seismometers are located at a great distance from the amplifiers. Remote control work often has been done with the seismometers as much as five miles from the amplifier; in these cases, it is necessary to separate the wires enough to prevent cross-feed from one line to the next due to mutual inductance between lines. (In difficult swamps this requires a considerable amount of labor.)

Figure 312 illustrates working conditions often encountered in laying cables in swamp areas.

FIG. 312.—Laying cables in swamp areas.

Another method which has proved satisfactory employs a single wire and a high frequency carrier system similar to that used in carrier telephony.

Magnitude and Depth of Explosive at Shot-Points

For economic reasons the charge of dynamite is made as small as consistent with adequate refraction or reflection energy.* Especially in reflection shooting, other reasons also may dictate this rule. With increasing charge, the useful wave energy does not rise proportionately to the magnitude of charge, due to increased mechanical pulverization of the earth in the vicinity of the shot-hole. Also, the energy of the surface waves generally increases more rapidly with the amount of dynamite than does the refracted or

* See pages 610 to 619 for detailed discussion of explosives and their characteristics.

reflected energy. The usual experience is that a charge that exceeds some certain amount has scarcely any advantage and may be a definite detriment. However, sufficient dynamite must be shot so that adequate amplitude of ground movement due to reflection or refraction energy is produced at the seismometers. It also is essential that the reflection or refraction energy be greater than the extraneous energy in the recorded band of useful frequency. Simultaneous explosion in several closely spaced shot-points is sometimes used to achieve sufficient energy. (See section on Multiple Shot-points.) Another consideration is that very small charges are difficult to load, causing time delays.

The depth of the shot is determined by the character of the subsection and must be below the low velocity, or so-called weathered zone, because absorption of energy is very great in the low velocity zone. The base of this zone frequently coincides with the top of the water table. A preliminary idea of the necessary depth for shot-holes in a new area can be obtained, therefore, from local water well records. When the medium immediately below the low velocity zone is stratified, it is preferable to seek out that stratum which yields highest efficiency of transmission of energy and to load the explosive at that depth. Shale usually transmits the shot energy more efficiently than sand or unconsolidated material, while the less porous rocks are still more effective.

The usual procedure for ascertaining the optimum shooting depth in a new area is: (1) to determine the depth of the low velocity zone from shallow refraction shooting, (2) to shoot light shots up-hole to obtain an up-hole low velocity diagram, (3) to shoot at various depths to obtain sample records. The drilled hole depth must exceed the depth of the bottom of the low velocity zone by an amount sufficient to insure that the top of the dynamite string is below the base of this zone and that the area of shattering is within the "consolidated" zone. Additional leeway is given when doubt exists as to the exact thickness of the low velocity zone.

Sufficient head of water must be maintained above the charge in order to keep the explosive from expending an excess of its energy up the hole. (The shooting truck usually carries a tank of water for this purpose.) Sometimes the subsurface water will maintain a sufficient head that no extra water need be added to the hole.

Seismometer Disturbance

The seismometers are generally buried in shallow holes or pits which are dug either with a post-hole auger or with spades. The depth of the hole should be at least sufficiently great to allow the top of the instrument to be flush with the ground in order to minimize wind disturbances.

The recent trend in reflection shooting is toward a large number of recording channels and the use of several seismometers on each channel. This has necessitated the use of numerous seismometers and has given an

impetus to placing the seismometers on the surface of the ground to conserve time and labor. When the seismometers are to be placed on the surface of the ground, their cases are usually designed with a flat extended bottom and streamlined upper portion. The surface of the ground is usually scraped to remove loose material, and the seismometer is then firmly seated or pressed in place.

If possible, the seismometers should be placed on firm ground and at a reasonable distance from trees, telegraph poles, irrigating ditches, pipe lines, concrete roads, and other structures that may transmit vibration to the nearby earth. Field men in the vicinity of the seismometers should remain motionless while the record is being made. In windy weather, trees, wheat, high grass, etc. often prove troublesome as they are a source of noise or vibration.

Extraneous vibrations are especially troublesome in the case of reflection prospecting. The higher the noise level in any particular location, the higher must be the energy level of the reflections in order to give a usable reflection record. This necessitates larger charges of explosive than would otherwise be necessary or even advisable. Conditions are comparable to radio reception: the incoming signal must be of a higher energy level than the static and local interference. Under conditions of high noise level, increased amplification usually does more harm than good. In areas of heavy traffic, or other disturbing factors, the operator must make his records during periods of minimum disturbance. (The operator should never give the signal for firing if any of the galvanometer traces is unduly disturbed.) Oftentimes, the difference between usable and useless records depends upon the vigilance and ingenuity of the operator.

Seismometer Spreads

The character of the records and the nature of the information which may be obtained is dependent to a large extent on the relative positions of the seismometers and shot-point. The type, length, and orientation of the seismometer spread are determined by such factors as magnitude and direction of strata dip, presence of particular geological features, character of low velocity layer, accessibility, disturbance energy, availability of permits, economic considerations, and degree of required accuracy, i.e., whether reconnaissance or detail shooting. Furthermore, in reflection shooting, the type, length and orientation of the spread are determined to a large extent by the quality of reflections, as shown by sample records.

Important types of *refraction* seismometer spreads have already been mentioned. †

† See also J. H. Jones, "A Seismic Method of Mapping Anticlinal Structures," *Proceedings World Petroleum Congress* (London), 1933, Part 1, pp. 169-173.

General classes of individual seismometer spreads used in reflection shooting include the split spread, the symmetric offset spread, the uni-directional spread, the offset uni-directional spread and the right-angle spread. (Figure 313.)

The split and uni-directional spreads indicated in this figure are used particularly when, due to limitation of accessibility or simply to greater ease of disposition of seismometers, the seismometers and shot-points are required to be on the same line. When the uni-directional spread has shot-points at *both* ends, it is usually called the "two-hole" spread.

The symmetric offset spread † is used to avoid the long gap between the seismometers nearest the shot-point. The effect of the offset spread is to displace reflection points on the strata along the cross dip, but in general this may be made sufficiently small to be negligible. The right-angle spread is designed to obtain line and cross dips simultaneously. The number of seismometers used in this spread is usually greater than that used in the spreads described previously.

For dip shooting, the symmetric rather than uni-directional spreads are preferred but sometimes the uni-directional spread must be used for a variety of reasons: such as, inaccessibility and, when due to excessive shot-point disturbances, the seismometer nearest to the shot-point must be located a great distance from that point, too far to solve the problem properly by offsetting the spread laterally. (See "open" spreads, p. 566.) In correlation shooting, the uni-directional spread is the most efficient when very short spreads are desired (p. 478). Continuous seismic profiling is another frequent application of uni-directional spreads (p. 513). (Velocity shooting (p. 527) is a special application of this type of spread.)

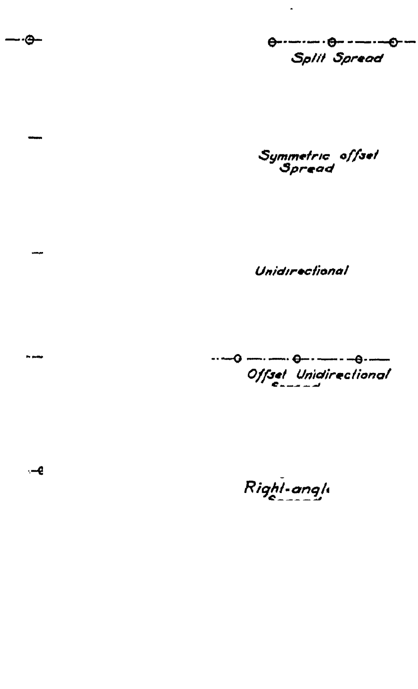


FIG. 313.—Types of individual spreads.

Also, in reflection shooting, spreads are sometimes shot in various combinations under certain circumstances. For example, when using equipment with relatively few traces, such as six, it may be advantageous

† Henry Salvatori, "Apparatus for Seismic Surveying," U. S. Patent 2,117,364, issued Dec. 24, 1936.

to place all the seismometers on one side of the shot-point for the first spread and then move to the other side for the second spread—a single equivalent long split spread will effectively be obtained. The reverse of this procedure is sometimes used when a single symmetrical disposition of seismometers at one shot-point serves also for shots from adjacent shot-points, the process being repeated for each disposition of seismometers. Here more than one set of dip data is obtained per seismometer disposition, and both "close" and "open" spreads are supplied. (See following section.)

Relation Between Surface Waves and Spread.—The ground surface waves usually have a velocity of about 1000 ft./sec.* The relative magnitudes of the surface wave velocity and the wave velocities of the deeper beds may be important in that they may govern the choice between "close" and "open" spreads in reflection shooting. In the "close" spread, seismometers are deployed near the shot-point. This spread is used where depth of investigation is sufficient to allow the reflections in question to arrive after cessation of the surface wave. In the "open" spread, seismometers are deployed far from the shot-point so that reflections are received prior to arrival of the surface wave. The consideration of "close" and "open" spreads is required only when the disturbance from the surface wave is prominent enough to require attention.

Field Methods for Improving Detection of Reflections

Multiple Detection

Where difficulty has been encountered in obtaining good reflections, it has become almost a general custom to use several seismometers in series or parallel instead of a single seismometer. This practice is called "multiple detection." † A group of seismometers which are connected in series feeds a single channel of the recording equipment and is represented by one recording trace. Usually there are two distinct advantages in such a

* van den Bouwhuijsen has investigated the velocity of surface waves by placing seismometers in very short spreads between 5 m. and 100 m. from the shot-point. He found that the time-distance curve of these waves is not linear. The slope of the curve at the origin corresponds closely to a velocity of 330 m./sec. The slope then decreases and finally attains a limiting value denoting a constant velocity. The final value of the apparent velocity in many regions is about 500 m./sec. It appears that the velocity at the surface is equal to the velocity of sound in air and that in the near-surface the air in the pore space plays a major part in the determination of the velocity of the longitudinal waves, while with increasing depth and decreasing porosity the bulk of the material which forms the upper layers becomes of greater importance in determining the velocity. In the experiments under discussion, where the top deposits were alluvial quartz sands, a definite break in the time-distance curve was observed, corresponding to a depth of about 10 m. The velocity of the second bed was of the order of 1200 m./sec.

† H. G. Taylor, "Method of Recording Seismic Waves," U.S. Patent 1,799,398, issued April 7, 1931.

method. First, the energy received by the channel is increased considerably. Second, the ratio of useful to extraneous energy recorded by the group generally is increased. The latter condition is realized because vertical waves strike the seismometers on the surface fairly well in phase whereas extraneous energy, such as the surface wave, will in general strike the seismometers in different phases at any given instant; that is, a discrimination is created in favor of vertical waves by a group of seismometers placed on a horizontal plane. Where reflected waves from strongly dipping beds are involved, however, the discriminatory powers of the seismometers may be a disadvantage unless the seismometers in each group are closely spaced or the line of seismometers is oriented in the direction of strike of the reflecting beds. Another disadvantage of multiple detection is that the time of arrival recorded by a channel fed by several seismometers often is not as sharp as that recorded by a single seismometer because, in general, the times of arrival will differ slightly for each seismometer, resulting in a "smoothing-out" of the wave characteristics.

It often has been found true that if records of seismometers placed a short distance apart, 2 feet for example, are appreciably dissimilar, the use of multiple detection is helpful; whereas, if they are similar, benefit is seldom obtained because the separation of the seismometers is relatively unimportant.

Overlapping Seismometer Output

Another proposed aid to the detection of reflections is an artifice known by the various titles: "composite," "inter-locking," "diversity," "overlapping," etc. recording.† In this type of recording, each seismometer output feeds two or more traces of the record in some specified manner. The seismic energy is thus overlapped across more than one trace and a particular seismometer cannot be associated with a given trace alone. The reason for using overlapping may be two-fold. First, for a given total number of seismometers, it permits a greater number of seismometers to operate in series and thereby increases the discrimination in favor of reflected waves (provided that the reflecting beds are not excessively steep). Second, reflections can be followed more readily across a record, because the difference between adjacent traces is less.

The procedure of overlapping is dangerous when carried too far, as will be evident from the following considerations. Spurious energy is usually disclosed by dissimilarities in traces. Hence a diminution of dissimilarities due to overlapping may yield a pattern or "line-up" which is due to spurious energy rather than reflections. Any overlapping

† Henry Salvatori, "Apparatus for Determining Subsurface Formations," U. S. Patent 2,064,385, issued Dec. 15, 1936.

at all, in fact, is condemned by some operators for this reason, while others approve its use to a limited and definitely controlled degree.

Variable Directional Discrimination

A method utilizing both the discriminating property of multiple detection and an extension of the overlapping principle is involved in the so-called Sonograph † system which has certain advantages for complex structures, particularly those involving steep dip.

It was observed that when seismometers are connected in series and are placed on a horizontal plane, a discrimination is produced such that waves from the vertical direction are favored. Similarly, a discrimination for waves from any other direction can be produced by locating the seismometers on a line at right angles to that direction.

In the Sonograph method, the response of each seismometer forms a trace on a multi-track variable density film. After the film is sent to the laboratory, it is passed through an analyzer and the response of all seismometers is combined by a photoelectric procedure, a delay being introduced for each trace to discriminate in favor of any particular direction of wave. In practice, the record is run through the analyzer for discrimination in successively different directions of specified interval, say 4°. The composite responses for each direction are recorded on a single sheet, one adjacent the other. (See Figure 339.) The angle of dip is determined by the direction of discrimination for which the highest composite amplitude is obtained. In regions where interference from waves arriving from two different directions is severe, the Sonograph often is advantageous for separating the component waves by discriminating first in one direction and then in the other.

Certain disadvantages, however, have been suggested with regard to the application of the Sonograph in general seismic prospecting. The equipment is more intricate than other seismic equipment and requires a considerably finer technique in overall operation. Records usually must be analyzed at headquarters, making it difficult for field operators to know with surety the condition of their records—a knowledge which is often necessary for planning of immediate field operations. Also, the analyzed composite record makes the differentiation between reflected and spurious energy somewhat more difficult.

Multiple Shot-Points

Beyond a certain maximum charge for a given shot-point the ratio of useful to extraneous response begins to decrease. (Compare p. 563.) Hence if the relative reflected energy of the "maximum" charge is

† F. Rieber, "A New Reflection System with Controlled Directional Sensitivity," *Geophysics*, Vol. 1, No. 1, Jan. 1936. "Visual Presentation of Elastic Wave Patterns Under Various Structural Conditions," *Geophysics*, Vol. 1, No. 2, July 1936. "Application of the Geo-Sonograph to Petroleum Exploration," *Petroleum Engineer*, Feb. 1937. "Complex Reflection Patterns and their Geologic Sources," *Geophysics*, Vol. 2, No. 2, March, 1937.

inadequate, more than one shot-point may be utilized, the charges in all shot-holes being fired simultaneously in as far as uniformity in detonation is possible. The effect of multiple shot-points is very much the same as that of multiple seismometers. Besides increased energy, therefore, there is also a discriminating effect which favors waves from an approximately vertical direction.

FIELD OF APPLICATION OF SEISMIC METHODS

Refraction profile methods are satisfactory for determining the depths of one or more well marked shallow boundaries, such as the low velocity layer, the water table, or the gravel beds in placer mining. Refraction profile methods are also useful in studies of soil dynamics designed to determine the suitability of soil for dam and building purposes, highways, etc.; the required thickness of concrete highways; the bearing capacity and depth of upper and lower beds, etc.† Fan shooting is most useful for finding shallow structures with quite definite velocity anomalies, for example, shallow salt domes and gravel deposits.

The reflection method has an advantage where depth is a factor, and generally is the only practicable method for depths in excess of about 5,000 feet. The method gives most accurate results when the low velocity layer is uniform and the structures under investigation have uniform and gentle relief. In broad areas where correlation is good, faults can be determined. The reflection method has not proved satisfactory in regions where the upper low velocity layer does not permit effective transmission of energy, where extraneous energy is excessive, and where the subsurface section is too complex to permit of an analysis, such as in severely fractured zones.

With time, as the more likely and readily investigated regions are explored, reflection shooting will be forced into greater refinements. The reflection shooting of the future will require greater depth of penetration, greater accuracy in detecting the low relief structural features previously overlooked, and more effective analysis to interpret data from complex structural and stratigraphic conditions, such as overlaps, stratigraphic traps, etc.

SEISMOMETERS

Seismometers usually comprise a mechanical system of desired frequency range which works in conjunction with some form of energy transfer device. The natural period of the mechanical system is usually within one of two ranges: (a) a much shorter period than that of the seismic waves to be received—which results in an approximately linear amplitude output—or (b) a period approximately equal to that of the seismic waves—which results in a considerably greater output

† Compare R. K. Bernhard, "Geophysical Study of Soil Dynamics," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. No. 834, Feb. 1938.

at frequencies close to the resonance frequency of the instrument. (In case (b) the output is controlled by some form of damping.) The energy transfer device may be of the electromagnetic, resistance, capacitative, or piezoelectric type, or combinations thereof, and will be described for various types employed in exploration seismology.

The essential parts of a seismometer are: a case which rests firmly on the earth, a heavy mass or weight which is attached to the case by means of a spring or other elastic arrangement, and a movable element which transforms the movement of the heavy mass to a form suitable for recording.

The arrival of seismic waves at the seismometer produces two momentary effects: (1) a displacement of the case which is in contact with the earth and (2) a displacement of the inertia reactor. That is, the seismic waves set both the case and the suspended inertia mass vibrating. The frequency of vibration of the case is approximately the same as that of the waves. The frequency of vibration of the inertia reactor depends on the natural frequency of the reactor system (mass plus suspension) as well as on the frequency of the seismic waves. In the absence of damping, the vibration of the inertia reactor system would continue even after the seismic waves ceased arriving. For this reason, the inertia reactor of practically all seismometers is provided with some form of external damping. The damping may be mechanical, electromagnetic, or both.*

Relation Between Absorption and Frequency of Seismic Waves and Design of Seismometers.—The amplitude of the elastic wave created by the explosive or mechanical disturbance decreases with increasing distance from the source. (Compare p. 457.) In particular, the change of amplitude with depth is especially marked, and, due to this factor, high sensitivity of seismometers and high amplification are necessary. (In this connection, it should be noted that the seismometer, amplifier and recording galvanometer all act together to produce the final overall response.)

The frequencies generally encountered in seismic exploration vary from 5 to 100 cycles per second. In the immediate vicinity of the disturbance the waves have a wide range of frequencies with the higher frequency components constituting a major portion of the energy. However, the absorption is greater for the higher frequencies; hence, as the distance from the disturbance is increased, the preponderance of wave energy shifts toward the lower frequencies.

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* For data on seismometer characteristics, particularly damping, see, for example, H. R. Prescott and A. P. Lipski, "System Sensitive to Vibrations," U. S. Patent 2,058,106, issued Oct. 20, 1936, and H. R. Prescott and K. C. Woodyard, "Seismophone," U. S. Patent 2,084,561, issued June 22, 1937.

Considering the detecting equipment (seismometer and recording system), one may note that the equipment must be so designed that the following three conditions are fulfilled: (1) The equipment must be sufficiently sensitive to detect and amplify the useful seismic disturbance; (2) the equipment must discriminate sufficiently against undesired energy to produce usable records; and (3) the system must have sufficient damping that successive seismic impulses can be recognized individually. In the matter of sensitivity, the energy output of the seismometer is of primary consideration. The seismometer must supply an impulse to the first stage of the amplifier which is sufficiently above the noise or microphone level of the first stage to allow effective amplification. (Above this level, amplifier units can be made to produce any degree of amplification desired.) Damping means must be present in both seismometer and amplifier units to avoid a sustained oscillation in either. The frequency response of the overall system must be sufficiently narrow that both the low frequencies characteristic of ground roll and the high frequencies (over 80 cycles per second) characteristic of wind and other disturbances are greatly attenuated. The amplifier and seismometer should be designed in such a way that their characteristics are complementary in order to obtain the desired overall response.

Classification of Seismometers.—Various arrangements are utilized for detecting the relative motion of the case with respect to the damped motion of the inertia reactor. Seismometers may be classified according to: (1) the type of detecting arrangement and (2) the aspect of the earth's motion which is detected. (Compare Table 21.)

TABLE 21

TYPES OF SEISMOMETERS

Type	*Type of Movement Measured*
Mechanical	Displacement
Electrical	
Electromagnetic reluctance	Velocity
Electromagnetic inductance	Velocity
Hot wire resistance	Velocity
Capacity (electrostatic)	Displacement
Photoelectric	Displacement
Piezoelectric	Acceleration
Carbon grain	Acceleration

The two types chiefly used in present day exploration work are the electromagnetic reluctance type and the electromagnetic inductance type.

Mechanical Seismometers.—At present, the mechanical seismometers are seldom used in prospecting. They are reviewed here chiefly because of their historical interest.

The Schweydar instrument † consists of two units so designed that one unit responds to the vertical component of the earth's motion and the other to the horizontal component. The inertia member of the vertical component unit is supported by a vertical spring. The movement of the seismometer case relative to the inertia reactor is first magnified by a light conical lever attached to the inertia member. At the free end of the lever is a "bow and string" mechanism which revolves about a small (3 mm. diameter) spindle supported between two jewelled bearings. The mirror is mounted on the spindle. A modified arrangement which has similar mechanical amplification but eliminates the difficulty due to the bearings is shown in Figure 314.

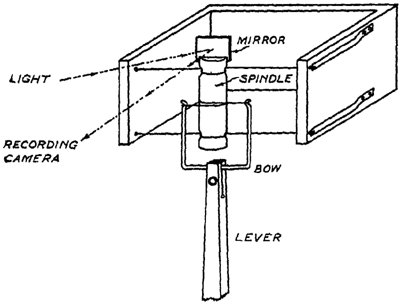

Fig. 314. — Diagram illustrating one type of mechanical and optical amplification.

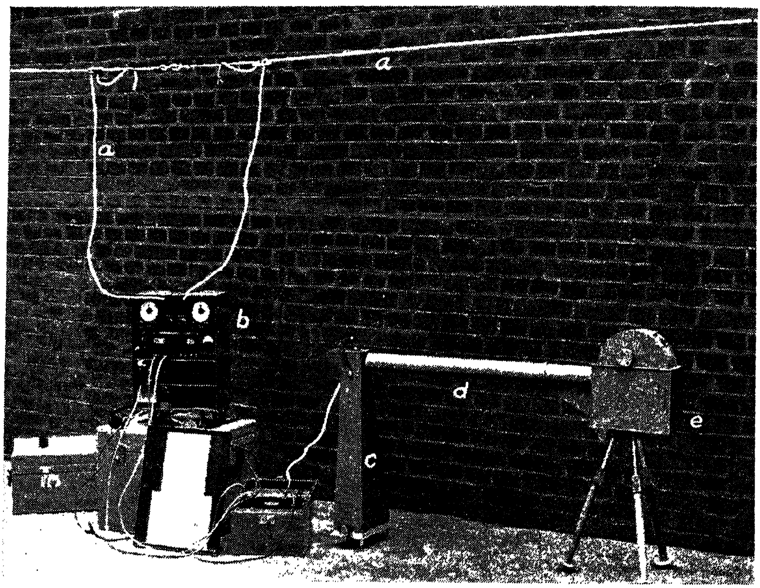

Fig. 315.—Early Askania mechanical seismograph. *a*, radio aerial; *b*, radio receiver for recording instant shot was fired; *c*, seismograph; *d*, light-tight tube for optical magnification; *e*, recording camera. (Courtesy of W. M. Rust, Jr.)

† W. Schweydar and H. Reich, "Künstliche elastische Bodenwellen als Hilfsmittel geologischer Forschung," *Gerland's Beiträge zur Geophysik*, 1927, 17, 121. Broughton Edge and Laby, *Geophysical Prospecting* (Cambridge Univ. Press, 1931), pp. 214-216.

The operation of the instrument may be summarized as follows. The arrival of seismic waves sets the seismometer case, and hence the bow, in motion. The movement of the bow rotates the mirror and hence causes a displacement of a beam of light which is reflected from the mirror to a photographic film in a camera located at a distance of about one meter.

A view of the Schweydar type seismograph is shown in Figure 315. At the left of the figure is a radio for receiving the shot-time from a radio transmitter connected to the firing circuit of the charge of explosive. The mechanical seismometer is separated from the recorder by a light-tight tube in order to obtain large optical magnification. This instrument has a combined mechanical and optical amplification of over 1500.

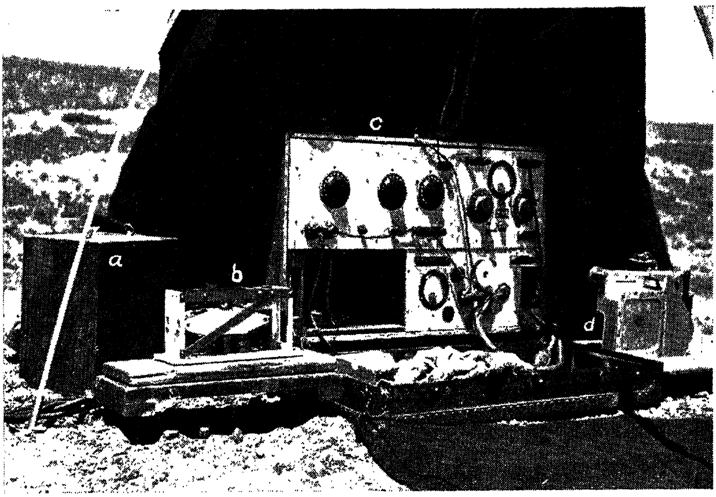

FIG. 316.—Layout of Taylor's early mechanical seismograph equipment in recording position. *a*, camera carrying case; *b*, seismometer; *c*, two-way radio; *d*, recording camera. (Courtesy of Continental Oil Co.)

The Taylor mechanical seismometer † was employed extensively in the Gulf Coast prior to the introduction of electric instruments. The characteristic feature of the seismometer is the provision of a thread suspension for the free end of the inertia mass. This suspension makes possible the conversion of the vertical motion of the inertia mass into rotary motion of a mirror which is readily adaptable to registration on a photographic film. The inertia mass comprises an aluminum frame filled with some heavy metal (usually a lead compound). The mass is pivoted by the end piece of the frame and supported by a steel spring. A cup partially filled with oil is provided to dampen the motion of the inertia mass when the instrument is in use. Light rays reflected from

† H. G. Taylor, "Seismometer," U. S. Patent 1,789,055, issued Jan. 13, 1931.

the mirror rigidly attached to the frame indicate the extent of the deflections of the inertia mass. The instrument has a combined mechanical and optical magnification exceeding 10.000.

Figure 316 shows a Taylor mechanical seismograph set up in recording position, tent open.

Disadvantages of Mechanical Seismometers

The mechanical type seismometer requires a solid foundation, such as hard ground or a heavy wooden platform. This greatly handicaps use of the instrument, particularly in the numerous areas where marsh and open waters prevail. In addition, the single trace record necessitates many instruments placed at different locations for each shot or else many shots for a single instrument.

Electric Seismometers.—The electric seismometer comprises two mutually coupled dynamical systems: viz., a mechanical system which is actuated by the wave motion of the earth's surface and an electrical system which is driven by the mechanical system. In contrast to most mechanical seismometers which respond to the displacement of the earth, electrical seismometers respond to the displacement, velocity, or acceleration of the earth. (Compare also Table 21.)

The generalized analysis of the reaction between the electrical and mechanical systems constituting a seismometer utilizes the Lagrangian equations for a dynamical system. Before introducing these equations, it will be convenient to describe an equivalence in energy terms for mechanical and electrical systems.

For linear motion in a mechanical system, the kinetic energy, equal to T_m, has the form: †

$$T_m = \tfrac{1}{2} m\dot{s}^2$$

where m is the mass, and $\dot{s}$ the velocity of the moving element. The potential energy V_m may be written as follows:

$$V_m = \tfrac{1}{2} \cdot \frac{s^2}{C_m}$$

where s is the displacement and C_m is a constant (the compliance) of the moving element. The dissipation function D_m which is a measure of the energy dissipated due to friction will be expressed by the relation:

$$D_m = \tfrac{1}{2} r_m\dot{s}^2$$

where r_m is a damping constant which depends on the side slip between the moving mass and a viscous medium.*

The equivalent expressions for an electrical system are:

$$T_e = \tfrac{1}{2} Li^2$$

where L is the inductance and i the current;

$$V_e = \tfrac{1}{2}\frac{q^2}{C}$$

where q is the electric charge and C the capacity of the system;

$$D_e = \tfrac{1}{2} ri^2$$

where r is the electrical resistance.**

† Compare H. Pender and K. McIlwain, *Electrical Engineers' Handbook*, Vol. V, Section 3 (Wiley and Sons, New York).

* This assumes that the damping is proportional to the square of the velocity, which is often a good approximation.

** The dissipation function as defined here differs from the Joule heating in an electric circuit by a factor of ½.

The equations of motion of the combined electrical and mechanical systems which constitute a seismometer may be obtained by using Lagrange's equations employing generalized coordinates.† In particular, most seismometers have two degrees of freedom, one corresponding to the displacement of the moving mass and the other to the flow of electrical charge or current, and the Lagrangian equations for a seismometer are:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{s}}-\frac{\partial (T-V)}{\partial s}+\frac{\partial D}{\partial \dot{s}}=f \tag{81}$$

and

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}}-\frac{\partial (T-V)}{\partial q}+\frac{\partial D}{\partial \dot{q}}=c \tag{82}$$

The symbols T, V, D, s and $\dot{s}$ have the meanings previously indicated; d/dt denotes as usual a time derivative; q is the electrical charge; $\dot{q}$ is the time rate of change of electrical charge; f is the externally applied force; and c is the E.M.F. in the electrical circuit.

The motion of the electromechanical system constituting the seismometer is specified when the quantities representing the kinetic energy (mechanical plus magnetic), the potential energy (mechanical plus electrical) and the dissipation function (friction plus electrical heating) are substituted into the two Lagrangian equations of motion.

The Lagrangian equations for an electromagnetic reluctance type seismometer are given in the following section. The Lagrangian equations for certain other types of electric seismometers will be found in an article by Scherbatskoy and Neufeld. ‡

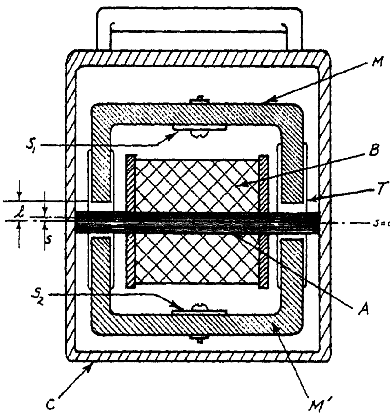

FIG. 317. — Reluctance seismometer, duplex-type. A, laminated armature; B, armature coil; C, aluminum case, waterproof type; M, permanent magnets; S_1 and S_2, suspension springs for the magnetic system (end view of springs); T, brass yoke.

† A discussion of Lagrange's equations is given in many texts on theoretical mechanics. See, for example, A. Zivet and P. Field, *Introduction to Analytical Mechanics*, pp. 352-359 (Macmillan Co., New York), 1926. A more generalized discussion is given by J. H. Jeans, *Mathematical Theory of Electricity and Magnetism*, 5th Edition, pp. 489-498 (Cambr. Univ. Press, 1927).

‡ S. A. Scherbatskoy and J. Neufeld, "Fundamental Relations in Seismometry," *Geophysics*, Vol. II, No. 3, July, 1937, pp. 192-212.

Electromagnetic Reluctance Seismometer

The reluctance-type seismometers are those wherein the reluctance of the mag- ath is caused to vary by the relative movement of the inertia reactor with ct to the seismometer housing. Numerous instruments of this type are used for ı earthquake and exploration

In one type of reluctance seismometer, two permanent magnets are supported by which are attached to the case. (Figure 317.) The laminated armature in the air gap between the magnets and is rigidly connected to the case. The motion rth's surface during the passage of seismic waves prod relative moti the E.M.F. in the armature coil.

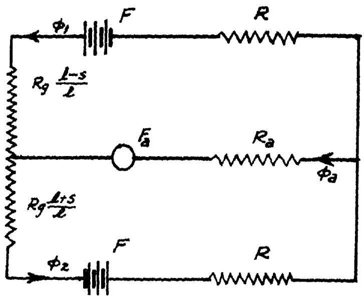

FIG. 318.—Magnetic circuit of an electromagnetic duplex reluctance seismometer. (After Scherbatskoy and Neufeld, *Geophysics.*)

To determine the quantities T, V, D for a reluctance type seismometer, it is convenient to refer to the equivalent magnetic circuit shown in Figure 318. The symbols denote the following quantities: †

F = magnetomotive force of each magnet.

R = reluctance of each magnet.

R_a = reluctance of the armature.

R_g = reluctance of each air gap when $s = 0$, where the reference line $s = 0$ is located midway between the magnets when they are undeflected. From symmetry R_g also equals the reluctance of the two air gaps in series above, or below, the neutral line $s = 0$.

l = one-half the vertical distance between the pole pieces of the two magnets when the magnets are undeflected.

$R_g \dfrac{l-s}{l}$ = reluctance of two air gaps in series above the armature corresponding to a displacement s of the reference line.

$R_g \dfrac{l+s}{l}$ = reluctance of two air gaps in series below the armature for a displacement s.

ϕ_1 = magnetic flux through upper magnet.

ϕ_2 = magnetic flux through lower magnet.

ϕ_a = magnetic flux through armature.

F_a = magnetomotive force created by the current in the armature coil.

† Scherbatskoy and Neufeld, *loc. cit.*

The application of Kirchoff's laws to the two meshes of Figure 318 yields

$$\phi_a = \phi_1 - \phi_2$$

$$? = \phi_1 \left(R_g \frac{}{l} \qquad + \phi_a R_a\right.$$

$$\frac{l+s}{l} \qquad - \phi_a R_a$$

Whence

$$\phi_1 = \frac{F}{\left(R_g \frac{l+s}{l} + R + R_a\right)\left(R_g \frac{l-s}{l} + R + R_a\right) - R_a^2}$$

$$\phi_2 = \frac{F\left(R_g \frac{l-s}{l} + R + 2R_a\right)}{\left(R_g \frac{l+s}{l} + R + R_a\right)\left(R_g \frac{l-s}{l} + R + R_a\right) - R_a^2}$$

$$\phi_a = \frac{2FR_g \frac{s}{l}}{\frac{l-s}{l} + R + R_a\Big) -}$$

The total kinetic energy of the electromechanical system is given by

$$T = \tfrac{1}{2} m\dot{s}^2 + T_{e1} + T_{e2} + T_{e3}$$

where

m = combined mass of magnets and springs (moving parts).
T_{e1} = energy stored in permanent magnetic field.
T_{e2} = magnetic energy due to current i induced in the armature coil.
T_{e3} = magnetic energy due to reaction of the current i on the permanent magnetic field.

If it is assumed for simplicity that the magnetic energy is contained only within the air gaps and the magnetic material and not in the space outside,

or

T_{e1} =

It may be shown by using Ampere's law (magnetomotive force $= 0.4\,\pi$ x ampere turns $= 0.4\,\pi\, n\, i$) that the magnetic energy T_{e2} associated with the current i is given by the equation

$$T_{e2} = \frac{0.4\pi\, n\, i\, \phi_i}{8\pi} \cdot 10^{-7} \qquad (85)$$

where ϕ_i, the flux created by the current i, is equal to the magnetomotive force around the magnetic circuit (Figure 318) divided by the reluctance of the circuit.

The magnetic energy T_{e3} associated with the reaction of the current i on the permanent magnetic field is given by the equation:

* In setting up this equation and various subsequent equations, it is assumed that T, V, and D are expressed in joules; s in centimeters; $\dot{s}$ in centimeters per sec.; q in coulombs; $\dot{q} = i$ in amperes; m in grams x 10^{-7}; r_m in dynes sec. per cm. x 10^7; C in farads; r in ohms; L in henrys; f in dynes x 10^{-7}, e in volts; F in gilberts; R in oersteds; ϕ in maxwells.

If each of the equations 84, 85 and 86 is expanded in a Maclaurin series and if all terms involving powers of s higher than the first are dropped, the *total* kinetic energy of the mechanical and electrical parts of the seismometer may be expressed by the relation

(87)

where

$$L_1 \equiv \quad \cdot 10^{-8} \quad \text{and} \quad K \equiv$$

The potential energy of the system is due entirely to the mechanical part, because the electrical circuit is assumed to have no capacity; that is,

$$V = V_m = \frac{\quad}{2C_m} \tag{88}$$

where C_m is the compliance of the spring supporting the horseshoe magnets.

The total dissipation function is

$$^{2} \tag{89}$$

where r_m is the mechanical resistance factor (determined by the side slip between the moving horseshoes and the air) and r is the resistance of the armature coils.

The equations of motion of the electromagnetic reluctance seismometer may be obtained by substituting the expressions for T, V, and D as given by Equations 87, 88, and 89 into the Lagrangian equations (81 and 82). In carrying out this substitution, certain terms will be zero and certain other terms will be constants which for the sake of simplicity may be set equal to zero. For example, T_e (equal to $T_{e1} + T_{e2} + T_{e3}$) is a function of s only. Hence, the derivative of T_e with respect to $\dot{s}$ vanishes; i.e.,

Also $\frac{\partial}{\partial s}\left(\frac{\quad_{e1}}{ds}\right)_{s=0}$ is a constant which may be set equal to zero, and $\frac{\partial}{\partial s}(L_1)_{s=0}$ is also a constant which may be set equal to zero. However, it will not be assumed that $\left(\frac{dK}{ds}\right)_{s=0}$ is zero, because that would be equivalent to assuming that there is no viscous drag. As a matter of convenience in notation, set

$$\left(\frac{dK}{ds}\right) \equiv K_m \tag{91}$$

Various derivatives of the energies with respect to the electric charge q and the current i (equal to $\dot{q}$) likewise vanish. Thus T_m equal to $\frac{1}{2}m\dot{s}^2$ is not a function of i or q and $T_e = T_{e1} + T_{e2} + T_{e3}$ is not a function of q. Hence,

$$\frac{\partial T_m}{\partial i} = \frac{\partial T_m}{\partial q} = \frac{\partial T_e}{\partial q} = 0 \tag{92}$$

Also

$$\partial \dot{q} \quad - \quad \partial i \qquad \overline{\partial i}$$

$$\partial T_e \; - \; ,$$

(93)

Setting

$$\imath \equiv L \tag{94}$$

$$\tag{95}$$

Also

$$\frac{\partial V}{\partial i} = \frac{\partial V}{\partial q} = 0 \tag{96}$$

On substituting into the Lagrangian equation (81), the expressions for T (Equation 87) for V (Equation 88) and for D (Equation 89), and making use of Equations 90 and 91, the following relation is obtained:

$$\frac{d}{dt}\frac{\partial T_m}{\partial \dot{s}} + \frac{d}{dt}\ \partial \dot{s} \qquad \frac{\partial T_m}{\partial s}$$

$$\tag{97}$$

where $\ddot{s} \equiv \frac{d^2s}{dt^2}$

Similarly on substituting the expressions for T, V and D and making use of Equations 92 through 96, the Lagrangian equation (82) becomes:

$$\frac{d}{dt}\frac{\partial T_m}{\partial i} + \frac{d}{dt}\frac{\partial T_e}{\partial i} \quad \partial T_e \quad \frac{\partial T_m}{\partial q} + \frac{\partial V}{\partial q} \qquad \partial q$$

$$di \tag{98}$$

Equations 97 and 98 are the fundamental equations of motion of the mechanical and electrical parts of the seismometer. Equation 97, which describes the motion of the mechanical part, contains one term, $-K_m i$, which depends on the electrical circuit, and Equation 98, which describes the conditions in the electrical circuit, contains one term, $K_m \dot{s}$ which depends on the relative *velocity* of the moving element in the mechanical system.

It is possible to solve Equations 97 and 98 so as to obtain the current in the electrical circuit as a function of the relative velocity of the moving element in the mechanical circuit, *provided* an explicit form is assumed for the mechanical force f. This is equivalent to determining the current in the electrical circuit as a function of the velocity of the ground at the seismometer station.*

Type Instruments.—Figure 319 is a schematic diagram of an early Benioff reluctance type seismometer designed to respond to the vertical component of the earth's velocity during the arrival of seismic waves.† The coils are wound around the pole pieces energized by a horseshoe magnet. The magnet path is divided between the gap RS and the double gap $R/2$, with reluctance of the latter varied by the armature. The pole pieces and magnet are attached to an inertia reactor which is supported by a helical spring attached to the seismometer case. The working air gaps

* The procedure for carrying out the simultaneous solution of Equations 97 and 98 is indicated by Scherbatskoy and Neufeld, *loc. cit.*

† H. Benioff, "A New Vertical Seismograph," *Bull. of the Seismological Society of America*, Vol. 22, No. 2, June, 1932, pp. 155-169. A critical discussion of the theory of the Benioff seismometer has been given by E. C. Bullard in the *Monthly Notices of the Royal Astronomical Society, Geophysical Supplement*, Vol. 4, No. 5, May, 1938, pp. 336-340. (Royal Astronomical Society, Burlington House, London, W. I.)

of reluctance $R/2$ each are included between the pole pieces and the armature. Displacements of the inertia reactor relative to the supporting system of the seismometer alter the length of the working air gaps and

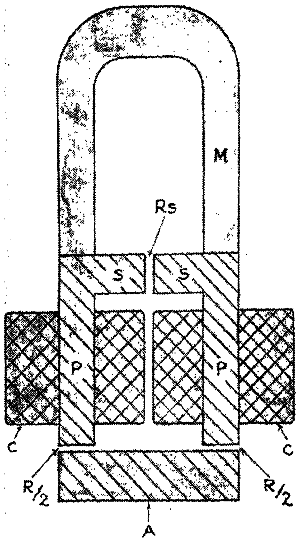

FIG. 319.—Electromagnetic reluctance seismometer. A, armature; C, coils; M, magnet; P, pole pieces. (Benioff, *Bulletin of the American Seismological Society.*)

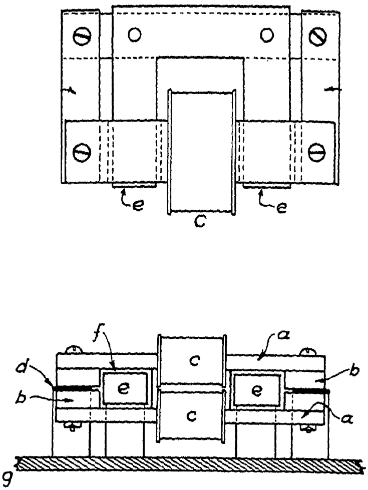

FIG. 320.—Electromagnetic reluctance seismometer. *a*, armature or pole pieces; *b*, spacer blocks (non-magnetic); *c*, coils; *d*, suspension springs; *e*, magnet poles; *f*, case of instrument; *g*, lid of instrument.

this in turn varies the magnetic flux. The changes in magnetic flux induce E.M.F.'s in the coils and the induced E.M.F.'s produce currents which may be measured by means of a galvanometer.

Another modification of the reluctance type seismometer utilizing a differential magnetic path is shown diagrammatically in Figure 320.† A photograph of the interior of the instrument is shown in Figure 321.

The horizontally suspended permanent magnet is of the U type, and is supported by a leaf spring. The two armatures or pole pieces are made of special low retentivity iron and carry the two coils. Vibratory movement of the case of the instrument is transmitted to the armatures, while the inertia of the magnets tends to hold them stationary, with a resultant change in flux through the coils.

† H. Salvatori, "Seismometer," U. S. Patent 2,111,643, issued Mar. 22, 1938.

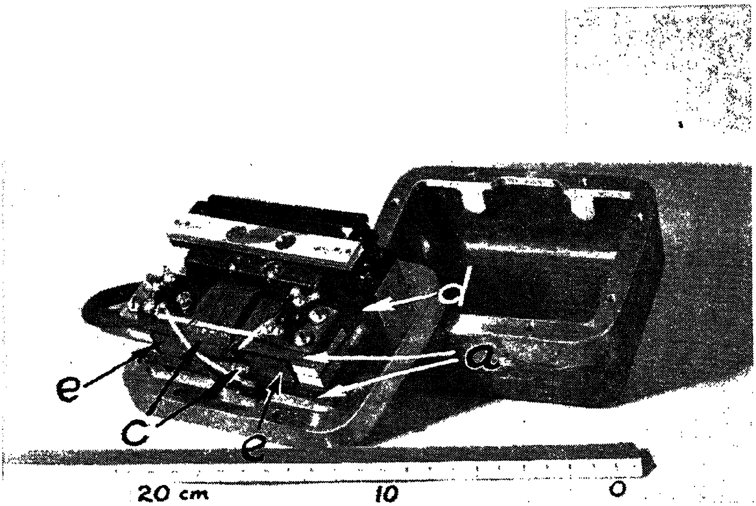

FIG. 321.—Reluctance seismometer. A centimeter scale indicates the size of the instrument. *a*, armature; *c*, coils; *d*, cantilever spring suspension; *e*, magnet pole pieces.

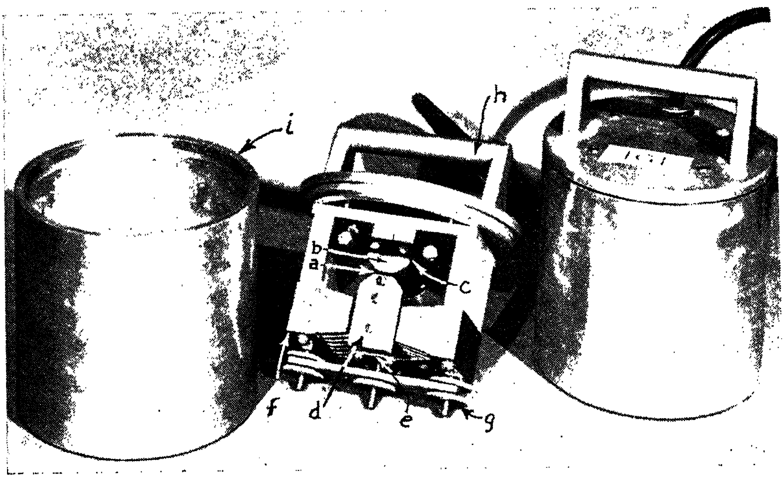

FIG. 322.—Reluctance type seismometer. *a*, air gap (greatly enlarged for photograph); *b*, laminated iron core; *c*, coil; *d*, moving armature, with permanent magnets; *e*, suspension spring, cantilever type; *f*, copper vanes for electromagnetic damping; *g*, amplitude adjustment and stops; *h*, carrying handle; *i*, waterproof case, aluminum.

Another reluctance type seismometer employing considerable electromagnetic damping is shown in Figure 322. The instrument is designed for surface work and is about five inches in diameter.

The reluctance type may have some mechanical damping as well as some electromagnetic damping. The disadvantage of mechanical damping is that it varies greatly with temperature. Consequently, the frequency response and sensitivity become functions of temperature.

Electromagnetic Inductance Seismometer

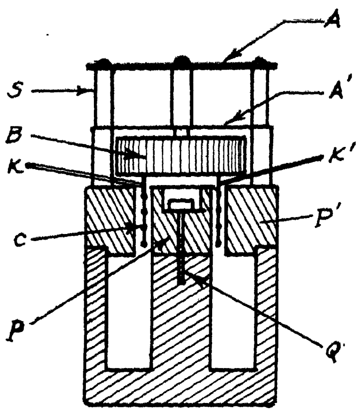

FIG. 323.—Electromagnetic inductance or moving coil seismometer. *A* and *A'*, suspension springs for supporting the mass *B* and the coil *C*; *K* and *K'*, flexible terminals for coil *P*; *P'*, cylindrical pole piece; *Q*, centering screw; *S*, spacer supports.

The inductance type seismometer utilizes a coil which can move in a magnetic field in such a manner as to generate a potential which is in phase with its motion. One form of inductance seismometer (Figure 323) comprises a circular conducting coil supported by elastic springs and a magnetic system rigidly attached to the case. The arrival of seismic waves produces a relative motion of the elastically supported coil with respect to the magnet. The relative motion, in turn, induces an electric current in the coil which depends on the velocity of the ground at the seismometer station. (Another form of inductance seismometer is shown in Figure 252.)

Piezoelectric Seismometer

The piezoelectric seismometer consists essentially of a crystal of anisotropic material such as quartz, tourmaline, Rochelle salt, etc., and an inertia reactor. The motion of the earth produces a relative acceleration of the inertia reactor with respect to the crystal. This relative acceleration in turn produces a potential difference between opposite faces of the crystal.

Several advantages are claimed for the piezoelectric pickup; chief of which are: (1) the seismometer does not have moving parts such as elements supported by elastic springs; (2) the voltages generated are proportional to the acceleration of the ground rather than to the velocity,

as in the case of the electromagnetic detector. The chief disadvantages of the piezoelectric detector are its low sensitivity and the variation of sensitivity with temperature and with moisture. The last factor makes it necessary to keep the inside of the case free of all moisture, and usually some type of desiccator is employed. The low sensitivity requires an additional stage of amplification which must be housed in the seismometer case in order to raise the signal energy above the noise level of the cable, which preferably is of the shielded type. Also, the fragility of the crystal necessitates some form of clamping device which will protect the crystal from mechanical damage during transportation.

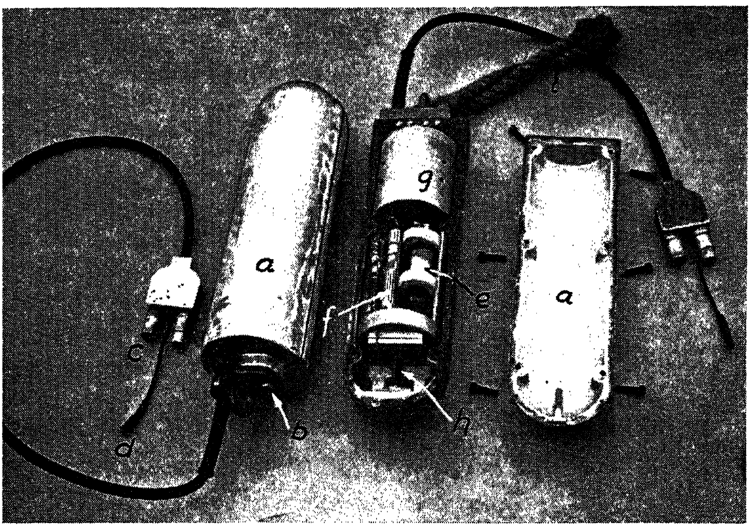

FIG. 324.—Piezoelectric type seismometer. *a*, waterproof case; *b*, clamp screw for locking instrument; *c* and *d*, double connector for filament and plate supply and pre-amplifier output; *e*, amplifier tube; *f*, grid bias battery; *g*, transformer; *h*, piezoelectric crystal; *i*, carrying rope.

Figure 324 shows the exterior and interior views of one type of piezoelectric seismometer. The mass acting on the crystal comprises the entire interior of the seismometer, including transformer, pre-amplifying stage, and chassis. A clamp screw is provided for locking the mechanism during transport to prevent injury to the crystal. The crystal is of composite construction having four separate crystals electrically connected in parallel, with their "Z" axes vertical. The output of the crystal is impressed across a high resistance in the grid circuit of the pre-amplifying tube.

Capacity or Electrostatic Seismometer

The coupling between the mechanical and electrical systems of a capacity seismometer (Figure 325) is accomplished by means of a condenser so designed that the displacement of the earth's surface during the arrival of seismic waves produces a relative displacement of the plates of the condenser. The varying capacity may be made to induce electrostatically a varying electromotive force in an auxiliary circuit.*

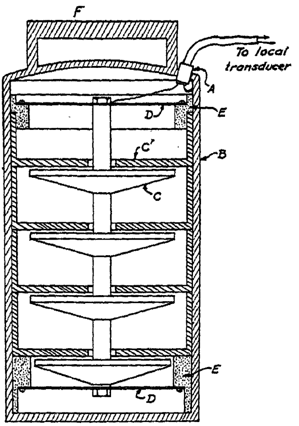

FIG. 325.—Condenser type seismometer. *A*, waterproof connector plug; *B*, case; *C* and *C'*, condenser plates; *D*, diaphragm; *E*, bakelite support ring; *F*, carrying handle.

In one type, the varying capacity causes a current flow in a high potential tuned circuit. In another type of capacity seismometer, the change in capacity is made to carry the frequency of an oscillating circuit. This circuit is usually heterodyned with another oscillating circuit to produce a beat frequency, which is amplified and rectified, and then recorded in the conventional manner.

Hot Wire Resistance Seismometer

The operation of the hot wire resistance seismometer depends on changes in the resistance of a hot wire (about 0.005 mm. diameter) when cooled by an air current which is produced by the motion of the inertia mass relative to the seismometer case. Because the changes in resistance depend on the motion of the air, the instrument responds to the *velocity* of the earth's surface at the seismometer station.

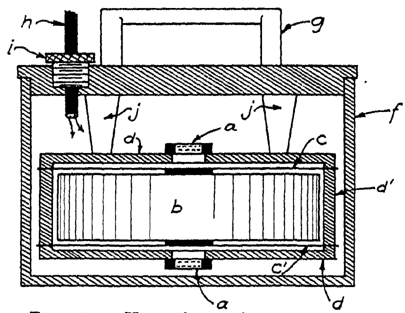

FIG. 326.—Hot wire resistance seismometer. *a*, fine wire grid; *b*, inertia mass; *c* and *c'*, upper and lower diaphragm supports; *d*, air chamber ring for diaphragms; *d'*, spacer ring; *f*, aluminum instrument case; *g*, carrying handle; *h*, cable (2 wire) *i*, water-tight lead in bushing; *j*, spacers.

An instrument of this type is illustrated schematically in Figure 326. It comprises a mass and diaphragm so mounted that they constitute a "bellows" arrangement, whereby relative motion of the mass and the case causes a movement of air past the hot wire grid. Two grids are employed, connected in series. A double diaphragm is employed to provide a good mechanical support for the mass and also to form a chamber for the oil serving as a

* The relation between the time rate of change of electric charge on the condenser and the relative displacement of the condenser plates may be obtained from the Lagrangian equations in a manner similar to that outlined above for the reluctance type seismometer.

damping medium. The milliampere current required for heating the grids is supplied by a central battery in the recording truck. By means of a transformer in series with the line to the seismometer, the variations in current are picked up and impressed on the amplifiers for recording in the conventional manner. Usually one or two stages of amplification are more than sufficient. This type of seismometer has been found to be especially suitable for the lower frequencies. (Also, it is better adapted for refraction prospecting than for reflection prospecting.)

Carbon Button Seismometer

This is one of the simplest forms of seismometers. The operation of the carbon button or carbon granule type seismometer depends on the changes of resistance which occur in carbon due to pressure variations. The resistance changes produce corresponding changes in current. Usually the carbon button is connected in series with the primary of an impedance matching transformer and a source of current (local dry cell). Variations in primary current cause corresponding variations in potential on the secondary of the transformer. The secondary is connected to the recording galvanometer. This type of seismometer is very efficient and has a high output. For shallow work it may be used without additional amplification. The carbon seismometer, however, is seldom used for deep work due to the high noise level caused by "frying" of the grains. These microphonic noises may be minimized to a certain extent by using two carbon buttons in a push-pull circuit in the primary of the transformer.

Photoelectric Seismometer

The photoelectric seismometer is a displacement type instrument wherein the relative motion of the inertia mass with respect to the seismometer case modulates the light which falls on a photoelectric cell. The instrument is shown schematically in Figure 327. The operation of the seismometer may be summarized as follows: The motion of the earth's surface produces a relative displacement of a mass, thereby actuating a pivoted mirror through a connecting link. Light rays from a small lamp are focused on the mirror by a condensing lens and are then reflected to a photoelectric cell. The output of the photoelectric cell depends on the displacement of the mass actuating the pivoted mirror.*

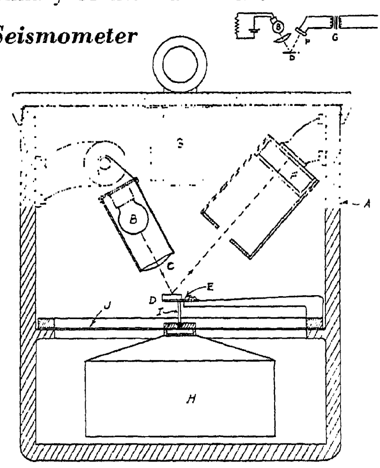

FIG. 327.—Schematic diagram of a photoelectric seismometer. *A*, case; *B*, light source; *C*, condensing lens; *D*, mirror; *E*, flexible support; *F*, photoelectric cell; *G*, impedance matching transformer between cell and line; *H*, stationary mass; *I*, connecting link between mass and mirror; *J*, diaphragm support for mass.

* The cell may be connected directly to a transformer for proper impedance match.

A seismometer of this type has certain definite advantages: namely constancy of response characteristics, negligible temperature coefficient, and a relatively high output level. One disadvantage of the instrument is the necessity for a local battery (usually placed adjacent the seismometer) or an extra pair of conductors to light the filament.

RECORDING EQUIPMENT

The output of the seismometer, obtained by converting the energy of the seismic wave into electrical energy, is amplified, filtered and then recorded by photographic means. In addition, it is usually necessary to employ manual or automatic gain control to prevent overloading by the stronger waves and to bring out the weaker waves.

The minute currents generated by the dynamic action of the seismometer are usually transmitted through an insulated two-wire cable to the recording truck which houses the filtering, amplifying, and recording instruments.

Filtering.—The seismic waves to be recorded are essentially transient disturbances and are composed of heterogeneous wave shapes of many frequencies and rates of attenuation. The predominant frequency range of the most useful bands is from 5 to 40 cycles per second for refracted waves and from 30 to 100 cycles per second for reflected waves. The quality of the record often may be improved by judicious selection of frequencies that are to be recorded. As an illustration, the filters and overall response range for reflection equipment is often chosen to cover the range from 20 to 100 cycles per second. The lower cut-off is preferably quite sharp, while the upper cut-off should be rather broad so as not to destroy the sharpness of the reflections. Surface disturbances caused by light winds can often be largely eliminated by attenuation of waves of frequencies above about 80 cycles per second.

In reflection work, the proper use of filtering can oftentimes make the difference between easily identified records and doubtful records. Excessive high frequency filtering, however, minimizes the first portion of a reflection and also masks the beginning of the reflection to such an extent that it is impossible to pick the instant of arrival reliably. In such cases it is necessary to pick corresponding peaks or troughs (or other characteristic portions) of the waves for determining relative arrival times on the different traces of the record.

Because filtering introduces a phase change, any variation in the degree of filtering causes a shift in phase with a resultant shift in time of recording of maximum amplitude. This shift must be considered whenever records obtained with different filtering are compared.

A high degree of damping generally should be employed in order to allow favorable response to the transient waves of different frequencies. Damping at or beyond the critical value is sometimes achieved by using multi-section filters to give the desired frequency range. A controversial question, at present, concerns the desirability of an over-damped multi-section filter versus an under-damped filter of fewer sections.

The degree of filtering is also dependent upon whether single seismometers or multi-detection systems are employed. If multi-detection is employed the low frequency surface waves arrive at each of the seismometers at different time phases, while the reflected (vertical) waves arrive at substantially the same time. As a result the low frequency waves tend to "smooth-out" while the vertical or reflected waves tend to reinforce each other. When a multi-seismometer hook-up is employed, less low frequency filtering need be employed than for the single seismometer per trace method.

A cross-feed system is advocated by some operators and gives good results under certain conditions. In this system part of the energy from each amplifier is fed to the other amplifiers, and a discrimination is obtained in favor of the desired vertical waves, which is similar to that described in an earlier paragraph on overlapping of seismometer output. As before, however, such methods are advantageously employed only with proper knowledge of their limitations and effects on the final record. (Compare also p. 566.)

One contracting company makes three separate records with discrimination curves covering approximately the ranges of 40-70, 30-50, and 20-40 cycles per second in difficult areas.

Amplication Control.—Following the strong burst of energy at the first part of a field record, the level of reflected energy will diminish down the record. The ratio of legible, reflected energy at the first and last part of a seismometer record may be from 10^3 to 1 up to 10^4 to 1. Consequently, some variable control of the amplification or response of the recording equipment is required in order that the entire wave train be legible. Three methods of overcoming this difficulty are in use.

A simple method from the point of view of recording equipment is to shoot several shots, setting the amplifier at different levels of gain to obtain legible amplitude at successively different reflection times. (An equally simple method is to use different amounts of explosive with the same level of gain.) A variation of this method is a manual control of gain by which the operator varies the gain during the shot, while watching the

incoming signal. Manual control is used in order to prevent the deleterious effects of automatic volume control which tends to smooth out peaks and minimize character of a record.

The second method is to have a semi-automatic device which varies the sensitivity with time in a predetermined manner.† In this method, the reflections are recorded through amplifiers whose gain varies at a governed, predetermined rate of increase.

The third method is an automatic volume control arrangement (Figure 328) which is essentially the same as that in a radio receiver. The rectified output voltage is applied to the grids of the preceding tubes through low pass filters so that as the output voltage decreases, the amplification of each tube is increased.‡

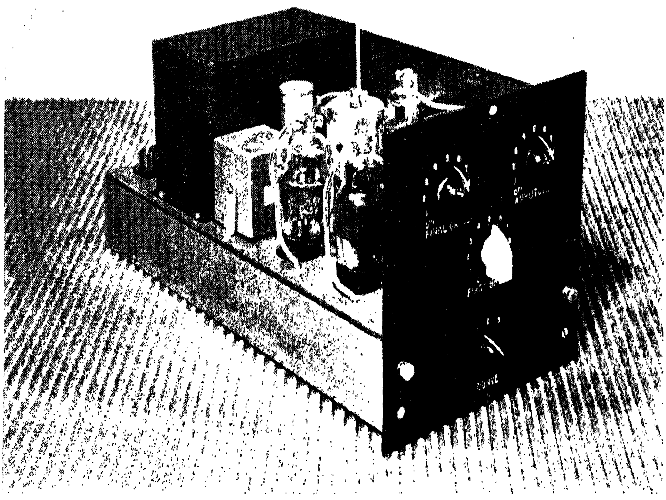

FIG. 328.—Seismic amplifier unit. (Courtesy of Western Geophysical Co.)

The automatic control is designed to be responsive to a group of wave trains or the envelope of the waves and not to each individual peak or transient. This is usually accomplished by utilizing some form of vacuum tube time delay and time averaging circuit.

† H. R. Prescott, "Method and Apparatus for Making Geological Explorations," U. S. Patent 2,158,198, issued May 16, 1939.

‡ A typical band pass filter, which is capable of making a frequency selection in the amplifier, is described by H. R. Prescott and F. L. Searcy, "Method of Geological Exploration," U. S. Patent 2,049,727, issued Aug. 4, 1936, and by H. R. Prescott, "Method and Apparatus for Making Geophysical Explorations," U. S. Patent 2,053,841, issued Sept. 8, 1936.

Amplifiers

The electrical amplification of the potentials generated by the seismometers is done by conventional equipment. The design of the amplifier follows the usual practices employed for high fidelity audio-frequency transformer or resistance coupled amplifiers. From three to five stages of vacuum tube amplification are frequently employed. The first input stage is usually a triode tube to minimize undesirable pickup and interference effects. The succeeding stages preferably utilize pentode tubes because of their superior operating characteristics.

In automatic volume control equipment developed by Nash and Palmer, each amplifier is coupled to a single triode tube which rectifies a portion of the output which then is utilized to furnish the necessary bias for amplification control.† A schematic diagram of connections for the equipment is shown in Figure 329.

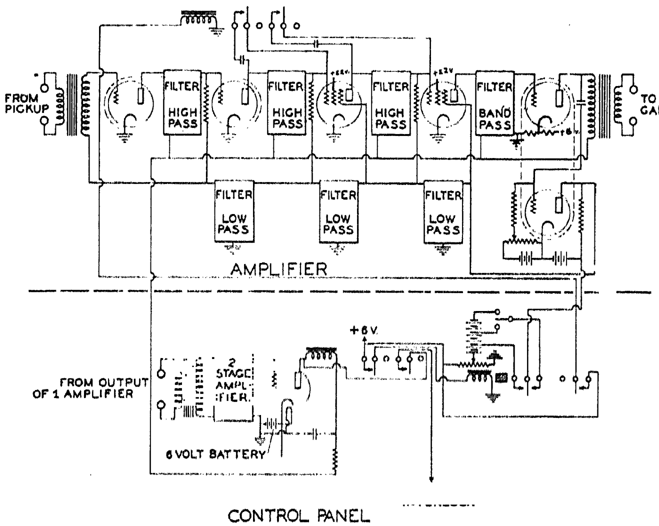

FIG. 329.—Schematic diagram of control panel and amplifier. (Nash and Palmer, *Amer. Inst. of Electrical Engineers.*)

The output of the amplifier is applied to the recording galvanometer through a transformer to allow proper matching of the impedance of the output tube with that of the galvanometer.

The amplifiers are of "unit" type construction to allow standardization in manufacture and the rapid interchange of units should operating difficulties develop. As may be seen from Figure 328, unit construction utilizes

† H. E. Nash and C. C. Palmer, "Electrical Equipment Used in Reflection Seismograph Prospecting," *American Inst. of Electrical Engineers*, April, 1939

a standard mounting chassis and individual control panel that may be inserted in a master control panel of the type shown in Figure 329.

Shielding.—All parts of the electrical system (seismometers, connecting cables, amplifiers, recording camera, and controls) are preferably shielded against disturbing magnetic and electrostatic fields. This is particularly advantageous when working in the vicinity of high tension power lines, street car systems, etc.* The circuits carrying the minute currents generated by the seismometers must be well insulated from the case of the seismometer and from the ground. Variable earth potentials are frequently present in the ground between the locations of the various seismometers and the recording truck. These earth potentials due to their varying magnitude may cause disturbances if picked up by the input circuit.

The case of the seismometer is usually of iron or aluminum, and serves as an electric shield as well as a mechanical protection for the instrument.

Photographic Recording.—The photographic method of recording, because of its sensitivity, reliability, and accuracy, has superseded practically all others: Recording apparatus for seismic field work should be: (1) rugged, (2) compact, (3) completely light-proof, (4) unaffected by extraneous vibrations and field transportation, (5) of sufficient recording paper capacity, (6) designed to use spools or rolls of recording paper which are commercially available, and (7) preferably equipped for visual observation of the recording traces while recording.

Galvanometers

The galvanometers are usually of the Einthoven type or of the D'Arsonval type. From a mechanical viewpoint, the Einthoven or string galvanometer is simpler than the D'Arsonval or moving coil galvanometer.

Einthoven or String Type.—This type of galvanometer consists of a single electromagnet with a number of wires strung through an air gap in the magnet. Each wire carries one of the amplified seismometer potentials to be recorded. A beam of parallel light rays passing through the air gap causes the strings to cast shadows on the photographic paper. These shadows reproduce as white traces when the record is developed. The strings may be oil damped.

Certain disadvantages of the instrument may be summarized as follows: In order to obtain sufficient optical amplification of the movement of the strings, a lens of short focal length is utilized. When a large number of strings is required it may be difficult to focus all strings sharply

* A method of eliminating the effect of extraneous electrical disturbances, with particular reference to power line interferences, ground motions, etc., is described by K. C. Woodyard, C. A. Putnam and H. R. Prescott, "Means and Method of Making Geophysical Explorations," U. S. Patent 2,164,196, issued June 27, 1939.

on the record. Occasionally, when excessively strong waves are recorded, the strings collide and may tangle. The black background makes it inconvenient to mark reference points and notes on the record. The chief disadvantage of the string galvanometer arises from the fact that if more than about 10 strings are desired, the construction becomes somewhat cumbersome.

The low impedance of this type of galvanometer necessitates a matching transformer, thereby making it unsuitable for D.C. potential work. However, the low impedance is not a disadvantage, because proper transformer ratios can be employed to match the output impedance of the amplifying tube with the low input impedance of the galvanometer.

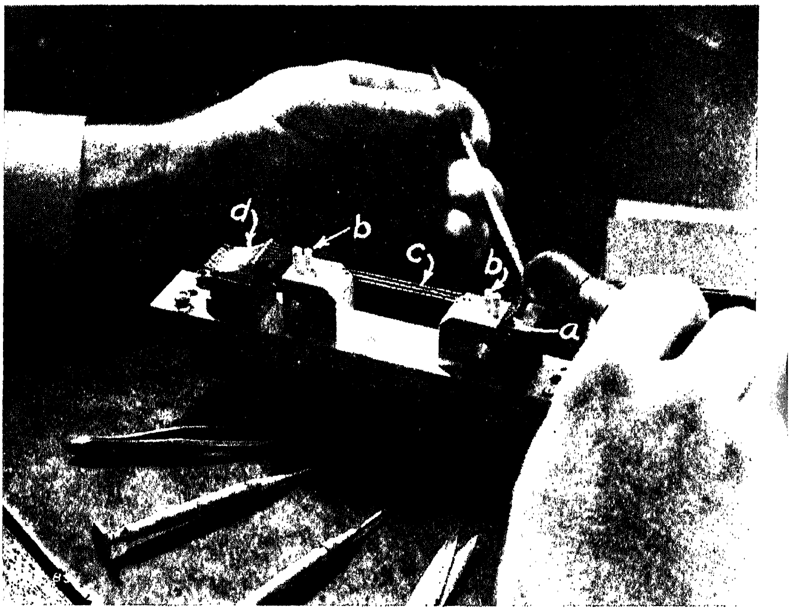

FIG. 330.—Stringing of an Einthoven galvanometer element. *a*, fixed terminals for galvanometer "strings"; *b*, ivory spacer posts; *c*, strings of fine copper wire; *d*, adjustable terminals for proper tension.

An Einthoven type of galvanometer element consisting of thirteen strings is shown in Figure 330. The individual strings consist of copper wire 0.00075 inches in diameter. They are separated from adjoining strings by means of accurately located grooves in the upright ivory posts and are soldered to batteries of connecting terminals at both ends. These terminals may be adjusted individually by set screws to obtain the desired tension in the strings after they are installed. The natural frequency of the elements is about 500 cycles per second, and the system of which they are a part gives a D.C. deflection of about 2 to 4 millimeters per microwatt at a distance of 1 meter.

Moving Coil or D'Arsonval Type.—In the D'Arsonval type galvanometer, a movable loop or coil of wire carrying the current to be recorded is suspended between the poles of a strong magnet. Light is reflected to the photographic recording paper from a small mirror attached to the coil.

Each trace on the record requires a separate galvanometer unit. Records obtained with a D'Arsonval mirror galvanometer can be distinguished from those obtained with an Einthoven string galvanometer because in the former black traces appear on a white background while in the latter white traces appear on a dark background.

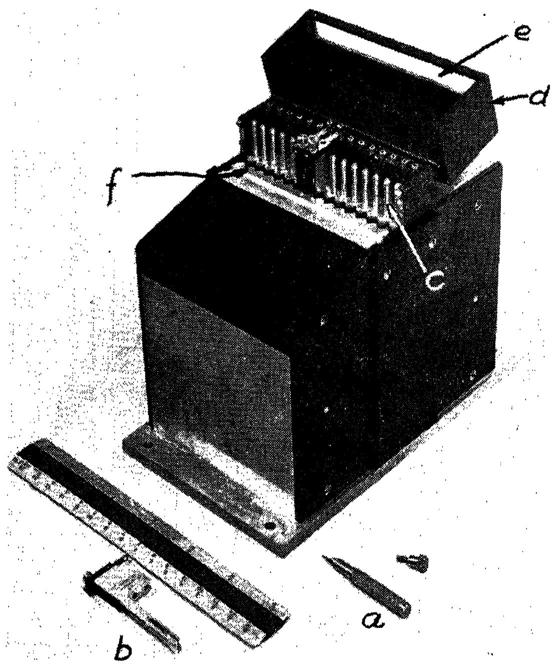

Fig. 331.—Fifteen element galvanometer, with cover open for inspection; two elements are connected. *a*, non-magnetic screw driver; *b*, single element; *c*, terminal supports for galvanometer units; *d*, dust-tight lid; *e*, plane glass window; *f*, air gap in permanent magnet for each galvanometer unit. (Courtesy of Wm. Miller.)

Figure 331 shows a fifteen element moving coil galvanometer with the cover opened for inspection, but with only two elements in place. In galvanometers of this type the frequency is determined mainly by the torsional stiffness of the suspensions and by the moment of inertia of the coil and mirror.

The galvanometers are supplied with critical electrical damping in natural frequencies up to 500 cycles per second. For higher frequencies, oil damping is used. Galvanometers with a low natural frequency should be critically or nearly critically damped. Much less damping need be employed for galvanometers with a high natural frequency. For seismic prospecting, galvanometers with natural frequencies of from 50 cycles per second to 250 cycles per second are most common.

In the galvanometer illustrated in Figure 331, the elements are individually removable and are held in place on accurately spaced defining pins mounted on the pole pieces. When they are in place in the galvanometer bank, the centers of the mirrors of adjacent elements are ¼″ apart. Thus, a twelve element galvanometer is only 3½″ wide, including the case; a thirty-six element galvanometer is but 9½″ wide, including the case. Each element carries an aluminum coated cylindrical glass mirror of the correct curvature to bring the light beam to a focus at the recording paper. The coils of the galvanometers are wound on accurately machined duralumin bobbins which weigh only 25 milligrams. This construction allows uniformity and also adds to the damping because the metal bobbin acts as a single turn short-circuited coil.

A galvanometer which is of the type shown in Figure 331 and which has a normal frequency of 100 cycles per second gives a D.C. deflection of approximately 10 millimeters per milliwatt at a distance of one meter. For other natural frequencies, the sensitivity is inversely proportional to the square of the frequency.

Cameras

One of the several types of individual recording cameras available for seismic work is shown in Figures 332 and 333. The front panel of the

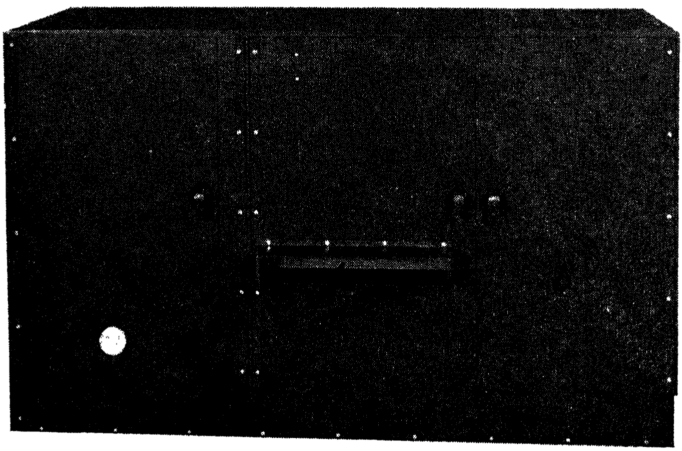

FIG. 332.—Exterior view of seismic recording oscillograph. *a*, ground glass viewing screen; *b*, galvanometer service door; *c*, supply and reroll magazine service door. (Courtesy of William Miller.)

camera is composed of three hinged doors. The center door carries the ground glass viewing screen. The right-hand door allows ready access to the galvanometers, while the left-hand door services the supply and reroll magazines for the recording paper.

The light source is housed in a separate compartment. A beam of light passes through a diaphragm, impinges upon the focusing mirrors of the multiple galvanometer, and is reflected from these mirrors in individual

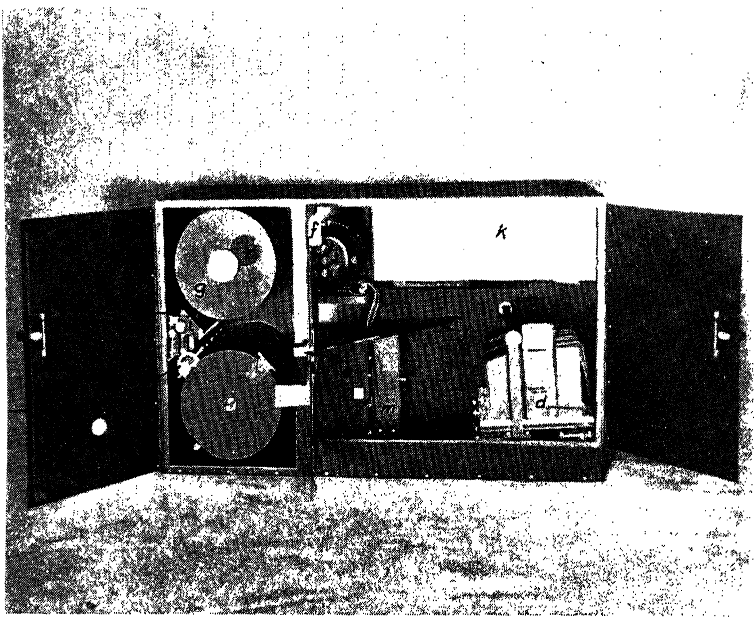

FIG. 333.—Interior view of recording oscillograph. *b*, galvanometer service door; *c*, photographic paper compartment service door; d, multi-unit galvanometer; *e*, light source housing; *f*, constant speed motor drive; *g*, new film magazine with inspection window *r*; *i*, paper guide; *j*, reflecting mirror for external viewing screen; *k*, thermally-insulated tuning fork compartment; *l*, paper cut-off and perforator; *m*, synchronous timing motor; *n*, cylindrical focusing lens; *o*, timing reflecting mirror; *p*, timing disc. (Courtesy of William Miller.)

pencils so as to come to a focus in sharp vertical image lines at the distance of the recording paper. Two centimeters ahead of the recording paper is placed a cylindrical lens having a horizontal axis and a length equal to the width of the recording paper. This lens focuses the individual galvanometer traces (vertical lines) to small bright spots on the recording paper. The lower portions of the pencils of light reflected from the galvanometer mirrors are intercepted by a narrow reflecting mirror and are projected on the ground glass viewing screen at the front of the camera, where they may be observed while a record is being taken.

A temperature-compensated 100 vibrations per second tuning fork is mounted in a thermally-insulated compartment and is equipped either with a carbon button or a vacuum tube drive. The output of the fork is amplified and operates a small synchronous motor which rotates a thin slotted timing disc. This disc rotates at 10 revolutions per second and has 10 radial slots, one of which is wider than the rest. Light from the timing

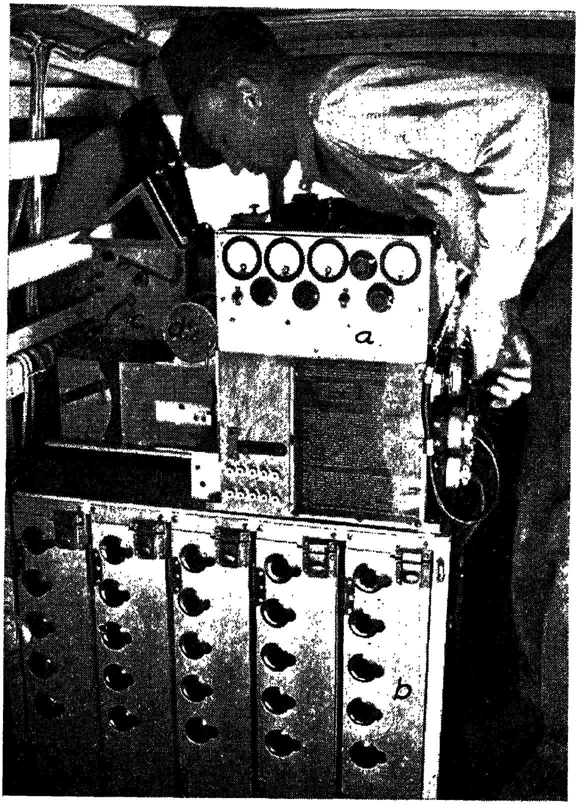

Fig. 334.—Complete seismic recording equipment. *a*, control panel for timer and recorder; *b*, amplifiers; *c*, film drive motor; *d*, used film magazine. (Courtesy of Continental Oil Company.)

light bulb shines intermittently through these slots as the disc rotates. This light is reflected from a mirror and is focused in parallel lines on the record strip. When the light shines through the wider slot, a heavier timing line is photographed, to indicate tenths of seconds.

The timing lines can be maintained to an accuracy of about 1 part in 2000 or better, the tuning fork being checked periodically for frequency.

Unexposed recording paper, in standard rolls of approximately 200 feet and of any width for which the oscillograph is constructed, usually from 3⅝" to 7⅛", is placed in the supply magazine. From there it is threaded through a paper guide and is drawn into a reroll magazine. On the reroll magazine is a paper cutter which cuts off the record and makes the magazine light-tight when it is removed to develop the record. The

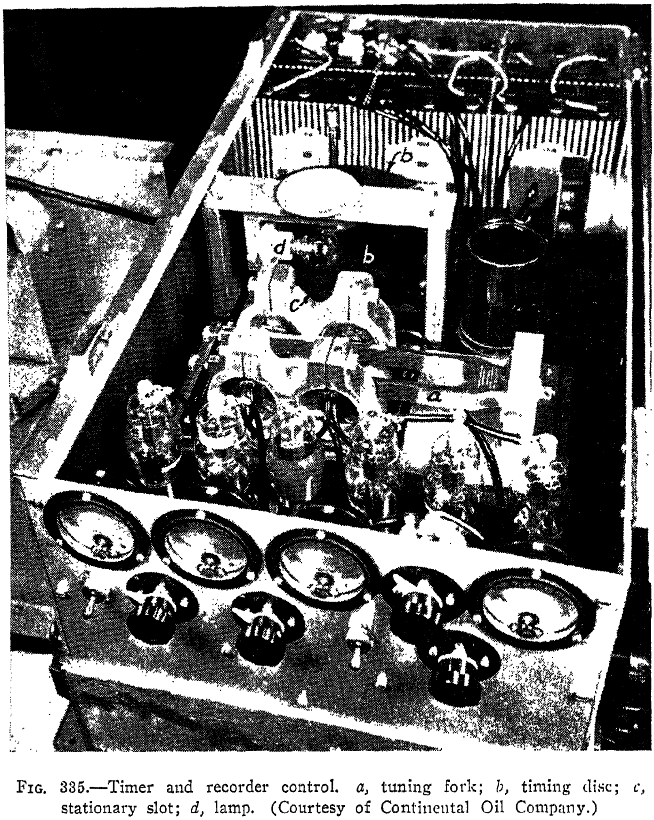

FIG. 335.—Timer and recorder control. *a*, tuning fork; *b*, timing disc; *c*, stationary slot; *d*, lamp. (Courtesy of Continental Oil Company.)

spool in the reroll magazine is driven by a constant speed, centrifugal contact governor motor. A small red glass window allows inspection of the quantity of paper in the supply magazine.

As an alternative design, the recording camera can be provided with a drive in which the paper is fed through rubberized rollers which rotate at a constant speed. Also, instead of the paper cutter mounted on the magazine, an externally-operated device is sometimes substituted which will either perforate or cut off the record, as desired by the observer.

The multiple galvanometer is carefully mounted so as to be practically shock-proof and free from disturbances by truck or other vibrations transmitted through the oscillograph case. The control switches usually are mounted on the lower edge of the front panel or on a separate panel external to the oscillograph. The recording lamps are so wired that their brightness is increased to the required intensity only when the drive motor is operated, which greatly prolongs their life.

Figure 334 shows the recording equipment developed by the seismic division of one of the major oil companies. The galvanometers, timer, and camera are contained in the upper compartment. The amplifiers are of the unit construction type and are housed in the lower compartment. The entire equipment is mounted upon a metal chassis and may be moved as a single piece of apparatus. Thus, complete calibration of the equipment may be carried out in the laboratory, and the entire unit then transferred to the seismic recording truck. An interior view of the timer compartment is shown in Figure 335. The tuning fork is driven by a vacuum tube electromagnetic drive. The output is amplified and drives the timing disc. When a slot in this disc coincides with the stationary slot, light from the lamp shines on a prism and is reflected to the film. On one of the tines of the tuning fork is mounted a small mirror from which light is reflected to the lower part of the record to provide an auxiliary timer to guard against breakdown of the synchronous motor timer.

A record (oscillogram) given by this apparatus is shown in Figure 336. Pertinent data relative to location, charge, depth of charge, elevation of the shot-point, and air temperature are recorded, together with amplifier volume and filter characteristics. The arrival-times of the main reflections have been marked on the record in seconds. The correlation of the wave trains may be seen by sighting across the record.*

A photographic view of another type of recording equipment is shown in Figure 337. The equipment is completely sectionalized, various units fitting into a single framework. The two lower sections consist of twelve amplifier channels and two switching units. The middle section consists of a circuit panel, master amplifier controls (including one spare set) and a camera. The upper sections consist of a loud speaker for communication with the shooting truck, spare amplifier channel, telephone unit, shot-point seismometer control unit, and microphone. The equipment embodies automatic amplitude control.

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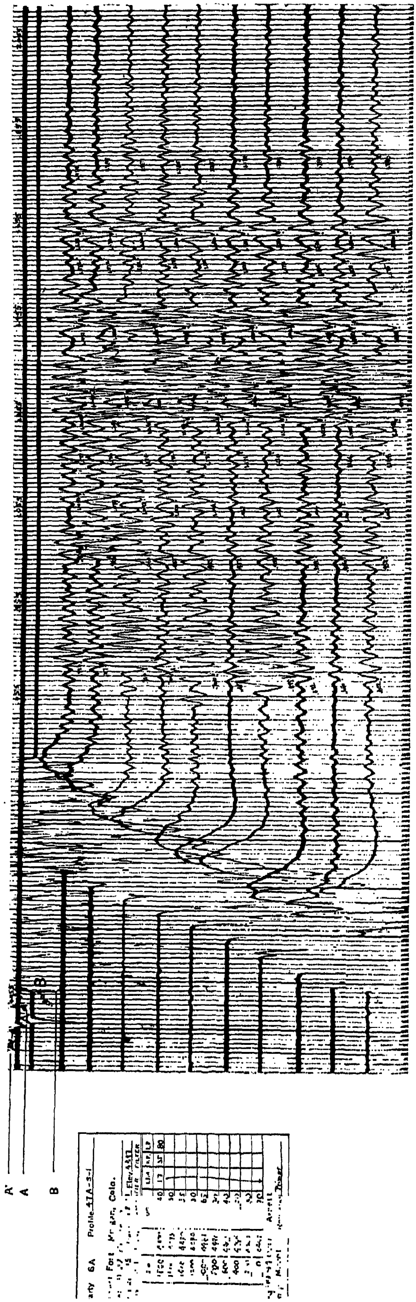
* The timing trace shows the inductive kick when current was first applied to the firing circuit and then the kick caused by the disruption of the firing circuit. The second trace shows the arrival of the explosion wave at the seismometer located adjacent the shot-point, and the time of unrest or ground roll at that seismometer.

The ten lower traces record the outputs of the galvanometers, which are connected to the field seismometers.

The vertical lines represent time intervals, the heavier lines being separated by 0.1 seconds and the finer lines by 0.01 seconds. The lower serrated record is the direct trace from the tuning fork and serves as an auxiliary check on the synchronous motor timer.



G. 336.—Reflection seismic record. *A′*, initial application of current; *A*, break in firing circuit; *B*, low velocity layer or "weathering" correction seismometer adjacent shot-point; 1 to 10, galvanometer traces. (Courtesy of Continental Oil Company.)

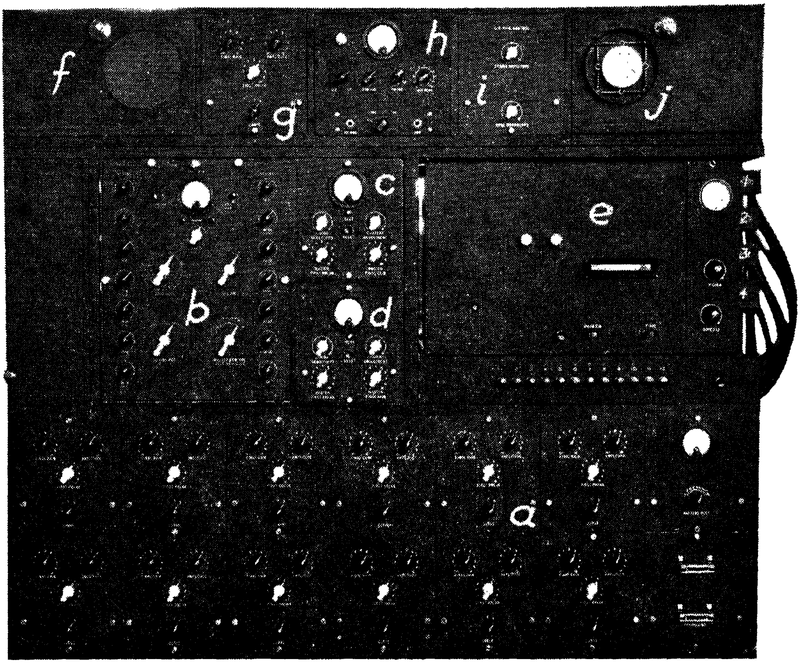


FIG. 337. — Automatic amplitude control recording equipment. *a*, unit type amplifier; *b*, switching unit; *c*, circuit panel; *d*, master amplifier controls; *e*, recording camera; *f*, loud speaker; *g*, spare amplifier, as shown at *a*; *h*, telephone communication panel; *i*, shot-point seismometer control unit; *j*, carbon button microphone. (Courtesy of Western Geophysical Co.)

Sonograph Equipment.—Although the use of this equipment † is as yet limited as compared to the conventional methods, it is described in detail because of its novel features and different operating principles. One characteristic feature is the elimination of the recording galvanometers. The amplified outputs of the seismometers are impressed on small over-heated incandescent lamps, having special filaments capable of heating and cooling in approximate synchronism with the wave motions being recorded. Twenty such lamps are used to record twenty channels, additional lamps being employed to make records of the tuning fork motion and the shot-break. Each lamp shines through a small well-defined slit, approximately 0.010 by 0.070 inches, twelve such slits being placed side by side to record on a strip of 35 mm. positive motion picture film.

In the twenty-trace recorder, two films are used for each shot, each film handling ten channels. The films, each about thirty inches in length, are wrapped side by side in film-wide grooves around a drum which is driven at a uniform rate of speed by a governed motor. The speed of the motor is such that the films are driven at the rate of five inches per

† F. Rieber, "A New Reflection System with Controlled Directional Sensitivity," *Geophysics*, Vol. 1, No. 1, Jan. 1936. See also p. 568.

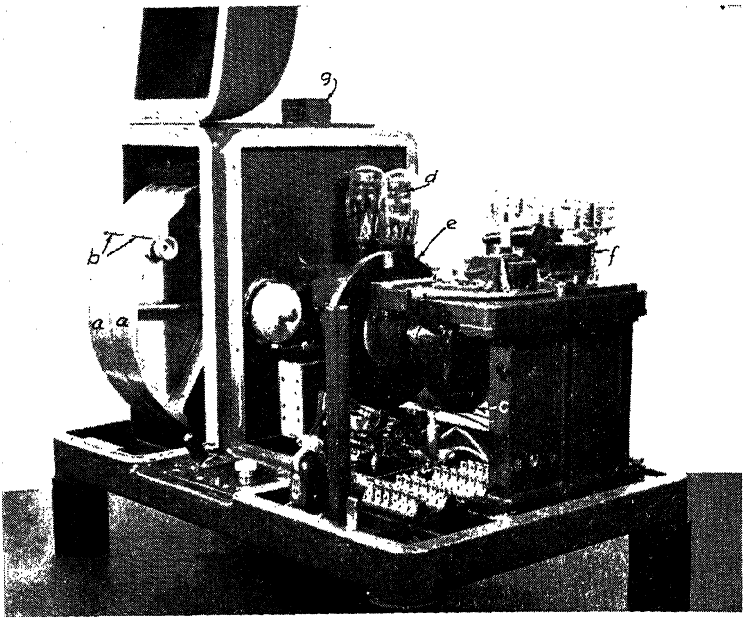


FIG. 338.—Recording oscillograph employing variable density light tracks. *a*, two film drums for 35 mm. film; *b*, clamping slots for film; *c*, motor drive; *d*, motor control tubes; *e*, commutator control for firing circuit to insure proper synchronization of firing and start of film record; *f*, relays for recording lamp circuits; *g*, housing for recording lamps. (Courtesy of Rieber Laboratories.)

second. A commutator attached to this recording drum synchronizes the firing of the shot with the beginning of the record. The speed of the drum, although maintained by a governor, is checked by a stroboscopic device operated by the timing fork.

The lamps are heated by a bias battery, with their temperatures adjusted to produce a mean track density. When the amplified impulses are superposed on this bias current, the track becomes alternately darker and lighter than the mean value. Thus, a variable density record, similar to that used in motion picture sound recording, is obtained. The recording camera for this system is illustrated in Figure 338.

Figure 339 shows four Sonograph films. The left two records carry five tracks each and were made on a ten-track recorder. The right two records contain ten tracks each. Although the prime purpose of using this type of film record is to permit later analysis, a direct visual examination often may be made by comparing the wave bands. Printing all the films from a continuous profile side by side on a single sheet facilitates the correlation. If desired, records of this type may be passed through a photoelectric light intensity recorder, which yields traces similar to those obtained from the conventional galvanometer systems. Usually,

however, the records are analyzed with the aid of a special device called the "analyzer."

The function of the analyzer is to pass a narrow beam of light across the traces. The angle of incidence of the light beam is controlled by a set of mechanically driven slots, and there is a separate slot for each trace. A photoelectric cell collects the light from all traces and passes on the resultant impulses to an amplifying and filtering circuit, and eventually to an electromagnetically-driven pen. A close-up view of the light slots in the analyzer head is shown in Figure 340.

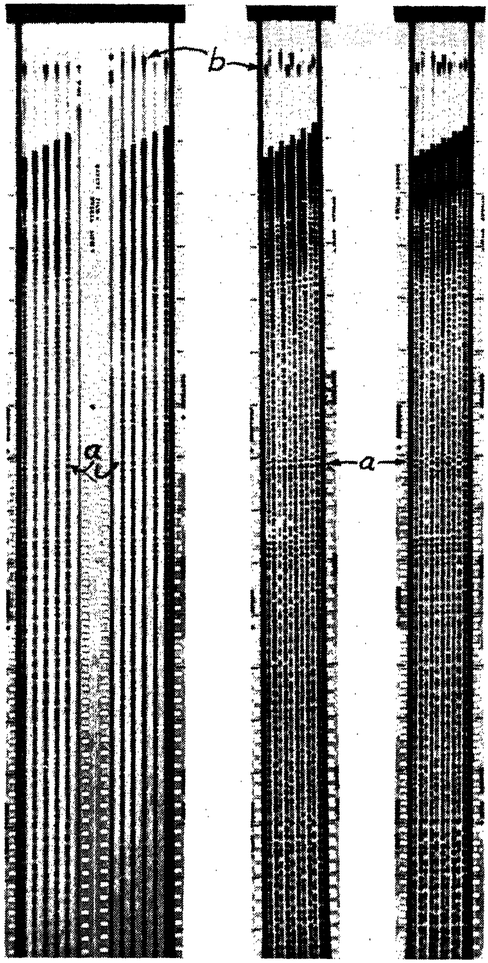


FIG. 339.—Sonograph records showing 2 five-track films and 2 ten-track films. *a*, typical correlations made by comparing light bands; *b*, shot-breaks. (Courtesy of Rieber Laboratories.)

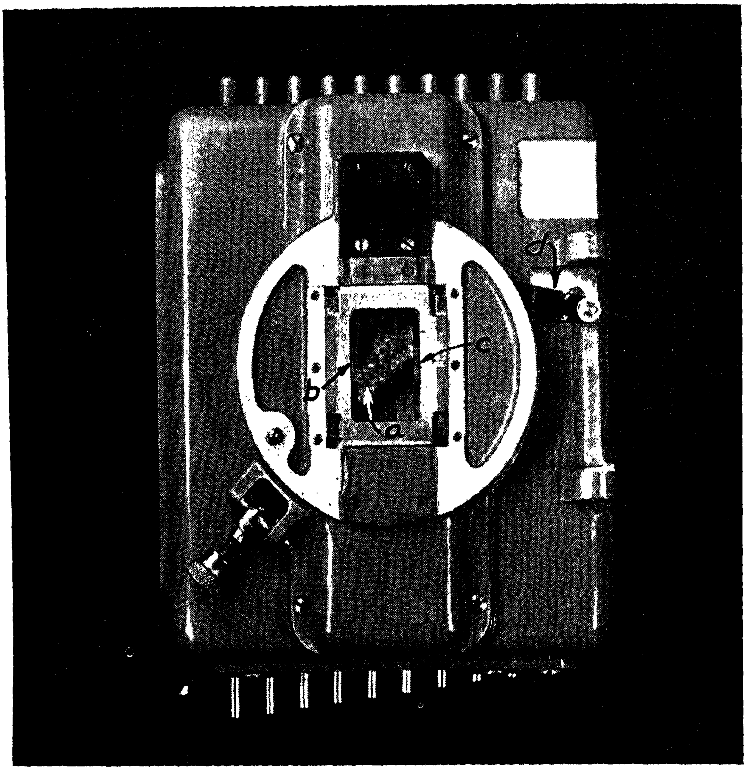


FIG. 340.—Sonograph analyzer head. *a*, apertures controlling angle of transverse light beam (one aperture for each seismometer trace). Slanting row of apertures corresponds to setting for extreme angle of arrival of the earth wave. *b*, apertures for tuning forks; *c*, aperture for time-break; *d*, lever for changing position of apertures. (Courtesy of Rieber Laboratories.)

A low velocity layer correction can be made by shifting the slots, each of which is individually adjustable for any desired time lead or lag. Furthermore, during the "analyzing process," the voltages from the separate traces can be combined in such a way that time differences are introduced that are equal to the time intervals which waves arriving from a certain chosen direction would require in traveling from the shot-point to the various seismometers. The resultant voltage is the sum of the voltages corresponding to the motions at the individual points, with phase differences so chosen that the effects of waves arriving along a certain direction are in phase, while the effects of waves arriving along other paths are out of phase. Thus, if a sufficiently large number of seismometers is used, the effects of waves other than those arriving along the chosen

path are greatly reduced while the wave form of the wave traveling along the chosen direction is unchanged.

Figure 341 shows an analyzed record made from the light intensity records. In this figure the north-south and the east-west components are shown. The optimum angle of recording is shown by the maximum amplitude of the reflection. Amplitude, therefore, becomes the chief criterion for judging a reflection, as compared to *character* and *amplitude* on the conventional type of record.

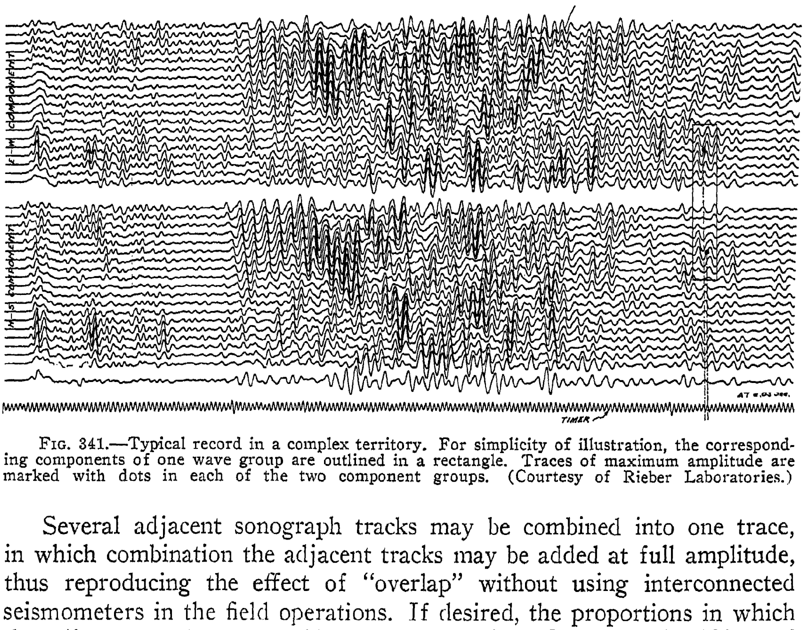


FIG. 341.—Typical record in a complex territory. For simplicity of illustration, the corresponding components of one wave group are outlined in a rectangle. Traces of maximum amplitude are marked with dots in each of the two component groups. (Courtesy of Rieber Laboratories.)

Several adjacent sonograph tracks may be combined into one trace, in which combination the adjacent tracks may be added at full amplitude, thus reproducing the effect of "overlap" without using interconnected seismometers in the field operations. If desired, the proportions in which the adjacent tracks are combined may be varied. For example, 100% of the outside track amplitude may be combined with 50% of the amplitude of the next track, and with 25% of the amplitude of the next track. Thus, any of the customary "overlap effects" may be reproduced in a form similar to that in which they would have been recorded by a seismograph using overlap connections.

Recording Papers and Photographic Developing.—The recording medium used in the photographic methods consists of a sensitized material deposited on a paper or film support. The paper should be of a good quality white stock. The film support is seldom employed in field work due chiefly to three factors: (a) greater cost; (b) developing difficulties encountered in warm weather due to separation of the sensitized emulsion from the film support; (c) greater bulk and weight. Occasionally when very high recording speeds are desired, film is used, because it usually can be obtained in a greater range of emulsion speeds than paper.

The sensitized emulsion employed for recording is a high contrast emulsion, usually of silver bromide in gelatin. This material is superior to the usual silver chloride emulsion employed in photography, because it is relatively unaffected by the processing procedure. Thus, records obtained with the silver bromide emulsion are uniform and not appreciably dependent upon the processing.†

The developer employed for processing the film should be a high contrast material, with sufficient potassium bromide to restrain fog. Practically all manufacturers of paper supply their own packaged developers. These frequently are found more satisfactory than the bulk materials, chiefly due to their convenience in use, their uniformity, and the elimination of waste and errors in weighing chemicals.

The developing time should be long enough to give a proper density in the portions of the record which have been properly exposed. The record may be examined occasionally during the development provided a safety lamp is used. The ordinary red light is seldom safe for the modern high speed recording papers. To determine the safeness of a red light, it is convenient to expose half of a test piece of the paper to the light for a few minutes and then develop and fix. Excessive fogging of the exposed portion of the test strip, as compared to the unexposed portion, will indicate that the light should be changed or moved further from the paper during the processing.

If the developing temperature is maintained at about 65° F., a more concentrated developer will usually give a more dense record. At higher temperatures, the more concentrated developer may produce less effective contrast due to the increase in fog density. The developing speed, also, is greatly dependent upon temperature; an increase to 85° F. will often cut the developing time in half, while a decrease to 50° F. may necessitate more than twice the normal developing time. The higher temperatures accentuate fog and also cause discoloration due to oxidation of the developer. For best results, the temperature should not exceed 70° F. Oftentimes, it is difficult in field work to control the developer temperature, and the best compromise is to vary the strength of the solution, i.e., use relatively strong solutions at the lower temperatures and diluted solutions at the higher temperatures. The most satisfactory method for uniform records is to employ a tank, the temperature of which is maintained constant by a surrounding water bath. Thermal insulation (cork or glass wool) prevents rapid temperature variations. An ice-pack in summer and a hot water bath in winter are used to adjust the temperature at the beginning of a day. Only minor attention is required thereafter to maintain it. Records are developed in accordance with time-temperature charts, with consideration given to the age of the developer.

† F. A. Tompkins, "Fundamental Photographic Processing Operations Influencing Production of Seismograph Records," *Geophysics*, 1936, Vol. 1, No. 1, pp. 107-114; "Effect of Development Time and Developer Temperature on the Production of Photographic Seismograph Records," *Geophysics*, 1936, Vol. 1, No. 3, pp. 313-318.

After it has been developed, the record should be washed in a stop solution consisting of 1½ ounces of 28% acetic acid dissolved in 32 ounces of water. Fixation of the record is accomplished by immersing it in a solution of sodium thiosulphate (trade term "hypo") which removes the unexposed silver. Prepared packages of this material, with the necessary hardener, are available from all manufacturers of paper. Fixation is usually complete within three to five minutes. After sufficient fixing, the film should be immersed in a fresh water bath to insure a permanent, non-fading, and non-stained record.

Although it is often neglected, proper washing of the records is one of the important steps in the processing. The removal of soluble chemicals remaining in the paper after the fixing operation can only be accomplished by thorough washing in clean water. If it is inconvenient to supply sufficient water for each batch of records, the films should be left in the water bath until the return of the operators to headquarters. The records can then be placed in a large container (the bath tub may be convenient) with cool fresh running water and washed for about thirty minutes, and then dried.

Fig. 342.—Processing assembly for photographic records. *a*, developer; *b*, stop bath; *c*, fixing bath; *d*, fresh water; *e*, rubber-sealed lids; *f*, absorbent paper toweling.

During the developing, fixing, and washing steps, the record must be carefully moved or the fluid agitated to insure uniform exposure to the solution of all parts of the record.

Caution must be exercised to prevent the stop bath and hypo from splashing or dripping into the developer. Among the best containers for solutions are glass (deep Pyrex cooking pans), glazed earthenware, or stainless steel jars. The containers should be provided with sealing lids to prevent "slopping" of the solutions when the truck is moved during the day.

Figure 342 shows a simple but effective assembly for field use where a limited number of records is to be developed. The jars are ordinary earthenware thermally-insulated jugs with large open mouth, of the type employed for food. These jars are provided with tight-fitting rubber-sealed lids that preserve the solutions and prevent splashing while the truck is in motion between set-ups. The lower ends of the jars are set into holes in a raised base board to allow their easy removal for cleaning and filling. Absorbent paper toweling will be found a convenience.

A procedure is often used which avoids the need of a dark room, particularly in milder climates where the normal outdoor temperature is suit-

able for developing of records. The chemical containers are placed in a light-tight tank. The cover of the tank consists of a frame to which a hood is tightly attached, light weight leather being generally used for the hood because of its ruggedness. The hood is equipped with a sleeve through which the operator inserts one of his arms. The operator's free hand is used to clasp the sleeve tightly to avoid light leaks. The magazine is unloaded, the record developed, washed, and placed in the fixing bath before the cover is raised from the tank.

An advantage of this type of equipment is that the recording truck need not be maintained light-proof, giving the operator greater freedom of action and permitting him to attend to other matters while his subordinate proceeds with the developing.

Fig. 343. — Complete photographic developing equipment. *a*, record storage compartment; *b*, drying racks; *c*, hot air circulating fan; *d*, oil-burning stove; *e*, processing compartment; *f*, developer; *g*, stop bath; *h*, fixing bath; *i*, opaque cloth cover for front of compartment; *j*, fresh water washing tank; *k*, safety light for developing. (Courtesy of Continental Oil Co.)

Figure 343 shows a complete photographic developing assembly with heater, wash water can, and record rack. The heater is a modified kerosene-burning stove, with a small fan for forced hot air circulation through the record drying compartment. This type of dryer practically eliminates the delays occasioned by cold weather and high humidity.

Three large mouthed one-gallon thermos jugs are placed in the lower cabinet. These jugs contain the developer, stop bath, and hypo solutions.

A small red lamp provides a means for inspecting the developing. Over the front of the cabinet is a black opaque cloth cover which is provided with two light-tight armholes and a small red observation window. The operator conducts the developing and fixing operations by inserting his arms through the armholes. Equipment of this type will handle the records of a fast working party conveniently and allow inspection of the records at each set-up without undue loss of operating time.

Recording Truck and Reels.—The recording truck not only transports the recording, telephone, and control apparatus, but may also serve as a darkroom. Where climate requires, the truck should be thermally insulated—especially the roof—for work in hot summer months; also, means should be provided for fan ventilation in summer and heating in

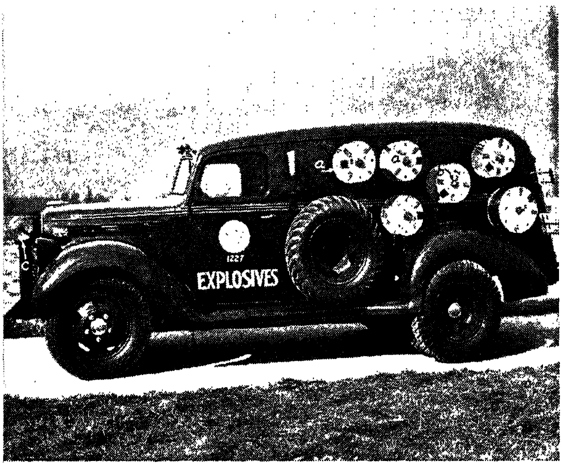


Fig. 344.—Seismic recording truck for average operating conditions. *a*, demountable reels for seismometer cables; *b*, spindle supports; *c*, power reel winder.

winter. The truck must be light-tight and provided with the necessary photographic paraphernalia for processing the records, including a red safety lamp. The recording truck also carries auxiliary equipment, such as augers or spades, pick axe, small tools, etc.

A full set of reels containing seismometer and telephone cables are fastened to the truck. The number of reels and the arrangement of cables vary considerably among operators and depend upon such factors as number of seismometers, length and type of spreads, provision for variation in spreads, efficiency in disposing and picking up of seismometers, and preservation of cable. The number of reels varies from two to eight or more. To avoid great cable strain, the cable often is reeled-in while the truck is driven along side of the cable. In any case, at no time should

cables be pulled with such force as to harm the conductors, particularly the splices and connectors.

The truck in Figure 344 illustrates the use of demountable reels in conjunction with a power drive. Reeling-in is accomplished conveniently by means of a power-driven shaft mounted on the front of the truck. The reels are locked on the shaft, and rotation speed is controlled by a hand clutch. As soon as the cable has been reeled in, the reel is removed from the shaft and placed on its spindle on the side of the truck.

Testing of Equipment.—At periodic intervals, as well as when suspicion is aroused, the following tests should be made to determine the operating characteristics of the equipment: (a) time parallax or phase relationship of the recording channels; (b) accuracy of the timing system; and (c) general circuit condition.

Phase lag between the channels can be detected by placing all of the seismometers at the same location and firing a regular reflection shot. If all channels give the same time of arrival and character of record, the equipment assembly is operating satisfactorily. If differences are recorded, tests should be conducted to determine whether the trouble is due to the location of the seismometers. This can be checked by interchanging the anomalous seismometer with one of the others. If a seismometer is faulty, it should be corrected or replaced. The character of the record is influenced considerably by the material at and adjacent the seismometer. If differences in character are noticed on the records, the anomalous seismometer should be interchanged with one of the others. If the trouble still exists, the anomalous seismometer should be replaced. Persistence of the trouble will then indicate difficulty in the amplifying, filtering, or galvanometer system.

The timing system should be checked for accuracy by comparison with a separate timing fork. The reference timing circuit should be connected to one of the galvanometer traces to obtain a direct check with the regular timer.

Miscellaneous tests on the circuits can be conducted. An example is test for cross-feed, for assurance that energy is not being distributed over more traces than expected and thus producing artificial line-up and distorting data.

GENERATION OF SEISMIC WAVES

Various methods are utilized in seismic exploration work for initiating elastic waves. The methods may be classified as (1) mechanical and (2) explosive. (Essentially similar seismograms are obtained for mechanical and explosive sources.)

Generation of Seismic Waves by Mechanical Means.—Mechanical methods are occasionally employed for shallow mining and bedrock in-

vestigations, chiefly to avoid the danger and liabilities incident to the use of the more convenient and efficient explosive methods.

Mechanical methods are usually of the impact type, utilizing a heavy weight dropped from a known height or the blow of a heavy sledge. For general bedrock determinations and similar shallow studies, a ten-pound sledge and an impact stake of the type shown in Figure 345 may be

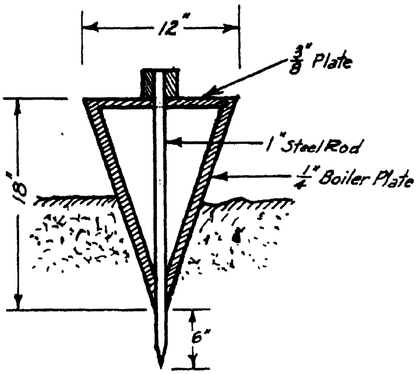


FIG. 345.—Impact stake suitable for shallow seismic investigations.

employed. The purpose of the large surface of the stake is to provide a means for relatively efficient transmission of energy from the hammer to the soil. In regions where rocks outcrop, the impact stake may be omitted and the blow delivered directly to the surface of the rock.

The time at which the impact occurs is usually obtained by placing a seismometer fairly close to the impact stake. This seismometer is connected to a special recording galvanometer or to one of the station galvanometers.

A discussion of seismic experiments utilizing falling weights will be found in the literature.† Also, L. G. Howell, C. H. Kean, and R. R. Thompson have described an electrodynamic ground shaker capable of initiating continuous seismic waves of moderately high frequencies and single frequencies over a range of 20 to 1400 cycles.‡ (The use of an oscillator for generating seismic waves was described by Fessenden as early as 1914.)

Generation of Seismic Waves by Explosives.—Elastic earth waves of sufficient amplitude for deep investigations are initiated by the detonation of a charge of explosive. The characteristics of the waves depend on the properties of the earth formations in the vicinity of the explosive, the degree of confinement of the explosive charge, and the weight and

† See, for example, V. G. Gabriel, "Some Experiments in Seismic Prospecting," *Ergänzungs-Hefte für angewandte Geophysik,* Vol. 2, 1931, pp. 122-130.

‡ *Geophysics,* Vol. IV, No. 3, 1939; Vol. V, No. 1, 1940.

type of explosive. Detailed theoretical studies of the desirable properties of explosives for this and similar purposes have been reported, but only certain factors of practical import for geophysical exploration need be considered here.

Types of Explosives

An explosive is a chemical compound or mixture of chemical compounds (usually in liquid or solid form, or both) which is capable of changing rapidly into simpler chemical compounds, largely of a gaseous nature, which occupy a much greater volume under standard conditions than the original material. This change is accompanied by a great evolution of heat. The useful or destructive effect of explosives is dependent upon the pressure developed by the sudden release of gases at high temperature and pressure, and the work done by these gases on the surrounding media.

Commercial blasting explosives may be grouped into two general classes: namely, high explosives, including dynamite and gelatin dynamites, and low explosives, i.e., explosives of the black powder type.

The explosive decomposition of high explosives is believed to proceed on a wave front along the explosive column at a high velocity, and is known as detonation. Low explosives burn or deflagrate and the decomposition proceeds inwardly from the surfaces of the individual particles. The difference in the observed effects produced by black powder and dynamite is due to the difference in rate of evolution of hot gas upon explosion, this being much less rapid for low explosives than for high explosives. Black powder is little used for seismic prospecting and only the high explosives need be considered here.

The most important types of commercial high explosives are: (1) the straight nitroglycerin dynamites, (2) the ammonia dynamites, and (3) the gelatin dynamites, which may be subdivided as (3-a) straight gelatin dynamites and (3-b) ammonia gelatin dynamites.†

The straight nitroglycerin dynamites form the standards with which high explosives are compared for strength. They should contain the actual amount of commercial nitroglycerin designated as their percentage strength. In addition to nitroglycerin these explosives contain an "active base" of absorbent carbonaceous material and an oxidizing salt, such as sodium nitrate. The straight nitroglycerin dynamites are extremely sensitive to propagation and find little use except where this property is specifically required, as for example, in ditch blasting by the propagated method.

Ammonia dynamites (also known as "extra dynamites") contain less nitroglycerin, grade for grade, than the straight nitroglycerin dynamites. Ammonium nitrate is substituted for part of the nitroglycerin in amounts sufficient to give them the same strength as the corresponding grades of the straight nitroglycerin dynamites.

Gelatin dynamites are manufactured using a colloidal solution of nitrocellulose in nitroglycerin instead of the nitroglycerin alone. Gelatin dynamites are dense,

† N. G. Johnson and G. H. Smith, "Explosives for Seismic Prospecting," *Geophysics,* June, 1936, pp. 228-238.

plastic, and highly water resistant, and are the principal explosives used in seismic shooting. They are classified into two general types: namely, straight gelatins, in which the nitroglycerin-nitrocellulose colloid is the explosive ingredient, and ammonia gelatins in which ammonium nitrate is substituted for part of the colloid. At the present time ammonia gelatins are the gelatins used almost universally for industrial blasting in this country, but both types of gelatin dynamites are used in geophysical prospecting.

For relatively shallow seismic work, the ammonia gelatins have been found entirely satisfactory and are to be recommended because they are usually sold at a lower price than the straight gelatins. While their resistance to water is not as good as that of the straight gelatins, it is still good enough for a great deal of work and in addition the ammonia gelatins are capable of being detonated by commercial electric blasting caps under moderately high heads of water. Special grades of 60% ammonia gelatin are available for seismograph use and because these grades are usually furnished with features such as extra heavy wrapping, special hardness and toughness, etc., to adapt them for seismic shooting, they are to be recommended instead of the 60% ammonia gelatin ordinarily used in industrial blasting.

For the more severe conditions where explosives must be detonated under very high heads of water or where "sleeper" charges must remain in the ground for a long time, the explosives manufacturers have developed special modifications of the 60% straight gelatin. These gelatins (sometimes referred to as "High Pressure" gelatins) are especially formulated by the manufacturers so that they will detonate with electric blasting caps under very high hydraulic pressure, even after immersion for considerable periods of time. 60% straight gelatin is also available in cartridges primed with a core of highly sensitive 60% straight nitroglycerin dynamite. This "cored powder" or "primed powder" has been found effective under the many severe conditions encountered in seismic operations in California. Provided the core of 60% dynamite is well protected against water by the surrounding gelatin, the primed grades may have advantages over the unprimed grades. However, their use is not to be recommended under conditions where the unprimed grades can be used. This is because of their somewhat higher cost and the fact that the more complicated construction of the cartridges offers certain disadvantages.

In addition to the 60% gelatin grades described, other gelatin grades have found some use in seismic shooting. † Modifications of 80% straight gelatin and 100% blasting gelatin have been developed for seismograph work, but their use has been limited. The lower strength gelatins down to 30% also have been used successfully. However, the 60% straight and ammonia gelatin grades seem to be the most popular, and certainly most of the intensive development work has been done on these grades by the manufacturers to make them suitable for seismograph use. Because of this, and because the 60% gelatin grades represent about the maximum explosives value per dollar over the entire range of gelatin strengths available, they are to be recommended as highly suitable grades for seismic prospecting.

An explosive, to be suitable for seismograph work, should be of proper consistency. (Usually, the seismograph gelatins must be harder and tougher than the gelatins used for industrial blasting.) If the explosive is to be shot in very small charges, it should be plastic enough so that it can be molded around the cap after being cut from the cartridge. Where the explosive is to be loaded in deep or partially blocked holes, it is essential that a stiff, firm cartridge wrapper be supplied.

It is held by some that the rate of detonation of an explosive is of great importance in seismograph work, but this question is probably controversial. Little is known of actual rates of detonation developed by charges of gelatin under the conditions of use in seismograph work. The rate of detonation may be a separate factor, but it is probably subordinate to other factors, such as the size, shape, and location of the charge, and the ability of the charge to detonate completely.

† H. E. Nash and J. M. Martin, *Geophysics,* June, 1936.

Various brands and sizes of seismograph explosives are offered by the different explosives companies, and the representatives of the manufacturers should be consulted before selection of the explosive to be used for an area.†

Electric Blasting Caps

Efficient operations in modern seismic technique require electrically fired blasting caps. Because in most firing systems the electric blasting cap becomes an integral part of the seismograph circuit, it is important that the operator have some understanding of the construction and functioning of an electric blasting cap. The caps offered by different manufacturers may be radically different in the details of their construction; however, the following general description is illustrative of the principles employed in all electric blasting caps.‡

Essentially, an electric blasting cap consists of a waterproof metal shell containing a charge of explosive material and a means of detonating this material by an electric current. The caps are equipped with insulated leg wires of various lengths. The explosive charge in the cap usually consists of a "base charge" of a highly efficient detonating material and one or more other explosive components referred to collectively as the "priming charge." A fine resistance wire, known as a "bridge wire," is in intimate contact with the priming charge and this wire is in circuit with the two insulated leg wires.

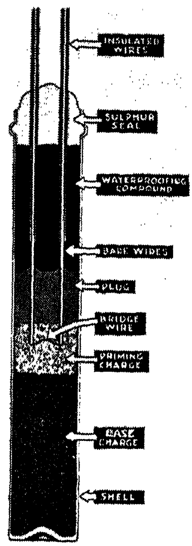


FIG. 346.—Section showing the construction of a typical electric blasting cap.

Figure 346 illustrates one of the most common types of construction employed in electric blasting caps.

Whereas electric blasting caps are commonly referred to as "instantaneous" caps, this term is inaccurate because the caps require measurable intervals of time for their operation. In industrial blasting this is of little or no interest to the consumer, but in seismic work even the very short time intervals involved become important.

When an electric current is caused to flow in the circuit of an electric blasting cap, heat is generated by conduction losses (I^2R losses) in the bridge wire. This heat is conducted or radiated to the priming charge. If a very low current is applied, the heat generated may be insufficient to raise the temperature of the priming charge sufficiently to initiate its decomposition. A current of 0.3 amperes usually is the minimum

† W. R. Farren and H. H. White, "Recent Developments in Explosives for Seismic Prospecting," *Geophysics*, Mar. 1937, pp. 114-119.

‡ G. F. Rolland and H. H. White, "Developments of Essential Characteristics in Electric Blasting Caps for Seismic Prospecting," *Geophysics*, March, 1937, pp. 119-126.

that will detonate a commercial cap. When sufficient current is passed through the bridge wire, it raises the temperature of the priming charge and starts decomposition. The decomposition of the priming charge progresses from "burning" to "detonation" very rapidly and detonates the base charge, which in turn detonates the dynamite.

The time elapsing between the application of an electric current to the cap and the explosion depends in part on the type of cap used, but chiefly upon the magnitude of current. Figure 347 illustrates the firing characteristics of two general types of electric blasting caps and shows the relationship between current and time of detonation.

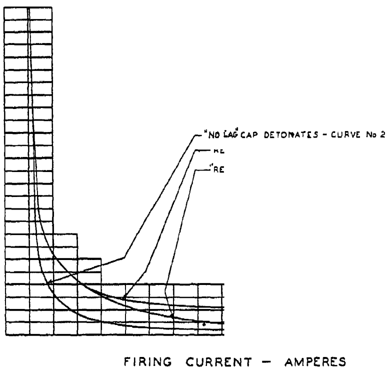


FIG. 347.—Curves showing firing characteristics of the typical "Regular" electric blasting caps (industrial type) and special "No-Lag" seismograph caps.

The time elapsing between the detonation of an electric blasting cap and the initiation of detonation in the charge of dynamite or gelatin is extremely short, and because it is far below the limits of accuracy of seismograph records, it may be neglected. For all practical purposes, the time of detonation of the cap may be accepted as the time of the explosion of the dynamite charge. In early seismic work the time-break was obtained from the breakage of a wire wrapped either around the dynamite charge or around a second cap connected in series with the cap used to detonate the charge. These methods have been largely supplanted by the single cap method, in which the time-break is determined from the interruption of the firing current occasioned by the breaking of the bridge wire circuit when the explosion occurs.† This method of determining the time-break is satisfactory provided the interruption of the firing current and the detonation of the cap are simultaneous. This is not always true, however, of all caps. In general, two types of electric blasting caps are

† G. H. Loving and G. H. Smith, "Explosives and Electric Blasting Caps for Geophysical Prospecting," *Journal S.P.G.*, Vol. 6, 1935, pp. 1-27.

H. R. Prescott and F. L. Searcy, "Method of Making Geophysical Explorations," U. S. Patent 2,046,843, issued July 7, 1936.

offered by explosives manufacturers for seismograph use. One is the type of cap used in industrial blasting. Typical firing characteristics of this "regular" type of firing cap are illustrated in Figure 347 by curves No. 1 and No. 1-A. For current values of about two amperes or less, the rupture of the bridge wire is simultaneous with the detonation of the cap. Above this current value, however, the interruption of the bridge wire circuit precedes the detonation of the cap by a time interval dependent on the firing current employed. These "regular" electric blasting caps are widely used for seismograph work and are found entirely satisfactory where low firing currents are used or where firing conditions are such that a correction can be made for the time lag between the breaking of the bridge wire and the detonation of the cap.

There are circuits, however, in which these conditions cannot be maintained, and some explosives manufacturers furnish caps in which the interruption of the cap current is practically simultaneous with the explosion of the cap regardless of the firing current used. In these caps the bridge wire does not fuse and is therefore not ruptured until the cap explodes. In general, the "no-lag" caps fire more rapidly at any current than do the "regular" caps. Typical firing characteristics of these special seismograph caps are illustrated in Figure 347, curve No. 2.

Electric blasting caps are usually furnished in two strengths: the No. 6 and No. 8 grades. When fulminate of mercury was the universally used detonating material these strengths were well defined, the No. 6 caps containing one gram of detonating material and the No. 8 caps containing two grams. At the present time, however, the different manufacturers use a wide variety of cap compositions. Nearly all of the No. 6 caps now offered are materially higher in detonating efficiency than the original No. 6 fulminate cap. No. 6 caps constitute a great majority of the caps used for seismograph shooting and are amply strong for most work. The present-day No 8 caps are more efficient than the fulminate No. 8 caps and offer an additional margin of safety where seismograph shooting is carried out in deep holes. Recently some manufacturers have furnished special high strength caps greatly exceeding even No. 8 caps in detonating efficiency. Such caps have been found advantageous under extremely severe shooting conditions.

Important properties of electric blasting caps may be summarized as follows: no time lag between rupture of firing circuit and detonation of cap; sufficient detonating strength to initiate the explosion under the conditions of use; and the various physical characteristics, such as waterproofing, efficient protective shunting and a method of packaging such that the wires can be "strung out" without excessive kinking or inconvenience. The electric blasting caps offered by the various manufacturers are furnished under various brand names and come in a variety of wire lengths. The manufacturers' representatives should be consulted for further particulars on any given brand.

Electric Firing Circuits

The electric firing current may be supplied to the caps either by batteries or blasting machines, the latter being considered safer.

Various arrangements may be employed for recording time-breaks. It is most common to use an inductive circuit which will produce an impulse at the time the cap is ruptured. An example of such a circuit is shown in Figure 348. The blaster is shunted by a high impedance circuit, usually the secondary of a transformer, the other winding of which feeds one of the galvanometer units in the camera. When the blaster plunger is forced down and the impulse switch closed, current flows to the cap and at the same time a small impulse is carried to the transformer and into the camera, recording the time of voltage application. When the cap current is broken at the time of detonation, voltage momentarily builds up at the blaster terminals and produces a strong impulse at the galvanometer. This strong second impulse is the time-break.

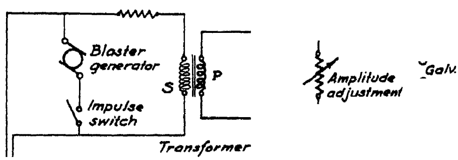


FIG. 348—Time-break circuit for seismic work.

Loading Procedure

Few general rules can be given concerning the best depth for the shot in seismic prospecting. Normally, the shot should be deep enough to avoid surface cracking. Likewise, it generally should be below the low velocity layer. (Compare p. 469.) In a particular area it may be found that best results are obtained when the shot is fired within the same stratum at all shot-holes—which stratum this is can only be determined by experiment. For the sake of accuracy in computing, it may be preferred to keep the bottoms of the holes at as nearly constant an elevation above sea level as field operating conditions will permit.

Various methods are used for placing the electric blasting caps in the cartridges of explosive. The cap may be inserted in a hole punched with a wooden or bronze punch, either in the end or obliquely in the side of the cartridge. The cap should be well seated at the bottom of the hole, which should not be punched to a greater diameter than is necessary to clear the cap. The explosive should be molded by hand over and around the cap. This aids in making a good mechanical assembly and also retards the access of water to the explosive immediately surrounding the cap. Occasionally, it is desirable to use tape for the further protection of the cartridge at this point.

Where more than one cartridge is to be loaded into a hole, it is desirable to fasten the cartridges into a unit to facilitate handling. For loading into shallow holes the cartridges can be dowelled or taped together at the ends. For deeper holes where more severe loading conditions are encountered the cartridges may be taped together to a lath. Insertion of the cartridges into tubes of light metal or heavy paper has been found effective under some conditions. The method to be used is dependent upon the conditions and will be more elaborate in deep or partially blocked holes than in shallow clear holes.

FIG. 349.—Method of loading explosives in a medium depth hole for seismic work. *a*, explosive cartridge; *b*, firing wires; *c*, spear or "schnozzle"; *d*, loading pole; *e*, extensions with link couplings. (Courtesy of United Geophysical Company.)

The cap wires should be secured to the charge in a manner that will prevent strain at the point where the wires enter the cap. The assembly also should be such as to prevent strain or pull on the cap wires where they enter the cartridge. The wires may be looped or hitched around the cartridge, sharp bends or kinks being avoided. In shallow clear holes the charge is lowered by the cap wires and care is taken to avoid damaging the wires during loading. When loading into a dry hole, it is desirable to confine the charge by filling the hole with water to a level well above the top of the charge.

A spear pole often is employed for placing the charge at the bottom of a hole filled with drilling mud or water. The procedure is illustrated in Figure 349. The charge is held on to the spear by keeping the wires under tension. The poles are usually 15 feet in length and are provided with link couplings to give the required depth of placement. When the charge has been pushed to the bottom of the hole the tension on the wires is released. A jerk on the pole then frees the spear and leaves the charge in place.

When loading charges into holes which are two hundred feet or more in depth or where caving and "sluffing" of the wall are common, a weighted brass "spoon" is operated by a steel cable and winch. (Figure 350.) The

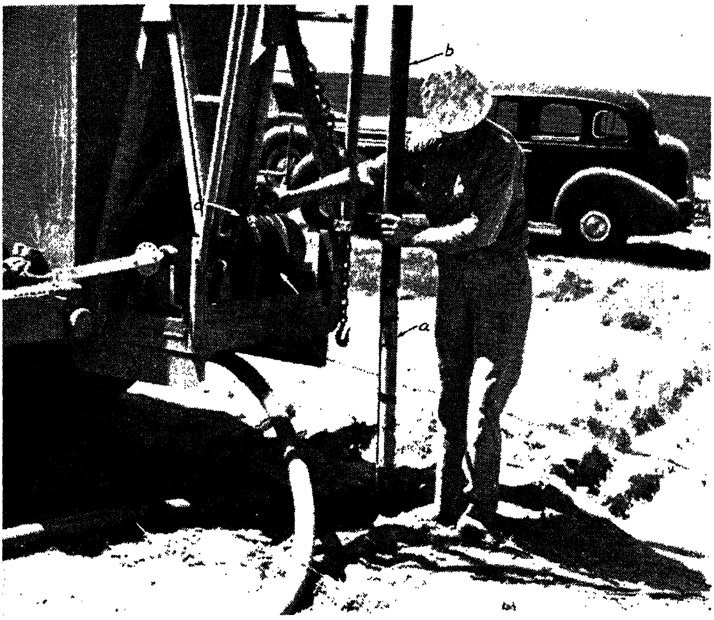


Fig. 350.—Loading deep seismic holes by use of weighted winch lines. *a*, explosive cartridge; *b*, weighted tube; *c*, winch; *d*, hand control on winch. (Courtesy of United Geophysical Company.)

charge is held in the spoon by two or three loose loops of wire which will allow it to slide off easily when the tension is released on the firing wires. The tubing above the spoon is filled with lead and the total weight is from 400 to 500 pounds. Using this method of loading, a charge may be pushed through "bridges" of material. The time required for loading deep holes is greatly decreased over that required for loading by use of hand poles.

The character of the record is influenced by the depth and size of shot. Each area behaves somewhat differently and the sound of the explosion

frequently is indicative of the shot characteristics. Generally, a muffled or dull explosion sound indicates poor energy transmission and the generation of low frequency waves. An explosion which has a sharp crack or snap usually produces a record of higher frequency waves and is the most efficient for energy transmission. Few general rules can be given for the size and placement of the charge. All this must be worked out empirically for each area and sometimes even for each hole.

The shooting truck often is equipped with a water tank of about 500 gallons capacity, dynamite box, cap box, shooting poles and rack, a water pump, and also a power winch for loading deep holes. Figure 351 shows a truck of this type, and the shot-point geyser following a shot.

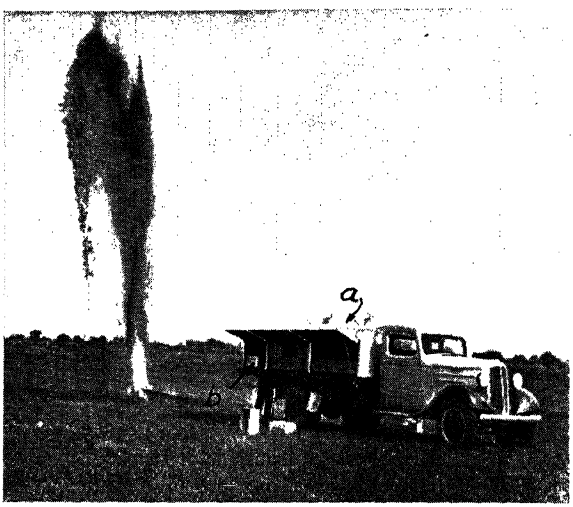


FIG. 351.—Shooting truck and shot-point geyser. *a*, water tank; *b*, explosive compartments.

Handling of Explosives

is the direct
occurred in seismic service.
to a failure on the part of
sponsible to appreciate the hazard, but rather to be an example of the old adage, "Familiarity Breeds Contempt." Constant vigilance is necessary on the part of the party chief to maintain proper control of this hazard in storage, transportation, and in field operations with explosives.

zines are a necessity for safe operations in order
reach of anyone not authorized to handle them.

Portable magazines are available and should be utilized wherever permanent storage facilities cannot be provided.†

Usually there will be found in each case of dynamite an instruction folder containing a list of "Don'ts" or safety rules. Each carton of electric blasting caps also contains a similar folder applying to caps. Every crew member having occasion to handle dynamite or caps or to be near the shot-hole or firing circuit should be required to familiarize himself with these safety rules and to re-read them at frequent intervals. These rules have been set up as the result of exhaustive studies of accidents from explosives, and their strict and literal observance is recommended. One of the most important precautions is to maintain a large separation in distance between the caps and dynamite, except for the single charge being prepared.

Wherever possible, wooden tools should be used. When metal parts are necessary for mechanical strength, bronze or other non-sparking metal must be used. Steel and iron should never be used, both because they can strike sparks and because they introduce a greater hazard from impact and friction than do the other materials mentioned.

There are on the market several explosives of a type differing radically from the usual varieties of commercial explosives. For some of these explosives, claim is made that they are much less sensitive to flame, impact, and friction than are dynamite and gelatin, and that these properties contribute to the safety of shooting operations where they are used. It must be borne in mind, however, that these products are designed to explode, and that under suitable circumstances they will do so. The party chief must, therefore, permit no relaxation in the safety precautions while these explosives are being used, since carelessness in handling these products may introduce hazards that can offset the safety advantages claimed for their relatively insensitive nature.

DRILLING EQUIPMENT FOR SEISMIC OPERATIONS

Types of Drill.—The drilling machines originally employed for drilling seismograph shot-holes were rotary diamond core drills of the same type as those used for mineral exploration. These diamond drills were mounted on automotive trucks together with a water or mud pump. A light derrick or mast was mounted on the truck platform over the drill.

Hydraulic Feed Type of Drill

The modern high speed portable drilling unit now used for seismograph survey drilling is essentially a modification and improvement of the early diamond core drill. These machines employ the rotary method of drilling rather than the percussion or reciprocating method which is known

† J. W. Flude, "Portable Dynamite Storage Magazines," *Geophysics*, July, 1935, pp. 61-65.

as cable-tool or spudding drilling. In the rotary method, drilling is accomplished by rotation of the hollow drill rod on the bottom of which is the cutting bit. Water or mud is pumped down through the drill rod and leaves through the bit to wash up the cuttings.

Upon arriving at the location where the hole is to be drilled, the mast or derrick is raised into a vertical position, usually by two hydraulic hoisting cylinders or by a power take-off operated by the truck motor. Figure 352 shows the setting-up procedure; in this view, the drill rod, swivel, and water hose are connected, and the derrick is being hoisted into position.

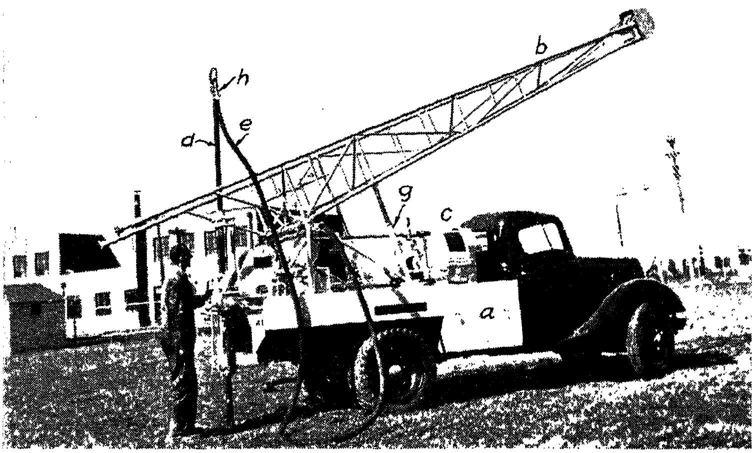


FIG. 352.—Setting up a portable rotary drill. The mast is being raised by hydraulic jacks. *a*, 1½ ton truck; *b*, mast; *c*, mud or water pump; *d*, Kelly; *e*, water hose; *g*, hydraulic jacks; *h*, swivel. The drill and mud pump are driven through a power take-off mechanism from the truck engine. (Courtesy of Sullivan Drill Co.)

The mast is about 25 feet high, which allows sufficient height for a ten-foot section of drill rod, the water swivel, the safety hook, and an additional 2 or 3 feet for spudding or reciprocating the rod string. At the top of the mast is mounted the sheave wheel for the wire line. If the drill is equipped with a cathead, a second sheave wheel is provided for the soft line (manila rope).

The drive rod or drill spindle is a hollow tube of steel through which the drill rods pass. The drill rods are attached to the drive rod by means of a chuck. The chuck, which is usually located at the bottom of the drive rod, contains four jaws which grip or release the drill rods by tightening or loosening four set screws. The outside of the drive rod is either fluted throughout its entire length or is hexagonal in cross section to allow it

to be fed up or down as it is rotated by the quill or driving bushing, through which the drive rod passes.

The piston rods of two hydraulic feed cylinders are attached by means of a yoke to the drive rod, thus providing a feed for the drill rod as well as a means for applying pressure to the drill bit which is screwed on to the bottom of the drill rods. The free movement of the hydraulic feed is usually 30″ advance. After the hydraulic feed has advanced, the set screws in the chuck are loosened; the hydraulic feed is reversed to raise the chuck to its top position and the set screws are again tightened, and the process is repeated.

The drill rods ordinarily used for seismograph drilling are described as the size "N" 10′ length diamond drill rods. (Figure 353I.) These are hollow steel rods 2⅜″ in outside diameter with a threaded coupling for screwing the rods together, providing an external flush joint.*

Numerous types of bits are available, but the type most commonly employed in seismograph drilling is the ordinary steel two-wing fishtail or drag bit. (Figure 353J.) The top end of the bit is provided with a socket and is threaded to screw on to the lower end of the drill rod. The cutting edge is faced with a tungsten carbide alloy, which produces a very hard and wear-resistant edge. The circulating mud is pumped down through the drill rod and leaves through holes in the bit. The function of the mud is to keep the cutting edges clear and to remove the cuttings from the hole.

The water swivel screwed to the top end of the drill rods is a hollow packed joint that provides a connection between the mud or swivel hose, which does not rotate, and the rotating drill rods. (Figure 353K.) The swivel is equipped with a bail for hoisting or lowering the rods.

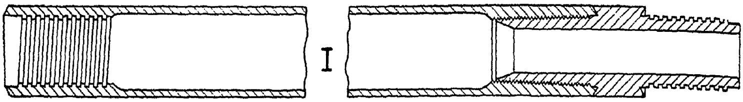
A rope or wire line is wound on the hoisting drum or draw works and is threaded over a sheave wheel mounted in the top of the mast and is attached to the water swivel bail by means of a safety hook.

After the drilling of a hole has advanced 10 feet (the length of the drill rod), it is necessary to disconnect the water swivel at the top of the rod string and add another 10-foot rod at this point. Before the new rod to be added is picked up, a hoisting plug with eye is screwed into the top end of this rod, and the hook from the hoisting line is attached to this eye in the hoisting plug. (Figure 353L.) The rod to be added is lifted up into position, and, before the hoisting plug is removed, the new rod is screwed into the top of the rod string. The hoisting plug is then removed and the water swivel is attached to the top of the new rod. When connecting or disconnecting rods, the rod string remaining in the hole is suspended either from the drill chuck or from a safety clamp set at ground level. (Figure 353M.)

The mud pump is usually a two-cylinder, double-acting, reciprocating pump with 4″ diameter pistons and a 5″ stroke. The foot valve on the end of the suction hose is submerged in the sump or pit which is ordinarily a shallow hole or trench dug in the ground. The mud flows from this sump through a suction hose, the pump, swivel hose, and water swivel into the top end of the rod string and then down through the hollow rods and the bit. After leaving the bit, the mud circulation washes the cuttings out of the hole. At the top of the hole, the mud flows back to the sump

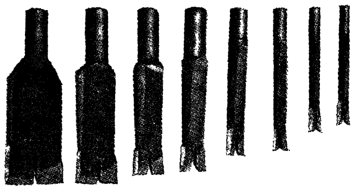
* Detailed dimensions of the "N" rods and couplings are listed in U. S. Department of Commerce, Bureau of Standards publication CS-17-30 on Commercial Standards.

YELLOW STRIPE "N" ROD



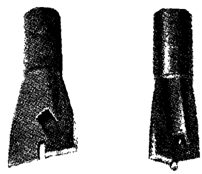
I

FISH TAIL BITS—HARD METAL FACED



J

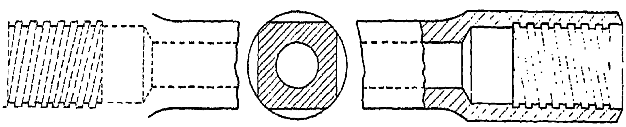
Forged Fish Tail Bits—Hard Metal Faced



Fish Tail 3 Way

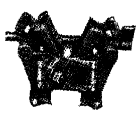
Cast Steel Bits—Hard Metal Faced

GRIEF STEMS OR KELLYS



N

SAFETY CLAMP



M

WATER SWIVEL

K

HOISTING PLUG

L

FIG. 353.

through a small trench. Cuttings settle in the trench and sump, and are shoveled out. Sometimes a heavy mud is circulated to help maintain the walls of the hole when drilling through caving materials, thus avoiding the necessity of casing the hole.

In the hydraulic feed system, the hydraulic pressure system for the feed is maintained independently of the mud pump and is included in the drill mechanism. When oil is introduced under pressure at the top of the two hydraulic feed cylinders, the drive rod is fed downward, and the oil in the hydraulic cylinders below the pistons flows back to the hydraulic oil tank under atmospheric pressure. When the process is reversed, the top of the cylinders is vented to the oil tank, and the feed of the drive rod is upward. The upward and downward feed or travel of the drive rod does not interfere with and is entirely independent of the rotation.

Between the power take-off and the drill is a multiple speed transmission or gear box which controls the speed of rotation of the drive rod. This transmission box also permits multiple speeds on the hoisting line so that heavier loads can be hoisted or lowered at slower speeds.

The rotating and feeding mechanism is mounted on guide rails running lengthwise along the truck, so that it can be shifted away from the hole when casing has to be set or pulled. The rotating mechanism is moved by a double-acting hydraulic ram which is actuated by the hydraulic pressure system.

The operating levers are placed at the rear of the truck so that they are readily accessible to the driller; from this position, he can raise and lower the mast, connect or disconnect the mud pump drive through a clutch mechanism, operate the rotating mechanism at any speed, control the operations of the hoisting drum and the cathead, and shift the rotating mechanism off the hole.

There is sufficient space on either side of the truck floor between the mast legs and the sideboards to stack several lengths of the 10-foot drill rods when moving from one location to the next.

The drive rod can be equipped with a chuck at the top, instead of at the bottom, and the top chuck can also be equipped with a Kelly drive plate for driving a Kelly or grief stem, where this method of operation is desired.

The Kelly or grief stem is either a hollow square rod or a hollow round rod with external splines extending along its entire length. (Figure 353N.) When drilling, the Kelly is kept in the drive rod but is not attached to it by the chuck. The rods are added below the Kelly.

In extremely soft drilling, where the hydraulic feed mechanism is not needed to force the bit into the ground, the Kelly type of operation is sometimes an advantage. The weight of the rods and the Kelly is sufficient to give satisfactory penetration of the bit when it is rotating. This eliminates the necessity of tightening and loosening the set screws on the chuck and permits the full 10-foot feed of a rod before rotation is stopped for the addition of another rod.

Where extra weight is not a disadvantage and comparatively deep holes are to be dug, or where cone type rock bits are used instead of the ordinary drag or fishtail bit, a separate engine is sometimes employed to drive the rod rotating mechanism and the hoist, and only the mud pump is driven by the power take-off from the truck engine, or else a single auxiliary engine is used for all drilling operations.

Various types of portable chassis mountings may be used. When operating conditions require it, the drills may be mounted on tandem conversion trucks, on crawler tread tractors and skids, or on barges.

Chain Feed Type of Drill

The chain feed type of drilling equipment is an alternative to the hydraulic feed type and is used as extensively. The hydraulic feed cylinders are omitted and in their place a chain driven by powered sprocket-

wheels applies the pressure on the drilling bit. The Kelly stem is used exclusively in the chain feed type. The chain is attached to a cross bar which is connected to the head of the Kelly stem through the swivel.

Figure 354 shows a photographic view of the top of a light chain feed drill built for areas which are inaccessible for larger equipment. Oversize tires are used to increase mobility. The truck with mast in upright position is shown in Figure 355. Various parts of the drill are labelled on the two figures.

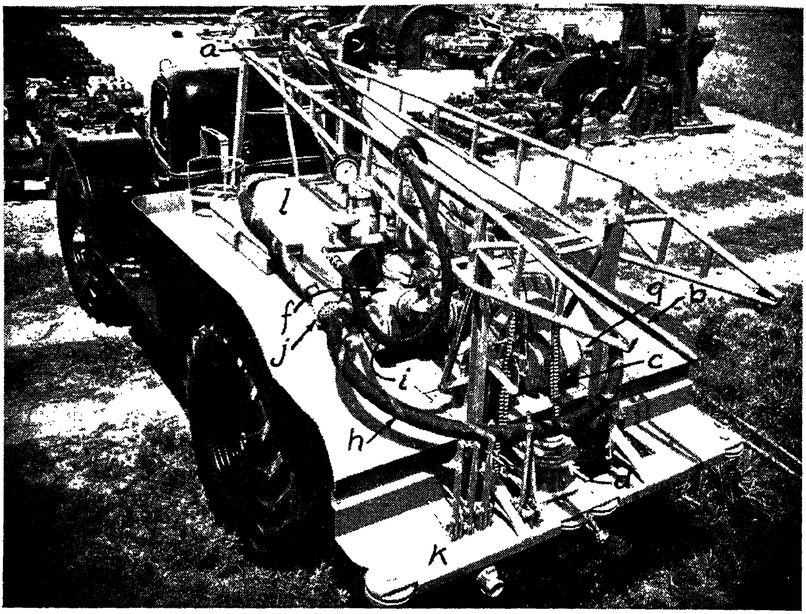


FIG. 354.—Light chain feed type drill. *a*, swivel head; *b*, Kelly; *c*, feed chains; *d*, rotary table; *f*, mud pump; *g*, cathead; *h*, suction hose; *i*, pressure hose; *j*, foot valve; *k*, control levers; *l*, line shaft cover. (Courtesy of Western Geophysical Company.)

Water Truck.—It is customary to have each drilling machine served by a separate water truck. Occasionally, when the holes are drilled on a closely spaced pattern, or when the distance for hauling water is short, it is possible for one water truck to serve two drill rigs, but this is the exception.

The water truck frequently is provided with a winch that is driven through a power take-off from the truck engine. This winch is useful for pulling the water truck or the drill truck over short stretches of wet or swampy ground or road. The rear end of the water truck is sometimes equipped with a small sturdy "A" frame which carries a sheave wheel. The winch line is passed over the sheave and can be used for pulling casing after the holes have been shot.

Four different pumping methods are available for filling the water tank. The first method utilizes a small rotary type pump driven through a power take-off from the truck engine. The second method employs a small single cylinder double-acting reciprocating type plunger pump that is driven through a power take-off from the truck engine and is also

FIG. 355.—Chain feed drill truck with mast in upright position for drilling. *a*, swivel head; *b*, Kelly; *c*, feed chains; *d*, rotary table; *f*, mud pump; *g*, cathead; *h*, suction hose; *i*, pressure hose; *j*, foot valve; *k*, control levers. (Courtesy of Western Geophysical Company.)

provided with means for hand operation. The third method (applicable where sealed tanks are employed) utilizes a vacuum system. The air in the tank is drawn out by connecting the tank to the vacuum or manifold system on the truck engine. This method makes it possible (at the expense of engine efficiency) to create a vacuum in the tank while the truck is traveling to the location of the water supply, so that upon arrival the tank can be very quickly filled by the vacuum already created.

The fourth method employs a small two or three horsepower air-cooled centrifugal pump. The pump is driven by a single-cylinder gasoline engine. This pumping unit is a completely independent portable unit and is light enough to be carried by one man. This method has an advantage when the water is to be elevated more than eighteen or twenty feet vertically above the surface of the source of supply and it is impossible to drive the water truck to a low enough position to pump by direct suction. In this case, the portable pumping unit is carried to a sufficiently low position that the water can be picked up by direct suction and then discharged through a hose under pressure into the water tank of the truck.

Drilling Crew Performance.—The usual drilling crew consists of three men: driller, helper, and water truck driver. When the water truck driver is not actually hauling water, he assists the other two members of the crew at the drill. It is general practice for the seismograph drill crew to work only the eight-hour daylight shift.

Over yearly periods, footage records of about 7,000 feet per drill per month may sometimes be expected. However, where boulders or particularly difficult formations are encountered, the footage will drop to a small fraction of this amount.

In soft formations, such as clay and most shales, it is often possible to drill a sixty-foot hole in twenty to forty minutes actual drilling time. The time required for moving from one hole location to the next varies with the distance between holes, the topography of the country, the condition of the roads, and the obstacles encountered, such as swamps, forests, fences, and weak bridges. After the drill truck has reached the location where the hole is to be drilled, it takes from 10 to 20 minutes to dig the mud pit or sump and prepare for drilling. It is sometimes possible for the water truck driver to have the pits already dug and filled with water at the location of the hole before the drill arrives, with a resultant saving in time.

The diameter of the hole varies from three to six inches. The size of the hole may be determined merely by opinion, but it is more likely to be determined by the size and quantity of explosive to be used, by the drilling characteristics of the ground, or by the need of casing in the hole. Casing sometimes is required in holes where the ground is caving or fitchery in order to make a deep enough hole to place the charge sufficiently below ground level. The casing often employed is second-hand, three-inch line pipe or boiler pipe which is commonly available in oil field communities.

In most cases, the drilling equipment is owned and operated by the organization making the seismograph survey. In some cases, the equipment may be owned and operated by an independent drilling contractor.

A drilling contractor is usually employed on a per drill per month basis. The charge for this work (where the contractor provides a three-man crew for single shift operation and furnishes all of the machinery, bits, gasoline, lubrication, supplies, insurance, and supervision) is about $1300 per month for light equipment provided the expensive roller bits are not required.

The actual cost per foot of hole drilled varies between wide limits, depending upon the hardness and character of the ground, the distance between hole locations, the type and condition of drilling equipment employed, the efficiency and experience of the crew, and the supervision. Minimum operating costs for labor, tools, lubrication, bits, replacements, and supplies have averaged as low as 6 cents or 7 cents per foot. If boulders or loose gravel is encountered, or if the holes are sparsely spaced, or if the water supply is inaccessible, or if casing is required in the hole, these direct costs will sometimes run to 50 cents per foot. A reasonable average direct cost for usual conditions of ground, hole spacing, and drilling equipment, is 18 cents to 20 cents per foot of hole drilled.

The initial cost of a drilling machine, including the truck, varies from $4,000 to $6,000. The cost of the water truck will vary from $1200 to $1800, depending upon the make, wheelbase, and size of truck, and upon the equipment.

In addition to the truck-mounted drilling machine and the water truck, the following operating equipment and tools are recommended for average daylight operation.

OPERATING EQUIPMENT

1—lightweight water swivel with size "N" pin connection
1—ball bearing type hoisting plug with "N" pin connection
200 ft. of 10′ lengths size "N" seismograph type drill rods
6—3″ fishtail bits
6—3½″ fishtail bits
6—4″ fishtail bits
2—4⅜″ fishtail bits
1—75′ length ½″ 6 x 37 extra pliable plow steel hoisting rope with safety hook
1—20′ length of 2½″ wire inserted suction hose with foot valve and connections
1—25′ length 1¼″ wire-wound swivel hose with connections at both ends

For **KELLY OPERATION** the following additional equipment is needed:

1—12′ length size "N" Kelly
1—top chuck assembly
1—Kelly drive plate assembly
1—L. H. bushing for water swivel to replace the R. H. bushing
1—5′ length "N" drill rod

TOOLS

1—18″ rigid pipe wrench
2—24″ rigid pipe wrenches
1—36″ rigid pipe wrench
1—12″ crescent adjustable wrench
1— 8″ crescent adjustable wrench
3—Cold chisels, ¼″, ½″, ¾″
1—¾″ x 24″ carpenter's wrecking bar
2—pipe tongs, 1″ to 6″ pipe
1—vise, 5″ jaws, with fittings for mounting on drill truck
1—hacksaw with six blades
3—12″ files—flat smooth, half round second cut, round smooth
1—hand oil can, 1 pt. size
2—#2 round point shovels
1—long handle single bit axe
1—7½″ pick with handle
1—wire brush
1—set mud pump liner and seat pullers
5 lbs. drill rod wicking

The additional cost of these tools, drill rods and operating fittings is approximately $1,000.

Hydraulic Drills.—Water jet or hydraulic drills are successfully employed in areas where shallow holes are to be drilled in fill material free from boulders and rocks. Drills of this type are employed quite extensively in the marsh and swamp areas of the Gulf Coast. Figure 356 illustrates one type. Power is supplied by a small two-cycle gasoline engine connected to a centrifugal pump. The discharge from the pump is carried by a 2″ woven cotton hose to a swivel head connected to the drill pipe. The drill pipe is of aluminum, usually about 1½″ in diameter. At the lower end of the pipe is a two-jet nozzle, which constitutes the "bit." During operation, the drill is rotated slowly by hand to allow uniform hydraulic action of the jets on the fill material.

SPECIAL APPLICATIONS OF SEISMIC GEOPHYSICAL METHODS

Investigations of Soil Dynamics.—Analysis of the dynamic constants of soil furnishes useful information for determining load-bearing capacity. In the past, foundation and load-bearing studies had been principally conducted by static load methods, which are relatively expensive and oftentimes inadequate or misleading in their results. The dynamic methods usually can be applied more economically to obtain data on the load-bearing capacity of the soil as well as its behavior under vibrations such as are produced by earthquakes.

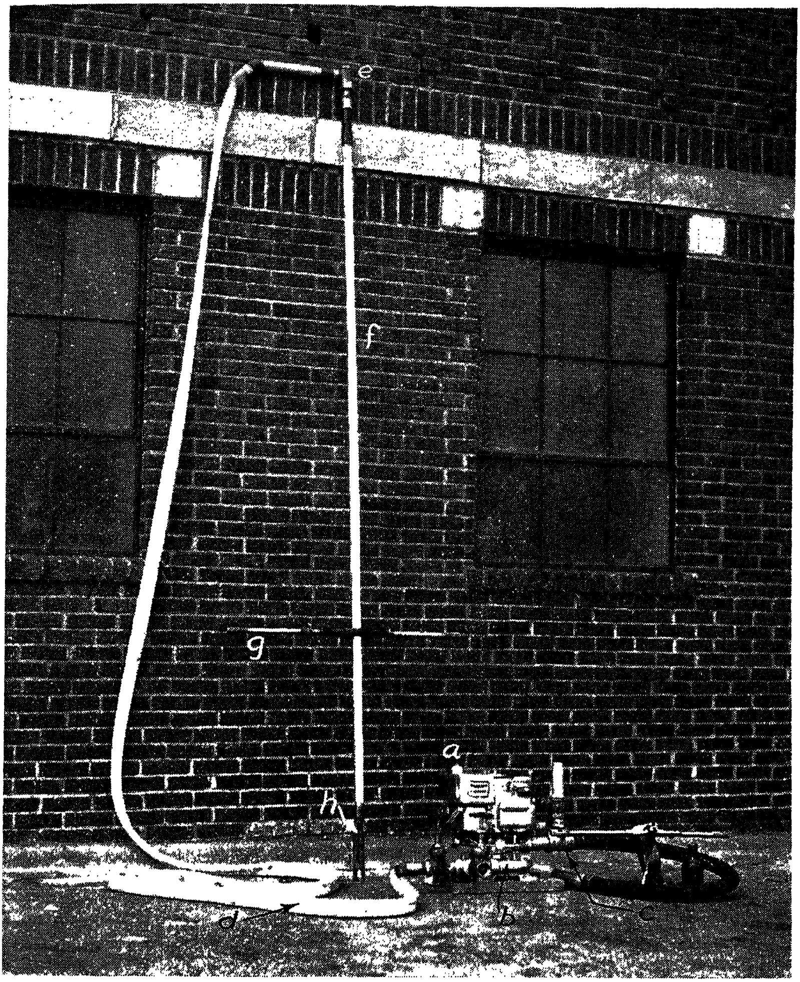


FIG. 356.—Hydraulic drill for fill material. *a*, two-cycle gasoline engine; *b*, centrifugal pump; *c*, pump intake; *d*, cotton hose; *e*, swivel head; *f*, drill pipe; *g*, handle for operator; *h*, nozzle.

To determine dynamic constants, the site under investigation is subjected to alternating mechanical forces by use of a special type of oscillator.† The soil is set vibrating in forced and damped oscillations, having any desired frequency and amplitude, by adjusting the frequency of alternation and the magnitude of the applied forces. Furthermore, by employing seismographs to record the motion of the soil, various characteristics of the motion are measured and the data thus obtained are used to draw inferences on the bearing capacity, composition, natural period, and compressibility of the soil.

Operating Principles

The oscillator for producing the artificial vibrations usually comprises two discs or cylinders revolving in opposite directions and provided with eccentric loading. (Figure 357.) The magnitude of the centrifugal

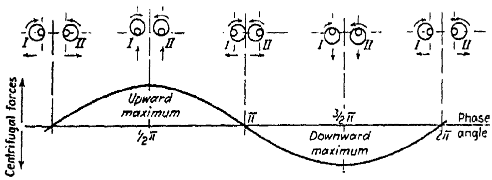


Fig. 357.—Oscillator for producing artificial vibrations. (Bernhard, *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 834.)

forces produced by the rotation of the discs depends on the eccentricity of the loading, and the frequency of alternation of the forces depends upon the speed of rotation. Usually, the horizontal centrifugal forces are neutralized and only the vertical forces are utilized in the studies.

The dynamic properties of the soil may be determined by measuring: (a) the amplitude of the soil motion at the oscillator; (b) the amplitude of the soil motion at various distances from the oscillator and at various depths; (c) the time phase difference between the applied (oscillator) forces and the vibration on or in the soil; (d) the settlement of the oscillator on the soil; and (e) the natural period of the site.

The soil motion characteristics enumerated above are related to the magnitude and frequency of alternation of the exciting forces. To simplify the analysis, the following assumptions will be made:‡ (1) The oscillator is a small mass vibrating on the surface of a semi-infinite elastic medium. (2) Vertical forces only are effective. (3) The damping is proportional to the speed of propagation of the disturbance. (4) The deformation of the soil varies linearly with load and settlement.

† R. K. Bernhard, "Geophysical Study of Soil Dynamics," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 834, Feb. 1938.

‡ Bernhard, *loc. cit.* See also, A. Hertwig, G. Angenheister, R. Koehler, and A. Ramspeck, "Application of Dynamic Soil Investigation," *Trans. German Soc. Research in Soil Mechanics* (Degebo) (1936) No. 4.

Corresponding to these assumptions, the equation of motion of the vibrating element is:

$$m \frac{d^2x}{dt^2} + K \frac{dx}{dt} + cx = P \sin nt$$

where m = mass of vibrating element = mass of oscillator plus a small amount of soil directly under the oscillator.

x = amplitude of vibration.

t = time.

K = coefficient of friction.

c = elastic coefficient (force in pounds required to deflect the system one inch).

n = frequency of oscillator.

$P \sin nt$ = centrifugal force of rotating disc oscillator.

The maximum amplitude a of the disturbance is given by the equation:

$$a = \frac{P}{m\sqrt{(n_0{}^2 - n^2)^2 + 4\delta\, n^2}}$$

where n_0 = natural frequency of the vibrating element

The phase difference ϕ between the exciting force and the amplitude of the motion is given by the relation

$$\tan \phi = \frac{2\delta n}{n_0{}^2 - n^2}$$

The resonance point of the soil adjacent the oscillator may be determined by measuring the amplitude of the motion—as recorded by a seismograph located near the oscillator—for various frequencies, i.e., for various values of n and by plotting an amplitude versus frequency curve. The *resonance frequency* may also be determined by measuring the settling or movement of the oscillator for various frequencies of the oscillator and plotting a settlement versus frequency curve.

The empirically established correspondence between natural frequency and bearing capacity or allowable soil pressure is given in Table 21.

The soil adjacent the oscillator will be set into vibration at its *natural* or *resonance* frequency for a particular value of the oscillator frequency. However, this soil, as well as the remainder of the soil constituting the

elastic medium, will undergo *forced* vibrations for any frequency of the oscillator.

Corresponding phases of the waves impinging on two seismometers located at different distances from the oscillator arrive at different times. Hence, the velocity of the wave in the soil under investigation can be determined by measuring the time phase difference of two corresponding maxima or minima, the velocity being obtained from the relationship:

$$V \qquad x_2 - x_1$$

where T_1 and T_2 are the times for corresponding maxima or minima on the traces and x_1 and x_2 are the distances from the seismometers to the oscillator. The correspondence between wave velocity and allowable soil pressure is also shown in Table 22.

Oscillators of this type may also be employed for shallow structural investigations to determine depths to bedrock, dip of contact, etc., by the techniques utilized in general refraction studies.

TABLE 22

CORRESPONDENCE BETWEEN WAVE VELOCITY, NATURAL FREQUENCY AND ALLOWABLE SOIL PRESSURE†

Test No.	Formation of Soil	Phase Speed, Miles per Hr. (Frequency 20 to 25 Cycles per Sec.)	Natural Frequency, Cycles per Sec.	Allowable Soil Pressure, Lb. per Sq. In.
1	10 ft. marshland on sand	180	4.0	0
2	Very fine sand	245	19.3	14
3	Tertiary clay, moist	290	21.8	
4	Clay sand	310	20.7	
5	Medium sand, moist	310	21.8	28
6	Jura clay, moist	335		
7	Old sand and cinder	355		
8	Medium sand in water	355		28
9	Medium sand, dry	355	22.0	28
10	Argillaceous sand	380	22.6	36
11	Gravel with stones	400	23.5	36
12	Clay, moist	420	23.5	
13	Marl boulder	420	23.8	42
14	Fine sand with 30 per cent medium sand	420	24.2	
15	Clay sand with lime inclusions	445	25.3	
16	Medium sand, undisturbed	490		57
17	Marl	490	25.7	57
18	Keuper sandstone, soft	560		
19	Diluvial loess, dry	580	23.5	
20	Gravel under 12 ft. sand cover	730		64
21	Gravel, dense	940	30.0	64
22	Sandstone, disintegrated	1120	32.0	⅔ of the allowable compression strength (rows 22–24)
23	Keuper sandstone, medium hardness	1450		
24	Sandstone, undisturbed	2450		

† Bernhard, *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 834.

Investigations of Pressures on Rock Pillars of Underground Mines.—A sonic method for determining the pressure on pillars in underground mines has been proposed which requires, first, laboratory determinations of the variation of the velocity of sound with pressure in samples of rocks taken from mine pillars and, second, measurements of the velocity of sound in the mine pillars from which the laboratory samples were taken. From a knowledge of the velocity of sound in a particular rock as a function of pressure (laboratory measurement) and the velocity of sound in an underground pillar composed of the same rock (mine measurement), the pressure on the underground rock pillar can be inferred.†

A direct and relatively simple method for measuring the variation of elastic wave velocity with pressure utilizes rocks cut in the form of rectangular prisms. Known amounts of pressure are applied to the ends of the rock specimen by means of a mechanical press. The prism of known dimensions and density is caused to vibrate in one of its fundamental modes of vibration and the frequency of vibration is determined. The elastic wave velocity for any pressure can be computed from well-known relationships expressing the velocity as a function of frequency, density, and geometrical dimensions. For example, if the prism is supported at its ends and if the driving force is applied at the center of the prism so as to cause the prism to vibrate as a unit with its two points of support as nodal points, the elastic wave velocity V of the prism material is given by the relation

$$V \qquad \pi t$$

where L is the length of the column, f the frequency of vibration, and t the thickness of the column.

Earthquake Insurance.—The appalling loss of life and the damage to property caused by earthquakes has led the governments of several countries, notably the United States, Japan and Germany, to institute investigations of methods of minimizing this damage.* Studies of the damage produced during the San Francisco earthquake of 1906, the Santa Barbara earthquake of 1925, and the Long Beach earthquake of 1933, have shown that the damage was due largely to the collapse of poorly constructed structures not designed to withstand earthquake forces and to fires which could not be controlled because of the failure of water supplies. These studies, therefore, indicated that one very important method of minimizing the damage is to construct buildings in earthquake areas which are as earthquake-resistant as feasible.‡ From an engineering design viewpoint, this is accomplished by so constructing the building that it will not be markedly affected by the natural frequency of vibration common to earthquakes in the area.

† L. Obert, "Measurement of Pressures on Rock Pillars in Underground Mines," U. S. Bureau of Mines, Report of Investigations 3444, April, 1939.

* It has been estimated that during the last two centuries on the average some thirty thousand persons were killed each year by earthquake phenomena.

‡ M. H. Gilmore, "Earthquake Investigations," *Geophysics*, Vol. II, No. 3, July, 1937, pp. 253-264.

An important economic problem associated with earthquakes concerns insurance. Logically, earthquake insurance rates for buildings in a given area should vary according to their susceptibility to earthquake damage. For example, if the natural period* of vibration of a building is the same as the ground period set up by most earthquakes in the area in which the building is located, a dangerous condition of resonance may be anticipated during the earthquake and the insurance premium correspondingly should be high. Furthermore, it is reasonable to expect that if the periods of the ground, the building, and the earthquake are approximately equal, a more dangerous resonance may be set up in the structure than would be the case if these periods were substantially different.

An evaluation of the earthquake-resistant properties of a particular building or structure requires, therefore, a knowledge of: (1) the dominant or most common periods of the earthquake waves in the area, (2) the natural period of the ground on which the building is located, and (3) the natural period of vibration of the building.

The first of the above factors can be deduced from a statistical analysis of earthquake records obtained at permanent seismological stations. Thus, Professor B. Gutenberg deduced from an analysis of numerous records that the following earthquake wave periods are most common in California: 0.2 to 0.3 sec., 0.5 to 0.6 sec. and 1.0 sec. The second factor (natural period of the ground) may be determined by an application of seismic methods, as described in the section on soil dynamics investigations. The third factor (natural period of vibration of the building) can be determined by putting controlled vibrations into the building. The oscillator used for this purpose is of the type previously described, except that much smaller energy is necessary. The natural frequency of the structure under test is determined by plotting a frequency-amplitude curve for the building and noting the frequency corresponding to maximum amplitude.**

To check buildings for damage, their periods must be redetermined after a severe earthquake. If the period has increased appreciably, the building probably has suffered considerable structural damage which may not be apparent always from a visual inspection of plaster or masonry cracks.

* It will be recalled that the period of a vibration is equal to the reciprocal of the frequency.

** Governmental agencies have measured the natural periods of several hundred buildings and many water towers and dams.

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CHAPTER VIII

GEOCHEMICAL METHODS

Geochemical prospecting is the technique of drawing inferences concerning subsurface petroleum deposits from chemical analyses of samples of soil or samples of gas contained in the soil at or near the surface of the earth. This method, unlike the other geophysical methods, does not work on structure, but attempts to locate petroleum deposits directly by detecting and measuring certain constituents of soil or gases contained in the soil which may be associated with subsurface petroleum deposits.

The commercial technique consists first in obtaining the samples in the field and second in determining the percentages of hydrocarbon minerals and related compounds. Interpretation or prediction is based primarily on the areal distribution of the materials.

HISTORICAL REVIEW

The forerunner of modern geochemical prospecting was the prospecting technique which utilized visible gas and oil seepages as indications o petroleum at depth. The success of the early method is attested by th fact that of 141 salt domes discovered in the Gulf Coast prior to 193 35 owe their discovery to macroscopic gas or oil seepages. †

Geochemical exploration as practiced at present is conveniently treated under two headings: viz., *gas analysis* and *soil analysis*. Gas analysis (analysis of samples of interstitial gas contained in the soil near the surface of the earth) is essentially a refinement on the gross seepage method. Records of this type of geochemical prospecting have been in the literature for several years. In 1929 Laubmeyer ‡ described a method for collecting samples of soil gas from bore holes and analyzing them for methane. The method was patented as U. S. Patent No. 1,843,878. In 1932, descriptions of methods for collecting and analyzing samples of soil gas appeared in the Russian journal, *Neftyanoe Khozyaistu*, the principal investigator being V. A. Sokolov. § The success of this work was sufficient to have the method officially adopted in Russia, and to encourage investigations in this country.

† George Sawtelle, "Salt Dome Statistics," *A.A.P.G.*, 1936 pp. 726-735.

‡ G. Laubmeyer, "A New Geophysical Prospecting Method." *Pet. Zeit.* 29, No. 18, pp. 1-4 (1933).

§ V. A. Sokolov, "Methods of Exploration for Natural Gas" (Monograph, 1932).

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Soil analysis (analysis of samples of near-surface soil) is the more recent type of geochemical prospecting. Analysis is made of the gaseous and so-called liquid and solid hydrocarbons and the mineral salts in the soil. This type has been exploited chiefly in the United States.

PHYSICAL PRINCIPLES

The applicability of the geochemical method derives from the fact that the hydrocarbon and related constituents of an oil or gas deposit are assumed to flow continuously, although at a very minute rate, from the reservoir through the overlying strata into the atmosphere.

Quantitative data on the relative content of hydrocarbons and related materials contained in the surface or near-surface soil are assumed to be diagnostic with respect to the probable occurrence of petroleum deposits in the regions being explored. The relative and total amounts of hydrocarbons discoverable at any locality depend not only on the mechanism of seepage and the nature of the soil from which the sample is taken, but also on the methods of sampling and of analysis. Furthermore, correlation methods that are successful in a given locality with a certain sampling and analytical technique are not necessarily applicable in another locality. The presence of hydrocarbons either in macroscopic or microscopic amounts is not necessarily sufficient evidence to go to the expense of drilling. Samples taken in a systematic fashion over a given area may show a hydrocarbon high in a restricted portion of the area. If this region should be marshy, and the only hydrocarbon constituent methane, the assumption of a commercial subsurface petroleum accumulation would be highly questionable.

Anaerobic fermentation of carbohydrate material yields methane quantitatively, and, as far as is known, no other hydrocarbon,† whereas the natural occurrence of higher paraffins is associated either with coal or petroleum. Thus the diagnostic value of methane is much less than that of the higher hydrocarbons. In the United States, methane is for this reason not considered important as a means for the discovery of new oil pools. In the U.S.S.R., however, the methane content is considered of great importance, due chiefly to a theory of Sokolov ‡ that the ratio of the methane content to that of the heavier hydrocarbons is indicative of the presence of oil or gas and furnishes a means to evaluate the depths of the accumulations and to decide between oil and gas as the hydrocarbon source. (Compare p. 649.)

Mechanism of Migration.—A true description of the assumed manner in which hydrocarbons migrate from a source bed to the surface would be extremely difficult. At any position along the path the motion would depend on many factors: the porosity at that point, the pressure gradient, the absolute pressure, the temperature, the con-

† Neave and Buswell, *J. Am. Chem. Soc.*, 52, 3208 (1930).
Symons and Buswell, *J. Am. Chem. Soc.*, 55, 2028 (1933).
‡ V. A. Sokolov, *Neft. Khoz.*, 1936, No. 5, pp. 18-33.

centration of the hydrocarbon, the viscosity of the moving medium, etc. If the hydrocarbon concentration is lower than a value determined by the temperature and pressure, no free gas phase will be present, and migration must be of the nature of solute diffusion. If the concentration exceeds this value, a free gas phase will be present and bouyant forces will be effective. Because the subsurface is neither homogeneous nor isotropic but contains intrusions, faults, fissures, ground-water, etc., an over-simplified description of the migration mechanism leads to invalid conclusions.

Some observers believe that the gases migrate upwards in the form of minute microscopic bubbles, others think that this upward movement is in great part a combination of absorption and capillary movement. †

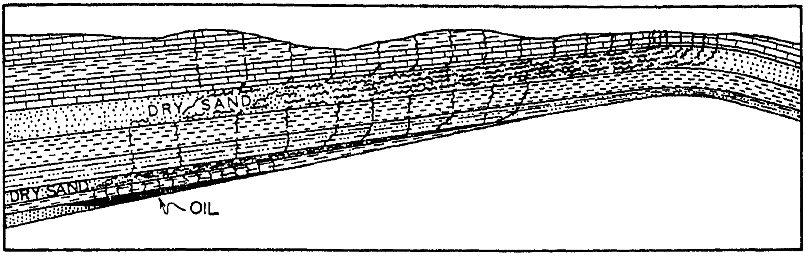

FIG. 358.—Hypothetical migration of hydrocarbon vapors from an oil sand in a stratigraphic trap. The migration follows the zones of greater permeability which are not necessarily vertical. (Howard, *Oil and Gas Journal.*)

The migration does not necessarily take place in vertical direction. It has been found that the usual sedimentary formations are more permeable along the bedding than they are across it. Migration upward from a stratigraphic trap containing oil might therefore follow inclined beds towards the top of the structure and from there follow a vertical path towards the surface. Consequently, the maximum content of hydrocarbons might be found over the dry top of a structure, while the oil accumulation is present at a lower point of the structure. (Figure 358.)† This does not condemn soil analysis as a means of finding off-structure oil accumulation. Results of this method combined with a knowledge of the geology of the area may well afford a satisfactory means of indicating the location of stratigraphic traps.

From a geological viewpoint, the principles governing migration may be restated briefly as follows:

1. The rock materials surrounding petroliferous deposits contain pore spaces or microscopic fissures which are large compared to molecular dimensions; hence, molecules of the hydrocarbons and related materials which constitute the petroliferous deposit can penetrate the "impervious" rocks and diffuse or spread out in all directions.

2. Besides diffusion, the migration of the hydrocarbons may be aided by pressure or gravity gradients and temperature gradients which are directed upward from the deposit to the surface of the earth. These vertical components aiding migration may account for the restricted areal distribution of the concentration patterns measured over many accumulations.

† W. V. Howard, "Some Factors Involved in Geochemical Prospecting," *Oil and Gas Journal,* July 18, 1940, pp. 50-53.

GAS ANALYSIS METHOD

The usual procedure in the process developed by Russian investigators is to obtain samples of "underground air," i.e., gas which is contained in the soil interstices, and to analyze these samples for traces of methane and "heavy" hydrocarbons.

A hole of suitable depth (10 to 30 feet) is dug by means of a hand auger so that its upper half has a larger diameter than its lower half.† The shoulder of the hole is then wetted, tamped, coated with clay, and sealed from the atmosphere by means of a bell-shaped sampling container. Next, the atmospheric gases are pumped out of the enclosure by means of a vacuum pump, and the soil gases are allowed to seep into the enclosure and partially evacuated sampling container.

Field Analysis of Gas Samples.—The sample is passed through a caustic solution to remove the CO_2 present. It then passes through a combustion chamber which oxidizes the hydrocarbons. The CO_2 formed on oxidation is measured by allowing the gas to bubble through a $Ba(OH)_2$ solution and noting the volume necessary to produce the first observable turbidity. This volume is compared with the volume required when CO_2 in known concentration is bubbled through the same apparatus. This method gives a measure-number of the *total* carbon content of the sample without regard to the types of hydrocarbons present. It is roughly quantitative.

Recently, an electro-thermal method has had considerable experimental application in the United States. This method is suitable for field use. It is claimed that the equipment can measure gas concentrations to within 100 parts of gas per one million parts of air. The equipment consists essentially of two containers, one of which contains a desiccator, such as calcium chloride, and the other calcium chloride and activated charcoal. The first container removes the moisture. The second container removes the remaining traces of moisture and the hydrocarbons.

The measuring mechanism of the instrument comprises two platinum wire filaments which are heated by an electric current. One of the filaments is exposed to the action of the hydrocarbons that are absorbed in the second container. Catalytic combustion of the hydrocarbon on the surface of the platinum wire causes an increase in temperature, with a resultant change in electric resistance. This change in resistance is measured on a D.C. bridge circuit, and a previously calibrated scale or chart gives direct readings of the hydrocarbon content.

A thermo-electrical method often is employed for differentiating qualitatively between methane and the heavier hydrocarbons. The method utilizes the fact that the ignition temperature of methane is several hundred degrees higher than that of the heavier hydrocarbons.* The temperature of the platinum filament is adjusted to allow partial combustion of the air-gas mixture, thereby burning the heavier hydrocarbons. The difference between this run and a second run at a sufficiently higher temperature to cause combustion of the methane and the other lighter hydrocarbons will be indicative of the concentration of methane.

† Sokolov, *loc. cit.*

* Personal communication, N. W. Hartz, Mine Safety Appliance Co., Pittsburgh, Pennsylvania, regarding MSA Benzol Indicator.

Laboratory Analysis of Gas Samples.—In the laboratory, separation is made into hydrocarbon groups. The method of separation depends on the difference in vapor pressures of the gases present in the sample at low temperature. The gases fall into two main groups: gases which are condensable in a liquid air trap, and those which are not. At the concentrations usually found in soil gas samples, and subject to the usual analytical treatment, hydrocarbons higher than ethane are condensable. The non-condensable portion includes nitrogen, oxygen, methane, and ethane. In the procedure described by Sokolov, the samples are freed from CO_2, water vapor, and basic constituents (e.g., ammonia) and circulated through a liquid air trap until the condensables are caught. This liquid air trap is isolated and the remaining gases are passed through a combustion tube, a dryer which removes the water, and then another liquid air trap which catches the CO_2 formed by the oxidation of the non-condensable hydrocarbons. The system is then evacuated to approximately a thousandth of a millimeter of mercury. The quantities of gas in each trap are then gauged by removing the liquid air and measuring the resulting pressure with a McLeod gauge. If advisable, further refinements are possible. The condensable fraction may be subjected to fractionation, and estimates made of the several constituents.

This procedure can detect hydrocarbons at a dilution of about 100 parts per million to an accuracy of about ± 10 parts per million.

Theoretical Interpretation of Gas Analysis Anomalies.—The development of interpretative technique by the Russian investigators has proceeded along the lines of attempting: (a) to deduce theoretical curves characteristic of petroliferous and gaseous deposits at various depths below the surface and (b) to solve the inverse problem: viz., infer the depth of the deposits from the observed anomaly curve.

Antonov[†] has published an article in which he assumes that diffusion takes place through a homogeneous overburden. In the stationary state, the diffusion equation reduces to Laplace's equation which may be written in the form:

$$\quad + \frac{\partial^2 M}{\partial y^2} + \qquad = 0$$

where M is the concentration or partial pressure of the hydrocarbon gas. Typical boundary conditions used by Antonov are: (a) the region below the source is completely impervious and (b) no diffusion takes place through the surface of the earth into the air. The validity of such boundary conditions is not self-evident. Also, reports on experimental, i.e., field, verifications of predictions based on this diffusion theory are not available.

In addition to this theoretical technique, the Russian investigators employ various empirical techniques derived from extensive investigations over many oil and gas fields. For example, investigations of various deposits have shown that the percentage of heavy hydrocarbons increases over an oil horizon and the percentage of methane generally increases over a gas deposit.[‡] The ratio of the light to the heavy hydrocarbon content is assumed therefore to furnish a useful diagnostic variable.

† P. L. Antonov, "Contribution to the Theory of Gas Surveying," *Neft. Khoz.* 26, No. 5, pp. 19-23 (1934).

‡ B. N. Victoroff, "Interpretation of the Nature of a Gas Survey," *Neft. Khoz.*, Sept., 1934.

In spite of the fact that more methane should leave a deposit of oil or gas than any other hydrocarbon, it is often found that the concentration of the higher hydrocarbons in the soil is greater than that of methane. There are three possible explanations of this phenomenon:

(1) The heavy hydrocarbons may diffuse more rapidly than the lighter hydrocarbons, although this appears extremely improbable.

(2) The heavy hydrocarbons do not originate in the oil deposits, but are formed in the soil from methane by oxidation and polymerization.

(3) The soil may adsorb or absorb the heavier hydrocarbons more readily or may have a greater retentivity for the higher hydrocarbons, while methane escapes towards the atmosphere.

SOIL ANALYSIS METHODS

Development of soil analysis techniques is due principally to American investigators. Analyses are made of the quantities of (1) volatile hydrocarbon constituents, (2) "solid" and "liquid" hydrocarbon constituents, and (3) mineral constituents.

The volatile hydrocarbons comprise those hydrocarbons which are adsorbed or occluded on the surface of the soil as opposed to constituents contained in the soil interstices. †

The gaseous hydrocarbons are recovered by evacuation and slight heating of the sample of soil. Numerous analytical methods have been proposed for this work, including the mass spectograph, special types of combustion analysis, selective freezing, vapor pressure studies, etc. The most successful work is conducted by basic methods well known in microchemical analysis. The adsorbed gaseous constituents of the soil are volatilized, and the non-diagnostic constituents (water, carbon dioxide, amines, ammonia unsaturates, alcohols, etc.) are removed by appropriate scrubbing agents. The scrubbed gas is then fractionated by selective refrigeration in a specially designed trap to allow quantitative separation of the minute condensables. After separation, the various fractions are mixed with air or oxygen which has been freed of carbon dioxide and hydrocarbons and are burned. The products of combustion are determined quantitatively by volumetric measurement, and then calculated back into equivalent weights of the original saturated hydrocarbons. The work must be done carefully with an accuracy which allows expressing the data in ratio form in parts of hydrocarbons per billion parts of soil on a weight basis.

The solid and liquid hydrocarbons are organic compounds which are not driven from the sample by the evacuation and gentle heating process employed in the gaseous or lighter hydrocarbon analytical work. These materials may be analyzed by one of two generally accepted procedures: namely, combustion analysis or selective extraction with appropriate solvents for the polymerized hydrocarbons. The extraction technique is more widely used. It commonly employs some form of hot solvent

† Compare Leo Horvitz, "On Geochemical Prospecting—I," *Geophysics*, Vol. IV, No. 3 (July, 1939), pp. 210-225.

reflux, similar to the Soxhlet apparatus. The sample is dried and then subjected to the action of the solvent for a sufficient period of time, usually eight hours or more. The recovered solute is then evaporated and the residual gum or solid material expressed in parts per million parts of soil, on a weight basis. Oftentimes, the results are more advantageously expressed in terms of parts per unit area of surface.

The mineral constituents which occur in soil samples are the inorganic salts or radicals, commonly referred to as carbonates, bicarbonates, chlorides, sulphides, sulphates, silicates, etc. The normal percentage of these materials in the soil varies with the areal distribution of surface and near-surface materials and rock types. It also is influenced by drainage factors and, naturally, ascending or descending solutions alter the ratio of the different materials. The ascending solutions, allegedly carrying the hydrocarbons to the surface, usually are high in dissolved salts which are deposited in the surface layers of the soil by evaporation.

Analysis of the mineral constituents of soil samples is conducted by various procedures, usually involving an extraction step with an aqueous solvent. The solute may be analyzed by a number of different procedures, chief of which may be mentioned: (a) chemical, to determine nature and percentage of salts; (b) physical, to determine weight and percentage of solids present when evaporated to dryness; (c) electrical, to measure changes in conductivity with bridge circuits. Success also is being had with modified micro-polarigraphic methods of the Heyrovsky type employing the dropping mercury cathode, whereby the polarization potentials of the ions in an aqueous solution are measured.†

The application of the mass spectrograph or mass spectrometer in soil analysis to date has been largely experimental.‡ The spectrometer comprises an ionization chamber, an analyzer tube, a collector, and an amplifier. The gas to be analyzed enters the ionization chamber through a capillary leak or gas inlet. The ionization chamber, analyzer tube, and collector are mounted in an air-tight container. This container is maintained at a high vacuum and is mounted in a uniform magnetic field.

The operation of the mass spectrometer includes: (1) ionizing the molecules of the gas; (2) impressing two uniform and mutually perpendicular fields on the ionized molecules: namely, an electric and a magnetic field; and (3) collecting ions of a particular mass or molecular weight in a collector where their quantity is measured by a suitable vacuum tube amplifier and galvanometer or recorder.

The molecules of the gas in the ionization chamber are ionized (made to acquire a positive charge) by bombardment by low voltage electrons emitted by a filament. The positive ions are then accelerated toward a slit which is maintained at a small negative potential with respect to the rest of the ionization chamber. The ions are then given a large velocity by means of a second slit which has a high negative potential with respect to the first slit. Due to the magnetic field, the ions will follow approximately circular paths, the radius of curvature for a particular ion depending on the ratio of the mass or molecular weight to the charge on the ion. Thus, ions of

† G. A. Perley, "Measurements with the Dropping Mercury Electrode," *Trans. Electrochemical Soc.*, Vol. LXXVI, 1939.

J. Heyrovsky, *Trans. Faraday Society* 16, 692, 1923.

‡ Herbert Hoover, Jr., and Harold Washburn, "Application of Mass Spectrometer to Petroleum Industry Problems," *Oil and Gas Journal*, Feb. 22, 1940, p. 48.

one mass or molecular weight will follow one circular path, while those of another mass will follow another circular path. By adjusting the magnetic field and/or the accelerating voltage, ions having a particular mass can be made to follow a predetermined path and pass through an exit slit into a collector where their quantity is measured by a vacuum tube amplifier and galvanometer.

It is claimed that a qualitative and quantitative analysis can be run on 1 cu. mm. of an unknown mixture of gases at atmospheric pressure with an accuracy of better than plus or minus 5 per cent of each of the various constituents.

Field Procedure.—Surveys are carried out by taking samples at designated locations so placed as to test the particular local condition. The points of sampling usually are chosen in a systematic manner such as: (1) along straight line traverses across the area, (2) along traverses radiating from a central hub, or (3) along parallel traverses crossed at right angles so as to form a grid of stations over the area.

One recommended field procedure for a *detailed* survey is to set up a grid of profile lines separated by less than one-half mile and locate the sampling stations every five hundred feet. For a *reconnaissance* survey, the profile lines may be spaced at intervals of one mile and the sampling stations at intervals of one thousand feet.

Soil samples are taken at depths varying from less than one-half inch to 10 feet by means of a sampling pick or soil auger. The samples are placed in containers suitably marked with the proper station location, and the containers are shipped to the laboratory.

To reduce the sampling error, some investigators collect three separate samples at each station and mix these samples prior to making the chemical analysis.

Samples are forwarded to the laboratory in glass jars which may be conveniently attached to the analytical equipment. The conventional pint fruit jar, with mason cap and rubber washer seal, is often used.

A careful record should be kept of the conditions at each sampling location, with particular reference to the following:

1. Name and number of survey.
2. Name of sampler and date.
3. Sample of location: traverse or station number.
4. Number of samples taken.
5. Method of sampling: surface scraper or auger.
6. Depth and/or surface area of sample.
7. Surface condition of ground: cultivated or undisturbed, dry, damp, or wet; type of soil.
8. Possible disturbing conditions, such as wells, pipelines, sumps or ditches.
9. Weather conditions and temperature.

At best, the data are erratic. Therefore, all possible information should accompany the field sample in order that the interpreter may be able better

to evaluate the data. If the samples are collected in areas covered with water or areas close to the water table, a sample of the water should be taken, especially for interpretation of inorganic constituents. (The adsorptive properties of wet or very damp soil usually are less than those of dry soil of the same type.)

Plotting of Data and Interpretation.—The sample results are either plotted as profiles or spotted as points on a map of the area and then contoured. The variables plotted or contoured depend on the area and the economic limitations placed on the analytical work. It generally is advisable to make plots or contours of all available data on methane, ethane, liquid hydrocarbons, surface wax, and mineralization.

The mean of the hydrocarbon content curve over a non-petroliferous area is usually a relatively flat regional trend. The profiles over petroliferous areas have been classified into three principal types: (1) the "halo" curve oftentimes observed over closed anticlinal structures; (2) the single anomaly curve sometimes observed over monoclinal stratigraphic traps; and (3) the abrupt high peak curve sometimes observed over faults associated with oil accumulation. Generally, the curves are very erratic and show many departures from a mean trend; hence, it is necessary to maintain a statistical viewpoint in interpretation and to give careful consideration to those local geological and surface conditions which may affect the accumulation of hydrocarbons.

The common "halo" type of curve generally found over an extended anticlinal structure usually shows lower hydrocarbon values over the crest of the structure and higher values around the margins. (Figure 359A.) Various theories have been advanced to explain this condition. It has been suggested that this condition may be caused by a cap of mineralization over the crest which diminishes the permeability of the strata over the top of the structure.† It has also been suggested that this effect may be caused by increased porosity or microscopic fractures at the edges of the structure due to greater flexure. This condition is not substantiated by known geology any more than the compaction or mineralization theory.

Another explanation which associates mineralization over structure with ground water movement may be given more consideration as experience with soil analysis methods increases. It is generally believed that the upward migration of water is greater along the flanks of the structure than over the top of the structure. This is particularly noticeable over intrusives, such as salt domes. The hydrocarbon anomalies which are sometimes found over surface traces of faults, rather than over the oil itself, when an angle of hade exists, again indicate that water movement may be the controlling factor. This theory on the influence of water on the hydrocarbon and mineral content of the surface oil is in agreement with

† E. E. Rosaire, *Geophysics*, Vol. III, No. 2, 1938, pp. 96-115; *Oil and Gas Journal*, December 22, 1938, pp. 43-56.

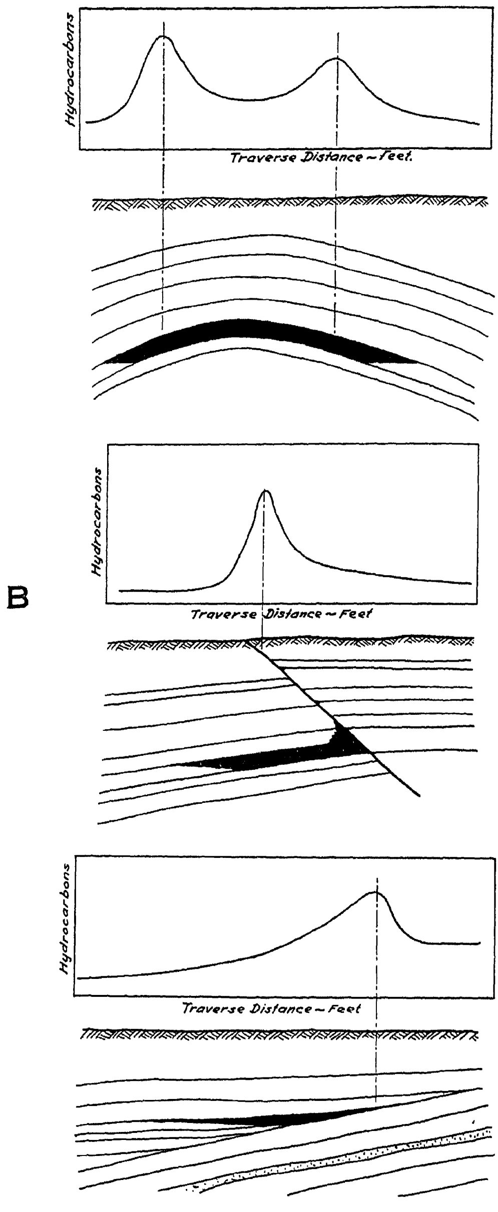

FIG. 359.—Ideal hydrocarbon anomalies. *A*, anomaly over extended closed structure; *B*, anomaly over fault zone; *C*, anomaly over stratigraphic trap.

the results obtained by the electrical methods over various structures. For instance, shallow resistivity measurements show that electrical anomalies usually occur over the surface trace of faults and are associated with increased moisture content.

Figure 359B shows the ideal anomaly curve sometimes found in areas containing fault zones. The location of the peak appears to be governed by the hade of the fault and the dip of the faulted strata.

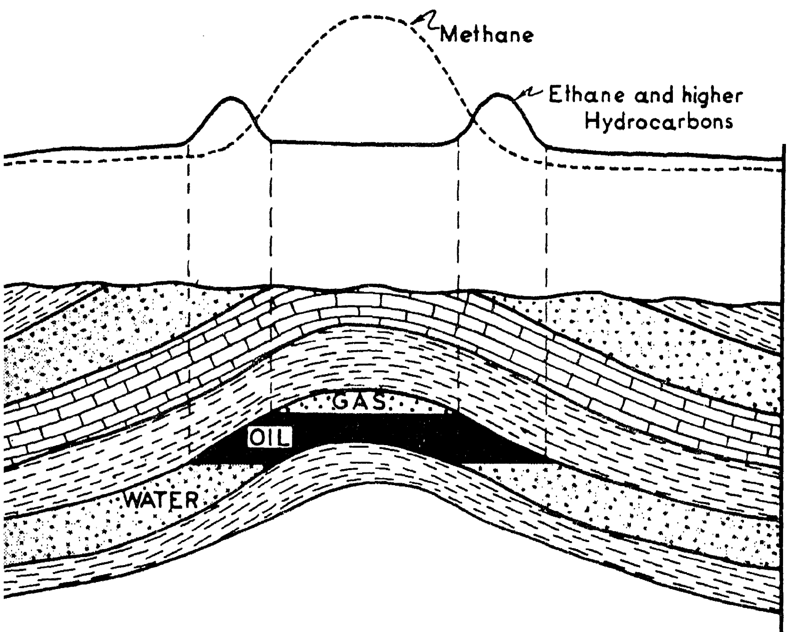

FIG. 360.—Sketch illustrating relatively high concentrations of methane and of higher hydrocarbons above a structure according to Russian geological theory. (Howard, *Oil and Gas Journal*.)

The single anomaly curve, such as that shown schematically in Figure 359C, has been found in surveys over small anticlinal structures and stratigraphic traps. The peak of the curve is occasionally shifted toward the up dip side of the accumulation.

Sokolov shows profiles which not only exhibit the regular "halo" of heavier hydrocarbons around the top of the structure, but also a methane maximum over the top. (Figure 360.) There are three possible explanations for this effect:

(1) Compaction and/or increased mineralization over the structure, which decreases the porosity of the strata and may allow better migration or cause selective absorption of the lighter hydrocarbons.

(2) Gas in the upper part of the reservoir, which prevents direct contact of the oil with the overlying rocks at the top of the structure. This would produce the effect of a methane peak over the top of the structure and a "halo" of heavier hydrocarbons along the flanks of the structure where the oil is in direct contact with the overlying rocks.

(3) A preferential absorption may exist for the lighter hydrocarbons, causing a saturation phenomenon.

The most logical explanation seems to be (2) because it is in accordance with established concepts and knowledge of conditions in oil reservoirs and does not require the introduction of assumptions whose validity is not immediately evident.

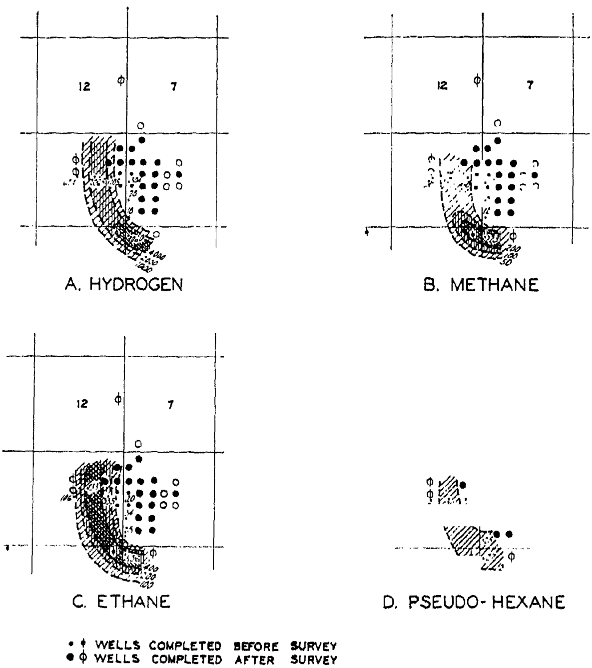

FIG. 361.—Graphical representation of soil analysis data for the Ramsey Oil Field, Payne County, Oklahoma. (Horvitz, *Geophysics*.)

Results of Hydrocarbon Soil Analysis Surveys.—Figure 361 shows the results from samples collected at depths of two to four feet at the Ramsey Oil Field, Payne County, Oklahoma. *A* represents the hydrogen

content of the samples, B the methane, C the ethane,* and D the "pseudo-hexane." The quantities of the diagnostic variables A to D are given in parts per billion by weight. The largest values of the diagnostic variables occured in samples taken from areas close to the edges of the producing zone.

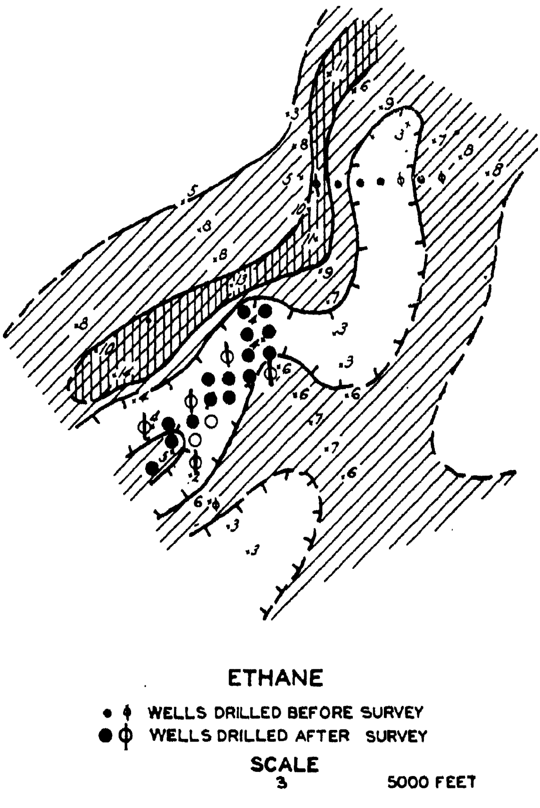

FIG. 362.—Graphical representation of soil analysis data over a sand lens structure in South Texas. (Horvitz, *Geophysics*.)

The application of soil analysis to stratigraphic prospecting is illustrated in Figure 362, which shows the ethane values of samples collected over a sand lens structure located in South Texas. The producing area is limited to the zone of low ethane values bordered by the higher concentrations. †

* The technique used in collecting these data was such that any propane present would be collected and measured together with the ethane.

† Horvitz, *loc. cit.*

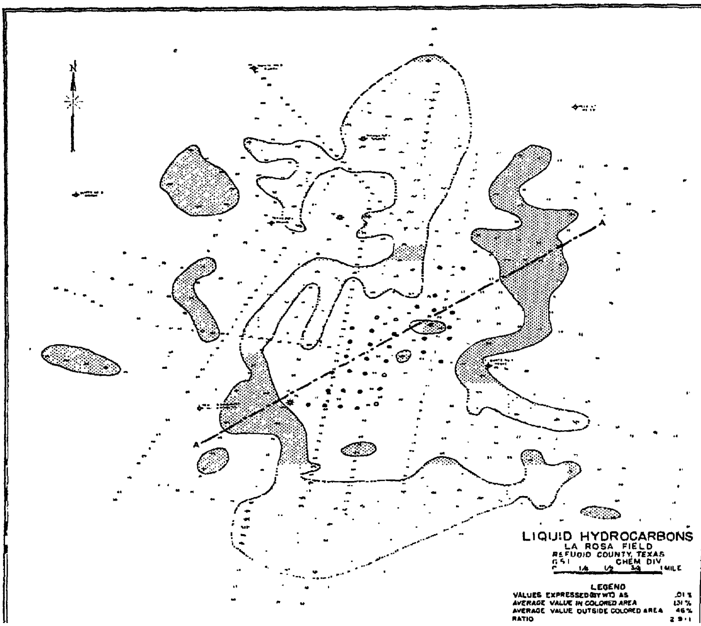

FIG 363.—Map of La Rosa Field showing liquid hydrocarbon concentration pattern and producing wells. (Courtesy of Geophysical Service, Inc.)

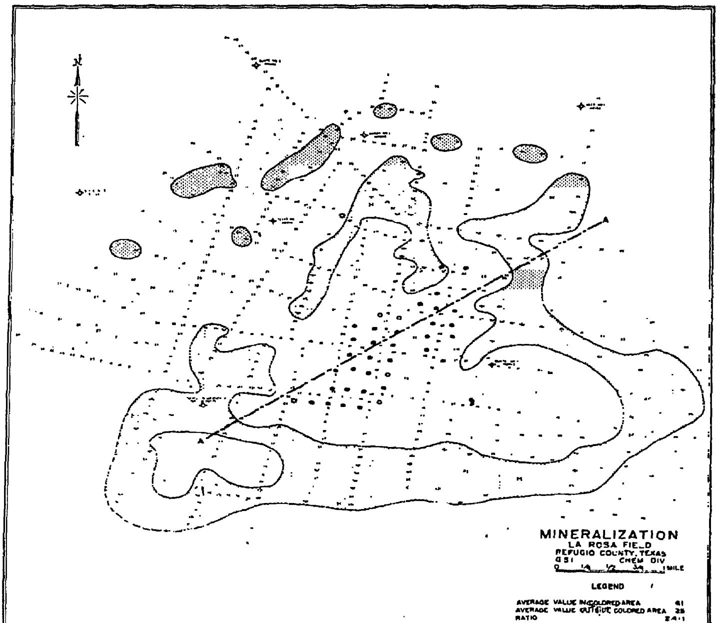

FIG. 364.—Map of La Rosa Field showing mineralization concentration pattern and producing wells. (Courtesy of Geophysical Service, Inc.)

The results obtained in a geochemical survey of the La Rosa Field are shown in Figures 363, 364 and 365. † Figure 363 shows the liquid hydrocarbon concentration pattern, and Figure 364 the mineralization pattern. (The samples were obtained at depths of several feet.) A profile along traverse *AA′* shows relative values of mineralization and liquid hydrocarbons. (Figure 365.)

The results of negative predictions by soil analysis during 1939 suggest that this method may prove to be a useful tool for reconnaissance. Because a large part of the soil analysis work carried out in 1939 was essentially experimental for the purpose of gathering empirical field data, negative findings were disregarded and over thirty wells were drilled in areas condemned by the soil analysis method.‡ Twenty-nine of the negative predictions proved to be correct. That is, drilling showed that the prospects did not have sands carrying commercial oil deposits, even though several of the prospects were structurally high. The single exception was a structure which had produced several years from a shallow formation and had been depleted. In this case, drilling disclosed a deep sand field where soil analysis showings were confusing and not conclusive.

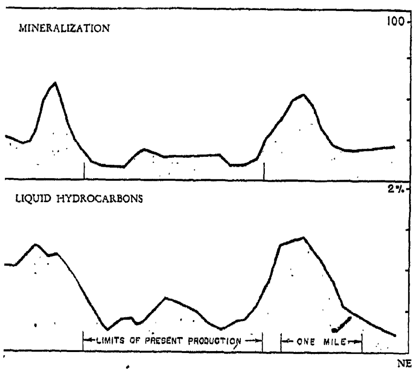

FIG. 365.—Section across La Rosa Field showing elative values of liquid hydrocarbons and mineraliza-on. (Courtesy of Geophysical Service, Inc.)

Cost of Soil Analysis Surveying.—The cost per sample of an analysis for hydrogen, methane, ethane, and "pseudo-hexane" (data presented in Figure 361) is about $7.50. The cost per sample for an analysis of liquid hydrocarbons and mineralization (data presented in Figures 363 and 364) usually averages about ten dollars. To each of the previous costs must be added the cost of surveying the ground and obtaining and shipping the samples. The cost of sampling varies with the area. In areas where the holes are hand dug with auger from five to ten

† Eugene McDermott, personal communication.
‡ H. F. Simmons, *Oil and Gas Journal*, Feb. 29, 1940, p. 54.

feet, and where the areas are accessible, the cost of sampling averages between one dollar and two dollars per sample. This includes the cost of field supervision and surveying. The net cost of the soil analysis method is comparable to that of the geophysical methods for mapping structure.

GEOCHEMICAL WELL LOGGING

A geochemical well log may be obtained by determining the hydrocarbon contents of well cuttings, and plotting these data versus depth. A log of this type may be of value for correlation purposes. Also, predictions may be made regarding the probable occurence of a pay formation prior to drilling into the producing zone. However, the practical value of such geochemical logs has yet to be established.

CRITIQUE OF GEOCHEMICAL PROSPECTING

Considering the fact that geochemical analysis does not depend on structural mapping, but is a possible means for providing direct evidence on the presence of gas and oil, it may well become a valuable tool to be used in conjunction with other geophysical methods to decide whether a structural "high" is commercially important. The difficulties of interpretation of the results of geochemical investigations and the still greater difficulties of correlation are ably brought forward by Sanderson: †

On the assumption that the gaseous paraffin hydrocarbons present in the soil have an unique source in subterraneous accumulations of petroleum, the value of soil analysis still depends to a great extent on the accuracy of the sampling technique and the subsequent analysis of samples. The presence of the hydrocarbons in the soil is connected with a problem of sorption, and the sorption is a function of many quantities. The sorption of any special type of gas depends on the affinity of that gas to the soil under observation and for different types of soil it is quite possible and even probable that the ratio of sorption of two gases will not be the same. The ability to sorb certain gas molecules depends, furthermore, on the surface conditions of the soil particles, the temperature and temperature changes, the moisture content, and a large number of other variables which change rapidly over small areas of surface and which may alter to a very large extent the ratio of hydrocarbons present in soil samples.

† R. Thomas Sanderson, "Some Neglected Aspects of Chemical Exploration," *Geophysics*, Vol. V, No. 3, July, 1940.

Even after proper corrections are made for the different factors enumerated above, the purely chemical side of the problem must be considered. A certain method of extraction might liberate the hydrocarbons of different soil samples in different ratios than those in which they are present in the soil, and there is no accepted method to evaluate the percentages of gases liberated by the chemical process.† It has been advocated that more effort be directed toward gas analysis than soil analysis, because the differences in air are so much smaller the world over than the differences in soil and the improved technique of gas sampling opens new possibilities for gas analysis which were not available when this form of geochemical prospecting was largely abandoned in favor of soil analysis.

Interpretation of a geochemical survey must be largely empirical. The diagnostic values of the various findings are not, as yet, well enough known. As confirmatory evidence to other geophysical data, a favorable hydrocarbon survey would be encouraging. Whether or not a favorable soil survey alone would justify the expense of drilling is impossible to say at present. Until more data are available and the method more accurately limited, it must be considered inchoate and statements made concerning it subject to revision to meet future developments.

† E. S. Shepherd, "Gases in Rocks and Some Related Problems," *Amer. Jour. of Science*, 35A, 311 (1938).

GEOCHEMICAL METHODS

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CHAPTER IX

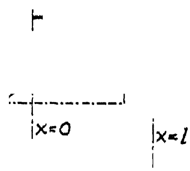
GEOTHERMAL METHODS

Temperature measurements of the earth's outer crust can be used to furnish fundamental information relating to the origin and the history of the areas under observation. In addition, temperature measurements in favorable cases will yield valuable information regarding the zonal distribution of ores; the configuration of intrusive bodies; the contacts between sedimentary and igneous rocks, or between (different) sedimentary rocks; the location, hade, and throw of faults; the ground water distribution and local regional subsurface flow. The practical application of geothermal measurements to the solution of various problems of economic geology is usually one of two types: (a) near-surface temperature measurements, to study the lateral variations of temperature; (b) subsurface measurements in drill holes, to study the vertical distribution of temperature along the drill hole.

Temperature measurements in drill holes are discussed at length in the chapter on drill hole investigations. The present discussion concerns methods designed to determine lateral variations of temperature at, or very close to, the surface of the earth.

MATHEMATICAL THEORY OF HEAT FLOW

As in the case of flow of electrical current through a solid medium, it is convenient to derive the differential equation for the uni-dimensional flow of heat and then generalize the equation for three-dimensional flow. Physically, uni-dimensional flow is illustrated by the flow through a sheet or slab of material having two dimensions considerably greater than the third dimension



Referring to Figure 366, assume that the planes $x = 0$ and $x = l$ are at temperatures T_0 and T_1 respectively and that $T_0 > T_1$. According to Fourier's law the quantity of heat which flows in the x direction across unit area in a time t is proportional to the product of the time and the temperature gradient.

$$ \tag{1} $$

where

q = quantity of heat
t = time
l = thickness of slab
$\frac{T_1}{l}$ = temperature gradient
k = a constant, called the thermal conductivity of the material.

In differential form, the equation for uni-dimensional heat flow in the x direction is:

$$\left(\frac{\partial q}{\partial t}\right)_x = k\,\frac{\partial T}{\partial x} \tag{2}$$

Equation 2 states that the time rate at which heat is transported across unit area of a plane perpendicular to the x direction is equal to the product of the thermal conductivity and the thermal gradient in the x direction.

The equation for three-dimensional flow may be obtained from Equation 2 by replacing $\frac{\partial T}{\partial x}$ by the temperature gradient appropriate to three dimensions. This yields:

$$\frac{\partial q}{\partial t} = k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right) \tag{3}$$

This equation can be simplified by transforming it into a second order differential equation in which T is the only independent variable. To carry out this transformation, it is necessary to introduce the *specific heat*, c, of the material in which the heat flow is assumed to occur. The specific heat is defined as the quantity of heat expressed in calories required to raise unit mass of the material by 1 °C.

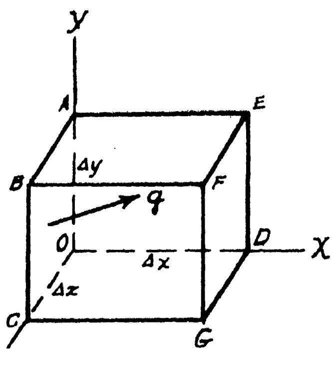


FIG. 367.—Sketch illustrating three-dimensional heat flow.

Consider a volume element having the shape of a rectangular parallelopiped and sides of lengths Δx, Δy, and Δz, located in an infinite, homogeneous, isotropic medium of density σ, specific heat c and heat conductivity k. (Figure 367.) When the temperature of the volume element increases by an amount ΔT, the quantity of heat absorbed by the volume element changes by an amount equal to the product of the specific heat, the mass, and the change in temperature. Hence, the time rate of change of absorption is

$$c\sigma\,\Delta x\,\Delta y\,\Delta z\,\frac{\partial T}{\partial t}$$

If it is assumed that the volume element does not contain any heat sources or sinks, the rate at which heat flows into the volume element must equal the rate at which heat flows out of the volume element. The rate at which heat flows into the volume element across the face $0ABC$ is:

$$\frac{-q}{\partial t}\Big)\,\Delta y\,\Delta z$$

From Equation 2

$$\frac{-q}{\partial t}\Big)_x \Delta y\,\Delta z = k\frac{\partial T}{\partial x}\,\Delta y\,\Delta z$$

For convenience, set

$$k\frac{\partial T}{\quad} = U_x;\; k\frac{\partial T}{\quad} = U_y;\; k\,\partial T \tag{4}$$

In terms of the new variable, the rate of heat flow into the parallelopiped across the face $0ABC$ is:

$$U_x\,\Delta y\,\Delta z$$

The rate of heat flow out of the parallelopiped across the face $DEFG$ is:

Therefore, the net rate of flow out of the element in the x direction is:

$$U_x\,\Delta y\,\Delta z - \left(U_x + \frac{\partial U_x}{\partial x}\,\Delta x\right)\Delta y\,\Delta z = -\frac{\partial U_x}{\partial x}\,\Delta x\,\Delta y\,\Delta z$$

Similarly, the net outward flow in the y and z directions are:

$$-\frac{\partial U_y}{\partial y}\,\Delta x\,\Delta y\,\Delta z \quad\text{and}\quad -\frac{\partial U_z}{\partial z}\,\Delta x\,\Delta y\,\Delta z$$

respectively.

The total rate of flow out of the element is:

Replacing U_x, U_y, U_z, by their equivalents from Equation 4, the total rate of flow out of the element is:

$$\partial^2 T \;+$$

But the outward flow is equal in magnitude and opposite in sign to the absorption of heat in the volume element; therefore,

$$\partial x^2 + \frac{\quad}{\partial y^2} + \frac{\quad}{\partial z^2}\Big)\,\Delta x\,\Delta y\,\Delta z = c\,\sigma\frac{\partial T}{\partial t}\,\Delta x\,\Delta y\,\Delta z$$

or

$$+ \quad \partial y^2 \quad \partial z^2) \; - \; \partial t \tag{5}$$

Equation 5 states that the time rate of the change of temperature at any point in an isotropic homogeneous medium is equal to $\frac{k}{c\,\sigma}$ times the Laplacian of the temperature. Thus, the temperature distribution expressed by Equation 5 involves the specific heat, the thermal conductivity, and the density of the material under investigation.† Hence, changes in any of these parameters result in changes in temperature and heat flow distribution. The observation of temperature changes, therefore, affords a means of mapping the contacts of rocks having different thermal coefficients and density.

Equation 5 does not cover the case of sources or sinks of heat inside the volume element. Hence, if chemical or radioactive processes play an important part in the temperature distribution in the area that is being surveyed by geothermal methods, Equation 5 should be modified to take into account such sources and sinks of heat near the geothermal stations.

Factors Affecting the Temperature Distribution Near the Surface of the Earth.—The two principal sources of heat flow through a volume element close to the surface of the earth are: (a) the high temperature nucleus of the earth and (b) the radiation of the sun. (The flow of subterraneous waters may be the cause of a flow of heat in a certain specified direction.) The first of the two sources causes for all practical purposes a constant radial flow of heat outward. The magnitude of this outward flow at a point close to the surface of the earth depends on the distance of the point from the surface and on the thermal properties of the strata between the point and the center of the earth. The average value of the vertical temperature gradient depends on the type of strata traversed by the hole as will be evident from Figure 368 ‡ which shows depth versus temperature data obtained in oil wells scattered throughout the United States. The data for the California oil wells yield a temperature gradient of 1° F./42 feet, approximately, and the data for the Louisiana oil wells yield a gradient of 1° F./100 feet, approximately. The constant flow of heat due to the high temperature nucleus of the earth would cause a constant temperature at any given point below the surface of the earth during the time of observation, but the variable effect of the sun's radiation causes a modulation of this temperature as described in the following paragraphs.

† Compare also D. O. Ehrenburg, "Mathematical Theory of Heat Flow in the Earth's Crust," Univ. of Colorado Studies, Vol. 19, No. 3, May 1932.

‡ Howard Pyle, Union Oil Company, Personal Communication.

The radiation of the sun causes a periodic change in the flow of heat through points close to the surface of the earth. The periodicity of the change in temperature is a double one and is associated with the change in position of the sun during the day in relation to the horizon and with the change in the path of the sun during the year. These two periodic changes in temperature are called the diurnal and the annual variation, respectively.

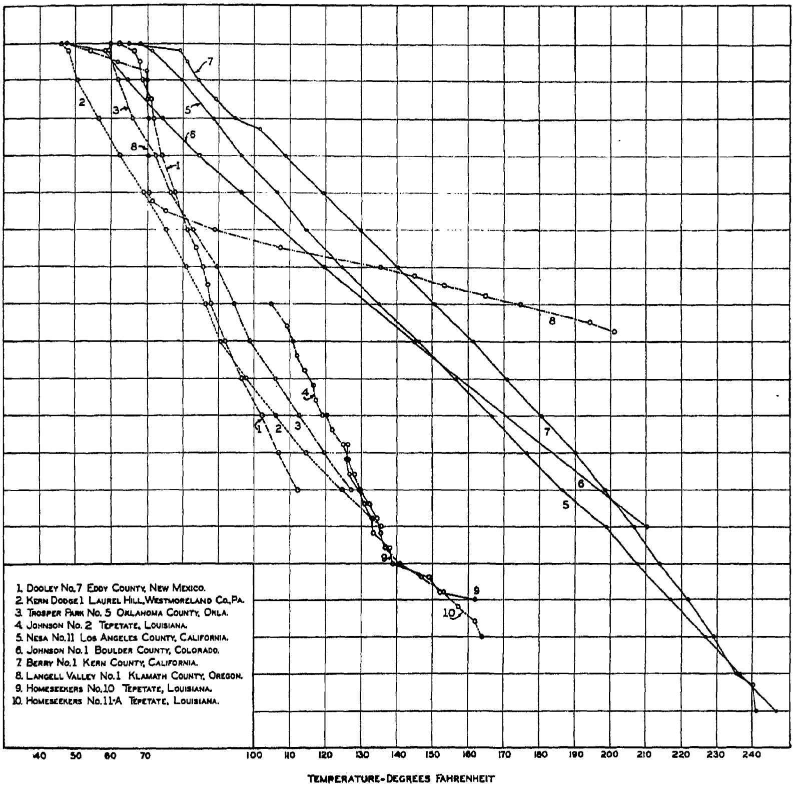


FIG. 368.—Graphs showing depth versus temperature for oil wells in various sections of the United States. (Courtesy of Howard Pyle.)

The heat of the sun reaches the surface of the earth mostly by radiation. The conductivity of the atmosphere is very poor.* The spectrum of the sun has a peak in the yellow portion, and the absorption of this

*The fact that the temperature of the air at higher altitude is generally lower than that at lower altitude shows that conduction of heat plays a minor part in the transport of the sun's heat to the surface of the earth.

radiation in the atmosphere is smaller than that of longer wave lengths. During the night, the earth, which has a much lower temperature and therefore a spectrum with a peak far in the infra red, loses a great part of the heat received during the day; but the atmosphere absorbs a great part of this radiation and therefore acts as a protective screen against loss of radiation, and this factor, combined with the poor conductivity of the atmosphere, allows the earth in the summer to retain during the night a part of the heat received from the sun. Hence, in the summer, the outer portion of the earth's crust gains heat over a 24-hour period. In the winter, however, the losses during the night exceed the gains during the day, and the balance is negative over a 24-hour period.

During the day the temperature is a maximum between 2 and 3 p.m. and a minimum just before sunrise. The plotting of temperature against time during a 24-hour period affords a means for evaluating the gains and losses over the whole or part of the period.

The diurnal variations manifest themselves only to a depth of a few feet below the surface. At greater depth they become rapidly smaller. The depth to which the changes are measurable depends on the character of the rocks close to the surface. In solid rock formations, the diurnal variations usually become imperceptible at a depth of about 3 feet. In loose sand and alluvial fill material, the variations escape measurement at a depth of 1 to 2 feet. In swamps and porous materials containing water and in areas where the water table is near the surface of the earth, the variations become imperceptible at a depth of 1½ to 2½ feet.

The annual variation for a given area is determined from data on measurements made intermittently throughout the year, or daily, at the same time of day.

The annual variation can be observed to a depth of between 75 and 100 feet, depending on the thermal properties of the rocks. The periodic variations in temperature may show local fluctuations due to changes in meteorological conditions and to differences in topography and overburden.

Below 100 feet, the temperature depends, in general, only on the flow of heat from the center of the earth.

In the foregoing paragraphs the possible influence on the temperature of moving subterraneous water and of chemical and radioactive processes has been neglected. In practical field work, these sources of heat must be taken into account for two reasons. One, if subsurface sources of heat exist in a region being surveyed by geothermal methods, they may cause anomalies in the temperature curves that might be interpreted, erroneously, as changes in the rock formations. Two, the anomalies produced by such heat sources may be used for the location of subsurface water courses and of mineral deposits which are undergoing chemical processes in which an exchange of heat with the surrounding material takes place.

Overcast skies during part of the observations, rain, snow, wind, and other factors affecting the distribution of temperature must be taken into account for proper correlation of observations when they are extended over a considerable period of time. Consideration must be given to the effect produced by varying coefficients of absorption of the surface at different stations. For example, sand or alkali patches have different coefficients of absorption than a surface covered with vegetation. Also, one must take into account that the distribution of temperature under a clump of shade-trees may be entirely different from that in the sun.

Great care should be taken to locate points of observation as far as possible from the exposed face of an outcrop or from the edge of a deep ditch filled with water or from any point where a sudden change in elevation can be observed. The field notes should show a complete record of all these local circumstances and, if the location of a station at an unfavorable point is imperative, proper care should be taken to detect changes in the distribution of temperature due to these local conditions by staking several stations in the vicinity of the discontinuity.

FIELD OPERATIONS

The crew usually consists of two men, one of whom drills the holes while the other makes the readings. With galvanometer and thermocouple of proper construction, readings at a station should not take more than 15 minutes. The cost of geothermal surveying is comparable to that of magnetic surveying with a two-man crew.

Field Apparatus.—The driller can usually drill the required holes with a hand drill. In one easily constructed hand drill, a rod equipped with a suitable handle is welded to a piece of pipe. The pipe is cut off slantwise (about a 45° angle) at the end and its edge sharpened and bent slightly in the form of a spoon. When the surface deposits are hard, some kind of power-driven drill may be used.

The measuring apparatus required for this type of work is very simple and consists of a sensitive galvanometer, potentiometer, and a thermopile or thermocouple of special construction. In addition, a couple of ordinary thermos flasks, a thermometer, reading to 0.01°C., and a 6-volt battery to supply the current for the galvanometer light are needed.

The sensitivity of the electrical detecting device should be such that a difference of 0.01°C. can be measured. The best thermocouples give an E.M.F. of 40 microvolts per °C. Measurement of 0.01°C. requires a voltage sensitivity of the galvanometer of 4×10^{-7} volts when one couple is used. For a thermopile this factor is cut down according to the number of junctions of the thermopile. The thermopile or thermocouple is constructed so that the hot junction can be lowered into the hole at the end of a supporting rod, while the cold junction can be kept at a constant temperature in a thermos flask at the surface of the earth.

The use of a thermopile has the advantage that the sensitivity requirement of the galvanometer is less than that for a thermocouple. The use of a thermocouple has the following two advantages: (1) A single thermocouple usually can be mounted to fit a hole of smaller diameter than a thermopile with many junctions, and this is important because the disturbance of the temperature equilibrium of the ground by a hole increases rapidly with the diameter of the hole due to convection air currents and the exposure of the hole to direct sunlight. (2) The heat capacity of a single thermocouple can be made very small compared to that of a thermopile, which allows the hot junction to take the temperature of the rocks with which it is in contact in the shortest possible time.

The cold junction, on the other hand, should have a large heat capacity to eliminate the influence of small changes in temperature of this junction. The cold junction is conveniently maintained at a constant temperature by placing it in a Dewar flask filled with ice if a thermopile is used, or filled with water at the temperature of the surrounding air if a thermocouple is used. The temperature of the cold junction is determined and constantly checked with the aid of a thermometer that may be read with an accurary of 0.01°C.

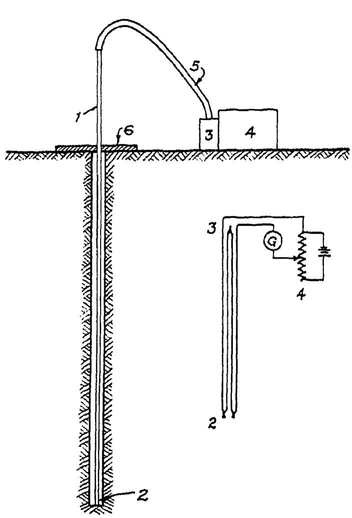


FIG. 369.—Schematic diagram of thermopile and associated apparatus in place for geothermal measurements. 1, sheath of low thermal conductivity; 2, hot junction; 3, cold junction; 4, potentiometer; 5, flexible rubber tube sheath; 6, cover plate of low thermal conductivity; *G*, galvanometer.

Figure 369 shows the arrangement of the apparatus ready for measurements.

In calibrating the galvanometer in terms of differences in temperature, the two junctions are first placed in a Dewar flask filled with water or ice to determine the zero of the galvanometer and then one junction is transferred to a flask containing water at another temperature. This is repeated for a series of temperatures and a plot is made of the deflections of the galvanometer against the temperature differences.

Field Technique.—A preliminary series of measurements is made at different depths in the same hole to determine the depth at which the

diurnal variations become imperceptible for the particular region under investigation. Anomalies in this critical depth should be discovered before a great number of observations is made at a depth insufficient to eliminate the diurnal variations. Furthermore, a determination of the minimum depth to which the holes need be drilled will tend to speed up the survey because the speed of drilling determines the speed of the observations. When the survey is made in a region where the water table is close to the surface of the earth, all holes must be drilled uniformly to the water table and measurements made at that depth.

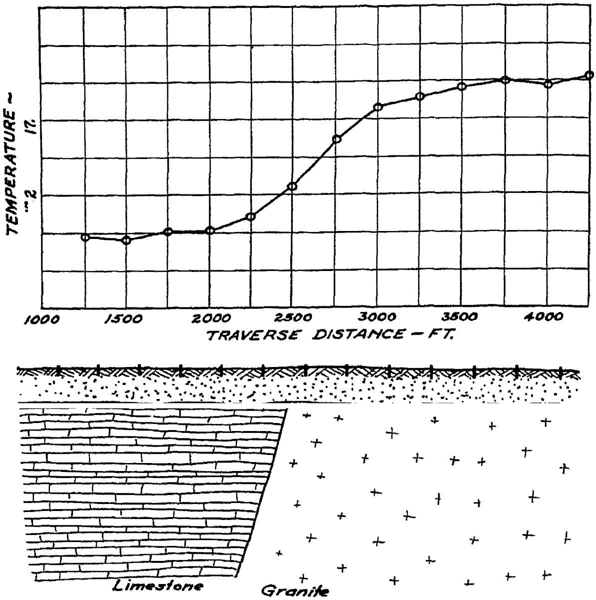
In order that thermal equilibrium may be reached, measurements should not be made for a period of 8 to 10 hours after drilling.

For purposes of mapping and correlation, a network of traverses is surveyed, the configuration of which depends on local conditions and on the particular features of the exploration problem. When the suspected structure has a well-defined strike, the traverses may be parallel lines perpendicular to the strike, and the stations 100 to 500 feet apart, depending on the rate of change in temperature to be expected. It is good practice to start a survey with a base line on which the stations are only a short distance apart and to determine the best distance between stations from the results obtained on this traverse. The measurements on this base line are repeated from time to time as the survey progresses to determine whether the annual variation has any influence on the observations.

When the measuring apparatus is in position close to the hole, the hot and cold junctions are placed in the same Dewar flask and the zero point of the galvanometer is determined by reading the thermometer and using the calibration curve. Then the hot junction is removed and lowered into the hole so that a firm contact is made with the earth. The galvanometer starts to drift toward its final deflection, which is reached as soon as thermal equilibrium is established. A bad contact will cause "jerky" behavior of the galvanometer. After the galvanometer has reached its final position, a small pressure on the end of the pole that is supporting the hot junction should not alter the deflection of the galvanometer.

After the reading, the zero point of the galvanometer is checked again as described above and the readings repeated several times. Generally, the readings should check within a couple of hundredths of a degree centigrade. If the deflection of the galvanometer keeps on increasing with every reading at the same point, the hole has not reached thermal equilibrium.

Special care should be taken that the connections between thermocouple and galvanometer are clean and tight. Plugs of the ordinary type generally do not meet these requirements. Care should be taken also that the thermocouple and the galvanometer are always connected in the same way. The small difference in temperature between the junctions makes it possible to introduce appreciable errors by reversing terminals.



:G. 370.—Temperature anomaly along a traverse over the co tact between limestone and an intrusive granite.

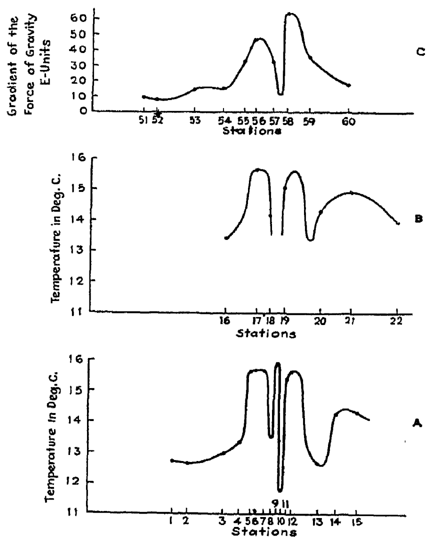


FIG. 371.—Comparison of gravity gradient and temperature anomalies over a fault. (van den Bouwhuijsen, *Eng. and Mining Journal.*)

ILLUSTRATIONS OF TEMPERATURE ANOMALIES

Figure 370 shows the temperature anomaly obtained across a traverse over a contact between a limestone series and an intrusive granite. The contact is clearly shown, even though the overburden is approximately 40 feet thick. The temperature anomaly along two traverses (*A* and *B*) over a fault is shown in Figure 371. The vertical displacement along the fault is approximately 2700 feet. †

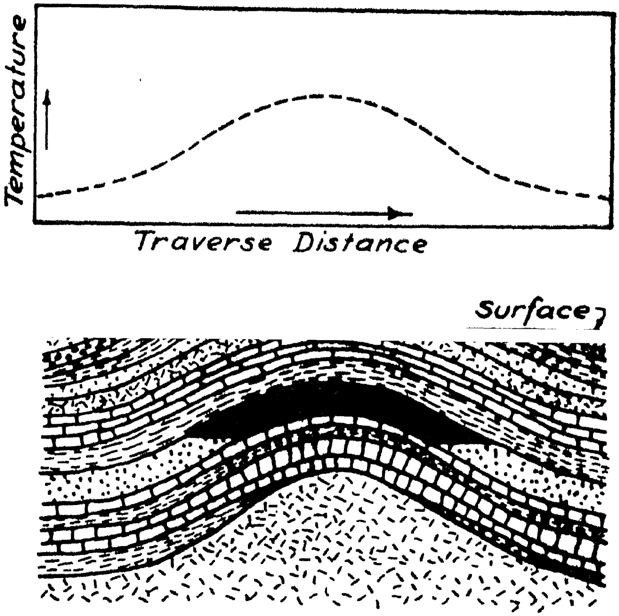


Fig. 372.—Temperature anomaly over a typical anticline.

The temperature anomaly, at depth, over a typical anticlinal structure is illustrated in Figure 372. It will be noted that the curves rise in passing over the crest of the anticline. Similar results have been obtained by several investigators. ‡

† J. N. A. van den Bouwhuijsen, "The Thermocouple Proves Useful on a Geophysical Survey," *Eng. and Mining Jour.*, Vol. 135, No. 8, Aug. 1934.

‡ C. E. Van Orstrand, "Normal Geothermal Gradient in the United States," *Bull. Amer. Assoc. Pet. Geol.*, Vol. 19, No. 1, Jan. 1935.

M. W. Strong, "Significance of Underground Temperatures," Proc. World Petr. Congress, Vol. 1, 1933, 124 pages, abstract, *Jour. Inst. Petr. Techn.*, Vol. 20, Feb. 1934, p. 63.

E. de Golyer, "The Significance of Certain Mexican Oil Field Temperatures," *Economic Geology*, Vol. 13, 1918, pp. 275-301.

James Fisher, L. R. Ingersoll, and H. Vivian, "Recent Geothermal Measurements in the Michigan Copper District," *A.I.M.E. Geophysical Prospecting*, Tech. Pub. 481.

GEOTHERMAL METHODS

UNITED STATES PATENTS

1,901,431 Issued March 14, 1933. Lewis A. Bond. "Method of and Apparatus for Determining True Rock Temperature in Rotary Drilled Wells."

CHAPTER X

DRILL HOLE INVESTIGATIONS

The geophysical exploration of drill holes is a separate field and quite distinct both in technique and results from the surface exploration methods. Drill hole investigations will in some cases supply more definite data than mechanical coring, particularly in areas where geological studies are difficult. Such cases occur when the formations are characterized by an irregularity in thickness of the sedimentary beds and an almost total absence of fauna and uniformity of color.

In common with surface exploration methods, drill or bore hole explorations may be classified according to two principles. One basic classification is concerned with the application of an artificial field to the formations penetrated by the drill hole. This field may be: (a) electrical; (b) magnetic; (c) thermal; or (d) seismic. The corresponding properties of the formations which may be investigated are: (a) resistivity, impedance, electrical anisotropy, and other electrical properties; (b) magnetic permeability and retentivity; (c) thermal conductivity and temperature; (d) velocity, attenuation, and absorption of seismic waves. The other basic classification is concerned with natural phenomena. These include: (a) electrochemical effects due to differences in the concentration of the fluids (electrolytes) in the hole and in the formations and electrofiltration effects due to flow of fluid through the pores of the formations; (b) magnetic anomalies associated with the formations penetrated by the bore hole; (c) variations of natural earth temperatures with depth and with the formations; (d) gravitational anomalies associated with the formations; and (e) radioactive effects.

The greatest commercial application and development in bore hole investigations has been in the measurements of electrical resistivities and electrolytic and electrofiltration potentials.

ELECTRICAL METHODS

In addition to the location of subsurface geological markers which are of value for correlation purposes, the proper combination of resistivity and potential measurements will oftentimes show the presence of oil sands, with particular reference to the boundary between oil and water-bearing formations. Although the electrical resistance of oil is very high, that of

the formations which contain oil is not correspondingly high, due largely to the presence of connate waters in the formations. This factor, and its effect on electrical resistance, has been discussed in connection with Figure 129. However, the oil-bearing formations generally do have a measurably higher resistance than the water-bearing formations which are usually adjacent to them. Hence, a relatively high electrical resistance *may* be indicative of an oil-bearing formation.

In certain oil fields, the parameters supplied by resistivity and potential measurements are quantitative and depend on the amount of oil in the formation; hence, such measurements occasionally are useful for ascertaining the general productivity.† However, due to the fact that there is no direct relation between the quantities measured (resistivity and earth potentials) and the information desired (production in barrels per day), inferences concerning productivity are entirely empirical and are valid only for the particular field and sand for which the empirical relationships were established.*

In mining work, drill hole measurements are often useful for determining the proximity of ore bodies and the extent of mineralization. In this application, electrical drill hole exploration may be considered as a means of extending the diameter of the drill hole. The history of mining exploration is replete with cases where commercial ore bodies have been missed by very short distances by drill holes. A contributing factor to this condition is the crookedness or drifting of drill holes.

The electrical records are similar to the common core and sample records, and they are, for that reason, termed "electrical logs." These logs generally may be obtained without an appreciable increase in drilling time and at a small fraction of the cost of mechanical coring. Usually, a complete and continuous electrical log of a drill hole may be obtained at a cost equivalent to only a few feet of mechanical coring.

Total Resistance Measurements.—The simplest method of measuring the electrical resistance of the formations traversed by a drill hole employs two electrodes. One electrode is lowered into the drill hole while the other electrode remains fixed at the surface of the ground or at some other convenient point. ‡ Referring to Figure 373, electrode A is arranged to traverse the drill hole and electrode B is fixed in position at the surface of the earth.

† M. Martin, G. H. Murray, and W. J. Gillingham, "Determination of the Potential Productivity of Oil Bearing Formations by Resistivity Measurements," *Geophysics*, July 1938, Vol. 3, No. 3, pp. 258-271.

*Production is often affected by conditions created during the drilling operations, for instance, the deposition of a mud sheath on the wall of the well opposite the oil- and gas-bearing formation. (Compare C. P. Bowie, "Hardening of Mud Sheaths in Contact with Oil, and a Suggested Method for Minimizing their Sealing Effect in Oil Wells," Bureau of Mines Report of Investigation No. 3354.

‡ C. Schlumberger, "Electrical Process and Apparatus for the Determination of the Nature of the Geological Formations Traversed by a Drill Hole," U. S. Patent 1,819,923, issued Aug. 18, 1931.

The resistance may be measured by use of a conventional bridge circuit, or voltmeter and ammeter. Intermittent readings or continuous recording may be employed as desired. The bridge may be of the slide wire or switch type for intermittent readings, or of the recording type for continuous readings.

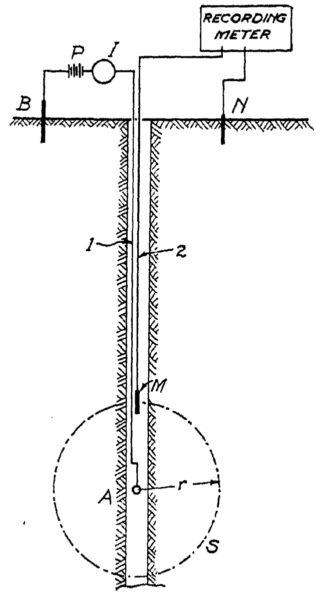


FIG. 373. — Schematic diagram of two-electrode arrangement for measuring electrical resistance of formations traversed by a bore hole.

The total resistance will depend on the size of the electrodes, the resistance of the connecting cables, and the salinity of the water or mud in the drill hole.* (It usually lies between 10 and 500 ohms.) The resistance offered to the current flowing between the two electrodes is directly proportional to the average resistivity of the conducting medium and to the distance between the electrodes, and inversely proportional to the cross section of the path. The earth immediately adjacent to the electrodes contributes a major part of the total resistance of the current path. That is, aside from the material within a few feet of the electrode surfaces, the main mass of earth included between electrodes A and B has a relatively low, or negligible, resistance due to its large cross-sectional area. Hence, the total resistance R of the circuit may be expressed by the relationship

$$+ R_3$$

where R_1 = resistance of the cable. This resistance is measured directly and is a function of the cable length.

R_2 = effective resistance of the electrode B. This resistance consists of the contact resistance between the electrode surface and the earth and the effective resistance of the earth material immediately adjacent to the electrode.

R_3 = effective resistance of the electrode A. This resistance consists of the contact resistance between the electrode surface and the drilling mud filling the drill hole and the effective resistance of

* The resistivity of the mud varies over fairly wide limits. Where the drill hole penetrates strata containing materials which are readily dissolved or where saline solutions from the strata enter the bore hole, the resistivity may be as low as 0.5 ohm-cm. In drill holes where the mud is relatively free from such effects or where fresh water enters the drill hole, the resistivity may be many hundred ohm-cm. Average drilling mud which is encountered in the usual drilling well varies in value from about 2 ohm-cm. to 20 ohm-cm.

> the earth material near the electrode. The contact resistance is relatively constant between electrode and drilling mud throughout the entire length of the drill hole, while the effective resistance of the earth materials near the moving electrode varies.

The resistances R_1 and R_2 remain relatively constant throughout the measurements. Hence, the measured total resistance R is equal to a constant resistance plus a variable resistance. The latter may be set equal to KR, where K is a coefficient which depends upon the geometrical shape and size of the moving electrode and the average resistance of the cable. K may be calculated or determined experimentally. The values of the resistance R_3 in the immediate vicinity of the moving electrode A may therefore be calculated from the measured total resistance.

A modification of the method employs an electrode system wherein two electrodes are lowered into the drill hole at a fixed distance apart. Variations in the total circuit resistance are due to changes in earth materials in the vicinities of both of the electrodes. For example, if the separation of the moving electrodes were such that at any depth the two electrodes were opposite two strata having different resistances, the total resistance measured would depend on both formations. Because these two variables cannot be separated where numerous anomalies occur within distances comparable to the electrode separation, it is necessary: (a) to use a small separation of the electrodes so they both will be affected by the same condition or (b) to make one of the electrodes sufficiently long (about 100 feet or more) so that it is not affected appreciably by the formations being passed.† If the electrodes are separated by too small a distance, the penetration into the strata will not be great enough to give resistance data which are sufficiently independent of variations in the size of the bore hole and the composition of the drilling mud. (The penetration is usually less than ¼ the effective separation of the electrodes.)

In a later development of the two-electrode method,‡ an electrode system is lowered into the drill hole and measurements are made of the alternating current properties of the subsurface included between the two electrodes. The electrodes are maintained a fixed distance apart, and the entire assembly is moved in the drill hole. Measurements are preferably made with a moderate or high frequency alternating current.

Blau and Gemmer (§) utilize a similar electrode arrangement wherein an alternating current may be passed between two electrodes in the hole or

† J. J. Jakosky, "Electrical Method and Apparatus for Determining the Characteristics of Geologic Formations," U. S. Patent 2,140,798, issued Dec. 20, 1938.

‡ J. J. Jakosky, "Method and Apparatus for Alternating Current Investigation of Uncased Drill Holes," U. S. Patent 2,038,046, issued April 21, 1936.

§ Ludwig S. Blau and Ralph W. Gemmer, "Method and Apparatus for Logging a Well," U. S. Patent 2,037,306, issued April 14, 1936.

between a single moving electrode and a stationary surface electrode. The variations in current which are caused by changes in the effective resistance of the strata are measured in a suitable auxiliary circuit. A modification of this method utilizes an electrical transient instead of an alternating current. †

McDermott ‡ has utilized the single moving electrode arrangement in a bore hole exploring method which does not require removal of the drill pipe from the bore hole before making the measurements. In this method, the drill and tubing are raised the desired distance off bottom, and an insulated conductor, on the end of which is a weighted electrode, is lowered through the inside of the drill pipe. The electrode passes out of the bit and is employed for mapping the resistance changes in its vicinity. The method is essentially one of mapping short intervals of formation, after which the drilling operations are resumed. The chief intention of the invention is to avoid the necessity of removing the drill pipe and bit from the bore hole before making the survey.

Karcher § has developed a modification of the single moving electrode method which is designed for exploring bore holes continuously during the process of drilling. This method utilizes an arrangement which causes an electric current to flow through a conductor, the drill bit, and thence into the earth. Both the conductor and the drill bit are insulated from the drill pipe. From an operator's viewpoint, the method has the advantage of allowing continuous exploration of the bore hole during the drilling process and the disadvantage of necessitating a special type of insulating cable passing through the drill pipe. In addition, it is necessary that the bit be electrically insulated from the drill stem by an insulating stem or section which is mechanically capable of taking the entire drilling load. An alternate arrangement consists in insulating the entire drill stem, thereby leaving only the bit exposed for contact with the formations.

Another continuous logging method†† has been developed wherein the measurements are made between the entire drill rod assembly, including the rotary equipment, and an electrode which is placed at a distance from the well comparable to the depth of the hole. In this system, the electrical

† L. W. Blau and L. Statham, "Electrical Transient Well Logging," U. S. Patent 2,165,213, issued July 11, 1939.

‡ E. McDermott, "Method of Electrically Exploring Bore Holes," U. S. Patent 2,070,912, issued Feb. 16, 1937.

§ John C. Karcher, "Method and Apparatus for Exploring Bore Holes," U. S. Patent 1,927,664, issued Sept. 19, 1933.

†† J. J. Jakosky, "Method and Apparatus for Continuous Investigation of Drill Holes," U. S. Patent 2,150,169, issued March 14, 1939. Reissue 21,102, May 30, 1939.

J. J. Jakosky, "Method and Apparatus for Continuous Exploration of Bore-holes," U. S. Patent 2,153,802, issued Apr. 11, 1939.

J. J. Jakosky, "Method and Apparatus for Continuous Exploration of Bore-holes," U. S. Patent 2,181,601, issued Nov. 28, 1939.

properties of the earth included between the entire drill mechanism and the distant electrode are measured. A recording is made of the variations in electrical properties as the drill stem traverses the strata. Measurements may be made with direct or alternating current. The electrical logs show variations of an electrical property versus depth. Usually, simultaneous continuous recording is made: (a) of the fluctuating potentials associated with the cutting action of the bit and (b) of the change in electrical resistance or impedance between the drill stem and the distant electrode.

Several modifications of this method are used. In one modification the drilling is done with oil or other poorly conducting fluid, such as is utilized in many areas to avoid sealing of the producing formations and other damaging effects of drilling muds.† Contact of the drill stem with the wall of the hole is prevented by use of the rubber protectors commonly employed during drilling operations. A second modification employs direct current for energizing the ground. The back E.M.F. due to polarization around the bore hole provides a sufficient potential drop to cause a major portion of the current to travel down the drill stem and flow into the earth at a point where polarization is not effective. This point is the cutting end of the bit, where new rock is continually being exposed by the bit. The resistance variations in the circuit, therefore, are predominately those encountered by the bit.

Numerous high frequency and radio methods ‡ have been proposed, but such methods have not come into commercial operation for deep bore hole work due chiefly to the enormous and unpredictable changes in capacity and inductance which occur when long lengths of cable are lowered into the hole; these methods are not capable of local determinations in the hole. As a matter of fact, up to the present, the commercially successful methods utilize direct current or alternating current having a frequency of less than 750 cycles per second.

The total resistance methods are relatively simple in their practical application but suffer from one defect: Variations in the earth or mud filling the drill hole in the immediate vicinity of the moving electrode, or electrodes, will sometimes exercise an undue effect on the readings, due to the fact that the total resistance between the electrodes is confined almost entirely to the immediate vicinity of the electrodes. (Compare p. 671.) The drilling mud itself is relatively uniform due to the circulation during the drilling process. On the other hand, because the mud is weighted to produce

† J. J. Jakosky, "Method and Apparatus for Continuous Exploration of Bore Holes," U. S. Patent 2,153,802, issued April 11, 1939, and Foreign Patents.

‡ T. Zuschlag, "Methods of Investigating the Nature of Subterranean Strata," U. S. Patent 1,652,227, issued Dec. 13, 1927.

H. Lowy, "Methods for Ascertaining the Nature of Subterranean Strata," U. S. Patent 1,092,065, issued March 31, 1914.

a hydrostatic head in excess of the probable formation pressures, it permeates and impregnates the porous formations.* Obviously, this invasion of the mud may influence the effective resistance of the permeable formations markedly.

The variations produced by changes in the effective diameter of the bore hole constitute another disadvantage of the single electrode method wherein the electrode is not in direct contact with the formation itself. This variation is particularly pronounced when drilling with excessive mud circulation through the relatively soft formations (shales, unconsolidated sediments, etc.) frequently encountered in the Gulf Coast and California areas.

There are certain advantages, however, which tend to offset the disadvantages. When the moving electrode is of small size (a few inches in length), the single moving electrode system will give much greater detail and will show the very thin formations better than the multiple moving electrode systems. In addition, the single moving electrode system utilizing a small moving electrode (or the multiple moving electrode system having a short separation between the current and the potential electrode) gives sharper structural breaks corresponding to the tops and bottoms of the resistivity variations than the measuring systems having a greater penetration. This greater detail is, of course, superposed on the non-structural variations previously mentioned.

It is advantageous to have both the shallow and the relatively deep penetration curves and it is common commercial practice to furnish a logging record which shows both curves. Interpretation is based on these two resistance curves and the potential curve.

Measurements may also be made between electrodes in two uncased wells.† Broadly, this method contemplates disposing at least one electrode in each of two wells and then adjusting the levels of these electrodes until the resistance between them reaches a maximum or a minimum.

Another arrangement utilizes resistance measurements along the bedding planes of the strata.‡ This method measures the resistance between an electrode in the bore hole and another electrode spaced a considerable distance from the hole. Another modification utilizes a distant electrode and two electrodes in the hole to determine the anisotropic properties of the strata by measuring vertically and along the bedding planes, and comparing the two sets of measurements.§

* The distance of permeation is primarily a function of the differential pressure, time of contact, viscosity and sealing properties of the mud, and the permeability of the formation.

† P. F. Hawley, "Two-well Method of Electrical Logging and Apparatus Therefor," U. S. Patent 2,183,565, issued Dec. 19, 1939.

‡ J. J. Jakosky, "Electrical Method and Apparatus for Determining Characteristics of Geological Formations," U. S. Patent 2,155,133, issued April 18, 1938.

§ J. J. Jakosky, "Electrical Method and Apparatus for Determining Character of Geologic Formations," U. S. Patent 2,140,798, issued Dec. 20, 1938.

Resistivity Measurements.—In order to remove almost entirely the effects of the potential drop adjacent to the moving power electrode, resistivity measurements are made over a short section of the total current path which is sufficiently distant from the moving power electrode to be unaffected by minor variations in its vicinity. (Figure 374.) † Equipment of this type has been developed and widely applied by Schlumberger.

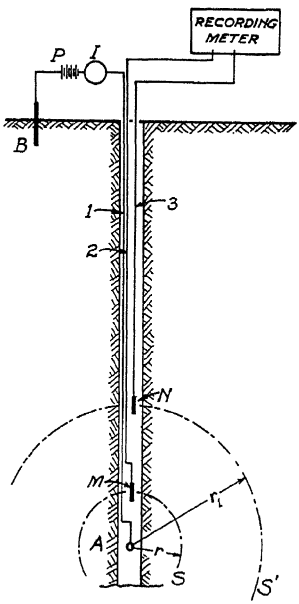


FIG. 374. — Schematic diagram of three-electrode arrangement for measuring electrical resistivities of formations traversed by a bore hole.

The measuring device utilizes three insulated wires 1, 2, and 3. These are suspended in the drill hole and terminate in three electrodes, A, M, and N, immersed in the water of the drill hole. The distance $AM = r$ and the distance $AN = r_1$ are large relative to the diameter of the hole; for example, they are 10 to 20 times the diameter. The insulated wire 1 connects the energizing electrode A with the power supply P. The other terminal of P is connected to some point B near the drill hole, or to the surface casing of the well. The potential electrodes M and N are connected by the insulated wires 2 and 3 to the two terminals of a recording device.

If the earth in the vicinity of A is approximately homogeneous, its resistivity ρ may be calculated from the following considerations. After a steady current flow is reached, all the equipotential surfaces in the vicinity of A, except the very near ones, are approximately spherical about the point A as origin. Let the equipotential passing through M be designated by the letter S and that through N by S'. An application of the principles developed in the chapter on resistivity methods yields the following expression for the electric intensity E at any point on a sphere of radius r

where I is the total current flowing out of the energizing electrode A. The potential difference between two points separated by a distance dr is related to the electric intensity by the equation

† C. Schlumberger, "Electrical Process and Apparatus for Determination of the Nature of the Geological Formations Traversed by a Drill Hole," U. S. Patent 1,819,923, issued Aug. 18, 1931.

C. and M. Schlumberger and E. G. Leonardon, "Electrical Coring: A Method of Determining Bottom-hole Data by Electrical Measurements," *A.I.M.E. Geophysical Prospecting*, 1934, p. 237.

$$dV = -$$

or

$$dV = \frac{\cdot \rho I}{4\pi r^2} dr \qquad (1)$$

The potential drop between M and N is obtained by integrating Equation 1 between the limits r and r_1; that is

$$\Delta V = \int_{r}^{r_1} \frac{I\rho}{4\pi r^2} dr = \frac{I\rho}{4\pi} \cdot \frac{r - r_1}{r r_1}$$

and

$$4\pi \Delta V$$

The distances r and r_1, the value I of the current (which is measured, for example, by a milliammeter), and the difference of potential ΔV between M and N (which is measured, for example, by a millivoltmeter) are known; hence, Equation 2 may be used to compute the average resistivity ρ of the rocks in the neighborhood of the electrode system.

A modification † of the last method which requires only two insulated cables is illustrated in Figure 375. The moving energizing electrode C is connected to the power supply by a cable. The other terminal of the power supply is grounded at some point B; for convenience, the metallic casing usually present in the upper part of a drill hole may be used as the ground. The moving potential electrode A is connected to a recording device by means of another insulated cable. The separation r of the moving electrodes A and C is made small relative to the distance CB.*

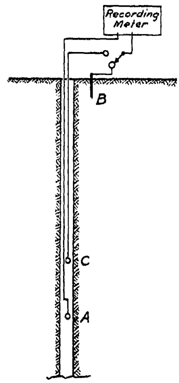


FIG. 375.—Schematic diagram of arrangement employing two movable electrodes (C, power electrode; A, potential electrode).

If it is assumed that the earth near A is approximately homogeneous, the calculation of the resistivity may proceed along the lines indicated in the previous paragraphs; that is, the potential drop between A and C may be obtained by integrating Equation 1. It may be demonstrated that 9/10 of the potential drop occurs between a sphere of radius r and a sphere of radius $10r$. Consequently, this method yields data similar to that obtained in the

† C. Schlumberger, "Electrical Device for the Determination of Specific Resistivity," U. S. Patent 1,894,328, issued Jan. 17, 1933.

* Although r is small relative to the distance CB, it is large relative to the diameter of the hole; for example, r is two to twenty times the diameter of the hole. The relative merits of short and long spacing between current and potential electrodes were discussed on page 671.

methods indicated in Figure 373. Also, the integration may be carried out between the limits r and ∞, because the current flow at distances greater than CB, and hence greater than $10r$, will not contribute appreciably to the total potential drop. Thus, the potential drop read on the potentiometer is:

$$\Delta V = \int_r^{\infty} \frac{\rho I}{4\pi r^2}\, dr$$

and

$$\rho = \frac{4\pi r\, \Delta V}{I} \tag{3}$$

The resistivity ρ of the rock adjacent to A may be determined, therefore, from observed values of ΔV, r, and I.

Slichter † has proposed a method in which four electrodes are lowered into the drill hole. The electrodes are spaced a fixed distance apart, and the entire electrode system is moved as a unit along the length of the drill hole. Means are provided for employing expanding prongs or contacts whereby electrical contact may be made with the wall of the drill hole, in the event the drill hole is not filled with water. (The electrodes may consist of many fine spring wires that extend radially outward and form a circular "brush" which sweeps along the wall of the drill hole.) The method is applicable, therefore, to dry drill holes frequently encountered in mining operations above the water table and to drill holes filled with non-conducting fluids, such as oil, etc.

The expression for the resistivity may be calculated from the fundamental relations expressing the flow of current in an infinite medium. The derivation for an arrangement in which the four electrodes lie in a line and are equidistantly spaced is as follows:

Assume that the electrodes A, M, N and B are located in a straight line and at distances of 0, a, $2a$, and $3a$, respectively, from A. Let I denote the measured current passing out of the electrode A and into the electrode B. The potential at M due to the source A and the sink B, is *

$$2a$$

and the potential at N due to A and B is

† Louis B. Slichter, "Apparatus for Exploring for Ore," U. S. Patent 1,826,961, issued Oct. 13, 1931.

* The expression for V_M is a modified form of Equation 5, p. 282.

Hence,

or

$$\rho = \frac{4\pi\, a\, \Delta V}{I} \tag{4}$$

It is evident from Equation 4 that the resistivity ρ may be determined from the observed values of the potential drop ΔV, the current I, and the electrode spacing a.

Huber † has described a method for obtaining two resistivity curves simultaneously. One curve shows the variations in the resistivities of the strata in the immediate vicinity of the moving current source, and the other curve shows the variations in the resistivities of the strata at an appreciable distance from the moving electrode. Thus, this method yields a direct comparison of the variations in electrical resistivity close to the bore hole where the formation may be permeated with the fluid of the bore hole, and the variations in electrical resistivity at a distance from the bore hole, the distance in question being greater than the distance of penetration of the bore hole fluid.*

Potential Measurements.—The importance of electrofiltration and electrochemical potential measurements is probably equal to that of resistivity measurements in commercial bore hole investigations. In some areas, e.g. in the Gulf Coast area, the potential measurements are of major importance.

In 1896 P. Bachmetjew ‡ described an experiment which he believed indicated that potentials could be created which were due not to mineral contacts but to the motion of ground waters through subsurface sands and porous materials. These potentials were believed to be due to an effect first discovered and described by C. Quincke § in 1859. The first practical application of these phenomena to drill hole exploration appears to have been made by Schlumberger, †† who received a patent for the location of permeable strata traversed by a drill hole.

The potential existing at the faces of the strata traversed by a drill hole consists, essentially, of two components: one, the potentials of electro-

† F. W. Huber. "Methods of and Apparatus for Electrically Exploring Earth Formations," U. S. Patent 2,072,950, issued March 9, 1937.

* Measurements of the same zones may also be obtained by use of variable radii, r and r_1, for the spheres of Figure 374, or by use of combinations of the various methods previously described.

‡ See article by W. M. Rust, Jr., "A Historical Review of Electrical Prospecting Methods," *Geophysics*, Vol. 3 (1938), pp. 1-6.

§ C. Quincke, *Annalen der Physik*, Series 2, Vol. 107 (1859), pp. 1-47.

†† C. Schlumberger, "Electrical Process for the Geological Investigation of the Porous Strata Traversed by Drill Holes," U. S. Patent 1,913,293, issued June 6, 1933.

filtration caused by movement of the fluid either into or out of the porous formations traversed by the drill hole; and two, the electrochemical potentials created at the contact of two solutions having different concentrations of dissolved salts. Although the two effects exist simultaneously, they are best described separately.

Electrofiltration Potentials

In most drill holes, the well is filled with water or mud in sufficient quantities to exert a pressure on the walls distinctly superior to the hydrostatic head of the fluid in the rocks. This is particularly true in the numerous cases where the muds filling the holes are purposely made relatively heavy to avoid the caving-in of the hole and blowouts of gas or oil. In these cases, the penetration of the circulating mud into the pervious layers generates electrofiltration potentials.

The electromotive force produced by the flow of an electrolyte through a pervious dielectric is directly proportional to the differential hydrostatic pressure and the electrical resistivity of the electrolyte and is inversely proportional to the viscosity of the electrolyte. It does not depend on the thickness of the filtering sheet nor on the radii and number of pores of the pervious medium. The magnitude of the E.M.F. produced by filtration may be expressed by the equation, †

(5)

where E = electromotive force; m = a constant which depends on the porous medium; ρ = resistivity of the fluid; p = the differential hydrostatic pressure; v = viscosity of the flowing electrolyte.

According to Poiseuille's ‡ law, the quantity Q of fluid which flows through a given capillary tube at a pressure p is:

(6)

where m' is a constant.

On combining Equations 5 and 6 one obtains

$$E = \frac{m \rho Q}{m'}$$

or

$$E = \text{constant} \cdot \rho Q \tag{7}$$

† C. and M. Schlumberger and E. G. Leonardon, "Electrical Coring: A Method of Determining Bottom-hole Data by Electrical Measurements," A.I.M.E. *Geophysical Prospecting*, 1934, pp. 237-272.

‡ Poiseuille, *Comptes Rendus* (1842), 18, 1167.

Equation 7 states that the electromotive force of filtration for a given electrolyte and pervious medium is proportional to the product of the amount of liquid which is filtered and the electrical resistivity of the liquid.

Generally, the potential opposite a surface of ingress will be negative with respect to that opposite a surface of egress. That is, in the usual case, a potential measured opposite a porous formation will be relatively negative if the bore hole fluid is flowing into the formation, and relatively positive if the formation fluid is flowing into the hole.

The electrofiltration potentials are a minimum at the boundaries of the porous zone and a maximum in the most permeable section. The *magnitude* of the electrofiltration potential differences may be of the order of 100 to 200 millivolts over a length of a few meters.

It has been suggested that formation pressures may be determined from electrofiltration potential measurements by using Equation 5 twice.† To develop the relevant mathematical theory, it is convenient to write Equation 5 in the form:

$$E = k\,(H - P) \tag{8}$$

where k is a constant for a given porous medium and electrolyte; H is the pressure of the well fluid; P is the pressure of the formation fluid. If the fluid level in the well is lowered, by bailing for example, then for the new hydrostatic pressure H'

Hence,

$$\frac{E}{E'} - \quad H' - \tag{9}$$

Theoretically, therefore, it is possible to determine the pressure P of the formation fluid by altering the fluid level and observing the quantities: E/E', H/H', and H or H'.

Potentials Due to Electrochemical Forces

Electrochemical potentials occur at the contact or boundary between two solutions of dissolved salts. For example, an electrochemical potential is created when the sweet water of a bore hole comes in contact with the salt water of a porous formation.

The physical principles involved may be indicated briefly by considering the phenomena which occur when two solutions having different concentrations of a dissolved salt, for example sodium chloride, are in contact but are prevented from actual mixing by some suitable porous membrane.

† C. and M. Schlumberger and Leonardon, *loc. cit.*, p.

(Figure 376.) The ions in the more concentrated solution C_1 tend to diffuse into the more dilute solution C. Also, negative ions generally have higher velocities than positive ions. Hence, the net number of negative ions diffusing from the more concentrated into the more dilute solution in a given interval of time will be greater than the net number of positive ions, and a preponderance of negative charges will accumulate in the more dilute solution. Thus, because the drilling mud of a bore hole is usually less saline than the formation fluid, this mud becomes negative with respect to the connate water of the formation and a recording device connected to a moving electrode will register a negative potential when the electrode is opposite a porous zone containing a high concentration of dissolved salts.

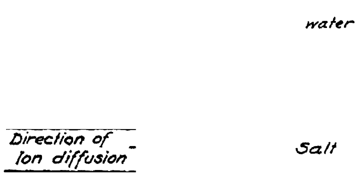


FIG. 376.—Sketch illustrating production of electrochemical potentials by ion diffusion between two electrolytes of different concentrations. (After Schlumberger and Leonardon, *A. I. M. E. Geophysical Prospecting.*)

The magnitude of the potential difference is given by the equation:†

$$E = \frac{RT}{nF} \cdot \frac{v-u}{v+u} \cdot \log_e \frac{C_1}{C_2} \tag{10}$$

in which the letters signify: E, electromotive force in volts; T, temperature in degrees absolute; n, valence of dissolved salt; R, gas constant for perfect gases; F, Faraday (96,500 coulombs); u and v, mobilities of negative and positive ions; C_1 and C_2, concentrations of the two solutions.

Applied to a sodium chloride solution for which

$$\frac{}{u+v} = 0.4, \quad \frac{}{u+v} = 0.6$$

Equation 10 gives the electromotive force in millivolts as

$$\frac{C_1}{}$$

and if $C_1 = 5\,C_2$, $E = 11.6 \times 0.699 = 8.11$ millivolts.

Total E.M.F. Produced by Electrofiltration and Electrochemical Forces

A complete circuit such as $A\ B\ C$ (Figure 376) contains two contacts between the impervious medium and the electrolytes filling the porous layer and the hole, respectively. Laboratory experiments indicate that the

† C. and M. Schlumberger and E. G. Leonardon, "A New Contribution to Subsurface Studies by Means of Electrical Measurements in Drill Holes," *A.I.M.E. Geophysical Prospecting*, 1934, pp. 273-288.

electromotive force E_{ABC} around the complete circuit A B C may be represented by the formula

$$E_{ABC} = K \log_{10} \frac{C_1}{C_2}$$

or because

$$\frac{C_1}{C_2} = \frac{\rho_2}{\rho_1}$$

$$E_{ABC} = K \log_{10} \frac{\rho_2}{\rho_1} \tag{11}$$

where ρ_2 and ρ_1 are the resistivities of the drilling mud and the salt water contained in the sand layer, respectively, and K is a constant which depends on the chemical compositions of the fluids in contact and the porosity of the impervious rock.

Operating Principles of the Potential and Porosity Measurements

The electrodes employed for commercial potential or porosity measurements are preferably made from a material having a low electrochemical contact potential, such as lead. The potential electrode is connected to the end of an insulated cable which may be lowered to various depths in the bore hole. The upper end of this cable is connected to one of the terminals of a recording potentiometer. The other terminal is connected to a second electrode grounded at a fixed point at the surface. The potential at this point is arbitrarily assumed to be equal to zero. Therefore, for each position of the movable electrode within the bore hole, the recording device gives the relative value of the potential at that particular point. Theoretically, all electrodes lowered into bore holes should be of the non-polarizing type to eliminate errors caused by electromotive forces arising from contacts between a metal and the water of the bore holes; practically, however, the error introduced by use of metal electrodes is negligible because of the homogeneity of the bore hole mud with which they are in contact.

An alternative method which has been proposed by Schlumberger,† but not employed commercially to a large extent, places two small or "point" electrodes within the bore hole, the electrodes being a fixed distance apart. The difference in potential between the two electrodes within the bore hole is measured and recorded at the surface of the ground. The readings obtained when employing two electrodes within the bore hole give the gradient of the potential, and it is necessary to convert this

† C. Schlumberger, "Electrical Process for the Geological Investigation of a Porous Strata Traversed by Drill Holes," U. S. Patent 1,913,293.

gradient to a potential difference along the bore hole. In areas where bad ground currents prevail, this method has the advantage over the method described above that it is much less affected by ground currents, due to the short distance between the electrodes. However, interpretation of the gradient curve is almost impossible due to the many thin strata encountered in the usual bore hole. The extended electrode method, with a small or "point" electrode spaced about 50 feet below it, has been found best in commercial work. Such a system is practically unaffected by extraneous ground currents and has the advantages of the two "point" electrode system. In practice, the metallic braid over the cable, if of the shielded type, will serve as the extended electrode.

Various modifications of the potential method have been proposed. For instance, Hummel † proposes to measure the potentials at null-point conditions without the use of a manually-controlled or automatically-controlled potentiometer. In Hummel's method, two electrodes are usually lowered into the bore hole; one is a current electrode and the other a potential electrode. Sufficient current is passed into the well through the current electrode to balance out exactly the natural ground potentials measured between the potential electrode in the well and the other potential electrode at the surface. A record is made of the variation in current necessary to create the zero potential condition.

The effects of extraneous earth potentials may also be eliminated or minimized by use of an auxiliary potential circuit. In this arrangement, the extraneous earth potentials are fed into the measuring circuit in such a manner that they oppose or neutralize the same variations in the measuring circuit.‡

Karcher § has developed a means for measuring polarization during drilling. The polarization in question is a back E.M.F. which opposes the voltage causing the current flow and persists after the voltage is removed. The method utilizes a bit which is insulated from the remainder of the drill stem and connected electrically to a recording potentiometer at the surface of the ground. It is claimed that because the electrical properties of the earth formations are affected by their porosities, the phenomena of polarization are also affected by the porosities; consequently, a variable which depends on the polarization is diagnostic with respect to the amount of the porosity of the earth formation in question. In operation, the current is allowed to flow for some definite period, for example, one minute, during which time the resistance to ground is recorded. †† The drilling bit is then disconnected from the current supply and connected to the potential measuring device. The instantaneous reading of the potential is, therefore, the diagnostic variable which depends on the amount of polarization produced by the original current applied to the measuring circuit.

It is contemplated that the above operation would be repeated as the drilling proceeds, and means are provided for performing this operation automatically.

† J. N. Hummel, "Process for Inspecting the Ground," U. S. Patent 2,084,143, issued June 15, 1937.

‡ J. J. Jakosky, "Method and Apparatus for Electrical Exploration of the Subsurface," U. S. Patent 2,162,086, issued June 13, 1939.

§ J. C. Karcher, "Method and Apparatus for Determining the Porosity of Rock Formations," U. S. Patent 2,085,664, issued June 29, 1937.

†† J. C. Karcher, "Method and Apparatus for Exploring Bore Holes," U. S. Patent 1,927,664, issued Sept. 10, 1933.

A means of conducting electricity through the drill stem to the bit has been developed by Hawthorne.† It should be noted that if the measurements are made while drilling is in progress these potentials are superimposed on the rapidly varying potentials generated by the bit. Measurement of the latter potentials when the entire bit and drill stem are in contact with the mud or earth has been described previously.

Field Technique — Resistivity and Porosity Measurements. — In general, electrical logging includes both resistivity and porosity measurements and appropriate equipment and technique are in use in most of the oil fields of the world. The electrode arrangement for resistivity measurements permits readings of the resistivity of formations over limited portions of the total path of the applied current, according to the discussions previously given with reference to Figures 373 and 374. The electrode spacings are arranged to give the resistivity either very near to the movable source electrode *A* or sufficiently far away to eliminate the influence of the mud on the formations. The self potential is measured by the single movable electrode described in the porosity discussion.

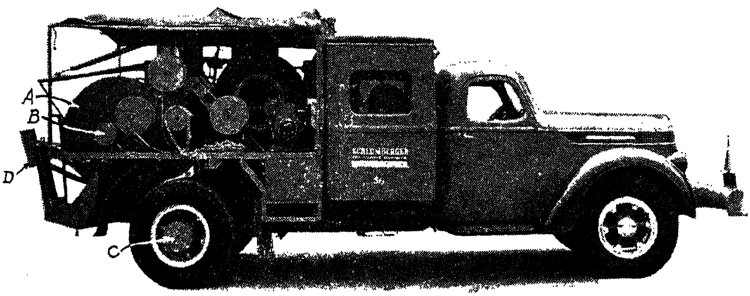


Fig. 377.—Equipment for logging of drill holes. *A*, winch for 15,000-foot, four-conductor cable; *B*, commutator housing; *C*, hoist drive sprocket; *D*, support shoe for truck during survey.

Commercial logging equipment usually comprises a multi-conductor insulated cable, a power winch for raising or lowering the cable in the hole, and the recording instruments. The winch and cable are mounted on a heavy truck. (Figure 377.) The recording instruments may be mounted either in the winch truck or in a separate panel truck. (Figure 378.) The resistivity equipment contains the necessary power supply for generation and control of the applied current and the oscillograph for recording two potential traces across measured portions of the current path. The equipment is so designed that the final readings are in resistivity units. A third oscillograph trace records the electrochemical and electrofiltration potentials.

† David G. Hawthorne, "Apparatus for Subsurface Surveying," U. S. Patent 2,096,359, issued Oct. 19, 1937.

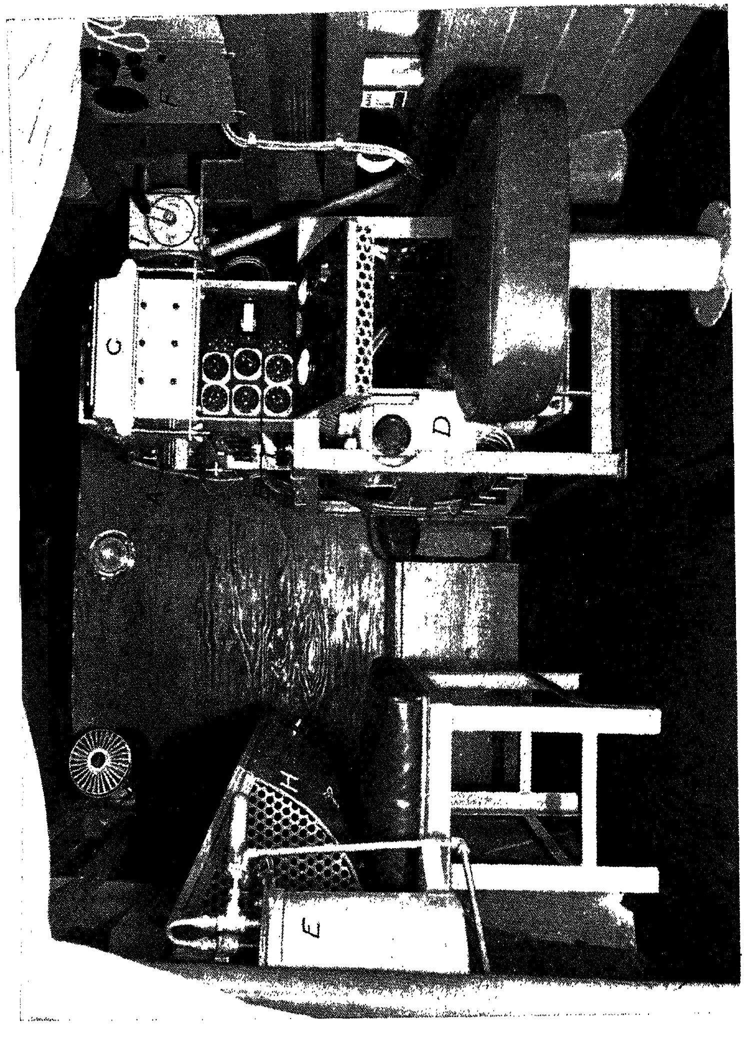


FIG. 378.—Recording equipment for electrical logging. *A*, depth counter; *B*, control panel; *C*, recording camera; *D*, power supply; *E*, film developing equipment; *F*, loud speaker and transmitter for communication *with operator* of winch truck controls; *H*, printing machine; *I*, auxiliary hand-cranked commutator. (Courtesy of Schlumberger Well Surveying Co.)

Resistivity and self potential * may be measured simultaneously by means of the double commutator apparatus illustrated in Figure 379. The

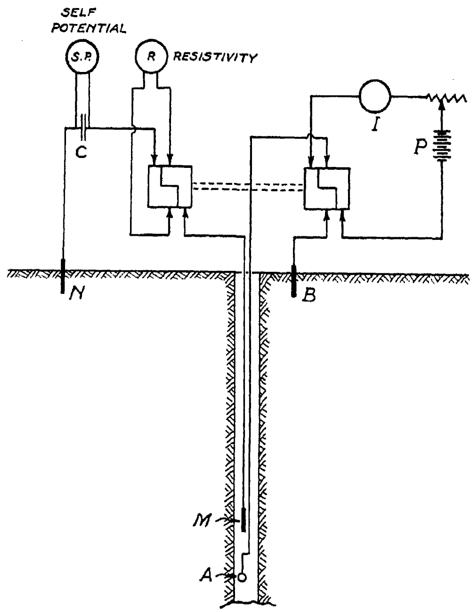


FIG. 379. — Electrical connections for simultaneous recording of self potential and resistivity by means of a double commutator. *S.P.*, self potential recorder; *C*, by-pass condenser; *R*, resistivity recorder; *A*, moving current electrode; *M*, moving potential electrode; *N* and *B*, stationary surface electrodes; *I*, current meter; *P*, power supply.

applied current is reversed periodically and in synchronism with the resistivity potentials which are to be measured by a second commutator. The direct current self potential recording meter is unaffected by the periodically reversed energizing current. The measured parameters are recorded on a sensitized film which is synchronized with the cable, so that a definite ratio exists between a unit of length on the film and a unit of length along the bore hole. For example, scales of 1″ to 100 feet, 2″ to 100 feet, and 5″ to 100 feet are available. Synchronization of film and cable may be accomplished by use of two interlocking synchronized motors, one being geared to a sheave or measuring wheel frictionally contacting the cable, and the other to the film drive. (Figure 380.) The speed of the film is varied by changing the film-drive gears.

* Abbreviated S.P., also termed natural potential or potential.

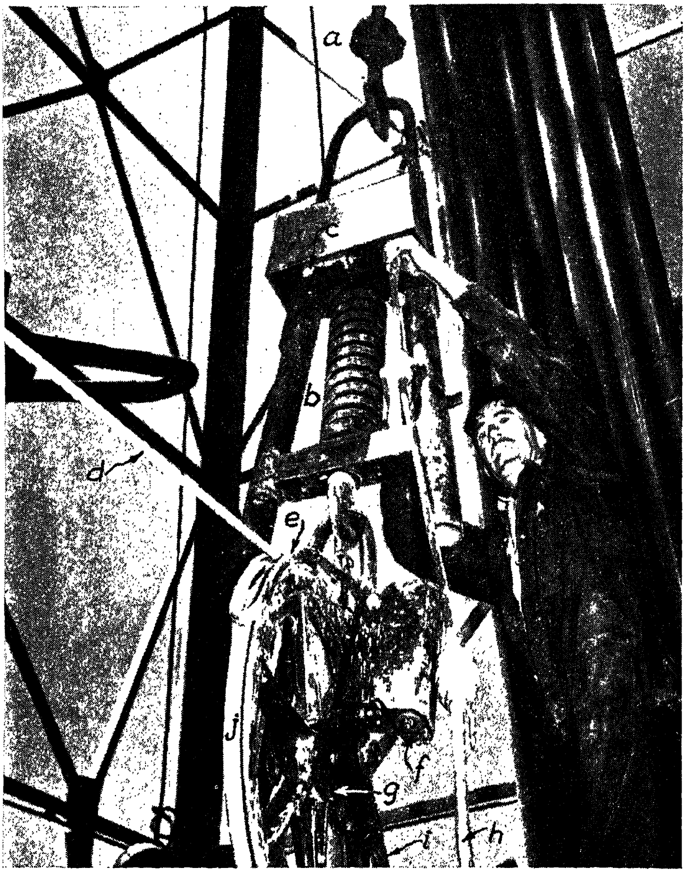


Fig. 380.—Measuring sheave wheel for electrical logging. *a*, supporting hook fastened to traveling block; *b*, weight spring; *c*, weight selsyn motor; *d*, cable to recording truck; *e*, cable guides; *f*, footage selsyn motor; *g*, selsyn drive gear; *h*, cable connections to selsyn motors; *i*, bail for tie-back chains; *j*, ground manganese steel sheave rim for measuring cable. (Courtesy of Lane-Wells Company.)

An alternate type of equipment is schematically illustrated in Figure 381. An alternating current, which is generated by a vacuum tube oscillator and has a frequency of approximately 700 cycles per second, flows in the cable to the main electrode and back through the formations to the return electrode at the surface. *Impedance* measurements are made

between the single electrode in the well and the surface electrode. At the same time over the same cable, it is possible to measure the D.C. self potential existing between the two electrodes. The single conductor cable therefore serves both for the D.C. and the A.C. measurements.

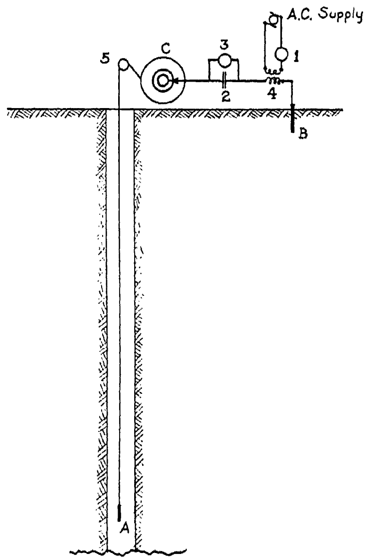


FIG. 381.—Method for simultaneous resistivity and potential logging with single moving electrode. *A*, moving electrode; *B*, stationary surface electrode; *C*, winch and commutator assembly; 1, A.C. recording meter; 2, by-pass condenser; 3, D.C. recording meter; 4, coupling transformer; 5, sheave wheel. (Halliburton Oil Well Cementing Company.)

Measurements are made between the moving electrode *A* in the hole and the stationary surface electrode *B*. Changes in the effective resistance or impedance are recorded on the meter 1. The direct current recording meter 3 is shunted by the large condenser 2, which by-passes the alternating current. The winch and commutator are shown diagrammatically at *C*.

Power Supply for Logging Operations

The characteristics and capacity of the power supply are dependent upon the type of recording and measuring equipment. When direct current, usually periodically reversed by a suitable commutator device, is employed

for energizing the ground, the power supply may be a bank of storage batteries or heavy duty dry cells. Such battery supply is usually employed with the mechanical type film feed. This type of feed utilizes a measuring sheave wheel mechanically connected to the film drive mechanism.

When the energizing current is alternating or when the power must be transformed to different potentials, as for supplying the plate current in amplifying tubes or oscillators, some form of alternating current power supply is most desirable. Alternating current is also required if the interlocking synchronous motor system is employed between the measuring sheave wheel and the film drive. The alternating current for these purposes may be supplied by one of two convenient methods. The first method utilizes a bank of storage batteries, usually 200 ampere-hours and 32 volts, which supply direct current to a suitable rotary converter. The output of the converter is usually 110 volts at 60 cycles, which may then be transformed to any desired potential. The storage batteries are charged by rotating or vacuum rectifier equipment whenever the truck is at its base station garage. The second method for power supply utilizes a small gasoline engine direct-connected to a 110-volt 60-cycle alternator. A 1000 watt ONAN outfit is illustrated in Figure 382. Numerous generating units of this type are available on the market. They may be obtained generally in sizes ranging from ¼ to 10 kilowatts capacity.

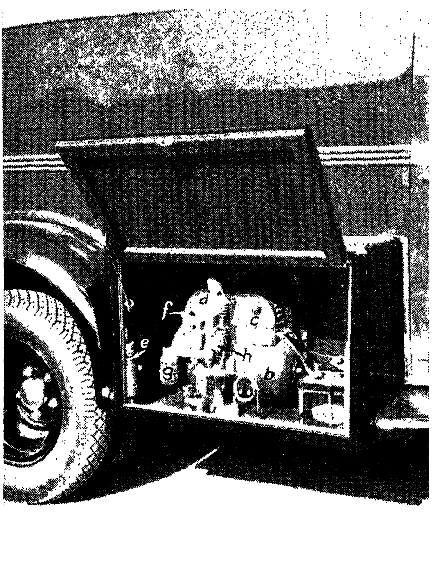


FIG. 382.—ONAN gasoline-driven alternating current power supply. *a*, storage batteries for starting; *b*, alternator; *c*, control panel and relays; *d*, two-cycle gasoline engine; *e*, demountable muffler; *f*, exhaust line; *g*, air cleaner; *h*, carburetor. (Courtesy of John C. Stick, Jr.)

Applications of Electrical Logging.—Identification of paleontological or lithological horizons is done by the examination of cores or samples secured during the course of drilling. These samples are obtained by coring over that section of the hole in which it is believed the studies should be made. Because the positions of these important horizons can only be estimated, coring must be done over a long section in order to be

certain that samples of the desired zone will be obtained. Such a process is extremely expensive and requires a large amount of drilling time. In addition, important horizons are sometimes missed entirely as a result of unsuspected faults or overlaps in the cored section. At best, therefore, the samples furnish only a partial and discontinuous record of the entire drill hole.

Electrical logging, on the other hand, provides a continuous record of the entire hole, plotted to scale. Most of the geological markers possess definite electrical characteristics which are easily correlatable. Each formation (limestone, sandstone, shale, chalk, etc.) is identified and placed according to depth. In addition, many purely electrical markers are found which furnish correlation points available in no other way. Furthermore, the electrical log furnishes indicative information on the porosity and fluid content of possible productive zones.

With use of the electrical log, the position of paleontological markers can be predicted. If actual samples of these zones are desirable, they can then be secured by use of the side wall sample taker. The electrical log will also furnish information on where samples of possible producing zones should be obtained.

Interpretation of Electrical Logging Curves.—Satisfactory correlations between various formations encountered in neighboring drill holes are relatively easy for areas where the sedimentary beds are continuous laterally over a large area. Generally speaking, each lithological division will yield a corresponding electrical variation; in addition, the variations of the electrical logs will maintain a similar appearance over large areas. As indicated by Deussen and Leonardon,† the reason for this is evident when one considers the conditions under which the sediments were deposited. These conditions are similar to those now present in many places at the surface.

Where lateral continuity is prevalent, as is likely to be the case for marine deposits, the sediments are identical in texture, porosity, and salinity at the time of deposition. Although these characteristics may have undergone important modifications since the deposition, the initial similarity still remains to a large extent because most of the modifications (variation of compactness due to varying pressures, alteration of the ion content of the connate waters, etc) are of a regional character. For example, the concentration of subsurface waters often exhibits a gradual and regular variation; this makes it possible to identify various horizons by means of isoconcentration curves covering areas of a large extent.‡ Consequently, if the electrical logs of two neighboring wells are similar or nearly identical, as is

† Alexander Deussen and E. G. Leonardon, "Use of Electrical Logs for Correlation in the Gulf Coast of Texas and Louisiana," *A.P.I.*, paper presented at the 16th Annual Meeting, Los Angeles, California, Nov. 14, 1935.

‡ L. C. Case, W. R. Berger, R. F. Fash, and H. E. Miner, *Problems of Petroleum Geology*, Part VI, published by *A.A.P.G.*, 1934.

often the case, the lithology and conditions of impregnation may be assumed to be very similar. On the other hand, decided changes in the form of the diagrams indicate corresponding lateral modifications of the geological sections.

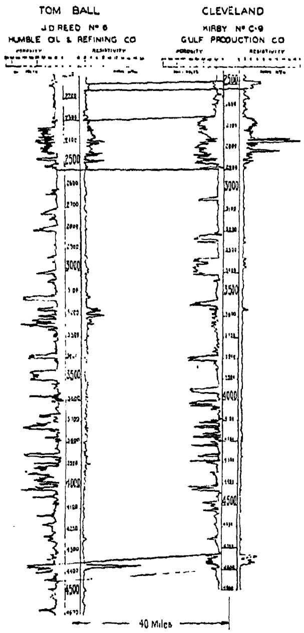


Fig. 383. — Correlations between two wells in the Tom Ball and Cleveland area along the Conroe trend. (Deussen and Leonardon. Paper presented before *A.P.I.* at the 16th Annual Meeting, Los Angeles.)

Correlation of electrical logs of wells in widely separated areas is usually confined to the more important changes or breaks in sedimentation. Continuous correlation, bed by bed, over long distances is possible, however, if the wells are spaced closely enough to indicate the presence of overlaps and faults which destroy the continuity of the geologic formations. Figure 383 shows the correlation of a well in the Tom Ball field with one in the Cleveland field. These two wells are forty miles apart on the Conroe trend. The group of sands at the top of the Marginulina zone is very distinct in both wells and, in spite of the distance, certain detailed features are recognizable. At greater depths, in a series of alternating beds of sands and shale of a uniform distribution, a characteristic sandy horizon located just above the Textularia Hockleyensis zone is apparent. This horizon is easy to identify in both wells.

Interpretation of Resistivity Measurements

Absolute resistivity values are not necessarily a distinguishing characteristic of a given formation because two formations of a very different geological age, or even of different lithological facies, may have the same resistivity. For this reason, a single resistivity measurement will not yield usable information. If, however, resistivity measurements are made continuously over a considerable distance in the bore hole, a resistivity diagram results which is characteristic of and in conformity with the sedimentation encountered during the measurements. Over that part of the section where electrical variations occur, the resistivity diagram will have characteristic changes or markers by means of which that particular sedimentary section may be identified in the different holes in the area. Due to the sensitivity of the electrical measurements, practically every definite lithological change

will be accompanied by an electrical variation. The number of electrical markers in an area will, therefore, be proportional to the number of definite lithological changes which are encountered in the stratigraphic column.

From a well logging viewpoint, higher electrical resistivities may be caused by (a) strata containing oil or gas, (b) strata containing fresh water (water of low salinity), or (c) highly compacted rocks having a very low percentage of conductive connate waters. Quite frequently, the water of the drilling mud will filter into the strata and replace the original more highly conductive connate water, with a resultant increase in electrical resistance. Under such conditions, resistivity measurements of low penetrating distance will give an erroneously high value, while measurements having greater penetrating distances will be more apt to give the true value.

Alteration of the physical properties of a porous stratum by changes in the liquid which permeates it furnishes a parameter which is indicative of the porosity of the stratum. Measurements of this type are made by logging the hole when filled with one liquid, and then changing the liquid and again logging the hole and noting the difference between the two curves.†

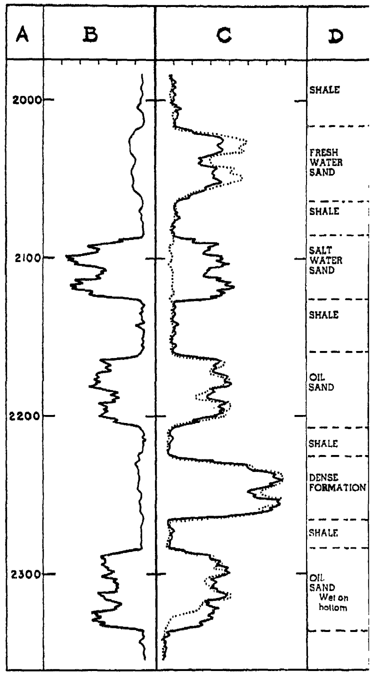


Fig. 384.—Comparison of geological, resistivity and porosity logs. *A*, depth; *B*, natural or self potential curve; *C*, resistivity curves; *D*, geological log. Solid resistivity line is shallow or normal curve. Dotted resistivity line is deep curve. (Courtesy of John C. Stick, Jr.)

Low electrical resistivity may be due to zones impregnated with highly conductive (saline) connate waters. From an economic standpoint, highly resistant zones are of primary interest, and it is desirable to have some means of differentiating the highly resistant zones caused by oil or gas impregnation from those caused by fresh water content and by low porosity. This is the chief purpose and function of the potential diagram obtained by the natural potential measurements. A comparison of typical geological, resistivity, and potential logs is shown in Figure 384.

† C. Schlumberger, "Process for Investigating Permeable Strata Traversed by Boring," U. S. Patent 2,172,625, issued Sept. 12, 1939.

Interpretation of Potential Measurements

The successful use of potential measurements depends largely on tak- … simultaneously with, or immediately following, drilling operations. … hole, has become equalized and if the migration of sal… rial to the other has caused equilibrium in concentration, the observed values of the potentials will be decidedly lower than they would have been if measured before such equilibrium had taken place. The curve made by a continuous recording of these natural potential measurements is termed a 'porosity curve" or "spontaneous potential curve" and is considered to be fundamentally related to porosity or permeability.

The relative values, as well as the absolute magnitudes of th … h and with the district. In general, for a … the observed potentials will increase with … principally to the increase in the relative salinity of the form … Electrochemical potentials for the same sand may also vary laterally from district to district due to variations in the formation fluid. For a given depth and fluid salinity, the observed potentials will vary with the porosity d permeability.

In practice, the secondary variations discussed in the last paragraph are usually neglected, and the observed variations in potential are assumed to depend on changes in porosity and permeability. The single exception to this rule is the fresh water zone near the surface; within this region, the potentials opposite porous zones are small, and frequently positive, in contrast to the usual negative potentials observed opposite porous formations below the fresh water zone.

As a consequence of the foregoing, a few general rules for interpretation may be made: *

Formation	*Resistivity*	*Potential*
Shale and clay	low	low
Sand, salt water	low	very high negative
Sand, fresh water	high	medium to low potential
Sand, oil	high	high negative
Sand, gas	very high	medium high negative
Limestone, non-porous	high	low
Limestone, porous, oil	very high	high negative
Limestone, porous	low	very high negative
Chalk	medium	slight negative
Anhydrite	very high	low
Rock salt	very high	medium low

* *Note*: These rules are to be interpreted only in a general way, and cover only a few of the actual examples. A wide experience is necessary in order to interpret the results obtained, due to the many variables introduced by the complexity of conditions encountered in practice. Interpretation should be guided by the past history of other wells in the field.

TEMPERATURE MEASUREMENTS IN DRILL HOLES

Temperature measurements offer an independent technique which frequently provides information not available from any other source. Such measurements may be used for determining the height of cement behind casing, logging the position of certain formations, and locating gas or water sands. In contrast to the electrical surveys, the presence of casing has little effect on the temperature measurements. Hence, temperature data may yield valuable information with regard to the rehabilitation of declining pumping wells.

Considerable work on thermal measurements has been done by Van Orstrand,† Fisher,‡ and others who used maximum registering thermometers of the mercury type. Considerable work has also been done with electrical thermometers.§

The technique of intermittent readings has not been commercially applied, however, because of the length of time necessary for the well to reach thermal equilibrium and because of the relatively few anomalies observed.

There are two methods of applying temperature measurements in drill holes: (a) measurements of the absolute temperatures throughout the hole (the well may or may not be conditioned by the circulating fluid); (b) thermal conductivity measurements based on differential temperature measurements.

Direct Temperature Measurements.—Direct temperature measurements are usually conducted with the aid of an electrical resistance thermometer which is suspended in the drill hole by a two-wire insulated cable. The thermal element of the resistance thermometer is composed of materials whose resistances change rapidly with the temperature. In commercial work, suitable materials are iron, nickel, silver, and certain alloys which have high temperature coefficients. The recording device may be similar to or identical with that used for electrical resistivity surveys.

When employing D.C. measuring or recording methods, the apparatus consists essentially of a small size wire wound on a cylindrical tube of small diameter in a single spaced layer. Because the electrical resistance

† C. E. Van Orstrand, "On the Estimation of Temperatures at Moderate Depths," *Trans. Amer. Geophys. Union,* 1937, pp. 21-23.

‡ J. Fisher, L. R. Ingersoll, and H. Vivian, "Recent Geothermal Measurements in Michigan Copper District," *A.I.M.E. Geophysical Prospecting,* 1934, pp. 528-536.

§ E. G. Leonardon, "Thermometric Measurements in Drill Holes," *Geophysics,* Vol. I, 1936, pp. 115-126.

of this unit is very high, the cable resistance constitutes a relatively small part of the total circuit resistance. The thermal unit is housed in an oil-filled copper container with a small separation between the winding and the walls of the container to minimize thermal lag. An alternating current instrument (Figure 385) utilizes 25 to 50-cycle alternating current, and measurements are made of the effective changes in resistance. The resistance of the element is made very low and therefore need not be protected from the short-circuiting effect of the water in the drill hole.

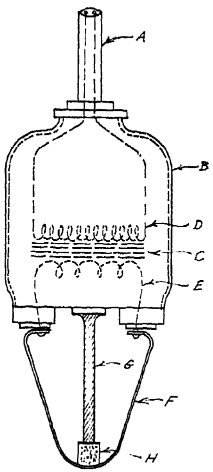


Fig. 385. — Impedance type alternating current thermometer for bore hole exploration. *A*, two-conductor cable; *B*, oil-filled or wax-impregnated container; *C*, closed core, low loss transformer; *D*, high impedance primary winding; *E*, low impedance secondary winding; *F*, resistance strip; *G*, strut; *H*, insulator.

This design is practically free from thermal lag and has proved most satisfactory in commercial operation. The input impedance of the transformer should match that of the cable and surface equipment, and the secondary impedance should match that of the exposed metal element. Any suitable type of recording bridge may be employed for recording the temperature variations. The speed at which the instrument is lowered into the drill hole will be dependent primarily upon the response speed of the recorder.

Method 1. Measurements Made After Thermal Equilibrium Has Been Established

In this method, a graph is made of depth versus temperature along a drill hole which has been undisturbed for a sufficiently long time to allow the hole and surrounding formations to be in thermal equilibrium. Because of differences in thermal conductivity and temperature, the gradient will tend to be different for different formations, each gradient change corresponding to the boundary between the major thermal zones. The general form of the temperature curve after thermal equilibrium has been reached is illustrated in Figure 386.

Probably the most successful application of temperature measurements has been the location of cement behind casing. The method † depends

† C. Schlumberger, "Thermometric Method of Locating the Top of Cement Behind a Well Casing," U. S. Patent 2,050,128, issued Aug. 4, 1936.

on the heat produced by the chemical reactions involved in the "setting" of the cement. It has been found that the temperature of the cement, and of the drill hole opposite the cement, is at a maximum between twelve and forty-eight hours after cementation, depending on the conditions and the nature of the cement. Consequently, it is only necessary to make one or more continuous temperature surveys during this period. The portions of the hole showing abnormally high temperatures, as compared to the normal curve, indicate the position of the cement behind the casing. (It is desirable to avoid any circulation immediately prior to the survey in order that the thermal conditions produced by the setting of the cement will not be disturbed.)

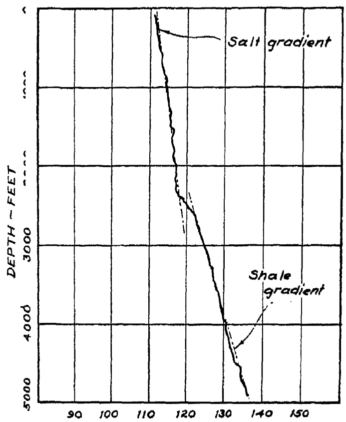


Fig. 386. — Temperature curve after thermal equilibrium has been reached. (Note change in slope at about 2300 feet.)

Method 2. Measurements Made Before Thermal Equilibrium is Established

This method, which is more commonly used than the one described in the last section, utilizes the evolution or change in the thermal gradient, measurements being made before the well is in equilibrium.

In practice, the mud of the well is circulated for several hours before the survey. During this circulation, the mud from the surface is forced down through the drill pipe and returns upward between the pipe and the formations.† At the beginning of this cycle, the mud is cooler than the normal formation temperature at the bottom of the hole and warmer than the formation temperature near the surface. During circulation, therefore, the formations near the bottom of the hole *give up* heat to the mud, and the formations near the surface *absorb* heat. Finally, when circulation is stopped, the temperature curve for the mud in the hole is approximately a straight line, having temperature values which are lower than the temperatures of the formations in the lower portion of the hole and higher than the temperatures in the upper portion. In Figure 387, the curve *B-B* represents the true temperatures of the formations, and *C-C* the temperatures of the mud immediately after circulation. It will be noted that the curves cross at point *A* where mud temperature and formation temperature are the same.

† E. G. Leonardon, "Thermometric Measurements in Drill Holes," *Geophysics*, Vol. I, 1936, pp. 115-126.

As the mud temperature and formation temperature seek equilibrium, curve *C-C* rotates about point *A*, tending to approximate curve *B-B*. However, certain formations, due to differences in heat capacity, tend to give up or acquire heat from the well mud faster than other formations. Thus, after a given time, a curve *D-D* is produced which shows deviations from the normal trend at the locations of the sand formations. After a longer time, the temperature curve for the mud approximates curve *E-E* and finally *B-B* wherein the anomalies due to the formations have disappeared.

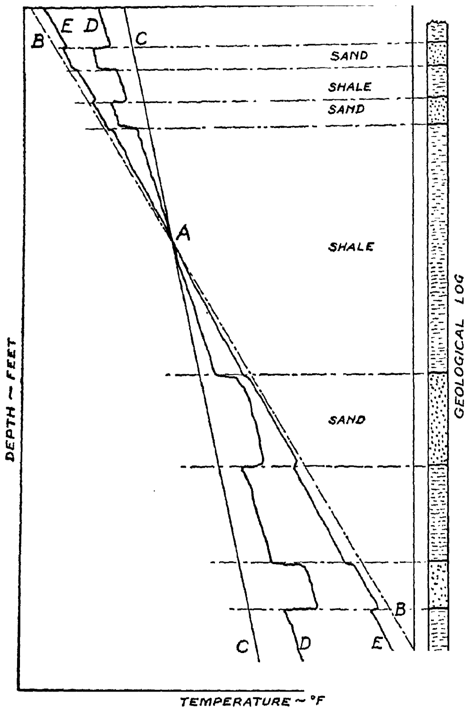


FIG. 387.—Evolution of temperature gradient. (After Leonardon, *Geophysics*.)

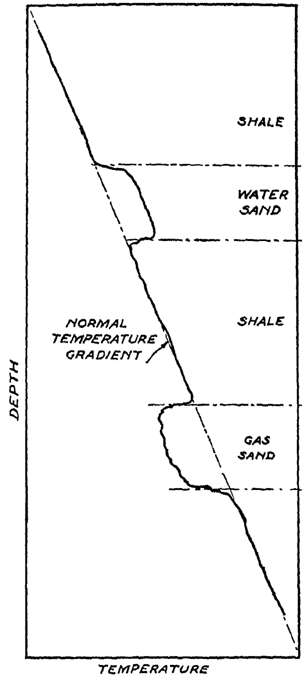


FIG. 388.—Temperature curve illustrating change in thermal gradient at gas and water sands.

The operation of this temperature method of logging formations may be summarized as follows. Use is made of a suitable mud circulation, following which a number of temperature curves are recorded in order to obtain curves *C-C*, *D-D*, and *E-E*. (Curve *B-B* is seldom obtained, due to the length of time involved in allowing the formations and mud to reach their original value.)

In the interpretation of the results, use is made of the fact that most gases absorb heat during expansion. Therefore, because a certain

amount of gas escapes from a sand during drilling and circulation, a *gas* sand will be relatively cool, and will produce a temperature drop on the temperature curves. (Figure 388.)

In certain areas, temperature data are more indicative of structure than resistivity data and hence constitute an important parameter in drill hole logging. Such a condition is illustrated in Figure 389 † which shows a graph of thermometric data obtained after casing was set, and graphs of original porosity and resistivity data obtained prior to setting of casing.

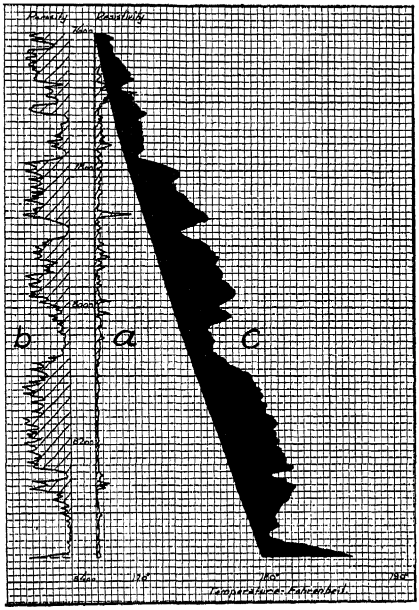


Fig. 389.—Comparison of original resistivity and porosity logs of the uncased well and a thermometric log obtained after setting of casing. (*The Petroleum Engineer*, Feb. 1937, p. 155.)

Differential Temperature Measurements.—The differential temperature method depends on the thermal conductivities of the various subsurface formations rather than on the absolute temperatures of the formations. Two advantages of this thermal method are: (1) It does not require prior conditioning of the hole; (2) variations in subsurface temperatures do not affect the results appreciably, because the measurements are governed by the thermal conductivities of the strata rather than their actual temperatures. The apparatus for this method consists essentially of a heating unit and two temperature measuring units. The heating unit, preferably electrical and of substantially constant current output, is mounted midway

† The Petroleum Engineer, Feb. 1937, pp. 155-159.

between two thermometric devices. (Figure 390.) This heater unit may comprise a means of heat transfer between heating coils or rods and the water, or direct contact electrodes may be employed and the heating produced by the I^2R losses in the mud itself.

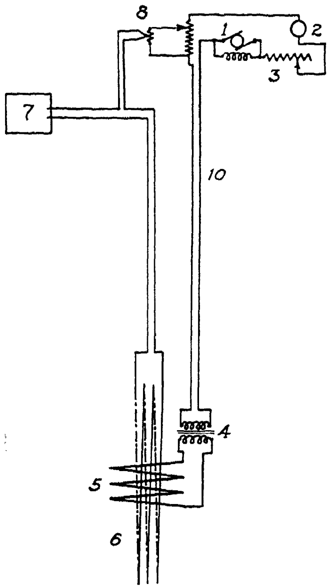


FIG. 390. — Differential temperature measurements. 1, alternator; 2, ammeter; 3, control rheostat; 4, step-down transformer; 5, heater; 6, thermopile; 7, recording potentiometer; 8, compensating couple.

In order to minimize the I^2R losses of the cable, the energy for the heater is transmitted at a relatively high voltage (500 volts); then the voltage is stepped down to approximately 25 to 50 volts by means of a transformer located at the heating element. The heater and two thermocouples are maintained at a fixed separation and lowered as a unit into the drill hole. Measurements are made of the difference in temperature between the two thermometric devices. The observed temperature gradient is principally a function of the thermal conductivities of the strata traversed by the drill hole, the rate of movement of the system, and the energy input to the heater. Continuous recording meters are employed for measuring the wattage input to the heater and the differential temperature between the two temperature measuring units. Variations in energizing current are compensated for by means of an automatic device which consists essentially of a special type thermocouple junction in the energizing circuit whose potential is opposingly connected to the potential of the thermocouple in the drill hole.

METHODS OF LOCATING WATER SOURCES IN A DRILL HOLE

The location of the source, or point of entrance, of formation water in oil wells is a problem of considerable economic importance in oil production. It is good engineering practice to keep the production of water at a minimum because this reduces the pumping, treating, and disposal costs, and in addition reduces corrosion damage to casing, pump, rods, and tubing. Also, in many fields, the maintenance of reservoir pressure is dependent upon the formation water, and a minimum water production will insure a longer flowing life for many wells.

Repair operations to stop the flow of water into a well must be preceded by a means for locating the point where the water is entering the well.

Several geophysical methods have been developed for determining the depth at which water is entering the hole. Usually, proper interpretative technique, based on production experience, will show the cause of water entrance and its remedy.

In contrast to the previously described electrical logging methods, the methods under discussion utilize means for causing the water to enter the drill hole from the formations, as well as means for locating the point of entry. Thus, for these methods, the hydrostatic pressure must be reduced by bailing, in order to allow the formations to produce into the hole.

Resistivity Methods.—In contrast to the use of resistivity methods for structural studies, this application requires that the sphere of measurement be confined substantially within the bore hole. The method of Figure 374 may be applied by reducing the lengths of r_1 and r_2 to values somewhat smaller than the radius of the bore hole.

During resistivity measurements, the hole should be conditioned with fresh water if the formation fluid is salty and with conductive salty fluid if the water source produces fresh or relatively non-conductive fluid. Immediately after the conditioning, a survey is made to verify that the column of fluid in the bore hole is of uniform resistivity. Following this, the well is bailed and a second survey made. The entire procedure (bailing and temperature surveying) is repeated until a change of resistivity, produced by the entrance of water into the well, is noted.

Practical work along these lines has been done by Schlumberger, Huber † and Elliott ‡.

Natural or Self Potential Method.—The spontaneous potential method utilizes the electrochemical potentials previously described. Obviously, if a well is conditioned with a fluid of different electrolytic concentration than that of the formation fluid and if water is then caused to flow into the hole from the formation, a potential will exist at the boundary between the entering fluid and the conditioning fluid.

This method uses two electrodes, situated only a short distance apart, and suspended on a cable. The parameters measured, therefore, are the potential gradients. Conditioning of the well is effected by circulating a suitable mud. For example, Ennis § has developed methods wherein the desired electrolytic concentration of the mud is produced by lowering a soluble electrolyte into the hole. The bailing process is

† F. W. Huber, U. S. Patents 1,536,007; 1,555,800; 1,555,801; 1,555,802; and 1,555,803.

‡ R. D. Elliott, U. S. Patent 1,537,919.

§ Geo. H. Ennis, U. S. Patents 1,725,979; 1,786,196; 1,865,847; 1,889,889; 1,994,761; and 1,994,762.

then used to cause a flow into the well. Measurements of the electrochemical potentials are made throughout the length of the hole, before and after bailing.

Thermometric Method.—The thermometric method does not require special conditioning of the mud. The only preparation necessary is circulation. Measurements of this type are particularly adapted to dynamic wells and to wells in which there is an interchange of fluid between two formations.†

In one actual case, a well was drilled to 6441 feet, with casing set at 6373 feet. When tested, the well produced no water and only a small amount of oil. A small nitroglycerine shot was then made, after which water appeared. It was believed that the shot had caused bottom water to break into the well. After standing for several days, the fluid level rose to about 2400 feet.

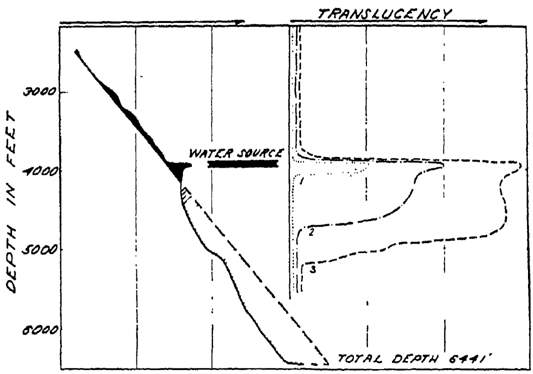


FIG. 391.—Comparison of temperature data and photoelectric data for the same well. (Courtesy of Schlumberger Oil Well Surveying Corp.)

A thermometric survey was then made. (Figure 391.) The temperature-depth curve showed an abrupt temperature change between 3913 and 3945 feet. The temperatures above the anomaly followed the normal curve, and the temperatures below the anomaly were less than the normal values. The data suggested that the temperature increase between 3913 and 3945 feet was due to entrance of water which had a slightly higher temperature than the normal temperature of the bore hole at that depth.

This water had a temperature of approximately 106° and flowed *down* the hole. Hence, it exerted a cooling effect on all formations below, because the normal temperatures of these formations are higher than 106°. This well will be discussed further in connection with the photoelectric method.

† C. R. Dale and B. L. Deleney, Personal Communication.

Photoelectric Method.—The photoelectric method locates water flows by measurements based on variations in the translucency of the drill hole fluid. Usually, the static column standing in the hole will have clear fluid in the upper part. Also, formation water entering a well is more translucent than muddy water or oil. Hence, it is necessary that the well fluid in the interval to be surveyed be made opaque by suitable conditioning in order to offer a contrast with the clear formation water that will enter when the fluid level is reduced by bailing or swabbing.

Conditioning of the well is done by one of two methods. In flowing wells, it usually is necessary to stop the flow and circulate a well-mixed muddy water. This usually gives the required opaqueness. In pumping wells, the interval to be surveyed can be conditioned by the use of a special dump bailer that can be tripped either at the bottom or at any desired depth.

The instrument for measuring the translucency of the liquid in the hole is about 2″ in diameter and 30″ long and consists, essentially, of a constant source of light, which is energized by dry cells, a selenium photoelectric cell, and louvers through which the fluid circulates. As the instrument is raised or lowered in the hole, the fluid passes between two windows which define the path through which the light rays reach the selenium cell. Variations in the translucency of the mud cause variations in the amount of light reaching the photoelectric cell, with corresponding variations in the current output. The cell is connected by means of an insulated cable to a continuous recording meter at the surface. The recording film, which is moved in synchronism with the cable, uses an arrangement similar to that described for the resistivity logging method. The photoelectric log is a plot of depth versus a parameter which is proportional to the translucency of the liquid passing through the instrument.

The practical application and interpretation of the photoelectric data* may be best illustrated by actual examples. The photoelectric data shown in Figure 392 were obtained in a well in which mud was dumped throughout the open hole. A conditioning run with the instrument gave curve number 1. The linear character of the curve showed that all the fluid in the zone that was being examined was opaque and would therefore offer a contrast to any translucent formation water which might enter the drill hole in that zone. Fluid was then bailed from the well and subsequent runs (Nos. 2, 3, 4) indicated that translucent water was entering from the bottom ten feet of the exposed formation. This determination allowed the operators to confine the cementation to the water-bearing

* The analysis is comparable to that employed in the electrical resistivity drill hole methods.

zone, thus preventing the loss of oil resulting from cementation of a portion of the oil-bearing zone.

Another example, Figure 391, shows the relation between thermometric measurements and photoelectric recordings on the same well. The thermometric data for this well have already been discussed. After obtaining the temperature anomaly between 3913 and 3945 feet, that section of the hole was conditioned by dumping several barrels of mud through the interval. The photoelectric instrument was then lowered into the well. A translucency versus depth graph (curve 1) obtained immediately after

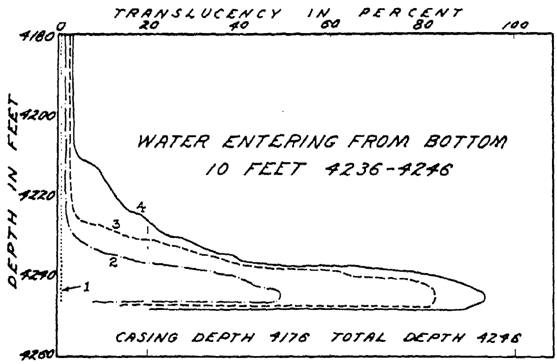


FIG. 392.—Photoelectric survey. Graph shows that water entered at bottom of well from exposed formation. 1, initial curve after conditioning well; 2, 3, 4, curves after formation water entered well. (Dale and Deleney, Personal Communication.)

the conditioning run showed that 150 feet of translucent water had entered the hole. (Note that the water was entering at 3915 feet and going down.) The instrument was pulled up, and 45 minutes later it was again lowered through the zone (curve 2). This curve showed 500 feet of clear fluid, and hence indicated a rate of flow of at least 400 feet per hour. This quantity of water entering at 3900 feet and going down into the producing formation would soon have flooded not only this one well but also the nearby producing wells had it not been shut off by cementing.

CORRELATION OF OIL WELL WATERS

An analysis of the salts of oil well waters oftentimes allows correlating the waters with regard to their origin.† Oil well waters commonly contain, in addition to various other elements, lithium, sodium, potassium, calcium,

† M. F. Hasler, "The Spectrographic Correlation of Oil Well Waters," *Geophysics*, Vol. 2, No. 2, March 1937, pp. 127-131.

J. S. Ross and E. A. Swedenborg, "Analysis of Waters of the Salt Field Applied to Underground Problems," *A.I.M.E. Petroleum Technology*, 1928-29.

strontium, and barium. The procedure is as follows: Well waters are evaporated to dryness and then volatilized in a direct current carbon arc. The light emitted by the discharge is photographed by a grating spectrograph.*

The statistical treatment of the data may be illustrated by a practical instance. Suppose a number of samples from each of several water zones are analyzed. Mean values are calculated from several samples of each zone. Suppose further that the mean value of the ratio of the potassium to sodium content is very different in each of the zones and the deviations from the mean are small, then this ratio is assumed to be diagnostic. It is postulated that by setting up several diagnostic factors, inferences can be drawn regarding the origin of the water zones.

RADIOACTIVITY OF SEDIMENTARY ROCKS

Radioactivity is the spontaneous disintegration of matter. A radioactive element is continually transforming into another element and the transformation is accompanied by emission of radiant energy. Several radioactive series have been discovered, the most prominent being the uranium series which includes radium. Uranium is continually transforming itself into several chemically distinct types of uranium, known as UX_1, UX_2 . . . UII. UII is transformed into ionium, ionium into radium; radium is transformed into radium emanation; this element in turn is transformed into radium A, radium B, and so on. The end product of every radioactive series is lead.

The transformations are due to the spontaneous ejection of an electron (beta ray) or of a helium nucleus (alpha ray) from the nucleus of the radioactive element. In the majority of cases, the transformations are accompanied by the emission of highly penetrating short wave length electromagnetic radiations (gamma rays). The gamma rays may have sufficient penetrating power to pass through iron having a thickness of several inches, and it is this property of the gamma rays which is utilized in well logging,† especially for identifying formations in cased holes in old producing fields.

Large portions of sedimentary rocks are radioactive and the activity varies from formation to formation. Usually, shales are the most highly radioactive, and sandstones and pure limestones are the least radioactive. Radioactive radiation measurements have been utilized in studies of the lateral extent of sedimentary rocks.‡ Studies have also been made in

* A grating spectrograph comprises a slit, a lens, a grating, and a camera. The camera records the "spectral lines," i.e., images of the slit, characteristic of the "excited" substances—in this case, the elements present in the salts.

† G. H. Westby and S. A. Scherbatskoy, *Oil and Gas Journal,* Feb. 22, 1940, p. 62.

‡ See J. N. Hummel, "Radioaktive Methoden," *Handbuch der Experimentalphysik* (1930), 25, part 3, pp. 532-533.

H. Landsberg and M. R. Klepper, "Radioactivity Tests of Rock Samples for the Correlation of Sedimentary Horizons," *A.I.M.E. Geophysical Prospecting,* Tech. Pub. 1103, 1939;

R. W. Clark and H. G. Botset, "Correlation between Radon and Heavy Mineral Content of Soils," *Bul. Amer. Assn. Petr. Geologists,* 1932, 16, pp. 1349-56.

drill holes to measure the radioactive variation from stratum to stratum.† Studies of the radioactivity of cores from drill holes have shown wide variations in the sedimentary layers. ‡

Recent studies in cased holes have indicated the possibility of logging formations behind the casing. § The most convenient means for making these measurements is by use of Geiger-Müller counters. The arrangement employed for measurements in cased holes is shown in Figure 393.

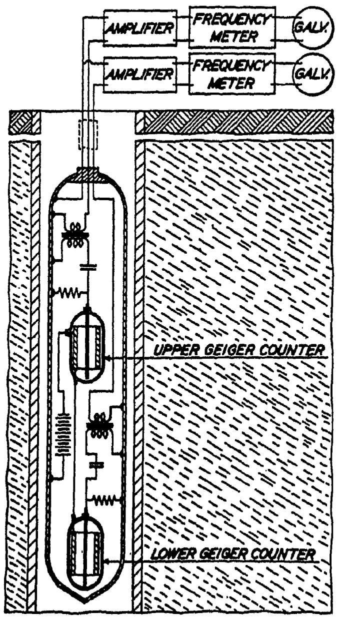


FIG. 393. — Geiger-Müller counters for measurements of radioactivity in bore holes. (Howell and Frosch, *Geophysics.*)

The gamma ray logging technique has also been applied to log the height of cement back of the casing. In this work, the cement slurry is "doctored" with a small percentage of a radioactive substance (usually carnotite containing about 10% uranium oxide) before it is pumped into the

† V. A. Spak, "Une nouvelle méthode de différencier les roches du sondage à l'aide du compteur des gamma-impulsions," *Compt. Rend. Acad. Sci. U.S.S.R.* (1937), 16, pp. 109-112.

‡ H. Landsberg and A. I. Ingham, "Core Testing by Radioactive Methods," *Oil Weekly*, 1938, 90 (4) pp. 26-28.

§ L. G. Howell and A. Frosch, "Gamma-ray Well Logging," *Geophysics*, Vol. 4, No. 2, 1939, pp. 109-112.

well for a squeeze job or casing-setting. Upon completion of the cementing operation, the logging apparatus is run into the hole and the height of the radioactive material back of the casing is logged.

DRILL HOLE SAMPLES

The analysis of core samples is one of the older methods of determining subsurface structure. More recent technique uses core samples to supplement the indications of the electrical log. Various mechanical samplers are on the market, and are usually operated hydraulically by mud pressure. The instrument is mounted on the bottom of the drill stem and is run into the hole to the depth at which it is desired to secure a sample. While traversing the hole, the sampling blades remain inside the body of the tool, but when the desired point is reached, pump pressure applied at the surface pushes down on a piston which expands the blades outward and upward against the formation, and with the pumps still running, the weight of the string of drill pipe is slowly and steadily placed upon the tool. This causes the blades to penetrate the formation, forcing cores of the formation into each of the sampler tubes.

At no time during the coring operation is the drill string to be rotated in either direction. When the pump pressure is removed and the string raised, the blades close into the body of the sampler, and the tool is removed from the hole with samples of the formation in the core tubes.

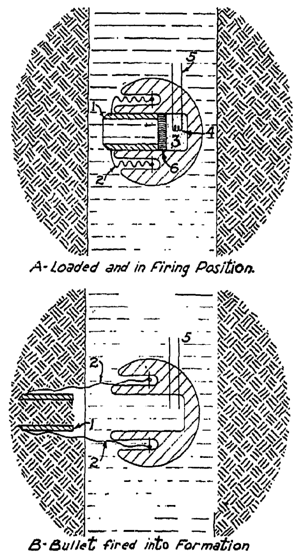


FIG. 394. — Schlumberger gun-type sample taker. 1, bullet; 2, retrieving wire; 3, explosive; 4, ignition wire; 5, firing cable; 6, removable cap.

The operating principles of the gun-type mechanical side wall sample takers † are illustrated in Figure 394. Part A shows the loaded gun in a well with the essential parts: bullet, fastening wires, powder, and igniting wire. By means of two electrical connections, a current is passed through the igniting wire, which thereupon becomes heated and ignites the explosive. The bullet is propelled into the formation as illustrated in Figure 394B. During this process, the bullet bottom cap is forced off so that the

† M. Schlumberger, U. S. Patents 2,055,506, issued Sept. 19, 1936, and 2,119,361, issued May 31, 1938.

mud cake on the wall of the hole, which is collected first, passes through. The bullet is recovered by lifting the gun, which in turn pulls the bullet, with sample, from the formation by the fastening wires. A view of a small gun is shown in Figure 395.* The gun is fastened to the lower end of a weight or stem suspended by the control cable.

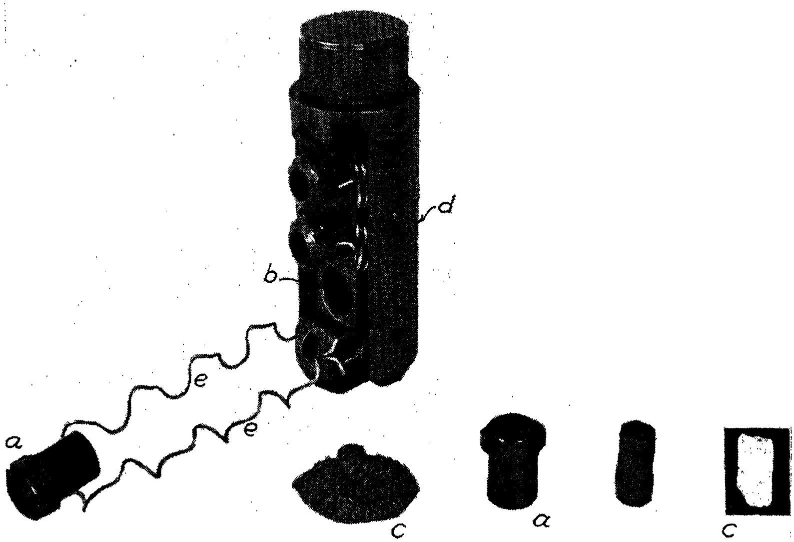


FIG. 395.—Small gun assembly. *a*, bullet; *b*, barrel; *c*, cores; *d*, body; *e*, retrieving spring. (Courtesy of Schlumberger Oil Well Surveying Corp.)

Figure 396 illustrates the discovery of a new sand in an old field from the combined data provided by multi-electrode recording and side wall sampling. The normal and third curves indicate a higher resistivity in the region from 5571 to 5622 feet. The fourth curve verifies the resistivity indication in this section and indicates further that the sandy shale from 5622 to 5632 feet is also oil-bearing. Cores, taken before the casing was set, are shown. The loose sand proved to be a medium grain oil sand of the

* A description of a similar gun for taking ocean-bottom samples is given by C. S. Piggot in the "News Science Bulletin," Carnegie Institution of Washington Pub., Vol. IV, No. 9, Sept. 6, 1936.

Marginulina with a very good odor. Casing was set and perforated from 5596 to 5606 feet with ten shots, resulting in an initial yield of five barrels per hour.

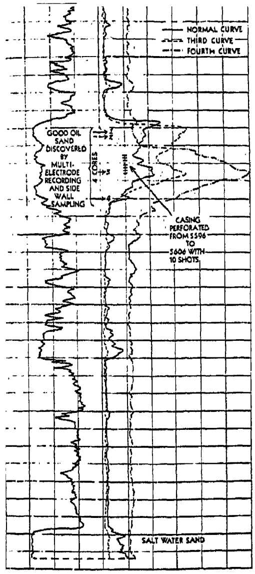


FIG. 396.—Correlations between electrical logging and side wall samples. (Courtesy of Schlumberger Well Surveying Corp.)

Paleontological Studies.—Paleontology is of course invaluable in the recognition of markers. Paleontological markers, in addition, are in general very definite. Studies of the foraminifera of the Gulf Coast have contributed widely to an advancement of the geology of that area. In much of the Gulf Coast, however, due to the scarcity of fossils in many of the formations (especially the younger ones) and the non-fossiliferous character of other formations, the number of horizons that can be definitely identified are limited. Identification of paleontological horizons is done by microscopic examination of cores or side wall samples. These samples are obtained by coring or taking side wall samples over that section of the hole in which it is believed the studies should be made.

Drill Core Measurements.—In the absence of continuous logging data, measurements are often made of the magnitudes of certain properties of drill core specimens. The magnitudes of the electrical resistivity†, fluid permeability‡, radioactivity§,

† M. W. Pullen, "Tentative Method for Making Resistivity Measurements of Drill Cores and Hard Specimens of Rocks and Ores," U. S. Bureau of Mines, Circular 6141, June, 1929.

‡ H. C. Pyle and J. S. Sherborne, "Core Analysis," *A.I.M.E. Petroleum Technology*, 1939, pp. 33-61.

§ E. Rothe and T. Kopcewicz, "Comparaison de la radioactivité des roches d'Alsace par la méthode des tubes compteurs," *Compt. Rend.* (1937), Vol. 205, pp. 165-166.

relative mineral content,† magnetic orientation and permeability,‡ fluorescence§, and other parameters have all been utilized as a means for obtaining supplementary data for guidance of the drilling and completion operations, and for structural correlation with other wells.

An illustration of resistivity measurements on core samples is shown in Figure 397. For this work, resistivity measurements are made on the cores and plotted

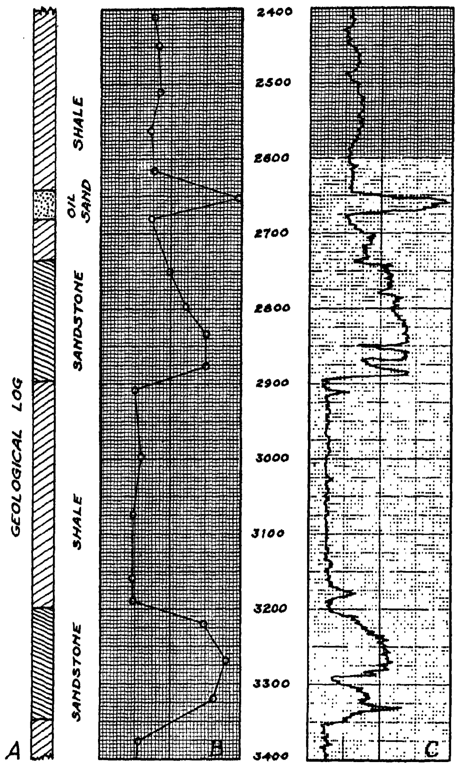


FIG. 397.—Comparison of *A*, geological log; *B*, resistivity measurements on core specimens; *C*, resistivity log in bore hole for a well in the San Joaquin Valley, California.

against depth (curve B) in conjunction with the geological log (A) determined by inspection of the cores. Upon completion of the well an electrical log is run (curve C). There is a good general agreement between the logs, although as would be ex-

† H. C. Pyle and J. S. Sherborne, *loc. cit.*

‡ H. N. Herrick, U. S. Patents 1,792,639, issued Feb. 17, 1931; 1,909,619, issued May 16, 1933; 2,104,746, issued Jan. 11, 1938.

H. N. Herrick and E. D. Lynton, U. S. Patent 2,104,752, issued Jan. 11, 1938.

§ J. A. Radley and J. Grant, *Fluorescence Analysis in Ultra Violet Light,* D. Van Nostrand Co., Inc., New York, 1935.

pected, the core log (B) lacks the good detail shown by the electrical log (C) of the hole. This lack of detail is due largely to (a) limited number of samples measured, (b) contamination of cores by the drilling mud, and (c) loss of cores during the drilling operations.

The most satisfactory method for measurement of core specimens is the potential drop method discussed in connection with Figure 141. Electrical connection to the ends of the core specimen may be obtained by wrapping a few turns of bare wire tightly around each end at right angles to the axis of the core. Two single turn coils are employed for making connection to the core for measuring the potential drop across any convenient length *L*.

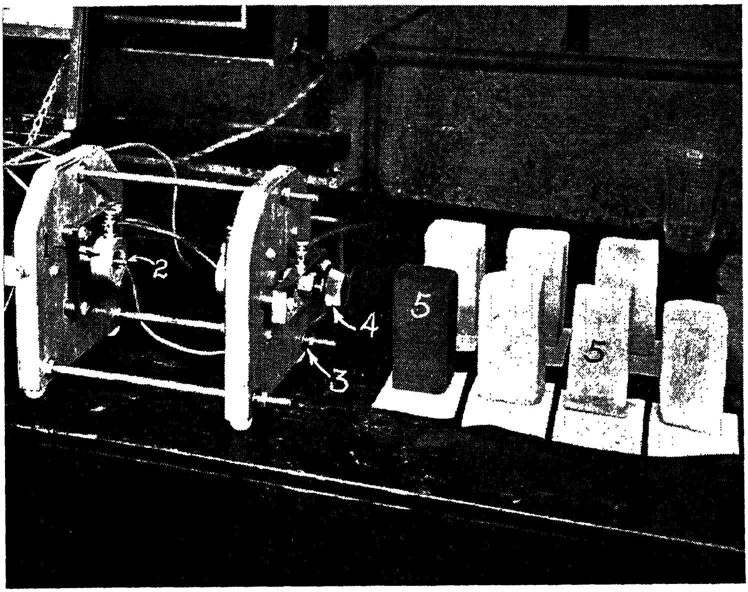


FIG. 398.—Apparatus for measuring resistivities of dressed specimens. 1, current contact ring; 2, potential contact; 3, nuts for clamping specimens between the contact rings; 4, knob for tightening potential contact; 5, test specimens.

When grinding facilities are available for dressing the ends of the cores, a more rapid and convenient method is that illustrated in Figure 398.† In this arrangement, metal rings are pressed against the ends of the specimen and serve as the energizing or current electrodes. The two potential electrodes are placed coaxially with the current rings.

Another arrangement of lower accuracy but greater convenience for irregularly shaped specimens is the four-prong resistivity method illustrated in Figure 399.

† J. J. Jakosky and R. H. Hopper, "The Effect of Moisture on the Direct Current Resistivities of Oil Sands and Rocks," *Geophysics*, Vol. 2, No. 1, January 1937.

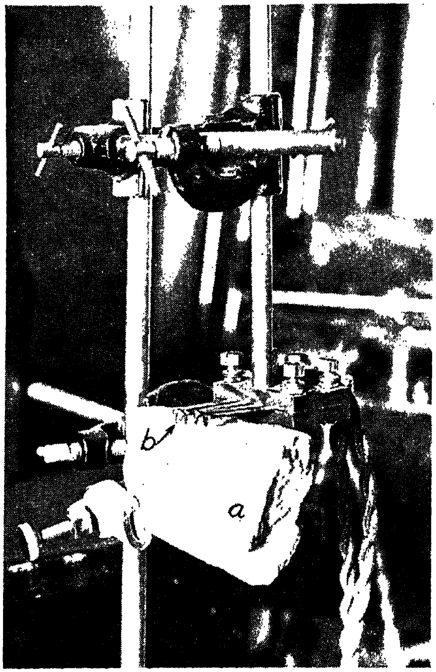


FIG. 399.—Method for determining the approximate resistivity of irregularly shaped materials. *a*, specimen; *b*, four-electrode system.

DRILLING TIME LOGS

It has been recognized for many years that the time required for drilling a unit depth of formation varies with the nature of the formation, as well as with the mechanical characteristics of the drilling equipment. Data may be plotted showing depth versus drilling time per unit depth of penetration.

Formerly, the proper correlation between time and depth was chiefly dependent on a log made by the driller, and many errors were common. Recently, it has been the practice to use mechanically actuated recorders which are geared to the drill stem. This is usually accomplished by means of a flexible cable connected to the swivel or the traveling block, which feeds over an idler near the top of the derrick to the recording instrument.† Movement of the cable, as the drill is raised or lowered, produces corresponding movement of the recording paper. In some cases, a ratchet device is employed to allow for continuous forward motion of the recording paper. In other devices, the chart is

† T. C. Heistand and P. B. Nichols, "Drilling Time Data in Rotary Practice," *Oil and Gas Journal*, July 13 and 20, 1939.

J. J. Jakosky, U. S. Patents 2,150,169, 2,153,802 and Reissue 21,102.

moved at a uniform rate of speed and each vertical trip of the Kelly produces a varying pressure recording on the chart. Drilling time is proportional to the length of chart between corresponding phases of the pressure curve.

Drilling logs supply valuable information for later correlation with sample logs and electrical and thermal logs. Proper correlation with sample logs is dependent on an accurate evaluation of the time required for cuttings to reach the surface sampling box. This time varies with the effective diameter and the depth of hole, the rate of mud circulation, and the viscosity of the mud. In a 5000-foot hole, the time lag is usually about one hour. Drilling time logs are finding increasing use in the industry, because they not only supply information which allows the operator to choose the optimum drilling technique, but they also supply a record showing time going in and out of hole, condition of bits, stand-by periods, etc.

Recent developments in mud analysis and logging electrically while drilling supplement the drilling time logs.

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790,904 Issued Nov. 29, 1935. Conrad Schlumberger. "Improvements of the Methods and Apparatus for Electrical Prospecting of the Ground Traversed by Boring."

791,227 Issued Dec. 6, 1935. Conrad Schlumberger. "Method and Apparatus for Electrical Prospecting of the Ground Traversed by Boring."

797,811 Issued May 5, 1936. Conrad Schlumberger. "Method for Studying Permeable Layers Penetrated by Boring."

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340,260 Issued Mar. 24, 1934. John C. Karcher. "Borehole Exploring Apparatus."

AUSTRALIAN PATENTS

1641/04 Issued Aug. 1, 1905. Hugh F. Marriott. "Improvements in Means for Surveying Bore Holes."

1642/04 Issued Aug. 1, 1905. Hugh F. Marriott. "Improvements in Instruments for Surveying Bore Holes."

3405/05 Issued May 8, 1906. Percy E. Lewis. "Improvements in Instruments for Surveying Bore Holes."

17,940/15 Issued Jan. 18, 1916. Charles Broe Galvin. "Improvements in Instruments for Surveying Drill Holes."

15,335/20 Issued Oct. 6, 1920. William Gallagher. "Improvements in Means or Instruments for Use in Surveying Bore Holes and the Like."

CHAPTER XI

PHYSICAL PRINCIPLES APPLIED TO PRODUCTION PROBLEMS

The present chapter deals with the application of sound or pressure waves for determining the distance (depth) to the fluid level and to partial obstructions in an oil well. Considerable work has been done on sonic means of determining fluid levels and also on sound or pressure wave methods for locating obstructions in tubings of various kinds.† The first practical applications of the wave method in the United States were probably made by Batcheller ‡ and by Stoddard * about 1900. A schematic arrangement of the apparatus is illustrated in Figure 400. The pressure wave was created by firing a pistol K, and the elapsed time between initiation n^1 of this wave and its reflection n^3 was measured on a recording chronograph, actuated by a diaphragm H^6. The trace M of a tuning fork of known frequency was recorded on the record simultaneously to provide a means for measuring time intervals accurately.

A commercial incentive for this early work was furnished by the difficulties encountered in operation of the underground pneumatic mail and transport tubes in New York City, Philadelphia, Boston, etc. These tubes were from six to eight inches in diameter and in some cases had lengths of 15 to 20 miles, with various stations along the route. The material to be transported was placed in a carrier tube and propelled to its destination by compressed air. Where the carrier became lodged between stations, it could be located by echo methods.

In applying this method in long tubes, it was found that the velocity of the wave in the tube varied with temperature, pressure, and humidity. It was noted further that where the tubes turned corners in following streets, and also where switch points or other partial obstructions were located, minor reflections were obtained, in addition to the main reflection coming

† Th. Vautier. "Secondary Waves Produced by an Aerial Wave," *Comptes Rendus*, June, 1925, p. 1919; "Experimental Researches on Propagation of Aerial Waves Through a Long Cylindrical Pipe"; *Annales de Physique*, Series 10, Vol. 6, pp. 311-364, 1926; Vol. 14, *Annales de Physique*, pp. 263-626, 1930.

‡ Birney C. Batcheller, "Apparatus for Locating Obstructions in Tubes," U. S. Patent 602,422, issued April 19, 1898.

* Personal communication from Chas. F. Stoddard, formerly Chief Engineer, American Pneumatic Service Company.

from the blocked carrier. Changes in the velocity of the wave therefore were minimized by applying correction factors between these partial obstructions at known locations, or by measuring from the last reco partial obstruction at a known location to the blocked carrier.

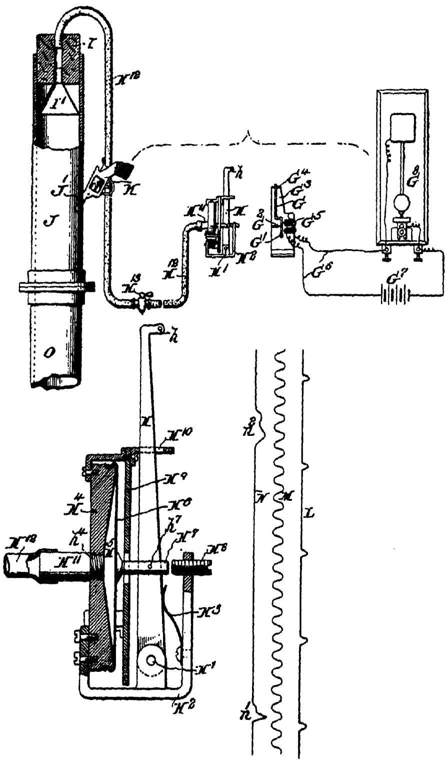

FIG. 400.—Early sound wave method of locating obstructions. (Batcheller, U. S. Patent 602,422.)

During the World War, and for some years afterward, an intensive application of sound wave methods was made in a variety of fields, † par-

† C. W. Rice, "Sound-Wave Apparatus," U. S. Patent 1,889,614, issued Nov. 29, 1932.
R. A. Fessenden, "Method and Apparatus for Determining Distance by Echo," U. S. Patent 1,853,119, issued April 12, 1932.
P. Langevin and C. L. Florisson, "Method and Apparatus for Sounding and for Locating Submarine Obstacles by Means of Ultra-Audible Waves," U. S. Patent 1,858,931, issued May 17, 1932.
H. C. Hayes, "Method and Apparatus for Sound Ranging," U. S. Patent 1,900,015, issued March 7, 1933.
E. E. Turner, U. S. Patents 2,009,460, 2,003,160, 2,044,820, on echo methods of measuring depths, issued July 30, 1935, Mar. 10, 1936, and June 23, 1936, respectively.
R. L. Williams, "Method and Apparatus for Measuring Distances and Depths," U. S. Patent 2,015,702, issued Oct. 1, 1935.

ticularly in military affairs. Many of the sound wave methods evolved during the war-time were later taken over into commercial navigation and aeronautic pursuits. All of the work utilized either loud noises, such as those made by firing guns, or sounds of extremely high pitch, such as whistles.

Before considering the application of sound waves in oil production problems, it will be desirable to indicate briefly: (1) certain physical principles of oil recovery in wells and (2) alternate methods for determining operating fluid levels or bottom-hole pressures.

PRINCIPLES UNDERLYING OIL RECOVERY IN WELLS †

The applications of bottom-hole pressure or fluid-level measurements are based on simple hydrodynamic principles. ‡ Under equilibrium conditions,

$$p_{bh} = hd + p_g + p_{ch} \tag{1}$$

where

p_{bh} = bottom hole pressure in pounds per sq. in.

h = height of fluid in the well in ft.

d = average fluid pressure gradient or fluid density in pounds per sq. in. per foot

p_g = weight in pounds of section of gas column one inch square from fluid level to casing head

p_{ch} = casing head pressure in pounds per sq. in.

In words, Equation 1 states that the bottom hole pressure is equal to the pressure of the column of fluid plus the pressure of the gas column plus the casing head pressure.

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† J. J. Jakosky, "Bottom Hole Measurements in Pumping Wells," *A.I.M.E. Petroleum Technology*, Tech. Pub. 1058, 1939.

‡ T. E. Swigert and C. R. Bopp, "Experiments in Use of Back-Pressure on Oil Wells," U. S. Bureau of Mines, Tech. Paper 322, 1924.

S. F. Shaw, "Well Back-Pressure and Fluid Levels," *Oil and Gas Journal*, Nov. 22, 1928.

S. F. Shaw, "Increasing the Ultimate Recovery of Oil," *A.I.M.E. Petroleum Technology*, 1931, pp. 178-193.

C. V. Millikan and Carrol V. Sidwell, "Bottom Hole Pressures in Oil Wells," *A.I.M.E. Petroleum Technology*, 1931, pp. 194-205.

C. V. Millikan, "Reservoir and Bottom Hole Producing Pressures as a Basis for the Proration of Crude Oil," *Oil and Gas Journal*, Aug. 11, 1932.

C. E. Reistle, Jr., and E. P. Hayes, "A Study of Sub-Surface Pressures and Temperatures in Flowing Wells in the East Texas Field and the Application of These Data to Reservoir and Vertical Flow Problems," U. S. Bur. of Mines Report of Investigations, 3211, May 1933.

H. C. Miller, E. S. Burnett, and R. V. Higgins, "Oil Well Behavior Based upon Subsurface Pressures and Production Data," *A.I.M.E. Petroleum Technology*, 1937, pp. 97-109.

M. Muskat, "Use of Data on the Build-up of Bottom Hole Pressures," *A.I.M.E. Petroleum Technology*, 1937, pp. 44-48.

M. Muskat, R. D. Wyckoff, H. G. Botset, and M. W. Meres, "Flow of Gas Liquid Mixtures Through Sands," *A.I.M.E. Petroleum Technology*, 1937, pp. 69-96.

M. L. Haider, "The Productivity Index," *A.I.M.E. Petroleum Technology*, 1937, pp. 112-119.

The rate of production Q depends on the reservoir pressure p_r, a "friction pressure" p_f, and the bottom-hole pressure p_{bh}. At moderate rates of flow, the rate of production Q may be expressed in the form

where

$Q =$ rate of production
$C =$ a constant of proportionality
$p_r =$ pressure of fluid in the formation at zero production
$p_f =$ "friction pressure." This so-called friction pressure is a back pressure in the formation and is a measure of the resistance which the fluid must overcome while flowing through the oil sand into the hole.

If it is assumed that the "friction pressure" is proportional to the rate of production ($p_f = kQ$, where k is a constant), the equation given above for Q may be written in the form

$$Q = Cp_r - CkQ - Cp_{bh}$$

On collecting the terms involving Q, one obtains

$$Q\,(1 -$$

or

$$(2)$$

where $c = \cdot \quad Ck =$ a constant.

The assumption that the pressure p_f varies linearly with the rate of production leads to a theoretical production rate that is always greater than the physical pumping rate. This is especially true at the higher rates of production where turbulent flow occurs in the vicinity of the well. Thus, at production rates approaching zero and also at high rates, where a transition takes place from stream-line to turbulent flow in the stratum adjacent the hole, Equation 2 may break down. However, in problems concerned with the theoretical maximum rates of production, it is customary to assume that the assumptions made in deriving Equation 2 are fulfilled approximately, and this convention will be followed here.

By combining Equations 1 and 2 the rate of production may be expressed as follows:

$$Q = c(p_r - hd - p_g - p_{ch}) \tag{3}$$

Equations 2 and 3 are the basic theoretical equations relating the rate of production Q with the bottom-hole pressure p_{bh} and with the fluid level h. Because p_r is assumed to be a constant, it follows from Equation 2 that a graph of the values of bottom-hole pressure p_{bh} as ordinate and rates of

production Q as abscissa is approximately a straight line. Also, it is evident from Equation 3 that a graph of the values of the fluid level h versus Q is approximately a straight line. Extrapolation of this line to the Q intercept ($h = 0$) yields the *theoretical* maximum rate of production, because $h = 0$ corresponds to a minimum back pressure of the fluid in the hole.* Thus, from Equation 2

$$Q_{\max} = c(p_r - p_g - p_{ch}) \qquad (4)$$

As will be illustrated later, this *theoretical* rate is ordinarily not achieved in practice, because a definite flow head is necessary to force the oil to flow into the pump.

The production characteristics of a well may be determined by measuring the bottom-hole pressure and using Equation 2 or by measuring the fluid height h and the average density d and using Equation 3. Technically, either set of factors is subject to direct measurement: bottom-hole pressures may be determined by means of bottom-hole pressure gauges; the fluid-level heights and fluid densities may be determined by sound or pressure wave methods.

BOTTOM-HOLE PRESSURE GAUGES

The bottom-hole pressure gauges in use at the present time are usually of a self-contained, continuous recording design.† The gauges are enclosed in a suitable case and consist essentially of two parts: (a) pressure element to record the hydrostatic pressure and (b) a clock-pressure or thermal-drive chart drum. A stylus attached to the free end of the pressure element records its movement on sensitized paper or metal affixed to the chart drum. The clock-drive comprises a small spring clock mechanism capable of driving the chart for periods of 24 to 72 hours. The pressure-drive depends upon a decrease in recorded pressure to rotate the chart. The thermal-drive comprises a bimetallic temperature-actuated system which produces a small angular rotation of the chart. The operation of the latter system depends on the increase of temperature with depth.

If pressures are to be taken while a well is pumping, a clock-driven gauge is attached to the bottom of the pump. This necessitates removing the pump, the rods, and at times even the tubing, from the well. The equipment is then run back into the hole, and the gauge records the pressure existing in the well at the depth of the pump. On conclusion of a run, the pump must be pulled a second time to recover the gauge chart. The record shows the variations in pressure versus time during the test.

* For thick sands and fluid level below the top of the sand a mean pressure should be used.

† C. V. Millikan and C. V. Sidwell, *loc. cit.*

E. K. Parks and C. W. Gibbs, "Instruments and Equipment for Recording Subsurface Pressures," *A.I.M.E. Petroleum Technology*, 1934, pp. 42-52.

Paul G. Exline, "A Precision Gage for Subsurface Pressure Measurements," *A.P.I., Drilling and Production Practice*, 1936, p. 116.

If the pressure-drive is used, pressure-depth measurements may be made by running the instrument into the tubing on a wire line. The gauge records a straight line as long as the pressure is increasing, but when the instrument is raised a ratchet arrangement causes the drum to rotate. This makes a convenient system for charting pressures at any desired depth.

The thermal-drive instrument, also run on a line, is allowed to remain stationary at each desired depth for about 2 minutes. During this time interval, the bimetallic system absorbs sufficient heat to cause rotation of the drum, with a resultant "step" in the depth-pressure curve.

The pressure gauge sometimes suffers a disadvantage in the time and expense required for each test, especially when used on pumping wells. Also, the effects produced on the bottom-hole pressures by altering the pump speed or the rate of production while the gauge is in the well are not available until after the gauge is recovered. In cases of pump-speed, well-proration, and similar tests, this time delay is often undesirable.

METHODS FOR DETERMINING FLUID LEVELS

Under certain conditions, various types of measuring lines, floats, and chalk-line systems may be employed for determining fluid heights. As a general rule, however, such methods are limited to open wells and cannot be employed successfully during pumping operations. The most practical methods for measuring fluid levels in pumping wells or wells under pressure utilize sound or pressure wave reflections.

The wave method of determining depths to fluid comprises: (1) a means for initiating the sound or pressure wave, (2) a suitable wave detecting and recording apparatus, and (3) a constant-speed recording system and/or timer.

Efforts to obtain more reliable data than those given by the original Batcheller procedure have yielded two commercial methods: (1) a pressure pulse method which employs a sub-audio frequency pressure impulse created by the release of compressed gas from a small tank and (2) a selected frequency method which employs the pressure wave initiated by a cartridge.

The first mentioned method was developed by Lehr and Wyatt.† The wave is created by releasing a small quantity of gas under pressure. The recorder attached to the casing head of the well comprises a diaphragm-driven system that actuates a mirror utilized in the photographic recording mechanism.‡ Acoustical tuning is employed to accentuate the weaker reflections.§ This may be accomplished either by setting up a condition of

† Paul E. Lehr and H. D. Wyatt, "Method and Apparatus for Measuring Well Depths," U. S. Patent 2,047,974, issued July 21, 1936.

‡ C. P. Walker, "Determination of Fluid Levels in Oil Wells by the Pressure Wave Echo Method," *A.I.M.E. Petroleum Technology,* 1937, pp. 33-43.

§ C. P. Walker, "Means for Measuring the Location of Obstructions in Wells," U. S. Patent 2,156,519, issued May 2, 1939.

resonance between the collar echoes coming out of the well and an oscillation that takes place in the tuning pipe between the recording instrument and the casing head, or by tuning the diaphragm to the frequency, or a harmonic thereof, of the tubing collar echoes.

Proper utilization of sonic resonance to pick up the weaker tubing collar reflections necessitates careful tuning of the detecting apparatus. This "cut and try" procedure, requiring considerable time, is repeated until the desired tuned frequency is obtained, after which the photographic record is made. The use of resonant or undamped tuned diaphragm systems is sometimes a source of error, because such devices are initially adjusted or tuned with the stronger tubing collar reflections coming from the upper portion of the well. If the average length of tubing or the velocity of sound transmission in the upper portion of the well differs from that in the deeper part, it is necessary for the deeper reflections to force the proper frequency on the undamped, tuned recording system.*

In the second method, a pressure wave is created by use of a cartridge and the reflections are detected by a suitable microphone. The mechanical part of the equipment is attached to the casing head of the well. (Figure 401.) It comprises: (a) a manually or automatically fired cartridge system; (b) fire trap; (c) manual control for swinging the grid or microphone into the gas stream after initiation of the pressure wave; (d) surge chamber; and (e) pressure gauge. The unit is connected to the casing head by means of a two-inch nipple. Units are constructed to withstand all pressures encountered in commercial operation.

The microphone is connected by a shielded cable to a high gain amplifier, suitably mounted inside a truck. A selector and automatic volume control circuit permits using high amplification to record the feeble echoes from tubing collars without putting an overload on the much stronger reflections from the fluid level. Undesirable phase shift in the amplifier is minimized as a consequence of the relatively narrow frequency band employed in the measurements.

The conventional type of photographic recording oscillograph may be employed for recording the reflections. However, a direct-writing ink recorder is considerably more convenient and has a sufficiently rapid response for work of this type. An exterior view of a recorder of this design, mounted beside the amplifier, is shown in Figure 402. This recorder employs a continuously moving paper tape passing under an ink stylus. Hunting and overshooting is avoided by proper negative feed-back in the

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* Obviously, a distortion of the true wave form in the casing head must result when a tuned harmonic system is employed for recording the aperiodic pulses resulting from the tubing collar reflections

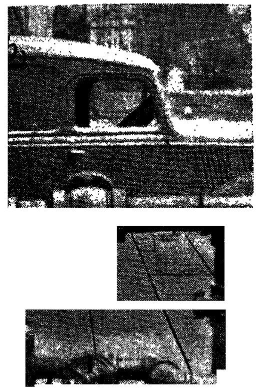


Fig. 401.—Casing-head unit for measuring depth to fluid level in wells. *a,* six-shot cartridge magazine; *b,* fire trap; *c,* handle for swinging grid into position after shot is fired; *d,* surge chamber; *e,* pressure gauge; *f,* gas flow line from casing head; *g,* oil flow line; *h,* sucker rod; *i,* recording truck; *j,* electric cable to truck.

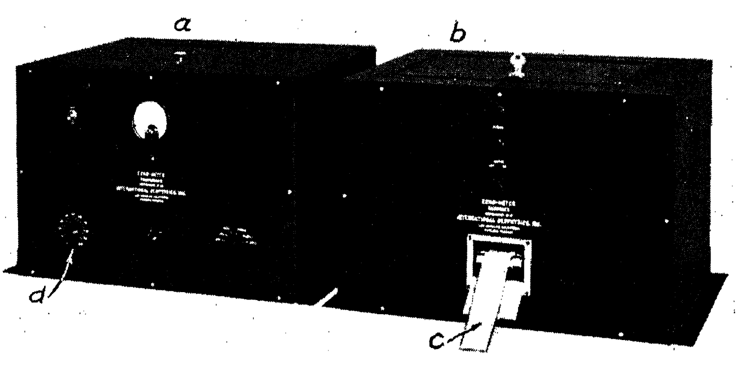


Fig. 402.—Wave reflection equipment for measuring depth to fluid level in wells. *a,* amplifier and automatic volume control equipment; *b,* direct-writing ink recorder; *c,* perforated paper tape; *d,* sensitivity control.

amplifier and electromagnetic damping in the recorder. The paper is moved at a constant speed by means of a small synchronous motor operating on the amplifier output of an electric tuning fork.

Operation of Wave Reflection Equipment.—When subsurface measurements are to be made, the instrument truck is stationed within 100 feet of the well and connected by a shielded cable to the casing head unit. The casing head unit is connected directly to the casing head of the well through existing valves or fittings.

As a preliminary step in operation, the recorder is started and the grid is swung out of the gas stream. In this position, sufficient energy is picked up to record the initial shot-time and the upper tubing collar reflections. Then the grid is swung into the main gas stream for recording the weaker reflections. When the cartridge is fired, a pressure wave is sent into the casing head and moves down the annular space between the casing and the tubing. The speed of travel varies with the composition and density of the gas; it also varies with the effective size of the space between tubing and casing and, to a slight extent, with the temperature.

As the wave travels down the annular space, small reflections are generated by tubing collars, while tubing catchers and liner tops usually send back much stronger reflections. The fluid surface reflects practically the whole wave front which has traveled to the bottom of the well. Figures 403 and 404 show typical ink records obtained by this method. In a normal day's work, single fluid level measurements can be made on from 20 to 3(

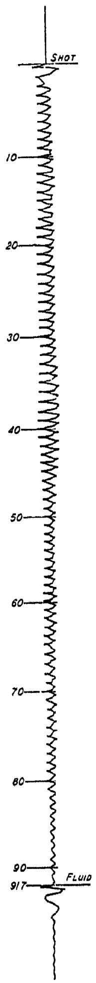


Fig. 403.—Typical reflection record without filter showing shot-point, tubing collar reflections, and fluid reflections.

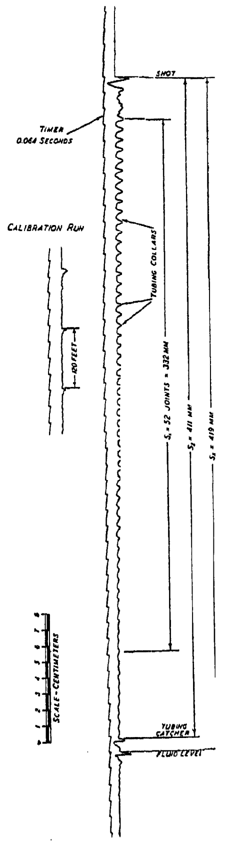


Fig. 404. — Record in which tubing collar reflections are partially observed. *a*, calibration run to determine actual gas velocity and tubing joint lengths; *b*, fluid level measurement to determine depth to fluid and other well obstructions. (Jakosky, *A. I. M. E. Petroleum Technology*, Tech. Pub. 1058, 1939.)

Corrections for Abnormal Initial Velocities

When the wave is initiated either by the release of gas which was confined at high pressure or by the firing of a shot, a certain volume of gas enters the casing head. This gas expands and travels down the annulus with an initial high velocity which gradually decreases as the pressure of the gas is reduced by its expansion. The velocity of the expanding gas is superimposed on the velocity of the wave motion in the gas, thereby producing an abnormally high velocity at the initiation of the wave. If this abnormal "apparent velocity" were used in the computations, an erroneously high fluid level would be obtained. The anomalous component of the velocity is dependent upon: (1) the volume of gas used in generating the pressure pulse, (2) the cross-sectional area of the annulus in the upper portion of the well, (3) the relative wave velocities in the original gas of the well and in the ejected gas, and (4) the casing head pressure of the well.

The error in measurement due to the expansion of the gas may be compensated by comparing the time interval for the first reflection with the intervals for the second and subsequent reflections, which will be longer. (Figure 405.) (All recordings should be carried at least to a second set of reflections when depths are determined solely from velocity calculations.)

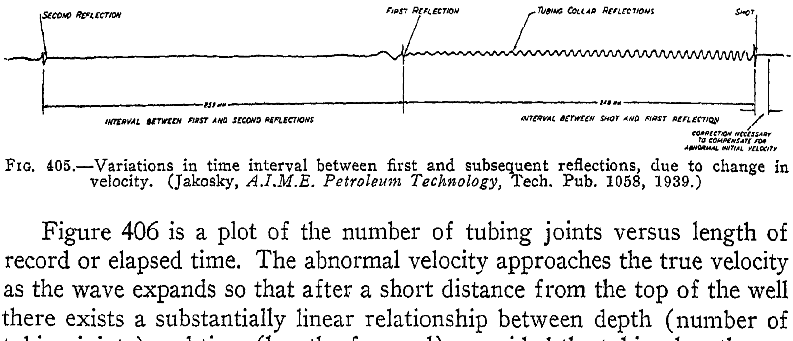


FIG. 405.—Variations in time interval between first and subsequent reflections, due to change in velocity. (Jakosky, *A.I.M.E. Petroleum Technology*, Tech. Pub. 1058, 1939.)

Figure 406 is a plot of the number of tubing joints versus length of record or elapsed time. The abnormal velocity approaches the true velocity as the wave expands so that after a short distance from the top of the well there exists a substantially linear relationship between depth (number of tubing joints) and time (length of record), provided the tubing lengths are uniform throughout the well. In this case (Figure 406) a portion of the abnormal velocity in the upper part of the well was caused by infiltration of air into the casing head.

Extension of the upper portion of the curve to length of record (or depth) equal to zero gives the correction necessary to compensate for the abnormal initial velocity. That is, the distance between the extrapolated intercept and the "observed" intercept is approximately equal to the difference in length of record between the first and second reflections.

Calculating Fluid Level Depths From Echo Measurements

The depth to the fluid level may be determined by various relationships. In the simplest case, when tubing collar reflections are shown on the record

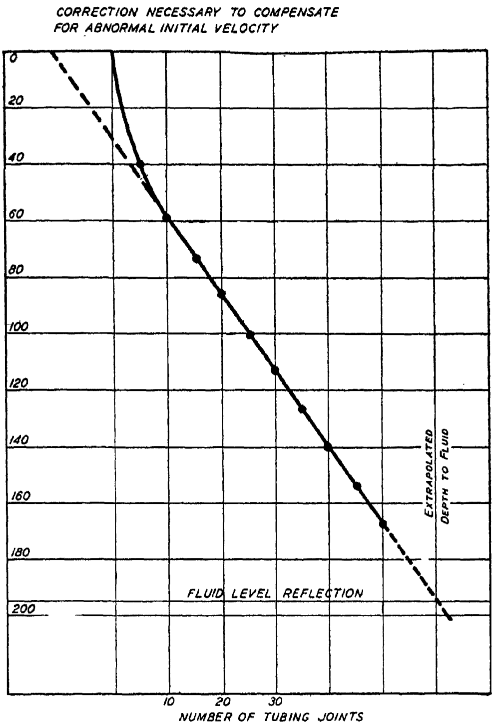


FIG. 406.—Graphic method for determining fluid level by plotting time versus number of tubing joints at known depths in the well.

all the way to the fluid surface, it is convenient simply to add the lengths of tubing (from the well records) above the fluid. If reliable tubing collar reflections are not obtained down to the fluid surface, the straight line or lower portion of the *true velocity* curve may be extrapolated, and the depth determined as shown in Figure 406.

Rapid determinations of the wave velocity can also be made by connecting to the casing-head unit a known length of high pressure tubing arranged, for convenience, in a coil and employing the recording equipment previously described. The system is purged entirely of well gas, and then is maintained at the well pressure. The time required for the *reflected* wave (which travels at the true well velocity) to traverse the tube is recorded in a calibration run. (Figure 404.) If the effective length of the calibration

tube and the time are known, the velocity may be calculated from the simple relation

$$V = \frac{2D'}{T} \tag{5}$$

where D' is the length of tube; T is the total travel-time; V is the wave velocity in gas from the well under test. (The factor 2 occurs because the wave must travel down the well and then back up to the microphone.)

The depth to the fluid may then be calculated by applying Formula 5 again. For the case at hand, Formula 5 may be written:

$$D = \frac{TV}{2} \tag{5a}$$

where D is the depth to the fluid level in feet; V is the true velocity of the wave in feet per second; T is the time in seconds required for the wave to travel to and from the *fluid surface.*

In an ideal gas the dependence of the velocity on temperature, pressure, and density may be expressed by the formula

$$\overline{1 + \alpha T)} \tag{6}$$

where V is the velocity, p_0 the pressure, ρ_0 the density at 0°C., α a constant, T the temperature in degrees C., and γ the ratio of the specific heats of an ideal gas.

After determining the true velocity and the corrected length of the record, the depth to the fluid may be determined by the simple proportion:

$$\frac{\text{length of calibration record}}{\text{corrected length of fluid level record}} = \frac{\text{length of calibration pipe}}{\text{depth to fluid}}$$

This same proportion may be employed in those wells where the tubing catcher, liner top, or an artificial reflector is located at a known depth not far from the fluid level. As an illustration, if the depth to the tubing catcher in Figure 404 could be taken directly from well records, the fluid level would be:

$$\frac{\text{411 mm. (to catcher)}}{\text{1325 feet}} = \frac{\text{419 mm. (to fluid)}}{\text{fluid depth}}$$

$$\text{fluid depth} = \frac{1325 \times 419}{411} = 1350 \text{ feet}$$

In another procedure, applicable only in those cases where the well records are reliable and show the total length of tubing and the number of joints, the "average" joint length may be calculated by dividing the number

of joints into the total length. (It will be noted that a true average value can be obtained only if the tubing is of uniform length.)

Thus in Figure 404 the record length S_1, with its known number of tubing reflections, is compared with the distances S_2 or S_3. This relationship or proportion may be expressed by the following equation:

$$\frac{D}{\ } = \frac{nL}{S} \quad \text{or} \quad D = \qquad (7$$

D equals the depth to fluid level in feet; S_3 equals the corrected distance millimeters to the fluid-level reflections; S_1 equals the distance in millimeters corresponding to a definite number n of tubing reflections; L equals the average joint length as obtained from the calibration data or the well records.

Fluid Density and Subsurface Pressures.—When wells of high productivity are pumped at rates considerably less than their potential, the fluid separates into its components when equilibrium is reached. The effects of this gravitational separation are illustrated in Figure 407. Because the pump is removing the fluid at the same rate that it is flowing into the well (the condition obtaining when equilibrium is reached and the fluid level is stationary), the fluid below the producing horizon is essentially of the same composition as that leaving the formation. In the annulus above the producing horizon, where direct flow of fluid from the formation is not taking place, a quiescent fluid condition exists. The oil-gas phase will comprise the upper portion of the fluid column, while the water phase will comprise the lower portion of the column. The degree to which this separation takes place will be chiefly dependent upon: (a) time during which equilibrium has existed and (b) the stability of the oil-water mixture in that particular well. In wells where the oil and water are readily separated by gravity, the water and much of the bottom settlings will be carried away by the fluid stream. In wells that make very little gas, the density of the fluid column will be substantially the same as the density of the clean oil. Therefore, the fluid density of the oil is used for the calculations rather than the density of the fluid being pumped from the well. When measurements are made in wells making appreciable quantities of

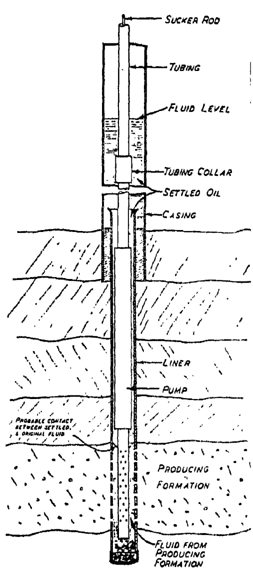


FIG. 407. — Illustration of gravitational separation of oil from fluid produced by a well. (Jakosky, *A.I.M.E. Petroleum Technology*, Tech. Pub. 1058, 1939.)

gas through the casing, the work must be conducted so as to compensate for the changes in density.

The fluid pressure gradient in the annulus of a well depends on the producing conditions and on the effective area of the annulus. For given producing conditions, the pressure gradient in the annulus is practically constant from the top of the fluid to the perforations, if there is no change in casing size. It has been found that the gradient varies from about 0.05 to 0.4 pounds per square inch pressure per foot of depth below the fluid level, depending upon the gas-oil ratio. Often, the density decreases almost proportionally to the effective area of the annulus. The reason for this will be apparent when it is recalled that the gas-oil ratio remains relatively constant at any given rate of production. As this gas is being evolved, it bubbles up through the oil in the annulus. The smaller the area of the annulus, the greater must be the percentage of gas present, with a resultant decrease in the weight of the fluid column, i.e., its effective density. Referring to Figure 407, the density in the liner will be much less than in the upper cased part of the hole, due to the change in area of the annulus.

Determinations of the Fluid Density or Fluid Pressure Gradient

To determine the average fluid density or fluid pressure gradient in a well, measurements may be made of the variation in fluid level as a function of casing-head pressure at a constant rate of production. If the rate of production is held constant, the bottom-hole pressure p_{bh} is constant. After equilibrium has been established, the rate of production is normally unaffected by the value of the casing-head pressure because the rate of production is dependent only upon the difference in pressure between the bottom hole producing pressure and the formation pressure. That is, for any given rate of production, a change in casing-head pressure causes a corresponding change in fluid elevation and after equilibrium has been established there is no resultant change in bottom-hole pressure. Hence, for a constant rate of production, Equation 1 may be written in the form:

$$p + hd = p_{bh} = \text{constant} \tag{8}$$

where p equals $p_{ch} + p_g$.

When Equation 8 is solved for h explicitly,

$$h = \frac{p_{bh}}{d} - \frac{p}{d} = \text{height above datum plane}$$

Differentiation of the last equation with respect to p yields

$$dp \; - \; dp \; \backslash \; d \qquad - \frac{d}{dp}\left(\frac{p}{d}\right)$$

is constant. Hence, $\frac{d}{dp}(p_{bh}) = 0$ and the last equation becomes

$$\frac{dh}{dp} = - \frac{p_{bh}}{d^2}\frac{d}{dp}(d \qquad \frac{1}{d}$$

Also, if the fluid density does not depend on the total gas pressure,

$$\frac{d}{\quad}(d) = 0 \quad \text{and} \quad \frac{dh}{\quad} = -\frac{1}{\quad}$$

or

$$d = -\frac{dp}{dh} = -\frac{\quad}{dh} \tag{9}$$

Equation 9 is a well known relationship for static fluid surfaces and other equilibrium systems. The equation states that the fluid density or fluid pressure gradient is equal to minus the slope of the total gas pressure versus fluid-level curve, provided the fluid density does not depend on the total gas pressure.*

In practice, an *average* value for the absolute magnitude of d may be obtained by using the relation:

$$d = \frac{\quad p_g)_1}{h_1 - h_2} \tag{10}$$

where h_1 and are the heights of the fluid corresponding to the total gas pressures $(p_{ch} + p_g)_2$ and $(p_{ch} + p_g)_1$.

It is of interest to point out that Equation 10 tacitly assumes that the total gas pressure versus fluid level curve is a straight line, that is, the fluid density is constant throughout the well.† (This assumption is usually valid under equilibrium conditions, except near the top of the well.)

The value of p_{ch} may be read directly on a pressure gauge connected to the casing head of the well. The value of p_g may be determined by use of charts or by the formula:

$$\log_{10} p_g = \frac{Shf}{122.82\,(t + 460)} + \log_{10} p_{ch} \tag{11}$$

where S is the specific gravity of the gas in the well, h is the length of gas column between top of fluid and casing head, f is the deviation factor from Boyle's Law, and t is average temperature in degrees Fahrenheit.

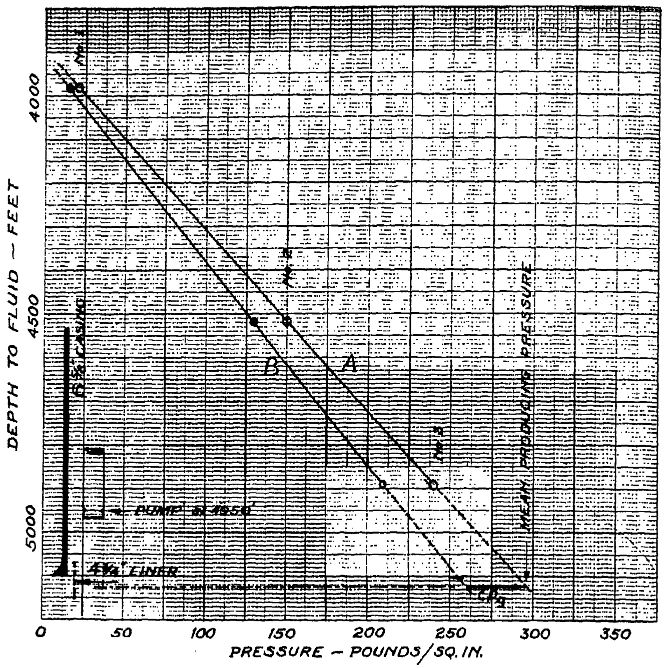
Hence, an average value of the fluid density may be obtained by keeping the rate of production constant and determining two values of h corresponding to two values of $(p_{ch} + p_g)$.

* The density might be expected to vary with gas pressure, temperature, producing gas-oil ratio, and diameter of casing—or annular spacing. Practically, it is usually observed that the fluid density is relatively constant provided the casing diameter does not vary. However, if there is a change in the size of the annulus, the fluid density changes almost proportionately to the effective areas of the annulus.

† C. P. Walker, "Method of Determining Fluid Density, Fluid Pressure and Capacity of Oil Wells," U. S. Patent 2,161,733, issued June 6, 1939.

Determination of Formation Pressure

The original bottom-hole pressure and its rate of decline as the well produces are of considerable importance. These data may be obtained by direct measurement with a subsurface pressure gauge or they may be calculated from the known relations between fluid level, casing-head pressure, and rate of production.



need be evaluated only once.

A plot of fluid-level data with h as ordinate and total gas pressure on the fluid as abscissa is a straight line, provided the production rate is held constant and the density does not vary with depth. Hence, the pressure at the reservoir depth, p_r, may be approximated by extrapolation. Curve A of Figure 408 shows a typical fluid level versus gas pressure curve. The data plotted in this curve are given in Table 23. The density of the fluid may be determined by applying Equation 10 to a small portion of the curve.

An alternative and more rapid method requiring only one determination of the weight of the gas column is illustrated in curve B. As before, the production rate is maintained constant and two or more measurements

TABLE 23

TYPICAL WELL DATA AND FLUID LEVEL DATA

Well Data

Total Depth	5,150′
Casing	6⅝″ Grade D cemented at 5,100′
Liner	80′ of 4¾″ Grade C hung at 5,150′
Perforations	Kobe 120 mesh perforated from 5,100′ to 5,150′
Depth to Pump Intake	4,950′
Temperature	182° at 5,150′

Production Data

Oil Rate	153 B/D Gross, 2% Cut
Gas Rate	170 M/ft. per day
Gas Gravity	0.782

Fluid Level Data	*Run* 1	*Run* 2	*Run* 3
Fluid Level	3,980′	4,515′	4,890′
Casing Pressure	16 lbs.	130 lbs.	209 lbs
Total Pressure on Fluid *	21 lbs.	150 lbs.	241 lbs

Calculations

Pressure at Pump	258 lbs. per sq. in.
Mean Producing Pressure	296 lbs. per sq. in.
Fluid Gradient in Casing	0.241 lbs. per sq. in. per ft.

* Includes casing head pressure plus weight of gas column from fluid surface to the casing head gauge. Weight of gas is calculated by formula

$$\log_{10} p_g = \frac{Shf}{122.82\,(t+460)} + \log_{10} p_{ch}$$

are made of fluid levels versus casing-head pressure. Extrapolation of the data to the depth of the mean producing horizon gives the theoretical casing-head pressure necessary to force the fluid level down to the producing horizon. The total pressure at this depth may be obtained by adding the weight of the gas column at this depth to the extrapolated casing-head pressure. Referring to the figure, the extrapolated casing head column was found to be 258 pounds, while the calculated weight of the gas column was 38 pounds, giving a total formation pressure of 296 pounds. (In this procedure, a direct determination of density cannot be obtained from the slope of the graph.)

PRODUCTIVITY DETERMINATIONS: WELLS WITH LOW CASING-HEAD PRESSURE

The "absolute potential" of a well is determined by conducting a series of measurements to evaluate two factors: (a) the effects of casing-head pressure and (b) the effects of rate of production. In most wells operating

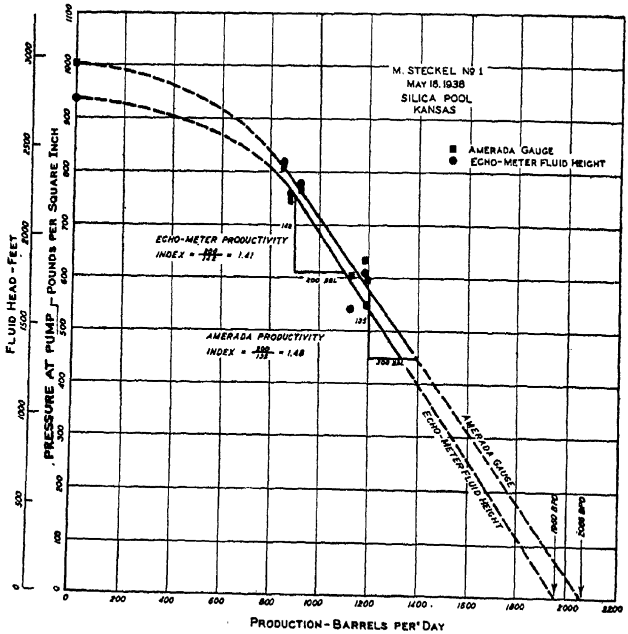


FIG. 409.—Determining productivity in wells making small quantities of gas from fluid-level or pressure-gauge data. (Jakosky, *A.I.M.E. Petroleum Technology*, Tech. Pub. 1058, 1939.)

at low casing-head pressures, or at atmospheric pressure, the casing-head pressure plus the weight of the gas column is sufficiently low to have a negligible effect on the potential. Measurements on such low pressure wells need therefore be conducted only to determine the effects of rate of flow on the fluid height or the bottom-hole pressure.

The producing characteristics of wells with negligible casing-head pressures may be determined by obtaining bottom-hole pressure or fluid-level data at two or more stable production rates. If fluid-level measurements are made, the casing-head pressure is held constant during the measurements at each of the production rates. The data are then plotted as shown in Figure 409.

It will be noted that the pressure-rate curve must deviate from a straight line in order to fit the static reservoir pressure value. Extrapolation of the curve to the Q-intercept gives the theoretical maximum production which the well is capable of yielding, provided flow friction, water coning or other variables encountered in practice do not change the slope of the curve. †

The productivity index is determined from the slope of the pressure-rate curve and is defined as "the barrels per day of gross liquid produced per pound per square inch pressure drop at a specified subsurface datum." ‡

Any condition tending to change the productive capacity of the well is indicated by a change in the productivity index or slope of the curve. § A decreasing productivity will be obtained when the gross production per pound (per square inch) drop in bottom-hole pressure decreases. This condition may be caused by increasing water, a decrease in permeability of the oil formation adjacent the well, or by drainage interference from offset wells. In some cases an increased productivity factor may be obtained, especially if the well is cleaning out or channeling. Periodic measurements of this type are a vital necessity in keeping a close check on a well's performance. The decline of a well's potential and its pumping performance can be intelligently followed, and in many cases controlled, by the use of bottom-hole pressure or fluid-level data.

† M. L. Haider, *loc. cit.*

‡ B. P. Kantzer and E. G. Trostel, "Oil Well Performance, Discussion and Proposed Terminology," *A.P.I., Drilling and Production Practice,* 1937.

§ T. V. Moore, *Proceedings Amer. Pet. Inst.,* 1930, 11 (4), 27.
M. L. Haider, *loc. cit.*
H. C. Miller, E. S. Burnett and R. V. Higgins, *loc. cit.*

PRODUCTIVITY DETERMINATIONS: WELLS UNDER CASING-HEAD PRESSURE

The producing characteristics of wells under pressure can best be determined as follows:

The bottom-hole pressure at some given datum point—usually the average depth of the producing formation—is plotted as ordinate and the production rate as abscissa. Methods of determining total pressures at a given datum point have been described in connection with Figures 404 and 408.

The productivity of the well may be determined by making a series of measurements with varying casing-head pressures at a given production rate to determine one of the pressure datum points. The rate of production is now changed and after equilibrium is established a second series of measurements is made to determine another pressure datum point. Two or more of these pressure datum points are then plotted, and the pressure-rate curve extrapolated to zero pressure (where no back-pressure will be exerted against the producing formation) to determine the theoretical maximum production rate for the well.

Another procedure allows the productivity to be obtained with no fluid above the pump. In this procedure, the pump is operated at a given production rate and the casing-head pressure increased to a value such that the well is pumped-off. Under this condition, the pressure at the pump intake is equal to the casing-head pressure plus the weight of the column of gas. The production rate is now changed by altering the speed of the pump, and the procedure is repeated. A curve is then plotted showing the total gas pressure versus production rate, and the curve is extrapolated to zero pressure to give the theoretical potential of the well at that particular depth setting of the pump.

SOLUTION OF PUMPING PROBLEMS

Dynamic Measurements.—It is possible to determine various important characteristics of a well, such as the maximum rate of production and the reservoir pressure, etc., by dynamic measurements wherein the bottom-hole pressure or the depth to the fluid level is determined as a function of the time before equilibrium has been established.† The theory of such measurements as applied to the determination of the reservoir pressure will be illustrated for a case in which the following

† Morris Muskat, *loc. cit.*

assumptions are made. (1) The well is open to the atmosphere so that the casing head pressure p_{ch} is zero. (2) The weight of the gas column p_g is negligible. (3) The free area a of the open flow string is uniform. (4) The fluid pressure gradient d is a constant, and the rate of production Q may be expressed by the equation

$$Q = a\frac{\partial h}{\partial t} \tag{12}$$

(5) The production is the same as that in a "dead well"; that is,

$$Q = c\,(p_r - p) \tag{2}$$

It follows from assumptions (1) and (2) and Equation 1 that the bottom-hole pressure is equal to the weight of the fluid, provided the friction drop in the well bore is negligible; that is,

$$p_{bh} = hd$$

Or if p_{bh} is set equal to p,

$$p \equiv p_{bh} = hd \tag{13}$$

On inserting this value of p into Equation 12 one obtains

$$Q = \frac{a}{d}\frac{\partial p}{\partial t} \tag{14}$$

Equating the right-hand members of Equations 2 and 14,

The last equation may be written in the form

$$p_r - p \qquad a$$

or

where k is a constant which may be evaluated as follows. At $t = 0$

$$p_r - p_0 = k$$

Hence

$$-\frac{cd}{-}t \tag{15}$$

In the same way, it may be shown that

$$h_r - h = (h_r - h_0) \cdot e^{-\frac{cd}{a}t} \tag{16}$$

where h_r is the equilibrium value of h and h_0 is the value of h at the instant chosen to represent $t = 0$.

Combining Equations 15 and 16,

$$p_r - p_0$$

or

$$c = \frac{a}{td} \log_e \frac{p_r - p_0}{p_r - p} = \frac{a}{td} \log_e \frac{h_r - h_0}{h_r - h} \tag{17}$$

Equation 17 may be related to the theoretical maximum rate of production as follows. From Equation 2

$$Q_{\max} = c \cdot p_r$$

Hence

Evidently a plot on semilogarithmic paper of $p_r - p$ or $h_r - h$ versus time should be a straight line, if the assumptions made in deriving Equation 18 are valid.

The practical use of the above analysis may be summarized as follows. Determinations are made of the bottom-hole pressure or the fluid level as a function of time. (Compare Figure 410.) An approximate value of the *equilibrium* bottom-hole pressure p_r is read off the curve. A plot is then made on semilogarithmic paper of $p_r - p$ versus time. (Figure 411.) If the estimated or approximate value of p_r is inaccurate, the plot (curve II, Figure 411) will deviate from a straight line. The value of p_r is then adjusted so as to make the data fall on a straight line. Thus, this method of observing the bottom-hole pressure as a function of time before equilibrium is reached yields an accurate determination of the reservoir pressure.

Maximum Pumping Efficiencies.—Fluid-level measurements supply information that will allow an operator to ascertain whether a well is pumping-off or is gas-locked. Oftentimes, it is impossible to tell by the pounding on the polished rod which condition exists in a well. Pounding due to the well being pumped-off is caused by a low fluid-level or by the pump being set too high in the well, while the pounding due to gas-locking may occur when a high fluid level exists above the pump.

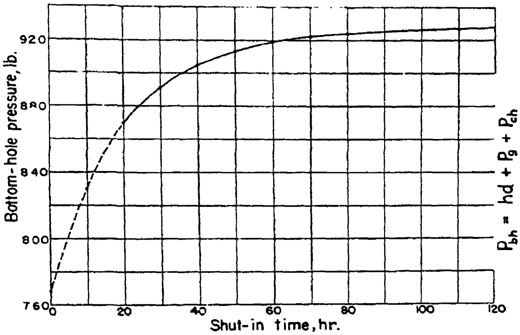


FIG. 410.—Average bottom-hole pressure rise for 20 wells in the Judkins Field, Ector County, Texas. (Muskat, *A.I.M.E. Petroleum Technology*.)

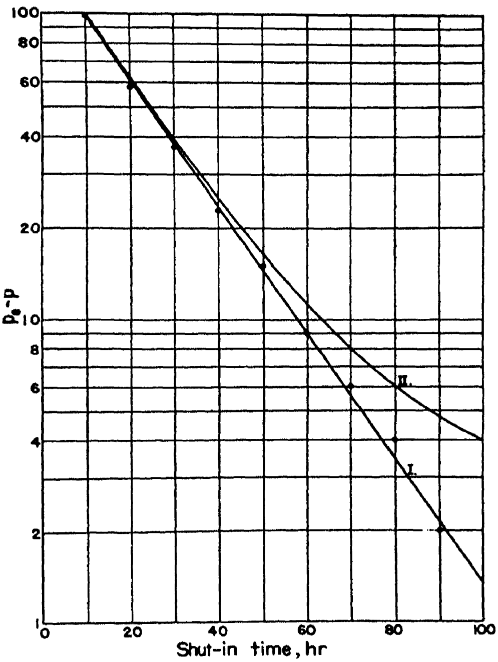


FIG. 411.—Plot of $p_r - p$ versus shut-in time. The data for this semilogarithmic plot were obtained from Figure 410. (Muskat, *A.I.M.E. Petroleum Technology*.)

For the maximum rate of recovery, the bottom-hole pressure usually should be reduced as much as possible, and this can be achieved by maintaining the lowest practical fluid level. In actual pumping operations, it is not possible to obtain the full theoretical production rate due to the hydrostatic head that must be maintained in order to feed the oil into the pump. The factors on which this head depends must be determined for each field and includes viscosity of the oil, pump speed and capacity, valve design on pump intake, etc.

After making changes in pump speed or capacity, it is advisable to run a subsequent fluid-level measurement to be certain that the proper draw-down is being obtained.

OTHER APPLICATIONS OF FLUID-LEVEL MEASUREMENTS

Evaluation Studies.—Considerable use is being made of fluid-level measurements for determining subsurface conditions in producing fields. Isobaric maps showing lines of equal bottom-hole pressure are plotted, generally expressed with reference to a common datum plane. Usually two sets of contours are plotted; they show: (1) shut-in fluid levels (static) and (2) pumping equilibrium fluid levels at some given rate of production. These isobar maps give important information on areas of higher permeability in the field and allow development work to be done in a better engineering manner. This phase of the work is becoming of considerable importance in connection with the various regulatory measures and the unit control trend of recent legislation.

Gas Lift Operations.—A recent application of fluid level measurements in Texas concerns existing or projected gas lift installations.† If a well is being converted from pumping or flowing to gas lift, it is convenient to be able to obtain the fluid level in the annular space in advance of the pulling job. Also, the draw-down at various producing rates will give valuable information regarding the size and location of the valves to be installed. If a flow-valve installation is made, its operation can be checked quickly and conveniently by fluid-level observations. If it is suspected that a valve is sticking, a close check of the fluid-level variation when the well begins to unload will show which valve is kicking off. Also, the correctness of valve spacings and their effective performance can be studied.

In wells where the conventional type insert valves are used, it is impossible to run a bottom-hole pressure bomb because the valves obstruct the tubing. In all gas lift installations using flow-valves the wave reflection measurements have the additional advantage that the fluid level can be watched continually and the input pressure properly regulated without the possibility of changed flow conditions as a result of running the pressure bomb in the flow string.

† Geo. Webber, "Fluid Level Indicator Useful in East Texas," *The Oil and Gas Journal*, Dec. 19, 1938, pp. 44-50.

Gas Repressuring Projects.—Periodic studies of formation pressure are the basis of many conservation programs designed to yield the optimum oil recovery. Conservation of natural gas, either by retention in the reservoir, artificial replacement, or both, is being accepted more and more widely as a standard production policy.

Gas replacement, or repressuring, is accomplished by returning to the oil-producing zones a certain per cent of the volume of gas produced with the oil during normal operations. Such a project requires numerous injection wells, together with compressor stations, pipe lines, and other surface appurtenances. Usually, repressuring directly precedes abandonment. In some instances, however, the repressuring program may be initiated during the early life of the field,* in which case the decline of subsurface pressures will be materially lessened and the flowing life of the field prolonged.

Irrespective of when wells go on pump, fluid-level measurements become a valuable tool in continuing the repressuring program. During the transitory stage, some wells are still flowing and will probably have their pressures recorded by the bottom-hole gauge, although fluid-level measurements are very practical. By using echo measurements, the bottom-hole studies may be continued on the pumping wells throughout the life of the field, at a minimum cost.

It is also necessary to check the gas-input wells periodically for fluid, to see that no condensate has accumulated in their bottoms. In a multi-zone field, gas may be injected into two or more pays from the same well. In this case the wave reflection instrument proves itself invaluable as a rapid and accurate means of measuring the depths to fluid or to bottom.

Cementing Operations.—The "squeeze" cementing technique has become a standard practice in completing and reconditioning wells whenever an undesirable water stratum is to be shut off. Such a procedure depends for its success upon the effectiveness of a protecting packer set above the stratum to be sealed off. Should any cement slurry which is forced down the tubing string under pressure escape around the packer, there is danger of contamination to the oil zone above. Also, this "leak" effectively lowers the pressure differential which would normally be applied to the face of the watered formation.

Such leaks during a cementing process can be detected by measuring progressively the depth to the top of the quiescent fluid which stands in the casing above the packer. If the wave reflection equipment shows a sudden change in fluid level, it indicates that some disturbance or leak has occurred, and it may be concluded that slurry is being pumped around the packer and into the casing annulus above. Such application of fluid level measurements is a valuable one.

Water Disposal and Supply Wells.—The problem of salt water disposal is one which receives considerable attention both of legislative and engineering bodies. Oil field brines, after being separated by treatment from the pipe line oil, must be cared for in some manner satisfactory both to the landowners and to the operator. One method of disposal is by the use of wells, either drilled expressly for such a purpose or abandoned from oil production and plugged back to some upper permeable

* E.g., Loudon Pool, St. Elmo, Illinois.

zone. The brine is flowed into the well, by gravity or under a pressure head, and then absorbed by the porous stratum.

Productivity indices can be determined for disposal wells in the same manner as for gas-free oil wells. Potential computations will show the amount of water which can be disposed of in any given well, and comparisons of indices computed periodically will show whether or not the thief formation is salting-up. If a treatment is necessary for the brine in order to keep the disposal wells clean and functioning properly, the effectiveness of different treating processes can be studied from fluid-level data.

In Michigan and other states, salt-water producing wells are drilled to obtain their brine contents for chemical manufacturing. Here again the same applications for fluid-level measuring devices are present. Productivities, pumping problems, fluid build-up studies, and pumping efficiencies are a practical part of the brine production program.

FUTURE PROGRESS OF FLUID-LEVEL MEASUREMENTS

The art of measuring fluid levels in oil wells by sonic methods is relatively old, but the widespread use of the method did not occur until it became a legal means of well proration. Under the impetus given by this, many developments took place which made the method a rapid and reliable means of measurement. As its use becomes more widespread, new applications will undoubtedly be found to increase the value of this tool in the production of oil.

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CHAPTER XII

PERMIT AND TRESPASS PRACTICE, AND INSURANCE

A general knowledge of the laws relating to trespass is extremely important to those engaged in geophysical field operations. Rarely does a field party complete its exploratory work without trespassing on the property of others. Covering, as a field party does, a very wide range of territory, with operations in states and localities in which the principal business of its concern is not located, it becomes of prime importance that the geophysical field party be instructed in certain basic principles of law relating thereto and the medium through which the employer can be protected from vexatious quarrels and litigation. Good-will of the public, and particularly of landowners, is essential to efficient geophysical prospecting with its allied leasing operations. In fact, it is only with the good-will and consent of landowners and lessees that most field operations can be carried on. It is the surface landowner's natural right to expect to be consulted before his fences are let down or fields crossed and if damage is done to his crops or other surface rights, he is entitled to just compensation for such loss. It is therefore necessary for every member of a party to be mindful at all times of the individual rights of others, not only because it pays to maintain the good-will of the public but from the legal standpoint such consideration prevents expensive and ofttimes protracted litigation.

The nature of this treatise on geophysics prevents any extended discussion of the law of trespass and of necessity this chapter must be confined to general rules of law which will be beacons or guides to the geophysicist and his field party. When operating in any locality, however, the general principles herein enumerated will apply, and in the event any special problem arises the field party should be instructed to consult with local counsel before acting to the detriment of the employer, as consultation is cheap but litigation is expensive.

A "trespasser" has been defined as:* "One who does an unlawful act, or a lawful act in an unlawful manner, to the injury of the person or property of another; one who makes an unauthorized entry of another's

* Little vs. State, 89, Alabama Reports, p. 89. Heller vs. New York, 265, U. S. Federal Reports, 192, Vol. 63, Corpus Juris, p. 887.

property; one who goes upon the premises of another without invitation, express or implied, and does so out of curiosity, or for his own purposes or convenience, and not in the performance of any duty to such owner; one who unlawfully enters or intrudes upon another's land, or unlawfully and forcefully takes another's personal property." From this definition of a trespasser one can readily define the word "trespass," and ascertain what acts or conduct fall within the category of trespass.

The gist of the action of trespass is disturbance of possession. Unlawful intent is not necessary; therefore, the intent or motive is immaterial as regards the trespasser's liability, except in so far as it may affect the measure of damages. The fact that the trespass was wilful or malicious is material only for the purpose of obtaining punitive damages.

From the foregoing it might appear that actual damage must be suffered by the party against whom the trespass is committed, but this is not the case. A trespass may be committed without any damage being done, as the act itself of entering unlawfully upon another's property is sufficient to support a cause of action for trespass.

In all jurisdictions a cause of action arises against the trespasser for all "actual" damages suffered by the landowner, the actual damages being based upon the amount of money which would compensate him for all damages suffered at the hands of the trespasser. This would include the cost of repairing fences, compensation for lost crops, replacing the soil so that it is in the same condition as before the trespass was committed, and other damages of like nature. In many jurisdictions exemplary or punitive damages are allowed in addition to the actual damages, and in most jurisdictions the landowner would be entitled to exemplary or punitive damages, although no actual damage of a substantial nature had been suffered.

Exemplary or punitive damages are given to punish the wrong-doer and to act as a deterrent to the commission of like acts in the future. The amount of actual damage which might be awarded is fairly well fixed by the ordinary rules of law and is always confined to the reasonable cost of restoring the landowner to his former estate and position, but exemplary or punitive damages may be awarded in the sound discretion of the court and for an amount which, in the opinion of the court, will accomplish the result which the court is seeking to accomplish. As a result, exemplary or punitive damages may be in a substantial amount and, having emanated from the mind of a court, it is sometimes difficult to review or obtain a reversal of that portion of the judgment.

It is a complete defense to the awarding of punitive or exemplary damages if permission is given by the landlord to the geophysical party to enter upon the property and conduct the exploration activities. As a result thereof the geophysical party should be instructed, wherever possible, to contact the landowner or the lessee of the landowner and

procure his signature to an instrument, which it is suggested should read about as follows:

> "The undersigned is conducting geophysical field operations in this vicinity and requests permission to enter upon property, of which we understand you are the owner (or lessee), to conduct such operations.*
>
> Will you kindly permit us to enter upon the parcel of real property described as: (here insert a description of the property involved).
>
> In consideration of the granting of this permission on your part we agree to commit no nuisance on your property and to leave your property in the same condition it was in before our entry thereon."

By use of a written authorization of entry such as this the geophysical party not only avoids the possibility of expensive litigation involving exemplary or punitive damages but the good-will of the public will be maintained. In most instances the landowner will cooperate in the work on his property and oftentimes will assist in locating other owners and securing like permissions from them.

After the exploration work has been completed the field party should be instructed to replace and repair all fences, and, in general, place the property in the same condition it was in before the entry was made. An improperly repaired fence or a gate left open by the exploration party may result in the loss of cattle or other stock, and accidents happening on the highway between vehicles and stray cattle may be directly traceable to this negligence on the part of the exploration party. For instance, holes left in the surface of the earth after seismic operations may be used as a basis for damages for injury to or loss of cattle, injury to persons, and other claims of injury, some of which may be of a very fantastic nature. It should always be remembered that regardless of how fantastic the claims may be there will be found unscrupulous practitioners who thrive on such litigation. To the geophysicist not trained in legal matters the ultimate chance of recovery may seem remote, but regardless of the possibility of recovery the expense of litigation still remains, and this expense will vary in the locality in which the geophysical party is working, with a possibility that several hundred dollars may be expended in defending such an action. In addition, there will be the expense occasioned by the loss of time preparing for and defending the action.

A complete defense to the possibility of an action involving "actual" damage is the procuring of a release from the landowner after the work of replacement or repair has been completed. This form should be as

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* The purpose of the field work, and such other information as the geophysical company may desire to give, can be explained verbally by the party chief or contact man.

short and concise as possible, as a lengthy legal document is more formidable and invariably results in "thinking it over" delays. This release may also serve as a receipt for payment of damages or trespass privileges. Proposed forms of release are set forth at the end of this chapter.

In many states the laws provide that a general release does not extend to claims which the creditor does not know or suspect to exist in his favor at the time of executing the release, which, if known by him, must have materially affected his settlement with the debtor.* In the event the field party has engaged in such type of exploration work as would give rise to the possibility of claims being asserted which the landowner did not know or suspect to exist in his favor, then a provision should be placed in the general release which would acknowledge receipt of satisfaction for such claims. Most statutes of this type can be circumvented in the event there is an express provision in the release by which the creditor waives such right.

In most states permission may be obtained to do geophysical work along public highways. It is good practice to obtain such permission from the proper public official before permitting the geophysical party to conduct its exploratory work. In seismograph work the shot-holes must normally be drilled beyond the limit of the highway and where there is any question shot-holes should not be drilled without obtaining information in relation thereto, as the drilling of such holes along public highways is a bad safety practice and when so drilled should never be left open or unattended.

The public official from whom permission must be obtained will vary in the different states. As an illustration, in Louisiana and certain counties in other states, a license must be obtained from the State Highway Commission or other local authority to work on public roads. Louisiana further requires that a separate license be secured for each parish in which work is done, and the District Road Commissioner of the specific parish in which work is contemplated must be advised when the work is initiated. Before the field party undertakes its work all of this information can be obtained as a general rule from the Division of Highways of the State in which the work is to be conducted. Highways which are within the incorporated limits of cities are generally supervised by municipal authorities, and in addition to obtaining the permission of the State Highway Commission a permit should be obtained from such local authorities.

* Civil Code of California, Sec. 1542.

RECEIPT AND DISCHARGE

Check No...

KNOW ALL MEN BY THESE PRESENTS: That,

The undersigned, in consideration of the sum of $......................................, and other valuable consideration, in hand paid by the ...., a corporation of ...., ..., receipt of which is hereby acknowledged, do hereby acknowledge full payment, settlement and satisfaction for, and do hereby forever discharge the said ....(company).............................. from any and all actions, causes of action, claims and damages of whatsoever kind, nature and character sustained by the undersigned that may have arisen by reason of, or founded upon the operation and acts of the ....(company).........................., its agents, servants and employees in carrying on exploration work, tests, surveys and operations by seismograph or other methods, on, in, over, under or about the following described land, of which the undersigned declare that I/we am/are the lawful owner, situated in the County of ...., State of ...., to-wit:

Dated this .... day of ..., 19...

....

....

....

....

....

....

RECEIPT AND RELEASE

19..

RECEIVED from ....(company).............................. the sum of ....
..............................DOLLARS ($..........................) which .... acknowledge to be in full settlement and discharge of a claim growing out of property damage sustained by .... on or about ...., 19........, for which damage .... have claimed the said ....(company).......................... to be legally liable, and in consideration of said sum so paid to ...., ... hereby remise, release and forever discharge the said ...., and its employees from any and all actions, causes of action, claims and demands for, upon or by reason of any damage or loss which heretofore has been or which hereafter may be sustained by .... in consequence of ....

WITNESS .... hand .... and seal .... the day and year first hereinabove written.

Witness..

Address..

Witness...

Address...

INSURANCE FOR GEOPHYSICAL OPERATIONS

One of the requisites in successful geophysical operations is adequate insurance protection which may serve as a protection against the expenditure of substantial sums of money in defense of actions for alleged trespasses. Inadequate protection or coverage is a hazard which no reputable geophysical operator should assume, yet unnecessary insurance or coverage is not only an added expense, but an economic waste.

The most common forms of liability encountered by an exploration company are: (1) injury or death of employees on the premises of the company, (2) injury or death resulting from operating equipment of the employer away from the premises of the employer, (3) damage to or destruction of the property of others while operating the equipment of the exploration company, (4) accident or liability as the result of operating motor vehicles owned by the company, (5) loss by fire, (6) theft of equipment, and (7) miscellaneous accidents to clerical employees and laboratory operators, mechanics, messengers, salesmen, etc.

Unfortunately insurance coverages can not be tabulated in a simple table due to the diversity of operating methods and conditions encountered in field operations.

The insurance rates vary with the duties of the employees. For example, the rate for an office employee may be eight cents per $100.00 of salary received, while for the field operators handling explosives it may be $10.00 per $100.00 compensation. Generally, the rating provides for a credit based upon past records of accidents. The employer, therefore, over a period of time more or less makes his own rates. For this reason, it is to the employer's benefit to teach his employees to think "Safety" at all times.

In all cases, careful consideration must be given to the reliability of the insuring company and the extent of its operations. For example, it is possible in some cases to effect a saving by placing the coverages with companies operating in one state only. This plan is satisfactory, provided the operations do not extend into other states. The standard form of policy limits not only the kind, but extent of operations as well. It is therefore necessary that a policy be endorsed to cover all operations and all states. It is for this reason that it is advisable to place the coverage with a company licensed to do business in the various states and countries where geophysical operations are contemplated during the term of the policy.

Caution must be taken to see that the coverage provided by the insurance company does not conflict with state laws. Some states, termed "monopolistic" states, require that certain forms of coverage, usually Compensation, must be placed with the respective state organizations.

To protect himself the operator must carry the following types of insurance:

Compensation Insurance. — Workmen's Compensation insurance covers the obligation imposed upon an employer, who is subject to the "Worker's Compensation Law" of a State, Territory of the United States and of the United States Longshoremen's and Harbor Workers' Compensation Act (with respect to off-shore operations), to pay the benefits prescribed by such law to his employee.

The form of policy issued by the standard insurance companies also provides Employer's Liability coverage, which protects the employer in those cases where the Workmen's Compensation Law is not the sole remedy at law of the employee. Appropriate state endorsements must be attached if the field operations are outside the state in which the policy is issued. In some states, individual policies are mandatory.

It is unnecessary to provide coverage for any states before the field operations are extended to them. However, an employee should never be sent to another state until the policy has been checked to determine if it covers operations in such state.

Public Liability and Property Damage Insurance. — 1. Public Liability (generally written on the contractors form for geophysical operations) covers bodily injuries to or the death of others (except employees) caused by or resulting from the ownership, maintenance or use by the employer of property, or from the operations conducted by him on or away from his premises.

2. Property Damage Liability covers damage to or destruction of property of others caused by or resulting from the ownership, maintenance or use by the employer of property, or from the operations conducted by him on and away from his premises (property owned, leased, rented, occupied or used by or in the care, custody or control of the employer or any of his employees being excluded).

Standard forms of policies are generally used when writing this form of coverage. Endorsements broadening or limiting the coverage to meet the assured's requirements should be made a part of the policy. If there is any ambiguity or doubt as to the coverage, the assured should make certain that the policy is properly endorsed to correct this ambiguity.

Under the definition of Property Damage Liability, "property in the care, custody or control of the assured" is not covered. Unless this exclusion is modified by endorsement, the coverage does not protect the

assured against a claim by the client for any damage to client's property caused by the field operations of the Geophysicist.

Another exclusion under Property Damage Liability not mentioned above is damage caused by "blasting or explosions of any character." Since explosives are used in certain operations, it is imperative that the equipment and method of operations be of the highest standard or the insurance company will hesitate to delete this exclusion from the policy. The proper endorsement should be attached to the policy to cover the use of explosives, if they are employed in the work.

Automobile Insurance. — 1. Automobile Public Liability covers bodily injuries to or the death of others caused by or resulting from the ownership, operation or use of automobiles owned by the assured.

2. Property Damage Liability covers damage to or destruction of property of others (including loss of use thereof) caused by or resulting from the ownership, operation or use of automobiles owned by the assured. The policy may be endorsed to cover automobiles hired by the assured, as well as automobiles owned by employees of the assured and used in his business. The type of automobile, its load capacity, its use and location, determine the premium for the coverages. At the present time, only the State of Massachusetts has a compulsory automobile insurance law. However, many of the states have stringent "Financial Responsibility Laws" which make it imperative for the owner to carry certain forms of insurance. These laws make certain limits of liability mandatory. It is advisable that the limits of liability should extend to possible serious claims, and usually a $25,000/$50,000 policy is deemed a minimum for adequate protection. Policies of double this amount are occasionally carried by the more prudent operators.

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